Fair Income Tax

Marc Fleurbaey* and François Maniquet†

March 2002

Abstract

In a model where agents have unequal skills and heterogeneous preferences, we look for the optimal tax on the basis of fairness principles and incentive-compatibility constraints. Our fairness principles lead us to construct new indices of individual well-being, and to apply the maximin criterion to those indices. The originality of our well-being measures is that they do not require any information about individual utilities, but only about non-comparable individual preferences. Our approach sheds light on the hotly debated issue of whether the optimal tax should focus on the hardworking poor or on the low incomes.

JEL Classification: D63, H21.

Keywords: optimal tax, fairness.

1 Introduction

Fairness is a key concept in redistributive issues. In this paper, we study how requirements of fairness can shed light on the design of the optimal income tax schedule.

We consider a population of heterogenous individuals (or households), who differ under two respects. First, they have unequal skills (that is, earning abilities). Second, they differ in terms of their preferences about consumption and leisure and, therefore, make different labor time choices. Both kinds of differences generate income inequalities. We study how to justify and compute a redistributive income tax in this context.

Redistribution through an income tax usually entails distortions of incentives, but the resulting efficiency loss has to be weighed against potential improvements in the fairness of the distribution of resources. We address this efficiency-equity trade-off here by constructing social preferences which obey the standard Pareto principle in addition to fairness conditions.

Two fairness requirements are introduced below. Briefly, the first requirement, a qualification of the Pigou-Dalton principle, states that transfers reducing income inequalities are acceptable, *provided* they are performed between agents having identical preferences and choosing identical labor time. This proviso makes the requirement much less demanding, and much more appealing, than the usual Pigou-Dalton transfer principle which

*CATT, THEMA, IDEP, Université de Pau et des Pays de l'Adour.

†Institute for Advanced Study (Princeton), FNRS and University of Namur (Belgium).

applies to all income inequalities. The second fairness requirement is that the laisser-faire (that is, the absence of redistribution) should be the social optimum in the hypothetical case when all agents have equal earning abilities. The underlying idea is that income inequalities would then reflect free choices from different preferences on an identical budget set, and that such choices ought to be respected.

The combination of these two requirements with the Pareto principle, and ancillary conditions of informational parsimony and separability (the idea that indifferent agents should not influence social preferences), lead us to single out a particular kind of social preferences, and enable us to derive some interesting conclusions about the optimal tax schedule. The social preferences obtained on this basis exclude a simplistic approach in terms of *income,* or a traditional welfarist measurement of *subjective utility*, and rely instead on individual consumption-leisure *preferences* in a sophisticated way, in order to assess how well-off the individuals are. An additional striking result is that these social preferences give an *absolute priority* to the worse-off individuals, even though the only redistributive fairness principle invoked here is the timid transfer principle, in the weak variant described above, which by itself is compatible with any degree of aversion to inequality. As explained in the paper, this result is due to the presence, along the above fairness and efficiency requirements, of an informational parsimony condition in the spirit of Arrow's independence of irrelevant alternatives (Arrow 1951).

As far as the optimal tax is concerned, the main result is that those individuals who have the lowest earning ability but work full time, namely, the *hardworking poor*, will be granted the greatest subsidy (a subsidy is simply a negative tax) of the whole population. This result may be rightly viewed as giving a certain legitimacy to some recent evolutions of the welfare system in several Western countries, where marginal tax rates for low incomes have been reduced. However, the result crucially depends on the second fairness requirement mentioned above (laisser-faire should be the social optimum in the hypothetical case when all agents have equal earning abilities) and the respect of individual choices of labor time it embodies. Indeed, we will show that if one replaces this requirement with one expressing the idea that no discrimination should ever be made between "deserving poor", who do not work because of low productivity, and "undeserving poor", who do not work because of labor-averse preferences, then one is led to different social preferences, and to a different optimal tax, namely, the tax which maximizes the minimum income.

This sheds light on an interesting ethical issue, which deserves to be the focus of public debates: The choice between an EITC-like[1] system and a more generous basic

[1]The Earned Income Tax Credit consists in giving a tax credit, or subsidy, to low incomes so as to provide incentives for labor participation to individuals with a low earning ability.

income hinges upon the degree to which individuals should be held liable for their choices about labor participation. The choice also depends on the legitimacy of a discriminating treatment in favor of the deserving poor. Our analysis pinpoints these issues in a vivid and transparent way.

