Feasible Mechanisms in Economies with Type-Dependent Endowments*

Françoise Forges
Université Paris IX-Dauphine

Preliminary (January 2004)

*I started this research at Cal Tech. I wish to thank the Division of Humanities and Social Sciences for its hospitality. I also acknowledge stimulating discussions with F. Koessler, J.-F. Mertens, E. Minelli and R. Vohra.

Abstract

We consider N.T.U. and T.U. exchange economies in which initial endowments and preferences depend on the agents' private information. Every agent knows his own initial endowment, but his utility over consumption bundles possibly depends on the others' information (common values). We describe general allocation games in which agents make non-verifiable claims about their types and effective deposits of consumption goods. The planner redistributes the goods that are deposited by the agents. In W-allocation games, the agents can *withhold* part of their endowment, namely consume whatever they do not deposit. In D-allocation games, the agents loose any part of their endowment that they do not put on the table, i.e., the agents can just *destroy* their endowments.

We introduce W- and D- incentive compatible (I.C.) direct allocation mechanisms, in which every agent must simply report a type and make a deposit consistent with his reported type. We show that the revelation principle holds in full generality for D-I.C. mechanisms but that some care is needed for W-I.C. mechanisms. We further investigate the properties of both classes of mechanisms under common assumptions in implementation theory, namely non-exclusive information and/or constant aggregate endowment, and argue that many existing results implicitly require D-I.C. mechanisms. We establish that in T.U. economies, W-I.C. and D-I.C. mechanisms are ex ante equivalent.

1 Introduction

Incentive compatible mechanisms are pervasive in microeconomic models in which agents have private information. If there exists a relevant set of outcomes X, which does not depend on the agents' information, a (direct, deterministic) mechanism ξ can be defined as a mapping from the set of states of nature (or agents' types) T into X. Such a mechanism ξ induces a revelation game $\Gamma(\xi)$, in which every agent is invited to report his information; ξ is called (Bayesian) incentive compatible if truthtelling is a Nash equilibrium of $\Gamma(\xi)$. The interest in incentive compatible mechanisms is justified by the well-known *revelation principle*. Suppose we consider any Nash equilibrium of a more general communication game, in which every agent sends a message in an arbitrary set and a planner chooses an outcome in X as a function of the message profile. Then the mapping, from T to X, induced by the agents' strategies and the planner's choice function is an incentive compatible mechanism.

The previous approach has been widely applied in exchange economies with differential information in which initial endowments are (at least in the aggregate) independent of agents' types (see, e.g., [3], [4], [13], [15], [17], [19]). In the more general situation where every agent privately knows his own endowment, allocating goods on the basis of the agents' reports can obviously lead to bankruptcy problems. It is natural to consider allocation games in which the agents not only send messages depending on their types (cheap talk) but also make deposits of consumption goods. This procedure solves both individual and joint feasibility constraints: the agent cannot deposit more than what he possesses and the planner can only allocate what has been put on the table.

Our first goal in this paper is to define precisely appropriate allocation games and to derive direct mechanisms, following the tradition of the revelation principle. Since the deposits that an agent is able to make depend on his privately known endowment, types are partially verifiable in allocation games. While this property sometimes impedes specific forms of the revelation principle, as shown by Green and Laffont (1986), it will on the contrary facilitate the fulfillment of incentive constraints in our framework, since agents cannot pretend to be richer than they are.

Hurwicz, Maskin and Postlewaite (1995) and Postlewaite (1979) proposed two classes of (direct) mechanisms in exchange economies with type-dependent initial endowments. They pointed out that agents, when invited to report their endowments, can be asked to put these on the table, so that they cannot over-report them. In W- (for "withholding") mechanisms, the agent can consume any part of his endowment that he conceals. In D- (for "destroying") mechanisms, any part of the endowment that is not displayed is lost. The latter class of mechanisms makes sense if the agents' initial endowments must be checked by some authority before being consumed[1]. Hurwicz, Maskin and Postlewaite (1995) focused on Nash implementation, without asymmetric information between the agents and thus did not investigate Bayesian incentive compatibility. They defined direct mechanisms without detailing the revelation principle behind them. Being not interested in incentive compatibility conditions, they did not use the fact that their mechanisms would make types partially verifiable.

Hong ([9], [10], see also [11]) studied Bayesian implementation with state dependent feasible sets. She did not address the question of the revelation principle and concentrated on a specific class of direct mechanisms, which, in the particular case of exchange economies with type-dependent endowments, allow the agents to withhold part of their endowment as the W-mechanisms of Hurwicz, Maskin and Postlewaite (1995). Furthermore, Hong's mechanisms recognize the a priori restrictions that the underlying Bayesian model imposes on the possible deposits. The associated Bayesian incentive compatibility conditions reflect the partial verifiability of types.

Forges, Mertens and Vohra (2002) considered W-mechanisms in T.U. exchange economies with asymmetric information, in which unlimited money transfers are allowed and utility functions are quasi-linear. By relying on an appropriate version of the revelation principle, they focused on simple, direct W-mechanisms. They established positive results on the non-emptiness of the ex ante incentive compatible core, which illustrate that asking the agents to deposit their initial endowment indeed facilitates incentive compatibility requirements.

In this paper, we define rigorously W- and D- allocation games for exchange economies with asymmetric information and type-dependent initial endowments. We show that the same direct mechanisms can be used in both frameworks, and capture Hurwicz, Maskin and Postlewaite (1995)'s concepts through the W- or D- incentive compatibility of these direct mechanisms. More precisely, we show (in propositions 1 and 6) that the set of all Nash

[1] In this paragraph, we obviously used the word "mechanism" in a wider sense than above. In the body of the paper, we will propose a precise terminology.

equilibrium payoffs of all W- (resp., D-) allocation game coincides with the set of payoffs to W- (resp., D-) incentive compatible direct mechanisms.

This representation result can be interpreted as a general revelation principle, but is not completely standard. For W-incentive compatible mechanisms in N.T.U. economies, it requires that we consider a large class of W-allocation games, in which agents can be forced to make a minimal deposit. If one focuses on W-allocation games with unrestricted deposits, the set of associated Nash equilibrium payoffs coincides with the set of payoffs to *interim individually rational* W-incentive compatible mechanisms (see proposition 3). This property, which is consistent with Hurwicz, Maskin and Postlewaite (1995)'s results on Nash implementation, is typical of W-allocation games (proposition 6) in N.T.U. economies (proposition 2).