Let us briefly describe the relationship between our approach and the literature. The theory of optimal taxation has focused mostly on social objectives defined in terms of welfarist social welfare functions, based on interpersonal comparisons of utility. It has obtained valuable insights into the likely shape of the optimal tax, as can be grasped from the outstanding works of Atkinson (1973, 1995), Diamond (1998), Ebert (1992), Mirrlees (1971), Sadka (1976), Seade (1976) and Tuomala (1990), among many others. Many results depend on the particular choice of individual utility function and social welfare function. The social marginal utility of an individual's income may thus reflect various personal characteristics (individual utility) and ethical values embodied in the social welfare function, including, potentially, fairness requirements. But, apart from the important relationship between inequality aversion and (Schur-)concavity of the social welfare function, the link between fairness requirements and features of the social welfare function are not usually made explicit. In contrast, our approach starts from requirements of fairness, and derives social preferences on this basis.

A remarkable feature of our approach is that it does not require any information about individual *utilities*, as it is able to construct social preferences solely in terms of ordinal, non-comparable individual preferences. Unfortunately, it has become almost a dogma, in welfare economics, that the construction of reasonable social preferences requires interpersonally comparable utilities. This view is usually buttressed on Arrow's impossibility theorem of social choice (Arrow 1951). However, our results prove this view to be mistaken. A detailed analysis of this point, in relation to Arrow's theorem, is made in Fleurbaey and Maniquet (1996b, 2001). The key point is that reasonable weakenings of Arrow's axiom of Independence of Irrelevant Alternatives, such as a condition proposed by Hansson (see Section 5 below), are compatible with the construction of social preferences. This should be taken as very good news for public economics. It *is* possible to construct social preferences by restricting ourselves to non-comparable individual preferences, and devising appropriate fairness requirements. It is therefore possible to study second-best issues without delicate interpersonal comparisons of utilities, and without restricting attention to efficiency considerations. This paper is a demonstration of how this can be done.

Our work focuses here on a model with unequal earning abilities and heterogenous preferences, and builds on early studies of this model, which dealt with first-best allocations (Fleurbaey and Maniquet 1996a, 1999) or with linear tax (Bossert, Fleurbaey and Van de gaer 1999), or focused on different fairness concepts (Fleurbaey and Maniquet 2000).

The paper is organized as follows. Sections 2 and 3 introduce the model and the concept of social preferences. Sections 4 and 5 list some requirements imposed on social preferences. A theorem describing the resulting social preferences is presented in section 6, with an intuitive proof in section 7. Section 8 introduces the tax redistributive system, while a preliminary analysis of the optimal tax, in a simple two-agent model, is expounded in section 9. The general case of a larger population is analyzed in section 10. Then, section 11 discusses how the results are modified if one wants to avoid any discrimination against the "undeserving poor". Concluding remarks are offered in the last section.

2 The model

There are two goods in our model, labor and consumption. A *bundle* for agent i is a pair $z_i = (\ell_i, c_i)$, where ℓ_i is labor and c_i consumption. The agents' *consumption set* X is defined by the conditions $0 \leq \ell_i \leq 1$ and $c_i \geq 0$.

The population contains n agents. Agents have two characteristics, their personal preferences over the consumption set and their personal wage rate. For any agent $i = 1, ..., n$, *personal preferences* are denoted R_i, and $z_i R_i z_i'$ (resp. $z_i P_i z_i'$, $z_i I_i z_i'$) means that bundle z_i is weakly preferred (resp. strictly preferred, indifferent) to bundle z_i'. We assume that individual preferences are continuous, convex and monotonic.[2]

Agent i's earning ability is measured by her *wage rate*, denoted w_i, and is measured in consumption units, so that $w_i \geq 0$ is the amount of consumption that agent i can afford when working $\ell_i = 1$, in the absence of tax and transfers, and, for any ℓ_i, $w_i \ell_i$ is the agent's pre-tax income. Wage rates are assumed to be fixed, as in a constant returns to scale technology.

3 Social preferences

An allocation is a collection $z = (z_1, ..., z_n)$. Social preferences will allow us to compare allocations in terms of their goodness. Social preferences will be formalized as a complete ordering over all allocations in X^n, and will be denoted R, with asymmetric and symmetric components P and I, respectively. In other words, $z \; R \; z'$ means that z is at least as good

[2]Preferences are monotonic if $\ell_i \leq \ell_i'$ and $c_i > c_i'$ implies that $(\ell_i, c_i) P_i (\ell_i', c_i')$. Strict monotonicity with respect to consumption is required here in order to exclude cases of satiation.

as z', $z\ P\ z'$ means that it is strictly better, and $z\ I\ z'$ that they are equivalent.