The previous paragraph suggests that choosing between W- and D- incentive compatible mechanisms might be delicate. Before elaborating on this question, let us point out that the problem can only arise in N.T.U. economies. We indeed prove (proposition 7) that in the T.U. case, W- and D- incentive compatible mechanisms are equivalent (in the sense that they both generate the same ex ante expected payoffs). In particular, except in section 6.2, Forges, Mertens and Vohra (2002) could as well have considered D-mechanisms.

All difficulties are also apparently avoided in the particular case of type-independent endowments: again, W- and D- incentive compatible mechanisms are equivalent and correspond to the mechanisms that have been used in the literature (see, among others, [3], [4], [13]). However, W- and D- incentive compatible mechanisms are *direct* mechanisms, and as mentioned above, they cannot be justified by the same revelation principle. If we think that W-allocation games with unrestricted deposits are appropriate when endowments are type-dependent, we should also use them in the limit case where endowments are type-independent, but then we must focus on interim individually rational mechanisms. In section 4, we illustrate the difficulties in economies with non-exclusive information (introduced by Postlewaite and Schmeidler (1986)).

Several articles (e.g., Postlewaite and Schmeidler (1986), Palfrey and Srivastava (1989), Serrano and Vohra (2001)) consider a slightly more general framework than type-independent endowments, but keep a set of feasible allocations that is independent of the state of nature, by assuming constant aggregate endowment. In this approach, the mechanism designer is allowed to re-allocate (with free disposal) the constant aggregate endowment whatever the reports of the agents. We show, in section 6, that this amounts to considering D-incentive compatible mechanisms, but is inconsistent with W-incentive compatible mechanisms.

2 Basic economy

We consider a pure exchange economy with asymmetric information described by the following ingredients:

n agents: $i = 1, ..., n$

l consumption goods: $j = 1, ..., l$

T_i: finite set of types of agent i, $T = \prod_{1 \leq i \leq n} T_i$, $q \in \Delta(T)$: probability distribution over T

$e_i(t_i) \in \mathbb{R}^l_+$: agent i's (initial) endowment, depending only on his own type t_i , thus measurable w.r.t. the agent's information.

$v_i : T \times \mathbb{R}^l_+ \to \mathbb{R}$: agent i's vN-M utility function, allowing for common values, increasing.

This corresponds to the **N.T.U. case**.

In the **T.U. case**, there is an additional good, money. Agents do not have any initial endowment in money. Utility functions are quasi-linear, namely take the form:

$$u_i : T \times \mathbb{R}^l_+ \times \mathbb{R} \to \mathbb{R} : u_i(t, x_i, m_i) = v_i(t, x_i) + m_i$$

The decisions to be made by the agents, the timing of information and actions, etc. will be detailed in the next section.

3 W-mechanisms and the revelation principle

A *W-allocation game* for the previous economy is played with the help of a planner. Such a game is described as follows:

- A chance move selects $t = (t_i)_{1 \le i \le n} \in T$ according to q; agent i is informed of t_i and endowed with $e_i(t_i)$.

- Every agent i sends a (cheap talk) message $c_i \in C_i$ to the planner, and makes a deposit $d_i \in D_i \subseteq \mathbb{R}^l_+$. The agents make their decisions simultaneously; let $c = (c_i)_{1 \le i \le n}$ and $d = (d_i)_{1 \le i \le n}$.

- Given (c, d), the planner selects, possibly at random, an allocation $x = (x_i)_{1 \le i \le n} \in (\mathbb{R}^l_+)^n$ such that $\sum_{1 \le i \le n} x_i \le \sum_{1 \le i \le n} d_i$. In the T.U. case, he selects in addition monetary transfers $m = (m_i)_{1 \le i \le n} \in \mathbb{R}^n$ such that $\sum_{1 \le i \le n} m_i \le 0$.

- Agent i receives the payoff $v_i(t, e_i(t_i) - d_i + x_i)$ in the N.T.U. case, $u_i(t, e_i(t_i) - d_i + x_i, m_i)$ in the T.U. case.

The sets C_i and D_i are part of the description of the allocation game. More precisely, the parameters of the allocation game are the sets C_i, D_i, $i = 1, ..., n$ and the transition probability[2] used by the planner to choose an allocation (and possibly, monetary transfers) as a function of the agents' messages and deposits. We will sometimes refer to the latter as the "planner's (mixed) strategy", even if the planner is not a player of the allocation game.

D_i is interpreted as a list of *allowed deposits*, which is fixed by the game. We assume that every agent is able to make a deposit, whatever his type, namely that the sets D_i satisfy

$$\forall t_i \in T_i \quad \exists d_i \in D_i : d_i \le e_i(t_i)$$

In other words, the set of possible deposits for agent i of type t_i, namely

$$D_i(t_i) = \{d_i \in D_i : d_i \le e_i(t_i)\} \tag{1}$$

is non-empty, for every i, t_i. Of course, the condition is satisfied as soon as $0 \in D_i$.

As typical examples of possible sets of deposits, one can take $D_i = \mathbb{R}^l_+$, $D_i = \left\{d_i \in \mathbb{R}^l_+ : d_i \ge \min_{t_i \in T_i} e_i(t_i)\right\}$ or the list of all possible initial endowments $\{e_i(t_i), t_i \in T_i\}$. The latter set is the most restrictive one. We say that

[2]As quite usual, we allow for randomization by the planner, as well as for mixed strategies of the agents, but the results also hold if one restricts to pure strategies (of course, the planner and the agents' strategies must obey the same restrictions).

deposits are *unrestricted* if $D_i = \mathbb{R}^l_+$. We will first consider *all* possible sets of deposits D_i that satisfy (1). We will thereafter turn to W-allocation games with unrestricted deposits.

In a W-allocation game, individual and joint feasibility constraints are automatically satisfied, even outside of equilibrium, since they are explicitly materialized. Agents possibly withhold part of their endowment (namely, $e_i(t_i) - d_i$), which they can consume (this is typical of W-allocation games, as opposed to D-allocation games, see below)[3].

Observe that the players of the allocation game are not asked (neither ex ante, nor at the interim stage) whether they want to participate in the game or not, but for some specification of the game, some moves in $C_i \times D_i$ could be interpreted as "no participation".

Even if D_i is specified independently of agent i's type (which the planner does not know), the set of possible deposits for agent i of type t_i is $D_i(t_i)$, which depends on t_i through $e_i(t_i)$. Hence, types are *partially verifiable* in the allocation game (see, e.g., Green and Laffont (1986), Forges and Koessler (2003)).

We will propose a *canonical representation* of all Nash equilibrium outcomes of W-allocation games. For this, we introduce simple *revelation games* $\Gamma(\xi)$ induced by a mechanism ξ, which will be defined formally below. In $\Gamma(\xi)$, agent i of type t_i must make a report in the following subset of T_i:

$$R_i(t_i) = \{a_i \in T_i : e_i(a_i) \leq e_i(t_i)\}$$

In other words, it is implicit that agent i of type t_i must show the endowment $e_i(a_i)$ corresponding to his reported type a_i. Hence, he cannot claim that he is richer than he really is, but can nevertheless withhold part of his endowment by pretending to be poorer than he is.

A mechanism ξ can now be defined as a mapping which associates with every type profile $t \in T$ a probability distribution $\xi(.|t)$ over

$$X(t) = \left\{ x = (x_i)_{1 \leq i \leq n} \in (\mathbb{R}^l_+)^n : \sum_{1 \leq i \leq n} x_i \leq \sum_{1 \leq i \leq n} e_i(t_i) \right\} \tag{2}$$

[3]The definition of the allocation games seems to depend on the implicit assumption that the planner knows the basic economy. However, the planner should be viewed as a device constructed by the players, so that the underlying assumption is rather that the basic economy is common knowledge among the agents.

The revelation game $\Gamma(\xi)$ induced by ξ starts with the move of nature selecting types. Then every agent i of type t_i makes a report in $R_i(t_i)$ and ξ selects an allocation x. As usual, we will say ξ is *incentive compatible* if truthtelling is a Nash equilibrium of $\Gamma(\xi)$. We thus compute the expected utility of agent i of type t_i when reporting $a_i \in R_i(t_i)$:

$$V_i(\xi|t_i, a_i) = \sum_{t_{-i}} q(t_{-i}|t_i) \int_{\mathbf{R}^l_+} v_i(t, e_i(t_i) - e_i(a_i) + x_i) d\xi(x_i|a_i, t_{-i}) \qquad (3)$$

We refer to these as *W-expected utilities*. Let us set

$$V_i(\xi|t_i) = V_i(\xi|t_i, t_i) \qquad (4)$$

ξ is W-incentive compatible (W-I.C.) iff

$$V_i(\xi|t_i) \geq V_i(\xi|t_i, a_i) \quad \forall i, \forall t_i \in T_i, \forall a_i \in R_i(t_i) \qquad (5)$$

Similar conditions can be given in the T.U. case. A mechanism is then a pair (ξ, μ) where ξ is as above and μ is a mapping from T into $\mathbb{R}^n$ such that $\sum_{1 \leq i \leq n} \mu_i(t) \leq 0$ for every t. Indeed, as noted in Forges, Mertens and Vohra (2002), one can restrict on deterministic money transfers w.l.o.g. Let us set

$$\begin{aligned} & U_i(\xi, \mu|t_i, a_i) & (6) \\ = & \sum_{t_{-i}} q(t_{-i}|t_i) \left[\int_{\mathbf{R}^l_+} v_i(t, e_i(t_i) - e_i(a_i) + x_i) d\xi(x_i|a_i, t_{-i}) + \mu_i(a_i, t_{-i}) \right] \end{aligned}$$

and

$$U_i(\xi, \mu|t_i) = U_i(\xi, \mu|t_i, t_i) \qquad (7)$$

(ξ, μ) is W-I.C. iff

$$U_i(\xi, \mu|t_i) \geq U_i(\xi, \mu|t_i, a_i) \quad \forall i, \forall t_i \in T_i, \forall a_i \in R_i(t_i) \qquad (8)$$

The *revelation principle* for W-allocation games can be stated as follows[4]

[4]Proposition 1 has a wider scope than a standard revelation principle, even in the case of partially verifiable types. For instance, Green and Laffont (1986)'s results only show (in our terminology) that any Nash equilibrium of a revelation game $\Gamma(\xi)$ is equivalent to a truthful Nash equilibrium of this game. Their "nested range condition" is indeed satisfied in our framework. Proposition 1 is essentially an application of Forges and Koessler (2003)' theorem 3, which holds in abstract Bayesian games. The "minimal closure condition" is verified here.

Proposition 1 *In both N.T.U. and T.U. economies, the set of all interim expected payoffs that can be achieved at a Nash equilibrium of some W-allocation game coincides with the set of interim expected payoffs to W-I.C. mechanisms.*

The proof relies on standard arguments and will be sketched below. Before that, let us point out that some care is needed to get the result in the N.T.U. case. Proposition 1 not only states that every equilibrium of a W-allocation game can be achieved by means of a W-I.C. mechanism but that the converse also holds. That is to say, one does not add new payoffs by considering W-I.C. mechanisms. This direction, which holds trivially in standard applications of the revelation principle, is obviously crucial for welfare purposes. In order to get a full equivalence between equilibria of W-allocation games and W-I.C. mechanisms, one has to allow for W-allocation games in which the sets of deposits D_i can be quite restrictive, as we did above.

Let us illustrate the problem with a simple example. Assume $l = 2$, agent 1 has two possible types s and t, $e_1(s) = (1, 0)$, $e_1(t) = (0, 1)$ and consider an allocation game such that $D_1 \supseteq \{(0, 0), (0, 1), (1, 0)\}$, e.g., $D_1 = \mathbb{R}^2_+$. Suppose that the planner selects an allocation such that $x_1 = (0, 0)$ whatever agent 1 says and deposits. Of course, at any equilibrium of the allocation game, agent 1 withholds his endowment (which is feasible with the above set D_1). But in a simple revelation game, agent 1 can either claim type s and deposit 1 unit of the first good or claim type t and deposit 1 unit of the second good. Hence he is forced to reveal his type. As a consequence, $(0, 0)$ can be allocated to agent 1. The revelation game corresponds to a W-allocation game in which $D_1 = \{(0, 1), (1, 0)\}$.

Sketch of the proof of proposition 1

The proof is similar to the one of theorem 3 in Forges and Koessler (2003). Let us call a W-allocation game *canonical* if C_i is (a copy of) agent i's set of types T_i and if D_i is agent i's list of possible initial endowments $\{e_i(t_i), t_i \in T_i\}$, $i = 1, ..., n$. A canonical W-allocation game is thus entirely described by the strategy of the planner: all the other parameters of the game are fixed by the underlying economy. In a canonical W-allocation game, a *canonical* strategy of player i consists of revealing his true type t_i and of depositing the corresponding initial endowment $e_i(t_i)$. By proceeding as in the standard revelation principle, one shows that the set of all interim expected payoffs that can be achieved at a Nash equilibrium of a W-allocation game coincides with the set of interim expected payoffs to canonical Nash equilibria (of canonical W-allocation games).