It must be emphasized that social preferences may depend on the population profile of characteristics $(R_1, ..., R_n)$ and $(w_1, ..., w_n)$, as in the theory of social choice. Formally, they are a mapping from the set of population profiles to the set of complete orderings over allocations. We do not introduce special notations for these notions in order to minimize the quantity of symbols in this paper. The domain of economies for which we want social preferences to be defined contains all economies obeying the description of the previous section, with $n \geq 2$.

4 Fairness

The main ethical requirement we will impose on social preferences, in this paper, is derived from the Pigou-Dalton transfer principle. Traditionally, however, this principle was somewhat questionably applied to all income inequalities. This entails that no distinction is made between two agents with the same income but very different wage rates and different amounts of labor. We will be more cautious here, and apply it only to agents with identical labor. In addition, we will also restrict it to agents with identical preferences. There are two reasons for this additional restriction. First, applying the Pigou-Dalton principle to agents with different preferences would clash with the Pareto principle (to be defined more precisely below), as proved by Fleurbaey and Trannoy (2000). Second, when two agents have identical preferences one may more easily defend the idea that they deserve to obtain similar incomes, whereas this is much less clear in the case of different preferences (as discussed below). This gives us the following, rather weak requirement:

Transfer Principle: If z and z' are two allocations, and i and j are two agents with identical preferences, such that $\ell_i = \ell_j = \ell'_i = \ell'_j$, and for some $\delta > 0$,

$$c'_i - \delta = c_i > c_j = c'_j + \delta,$$

whereas for all other agents k, $z_k = z'_k$, then $z\ R\ z'$.

The second important requirement we will introduce has to do with providing opportunities and respecting individual preferences. Although reducing income inequalities is a generous goal, it is not obvious how to deal with agents who "choose" poverty out of a budget set which contains better income opportunities. In particular, when all agents have the same wage rate, it can be argued that there is no need for redistribution, as they all have access to the same labor-consumption bundles (Dworkin 1981). Any income difference is then a matter of personal preferences. The laisser-faire allocation z^* is such that for every agent i, z_i^* is the best for R_i over the budget set defined by $c_i \leq w_i \ell_i$. The following requirement says that the laisser-faire allocation is, in this particular case of uniform earning ability, the best among all feasible allocations.

Laisser-Faire: If all agents have the same wage rate w, then for any allocation z' such that $\sum_i c'_i \leq w \sum_i \ell'_i$, one has $z^* \; R \; z'$, where z^* denotes the laisser-faire allocation.

5 Efficiency, parsimony, separability

The other requirements are basic conditions derived from the theory of social choice. First, we want social preferences to obey the standard Pareto condition. This condition is essential in order to take account of efficiency considerations. Social preferences satisfying the Pareto condition will never lead to the selection of inefficient allocations. In this way we are preserved against excessive consequences of fairness requirements, such as equality obtained through levelling-down devices.

Weak Pareto: If z and z' are such that for all i, $z_i \; P_i \; z'_i$, then $z \; P \; z'$.

Second, we want our social preferences to use minimal information about individual preferences, in the spirit of Arrow's condition of independence of irrelevant alternatives. Arrow's condition is, however, much too restrictive, and leads to the unpalatable results of his impossibility theorem. Arrow's condition requires social preferences over two allocations to depend only on individual preferences over these two allocations. This condition makes it impossible to take account of marginal rates of substitution at the two allocations, of whether an agent envies another's bundle, of whether two agents have similar or very different preferences, etc. For instance, it destroys the power of the restriction that agents must have identical preferences in the above Transfer Principle axiom, because for any monotonic preferences it is always the case that i prefers z' and j prefers z, for the allocations z and z' described in this axiom. Therefore Arrow's independence is incompatible with Weak Pareto and Transfer Principle, since, as mentioned above, Transfer Principle applied to agents with different preferences runs against Weak Pareto. For extensive discussions of how excessive Arrow's independence is, see Fleurbaey and Maniquet (1996b, 2001) and Fleurbaey, Suzumura and Tadenuma (2000). We will instead follow Hansson (1973) and Pazner (1979) who have proposed a much more reasonable condition, which is ethically more acceptable than Arrow's independence, and requires social preferences over two allocations to depend only on individual *indifference curves* at these two allocations.