The previous canonical representation can be further simplified. Agent i can just be asked to report a type a_i, with the understanding that he has to deposit $e_i(a_i)$ when he claims to be of type a_i. Every canonical equilibrium yields a distribution over outcomes as a function of types that can be viewed as a W-I.C. mechanism (with the same interim payoffs). Conversely, a W-I.C. mechanism generates a canonical equilibrium by treating any report of the form (a_i, d_i) with $d_i \neq e_i(a_i)$ in the same way as $(s_i, e_i(s_i))$ for some s_i such that $d_i = e_i(s_i)$.

Q.E.D.

As implicit in Forges, Mertens and Vohra (2002), in T.U. economies, proposition 1 holds even if one allows only for W-allocation games with unrestricted deposits, in which any amount of collateral is allowed a priori. The reason is simple: in a T.U. economy with free disposal of money, the planner can always decide to impose a large negative money transfer to any agent whose deposit does not belong to some prevailing list. Hence, one can restrict D_i to a minimal set, w.l.o.g. We thus have

Proposition 2 *In T.U. economies, the set of all interim expected payoffs that can be achieved at a Nash equilibrium of some W-allocation game with unrestricted deposits coincides with the set of interim expected payoffs to W-I.C. mechanisms.*

One may argue that the W-allocation games with unrestricted deposits are the natural ones: agents face liquidity constraints depending on their initial endowments but do not have to place some minimal amount of consumption goods on the table. The next proposition characterizes the interim expected payoffs that can be achieved by means of such allocation games in the N.T.U. case.

Let $\alpha_i(t_i)$ denote the *interim individually rational level* of agent i of type t_i

$$\alpha_i(t_i) = \sum_{t_{-i}} q(t_{-i}|t_i) v_i(t, e_i(t_i)) \tag{9}$$

A profile of interim payoffs is interim individually rational if every agent i gets at least his interim individually rational level $\alpha_i(t_i)$ for every type t_i. In particular, a mechanism ξ is *interim individually rational* (int.I.R.) if $V_i(\xi|t_i) \geq \alpha_i(t_i)$ for every i, t_i.

Proposition 3 *In N.T.U. economies, the set of all interim expected payoffs that can be achieved at a Nash equilibrium of some W-allocation game with unrestricted deposits coincides with the set of interim expected payoffs to int.I.R. W-I.C. mechanisms.*

Sketch of the proof of proposition 3

Starting from a Nash equilibrium of a W-allocation game with unrestricted deposits one constructs a W-I.C. mechanism with the same interim expected payoffs by proceeding as in the proof of proposition 1. Since every agent can always withhold his full initial endowment in the allocation game, these payoffs are necessarily int.I.R.

Consider now an individually rational W-I.C. mechanism. We have seen in the proof of proposition 1 that it induces a canonical equilibrium in a canonical W-allocation game, in which agent i makes reports in $T_i \times \{e_i(t_i), t_i \in T_i\}$. The canonical equilibrium can be retained as an equilibrium even in a game with unrestricted deposits provided agent i gets 0 whenever he makes a report of the form (a_i, d_i) with $d_i \notin \{e_i(t_i), t_i \in T_i\}$. Agent i will not benefit from such a deviation since his original payoff is int.I.R.

Q.E.D.

Let us close this section by investigating the W-allocation games in the (extensively studied) N.T.U. case with *type-independent* endowments, i.e., $e_i(t_i) = e_i$ for every i and $t_i \in T_i$. Types are then completely unverifiable, and the standard approach is to define allocation games without any deposit requirement: the agents just send a cheap talk message at the second stage of the game, and the planner selects an allocation in

$$X = \left\{ x = (x_i)_{1 \le i \le n} \in (\mathbb{R}^l_+)^n : \sum_{1 \le i \le n} x_i \le \sum_{1 \le i \le n} e_i \right\}$$

with the understanding that he knows the type-independent initial endowments. If one starts with such allocation games (without deposits), the standard revelation principle leads us to incentive compatible mechanisms ξ satisfying the W-I.C. conditions (5), with of course, $e_i(t_i) = e_i(a_i) = e_i$ for all t_i, a_i and $R_i(t_i) = T_i$. These are exactly the conditions imposed in, e.g., Forges (1999), Forges and Minelli (2001), Forges, Minelli and Vohra (2002), Postlewaite and McLean (2002).

We thus recover the traditional I.C. mechanisms when we apply W-allocation games to environments with type-independent endowments if, as proposed above, we consider a large class of possible sets of deposits in the W-allocation games. For instance, a set of deposits of the form $D_i = \{d_i \in \mathbb{R}^l_+ : d_i \geq \min_{t_i \in T_i} e_i(t_i)\}$ forces agent i to deposit his constant endowment. However, focusing on $D_i = \mathbb{R}^l_+$ for every i leads to int.I.R. mechanisms.

To summarize this section: with allocation games allowing for all D_i's satisfying (1), the revelation principle justifies that we use W-I.C. mechanisms, both in N.T.U. and T.U. economies. However, if we focus on allocation games with unrestricted deposits, we give the agents the opportunity of not participating in the game at the interim stage and thus generate only int.I.R. payoffs in the N.T.U. case.

4 W-mechanisms and non-exclusive information

In this section, we study the effects of W-mechanisms in the N.T.U. economies with *non-exclusive information* (N.E.I.) introduced by Postlewaite and Schmeidler (1986). Recall that N.E.I. means that

$$\forall t \in T : q(t) > 0, \forall i = 1, ..., n \quad q(t_i | t_{-i}) = 1$$

This condition is known to imply that any allocation can be made incentive compatible. Let us illustrate that if endowments depend on types, and incentive compatibility is interpreted as W-incentive compatibility, such a property does not hold.

Example: Consider two agents 1 and 2, a single good ($l = 1$), and two equiprobable states of nature ω^1 and ω^2. At ω^1, agent 1 is rich and agent 2 is poor. At ω^2, it is the reverse. Information is *complete* at the interim stage: both agents observe the state of nature (hence N.E.I. obviously holds). This fits in the basic model by taking $T_1 = \{\omega_1^1, \omega_1^2\}$, $T_2 = \{\omega_2^1, \omega_2^2\}$, $q(\omega_1^1, \omega_2^1) = q(\omega_1^2, \omega_2^2) = \frac{1}{2}$, $q(\omega_1^1, \omega_2^2) = q(\omega_1^2, \omega_2^1) = 0$. The initial endowments are $e_1(\omega_1^1) = 2$, $e_1(\omega_1^2) = 0$, $e_2(\omega_2^1) = 0$, $e_2(\omega_2^2) = 2$. The utility functions are the same for both agents and do not depend on types: $v_i(t, x) = \sqrt{x}$, for every $t \in T$ and every $x \in \mathbb{R}_+$.

Consider the "full insurance allocation" allocating 1 unit of good to each agent in each state. It looks like a reasonable ex ante allocation. But it cannot be achieved as an equilibrium outcome of a W-allocation game as defined in the previous section. Indeed, suppose agent 1 is rich (state ω^1). He should deposit his 2 units in order to give 1 to agent 2. If he pretends to be poor (i.e., of type ω_1^2) and deposits nothing, he consumes his 2 units, which is better than 1. Of course, if agent 2 reveals truthfully his type, the planner sees that one of the agents has lied, since a state of zero probability has been reported. But he cannot figure out which agent has cheated, and there is nothing to be confiscated on the table.

In this example, even allowing for the large class of W-allocation games introduced above, incentive compatibility implies interim individual rationality. We go on by recalling well-known results which show that the crucial features of the example are: N.T.U. economy, type-dependent endowments and allocation that is not int.I.R.

The key to the use of non-exclusive information is the existence of a "worst outcome": if a state of zero probability is reported, than the planner punishes all agents by selecting the worst outcome. In the T.U. case, with free disposal of money, a worst outcome is easily constructed. For int.I.R. mechanisms, one can establish the following result:

Proposition 4 *Under non-exclusive information, for every int.I.R. mechanism ξ, there exists a W-I.C. mechanism ξ' achieving the same interim expected payoffs as ξ.*

The proof is similar to the one of lemma 3.1 in Vohra (1999), but, since Vohra does not consider W-allocation games[5], we briefly recall the argument. Define ξ' by: $\xi'(.|t) = \xi(.|t)$ if $q(t) > 0$, $\xi'((e_i(t_i))_{1\leq i\leq n}|t) = 1$ if $q(t) = 0$. The interpretation of the latter is that if the agents' reports are inconsistent, the planner returns to every agent i the endowment $e_i(t_i)$ corresponding to his claimed type t_i (i.e., the deposit which is on table). If agent i of type t_i reveals his true type t_i, he gets at least his interim I.R. level $\alpha_i(t_i)$ (recall (9)). If he claims to be of type $a_i \neq t_i$, $e_i(a_i) \leq e_i(t_i)$, for every truthful report t_{-i} of the other agents, $q(a_i, t_{-i}) = 0$, so that agent i gets $e_i(t_i)$ $(= e_i(t_i) - e_i(a_i) + e_i(a_i))$. By lying, he cannot expect more than $\alpha_i(t_i)$.[6]

The following result is also easily established:

[5] See remark 2, section 7.

[6] One can also take $\xi'(0|t) = 1$ if $q(t) = 0$, in which case agent i just keeps $e_i(t_i) - e_i(a_i)$ when he lies.

Proposition 5 *If endowments do not depend on types and information is non-exclusive, for every mechanism ξ, there exists a W-I.C. mechanism ξ' achieving the same interim expected payoffs as ξ.*

The proof is similar as above, provided that one adopts the variant in the footnote: nothing guarantees here that ξ is int.I.R. (observe that ξ does not have to be ex ante I.R.). When endowments do not depend on types, they can be confiscated, which determines a "worst outcome"[7].

Obviously, the analysis is extremely sensitive to the details of the W-allocation game. We may imagine a number of scenarios, e.g., inspired by recent contract theory, that would allow the planner to check the agents' endowments in some circumstances. We will just stick to the general equilibrium literature, and consider, in the next section, the D-mechanisms used by Postlewaite (1979), Hurwicz, Maskin and Postlewaite (1995), and implicitly, all the papers assuming constant aggregate endowment.

5 D-mechanisms

The only difference between a *D-allocation game* and a W-allocation game is that agents cannot consume any part of their endowment that they do not put on the table. In other words, they can destroy some of their endowment, but cannot withhold anything. A D-allocation game can thus be defined exactly as a W-allocation game, except for the last stage, which becomes:

- Agent i receives the payoff $v_i(t, x_i)$ in the N.T.U. case, $u_i(t, x_i, m_i)$ in the T.U. case.

Canonical D-allocation games and canonical equilibria can be defined in the same way as in section 3. In particular, we can characterize them by means of the same mechanisms ξ as above. The expected utility of agent i of type t_i when reporting $a_i \in R_i(t_i)$ becomes now, in the N.T.U. case,

$$V_i'(\xi|t_i, a_i) = \sum_{t_{-i}} q(t_{-i}|t_i) \int_{\mathbf{R}^l_+} v_i(t, x_i) d\xi(x_i|a_i, t_{-i}) \tag{10}$$

[7] The same argument is used in the proof of proposition 4.1 in Vohra (1999). Hence, endowments are implicitly assumed to be constant in that result (unlike in the rest of the paper, see remark 2 in section 7).

and in the T.U. case,

$$U_i'(\xi, \mu | t_i, a_i) = \sum_{t_{-i}} q(t_{-i}|t_i) \left[\int_{\mathbf{R}_+^l} v_i(t, x_i) d\xi(x_i | a_i, t_{-i}) + \mu_i(a_i, t_{-i}) \right] \quad (11)$$

We refer to these expressions as *D-expected utilities*. By proceeding in a similar way as in section 3 and recalling (3) and (4), we get in the N.T.U. case

$$V_i'(\xi|t_i) =_{def} V_i'(\xi|t_i, t_i) = V_i(\xi|t_i, t_i) = V_i(\xi|t_i)$$

and similarly, in the T.U. case, recalling (6) and (7),

$$U_i'(\xi, \mu|t_i) =_{def} U_i'(\xi, \mu|t_i, t_i) = U_i(\xi, \mu|t_i, t_i) = U_i(\xi, \mu|t_i)$$

D-expected utilities and W-expected utilities thus coincide at equilibrium. *D-incentive compatibility* can be defined as in (5) and (8). In the N.T.U. case,

$$V_i'(\xi|t_i) \geq V_i'(\xi|t_i, a_i) \quad \forall i, \forall t_i \in T_i, \forall a_i \in R_i(t_i) \quad (12)$$

and in the T.U. case,

$$U_i'(\xi, \mu|t_i) \geq U_i'(\xi, \mu|t_i, a_i) \quad \forall i, \forall t_i \in T_i, \forall a_i \in R_i(t_i) \quad (13)$$

A revelation principle as in proposition 2 (with full freedom to the agents, namely, $D_i = \mathbb{R}_+^l$, $i = 1, ..., n$) holds for D-I.C. mechanisms, even in the N.T.U. case.

Proposition 6 *In both N.T.U. and T.U. economies, the set of all interim expected payoffs that can be achieved at a Nash equilibrium of some D-allocation game with unrestricted deposits coincides with the set of interim expected payoffs to D-I.C. mechanisms.*

Obviously, an analog of proposition 1 also holds. To see that we need not consider all possible sets of deposits when withholding is not possible, consider a Nash equilibrium of some D-allocation game, with sets of deposits D_i. We can get the same expected payoffs at an equilibrium of a D-allocation game with unrestricted deposits by allocating the bundle $x_i = 0$ to agent i whenever his deposit is in $\mathbb{R}_+^l \setminus D_i$ and proceeding as in the original game otherwise.

We now investigate the relationships between W-I.C. and D-I.C. mechanisms. Consider a W-I.C. mechanism ξ in an N.T.U. economy. ξ can be used in a D-allocation game. Since utility functions are increasing, we have for every t_i, a_i such that $e_i(t_i) - e_i(a_i) \geq 0$,

$$V_i(\xi|t_i, a_i) \geq V_i'(\xi|t_i, a_i)$$

Hence, every W-I.C. mechanism ξ is automatically D-I.C. This property obviously holds in the T.U. case as well. This immediately implies the first part of the next proposition:

Proposition 7 *In both N.T.U. and T.U. economies, the set of interim expected payoffs to W-I.C. mechanisms is included in the set of interim expected payoffs to D-I.C. mechanisms. In T.U. economies, for every D-I.C. mechanism* (ξ, μ)*, there exists a W-I.C. mechanism* (ξ, μ') *achieving the same aggregate ex ante expected payoff as* (ξ, μ)*.*

Before proving the second part of the statement, we illustrate that the inclusion in the first part may be strict by going back to the **example** at the end of the previous section.

The "full insurance allocation" allocating 1 unit of good to each agent in each state can be achieved by a D-I.C. mechanism. Indeed, let us allocate goods as follows, as a function of the agents' reports (and corresponding deposits):

$$\begin{aligned} x(\omega_1^1, \omega_2^1) &= x(\omega_1^2, \omega_2^2) = (1,1) \\ x(\omega_1^1, \omega_2^2) &= x(\omega_1^2, \omega_2^1) = (0,0) \end{aligned} \tag{14}$$

The mechanism is feasible: if the agents report (ω_1^1, ω_2^1), agent 1 puts 2 units of good on the table, 1 unit of which can be given to agent 2 (who deposits nothing), etc. The mechanism also satisfies the I.C. conditions (12). When poor, agent 1 has no way to misrepresent his type. If he is rich, he can pretend to be poor, but, given that agent 2 tells the truth, this results in a reported state of zero probability, and agent 1 gets nothing (since in the present model, he cannot consume his 2 units if he does not put them on the table). If he tells the truth, he keeps 1 unit, which is better than nothing.

Proof of the second part of proposition 7 (*equivalence of D-I.C. mechanisms and W-I.C. mechanisms in the T.U. case*)

Let (ξ, μ) be a D-I.C. mechanism. The aggregate ex ante (W- or D-) expected payoff from (ξ, μ) is

$$v(\xi, \mu) = \sum_{1 \leq i \leq n} \sum_{t_i \in T_i} q(t_i) U_i(\xi, \mu | t_i)$$

We first construct monetary transfers λ such that $(\xi, \mu + \lambda)$ is W-I.C., namely satisfies (8).

Define, for every i and $t_i \in T_i$,

$$\begin{aligned} K_i &= \max_{t_i \in T_i} \max_{r_i \in R_i(t_i)} \left[U_i(\xi, \mu | t_i, s_i) - U_i(\xi, \mu | t_i) \right] \\ \eta_i(t_i) &= \left| \{ s_i \in T_i : e_i(s_i) \leq e_i(t_i) \} \right| \\ \lambda_i(t_i) &= K_i \eta_i(t_i) \end{aligned}$$

Let us show that $(\xi, \mu + \lambda)$ is W-incentive compatible. Consider agent i of type t_i. By telling the truth (and thus depositing $e_i(t_i)$), he gets $U_i(\xi, \mu | t_i) + \lambda_i(t_i) = U_i'(\xi, \mu | t_i) + \lambda_i(t_i)$. This agent contemplates to report $a_i \in R_i(t_i)$, i.e., to deposit $e_i(a_i)$ and withhold $e_i(t_i) - e_i(a_i) \geq 0$. Suppose first that $e_i(a_i) = e_i(t_i)$. Then $\lambda_i(a_i) = \lambda_i(t_i)$ and $U_i(\xi, \mu | t_i, a_i) = U_i'(\xi, \mu | t_i, a_i)$. Hence the W-I.C. of $(\xi, \mu + \lambda)$ at (t_i, a_i) results from the D-I.C. of (ξ, μ) and the construction of λ.[8] Suppose next that $e_i(a_i) \neq e_i(t_i)$; then $\eta_i(a_i) \leq \eta_i(t_i) - 1$ and $\lambda_i(t_i) \geq \lambda_i(a_i) + K_i$. The W-I.C. of $(\xi, \mu + \lambda)$ at (t_i, a_i) just results from the construction of λ:

$$U_i(\xi, \mu | t_i, a_i) - U_i(\xi, \mu | t_i) \leq K_i \leq \lambda_i(t_i) - \lambda_i(a_i)$$

At this point, nothing guarantees that $\mu + \lambda$ is feasible. λ has the particular feature that the transfers λ_i to agent i only depend on his own type in T_i. Hence, by a well-known procedure (see, e.g., Johnson, Pratt and Zeckhauser (1990)), one can replace λ by λ' such that λ' is exactly balanced (i.e., $\sum_{1 \leq i \leq n} \lambda_i'(t) = 0$ for every $t \in T$) and $(\xi, \mu + \lambda')$ is W-I.C.:

$$\lambda_i'(t) = \lambda_i(t_i) - \frac{1}{n-1} \sum_{j \neq i} \lambda_j(t_j)$$

[8]Except for this part of the reasoning, the proof is similar to the one of theorem 1 in Forges, Mertens and Vohra (2002), which assumes injective endowments, i.e. $e_i(a_i) \neq e_i(t_i)$.

$\mu' = \mu + \lambda'$ satisfies all our requirements, since by the exact balancedness of λ', the aggregate ex ante (W- or D-) expected payoff satisfies $v(\xi, \mu') = v(\xi, \mu)$.

Q.E.D.

6 Constant aggregate endowment

Several papers on implementation (Postlewaite and Schmeidler (1986), Palfrey and Srivastava (1989), Serrano and Vohra (2001)) assume that the aggregate initial endowment $\sum_i e_i(t_i)$ does not depend on t, i.e., is equal to, say, $\overline{e}$ for every $t \in T$: $q(t) > 0$.[9] Mechanisms ξ are then defined over

$$\overline{X} = \left\{ x = (x_i)_{1 \leq i \leq n} \in (\mathbb{R}^l_+)^n : \sum_{1 \leq i \leq n} x_i \leq \overline{e} \right\}$$

Note that the above papers do not make any assumptions on a possible deposit scenario. They proceed as if $\overline{e}$ was available to the planner in the allocation game, so that in particular $\overline{e}$ can be confiscated for some reports of the agents.

More precisely, they focus on deterministic mechanisms, which are thus mappings $y : T \to \overline{X}$; let us call them C-mechanisms. Such a mechanism is I.C. if

$$\sum_{t_{-i}} q(t_{-i}|t_i) v_i(t, y_i(t)) \geq \sum_{t_{-i}} q(t_{-i}|t_i) v_i(t, y_i(a_i, t_{-i})) \quad \forall i, \forall t_i, a_i \in T_i \qquad (15)$$

We will refer to this condition as C-I.C. It is similar to (12), but must hold for *all* a_i's. On the other hand, y is a C-mechanism, defined over $\overline{X}$ whatever the reports of the agents, while the mechanisms derived from canonical equilibria of canonical D-allocation games are defined over $X(a)$ when the agents report a.

Consider an N.T.U. economy with type-independent endowments, i.e., $e_i(t_i) = e_i$ for every i, t_i. In this case, one checks immediately that W-I.C. (see (5)) is equivalent to D-I.C. (see (12)).

[9] Given our basic measurability assumptions (expressing that every agent knows his own initial endowment), if $\sum_i e_i(t_i) = \overline{e}$ for all t's, then $e_i(t_i)$ must be independent of t_i for every t_i. We will consider this particular case. Our formulation of constant aggregate endowments allows for other situations.

This property does not extend to the case of constant aggregate endowment, as illustrated by the **example** of section 4, in which $\sum_i e_i(t_i) = 2$ for every t of positive probability. We saw in section 5 that the full insurance allocation could be implemented by a D-I.C. mechanism, but could not be implemented by a W-I.C. mechanism. (14) can be viewed as a C-I.C. mechanism: for the rich types, we can proceed as before; we now have an I.C. condition for every agent when he is poor, but this is clearly satisfied.

We will show that the pattern of the example is typical: C-I.C. mechanisms are essentially equivalent to D-I.C. mechanisms. We focus on deterministic mechanisms, since C-I.C. mechanisms have been defined under that further assumption in the literature.

Proposition 8 *If the aggregate initial endowment is type-independent, the set of interim expected payoffs to deterministic C-I.C. mechanisms coincides with the set of interim expected payoffs to deterministic D-I.C. mechanisms.*

Proof: Let us start with a C-I.C. mechanism y. $\sum_i y_i(a) \leq \bar{e}$ for all a's, even if $q(a) = 0$. Let us construct y' from y, as follows, for every $a \in T$:

$$\begin{aligned} y'(a) &= y(a) \quad \text{if } \sum_i y_i(a) \leq \sum_i e_i(a_i) \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

This modifies y only on a's such that $q(a) = 0$, since $q(a) > 0 \Longrightarrow \sum_i e_i(a_i) = \bar{e}$. Hence y and y' yield the same interim expected payoffs. y' is still C-I.C., since we lower the payoffs from y in case of deviation. y' never allocates more than $\sum_i e_i(a_i)$ when the agents report a and is D-I.C.

Conversely, let us consider a D-I.C. deterministic mechanism ξ, namely a mapping defined on T, such that $\xi(a) \in X(a)$ for every $a \in T$, satisfying (12). We modify ξ in order to fulfil (15), at *every* a_i.

$$\begin{aligned} \xi'(a) &= \xi(a) \quad \text{if } q(a) > 0 \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

ξ' is a D-I.C. mechanism achieving the same interim expected payoffs as ξ. Let us show that ξ' satisfies (15). Consider agent i of type t_i but claiming to be of type a_i. He gets $\xi'_i(a_i, t_{-i})$ depending on the others' types. Let t_{-i} be such that $q(t_{-i}|t_i) > 0$; if $q(a_i, t_{-i}) = 0$, $\xi'_i(a_i, t_{-i}) = 0$; if $q(a_i, t_{-i}) > 0$, we must have $e_i(a_i) + \sum_{j \neq i} e_j(t_j) = \bar{e}$, but since $q(t_{-i}|t_i) > 0$, we also have

$e_i(t_i) + \sum_{j \neq i} e_j(t_j) = \bar{e}$ so that $e_i(t_i) = e_i(a_i)$. If the type a_i claimed by agent i of type t_i is such that $e_i(t_i) = e_i(a_i)$, then ξ' satisfies C-I.C. at t_i, a_i because it satisfies D-I.C. at t_i, a_i. Otherwise, for any t_{-i} such that $q(t_{-i}|t_i) > 0$, $q(a_i, t_{-i}) = 0$ and $\xi'_i(a_i, t_{-i}) = 0$, so that ξ' also satisfies C-I.C. at t_i, a_i in this case.

In words: D-I.C. seems easier to fulfil than C-I.C, since one cannot pretend to be richer than one really is in D-I.C. However, under the assumption that the aggregate endowment is constant on every type profile of positive probability, a player cheating on his endowment is automatically detected if the other players tell the truth. And the planner can then confiscate the whole aggregate endowment.

Q.E.D.

Remark: In the first part of the proof, we could have modified y in the same way as ξ.

7 Concluding remarks

Remark 1:

The fact that D-I.C. mechanisms allow to implement more allocations than W-I.C. ones does not depend, as our previous example might suggest, on non-exclusive information. Furthermore, there may exist int.I.R., D-I.C. mechanisms which are not W-I.C. Take an economy in which agent 1 is the only agent with private information and cares only about the first good[10]: $T_1 = \{s, t\}$, $q(s) = q(t) = \frac{1}{2}$, $e_1^1(s) = 0$, $e_1^1(t) = 1$, $v_1(s, x) = v_1(t, x) = x^1$. Consider a mechanism ξ such that $\xi_1^1(s) = 1.5$, $\xi_1^1(t) = 2$. In state s, agent 1 cannot lie. In state t, if he cannot withhold his unit of good, he prefers to deposit it and get 2 than pretending that the state is s, destroying his endowment and getting 1.5. However, if he can withhold his endowment, he lies in order to get 2.5.

Remark 2:

Several papers (e.g., [18], [7], [20]) deal with incentive constraints in type-dependent endowments economies without making any allocation game explicit. Vohra (1999) defines direct mechanisms which, except for being deterministic, are similar to the mechanisms ξ introduced above. His incentive

[10]Good 1 appears as a superscript. The example can be completed into an exchange economy by adding non-informed agents who care about all goods but good 1.

compatibility conditions consist of requiring (5) for every $a_i \in T_i$: Vohra (1999) uses W-mechanisms but does not take into account that types are partially verifiable. He assumes that utility is $-\infty$ outside $\mathbb{R}^l_+$. His incentive compatibility conditions are stronger than ours.

In defining incentive compatible trading plans, Kehoe, Levine and Prescott (2000) explicitly take into account the feasibility of agents' reports. Our sets $R_i(t_i)$ typically satisfy their assumptions on sets of feasible reports (they correspond to "voluntary public endowments"). Kehoe, Levine and Prescott (2000) assume that the utility function of a given agent does not depend on other agents' types (private values) and restrict themselves on privately measurable net trades (i.e., they do not allow for mechanisms relying on all agents' reports). Furthermore, in their framework, the type dependent utility function of an agent is defined over *net trades*. Hence, by interpreting their utility functions in an appropriate way, their I.C. conditions can accommodate W-I.C. as well as D-I.C. As the authors make clear, "they treat sets of feasible reports as data". Unlike in the present paper, they thus do not attempt to deduce the sets of reports from a relevant version of the revelation principle.

Remark 3:

Some more elements on the choice between W-mechanisms and D-mechanisms in N.T.U. economies. As mentioned in section 3, consistency with the by now standard mechanisms in economies with type-independent endowments (or type-independent aggregate endowment) is in favor of D-mechanisms. These mechanisms ensure that a result like the non-emptiness of the ex ante I.C. core in economies satisfying N.E.I. holds even if endowments depend on types. On the other hand, Forges, Mertens and Vohra (2002) used W-mechanisms in an N.T.U. example (in section 6.2), in order to illustrate the possible emptiness of the ex ante incentive compatible core in a Walrasian economy with injective type-dependent endowments (thus proving the necessity of transfers for the positive result stated as theorem 1).

Remark 4:

Participation constraints are important in mechanism design, and have perhaps been overlooked in the implementation literature (see, e.g., Palfrey (2002)). However, we would like to separate these participation constraints from the feasibility constraints. Incentive compatibility constraints are usually conceived as feasibility constraints. It is thus disturbing that W-incentive compatibility easily generates interim individual rationality, because we may very well want to use W-I.C. mechanisms in a context where only ex ante individual rationality matters.

We mentioned in the introduction Hong's extensions of the results on Bayesian implementation ([5], [15], [17], etc.) to economies with type-dependent endowments, or adopting another point of view, of [12] from Nash to Bayesian implementation (see [9] and [10]). Using our terminilogy, Hong relied on W-I.C. mechanisms which are not necessarily int.I.R., an approach which may be discutable, as we showed.

References

[1] Forges, F. and F. Koessler, "Communication equilibria with partially verifiable types", Document de Travail du THEMA 2003-10

[2] Forges, F., J.-F. Mertens and R. Vohra (2002), "The ex ante incentive compatible core in the absence of wealth effects", *Econometrica* 70, 1865-1892.

[3] Forges, F. and E. Minelli (2001), "A note on the incentive compatible core", *Journal of Economic Theory* 98, 179-188.

[4] Forges, F., E. Minelli and R. Vohra (2002), "Incentives and the core of an exchange economy: a survey", *Journal of Mathematical Economics* 38, 1-41.

[5] Jackson, M. (1991), "Bayesian implementation", *Econometrica* 59, 461-477.

[6] Johnson, S., J. Pratt and R. Zeckhauser (1990), "Efficiency despite mutually payoff-relevant private information: the finite case", *Econometrica* 58, 873-900.

[7] Kehoe, T., D. Levine and E. Prescott (2000), "Lotteries, sunspots and incentive constraints", mimeo, University of Minnesota.

[8] Green, J. and J.-J. Laffont (1986), "Partially verifiable information and mechanism design", *Review of Economic Studies* 53, 447-456.

[9] Hong, L. (1996), "Bayesian implementation in exchange economies with state dependent feasible sets and private information", *Social Choice and Welfare* 13, 433-444.

[10] Hong, L. (1998), "Feasible Bayesian implementation with state dependent feasible sets", *Journal of Economic Theory* 80, 201-221.

[11] Hong, L. and S.E. Page (1994), "Reducing informational costs in endowment mechanisms", *Economic Design* 1, 103-117.

[12] Hurwicz, L., E. Maskin and A. Postlewaite (1995), "Feasible Nash implementation of social choice rules when the designer does not know endowments or production sets" in *The Economics of Informational Decentralization: Complexity, Efficiency and Stability*, ed. by J. Ledyard. Amsterdam: Kluwer Academic Publishers, 367-433.

[13] McLean, R. and A. Postlewaite (2002), "Informational size, incentive compatibility and the core of a game with incomplete information", forthcoming in *Games and Economic Behavior*.

[14] Palfrey, T. (2002), "Implementation theory", in *Handbook of Game Theory*, vol III, ed. by R. Aumann and S. Hart, Amsterdam: Elsevier, 2272-2326.

[15] Palfrey, T. and S. Srivastava (1989), "Implementation with incomplete information in exchange economies", *Econometrica* 57, 115-134.

[16] Postlewaite, A. (1979), "Manipulation via endowments", *Review of Economic Studies* 46, 255-262.

[17] Postlewaite, A. and D. Schmeidler (1986), "Implementation in differential information economies", *Journal of Economic Theory* 39, 14-33.

[18] Postlewaite, A. and D. Schmeidler (1987), "Differential information and strategic behavior in economic environment: a general equilibrium approach" in *Information, Incentives and Economic Mechanisms*, ed. by T. Groves, R. Radner and S. Reiter, University of Minnesota Press.

[19] Serrano, R. and R. Vohra (2001), "Some limitations of virtual Bayesian implementation", *Econometrica* 69, 785-792.

[20] Vohra, R. (1999), “Incomplete information, incentive compatibility and the core”, *Journal of Economic Theory* 86, 123-147.