Hansson Independence: Let z and z' be two allocations, and R, R' be the social orderings for two profiles $(R_1, ..., R_n)$ and $(R'_1, ..., R'_n)$ respectively. If for all i, and all $q \in X$,

$$z_i \; I_i \; q \quad \Leftrightarrow \quad z_i \; I'_i \; q$$
$$z'_i \; I_i \; q \quad \Leftrightarrow \quad z'_i \; I'_i \; q,$$

then

$$z \; R \; z' \Leftrightarrow z \; R' \; z'.$$

Finally, we want our social preferences to have a separable structure, as is usual in the literature on social index numbers. The intuition for separability requirements is that agents who are not concerned by a social decision need not be given any say in it. This is not only appealing because it simplifies the structure of social preferences, but also because it can be related to a standard conception of democracy, implying that unconcerned populations need not intervene in social decisions. This is often called the subsidiarity principle. We retain the following condition.

Separability: Let z and z' be two allocations, and i an agent such that $z_i = z'_i$. Then

$$z \; R \; z' \Rightarrow z_{-i} \; R_{-i} \; z'_{-i},$$

where $z_{-i} = (z_1, ..., z_{i-1}, z_{i+1}, ..., z_n)$, and R_{-i} is the social preference ordering for the economy with reduced population $\{1, ..., i-1, i+1, ..., n\}$.

6 A maximin result

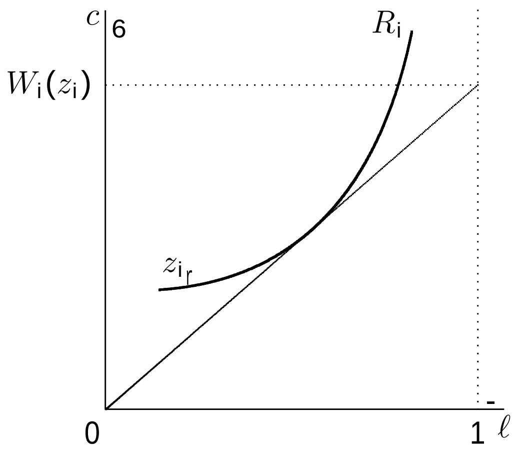
The fairness conditions introduced here are rather weak and, in particular, do not convey a strong aversion to inequality. Actually, the only redistributive condition here is the Transfer Principle, which, in the above weak formulation, is compatible with any degree of inequality aversion, including zero. A remarkable fact is that, nonetheless, the combination of these properties entails an infinite aversion to inequality, and forces social preferences to rely on the maximin criterion. This result is stated in the following theorem, which gives a quite precise description of social preferences.

Theorem 1 *If social preferences satisfy Transfer Principle, Laisser-Faire, Weak Pareto, Hansson Independence and Separability, then for any allocations z, z', one has*

$$\min_i W_i(z_i) > \min_i W_i(z'_i) \geq 0 \Rightarrow z \; P \; z',$$

where $W_i(z_i) = \max\{w \in \mathbb{R}_+ \mid \forall \ell, \; z_i \; R_i \; (\ell, w\ell)\}$.

The computation of $W_i(z_i)$ is illustrated on Fig. 1.[3]



- Fig. 1 -

The above theorem does not give a full characterization of social preferences. First, it does not say how to compare allocations for which $\min_i W_i(z_i) = \min_i W_i(z'_i)$. But for the purpose of finding the optimal tax, the description given in the theorem is quite satisfactory and yields precise results in most cases. Second, it does not say anything about allocations with some agents for whom $(0,0)\ P_i\ z_i$. Indeed for such allocations $\min_i W_i(z_i) = -\infty$. But, again, this is not a serious limitation for optimal taxation, since allocations obtained via tax redistribution are always such that $z_i\ R_i\ (0,0)$ for all agents, under reasonable tax schemes. As a consequence the above theorem gives us all we need to study the optimal tax.

7 Intuitive proof

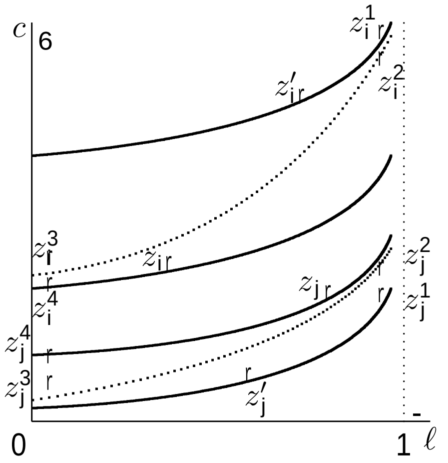
The proof of the theorem is in the appendix. We provide the intuition for it here (the uninterested reader may skip this section). Let us first show how the combination of Weak Pareto, Transfer Principle and Hansson Independence entails a strong aversion to inequality. Consider two agents i and j with identical preferences R_0, and two allocations z and z' such that

$$z'_i\ P_0\ z_i\ P_0\ z_j\ P_0\ z'_j.$$

The related indifference curves are shown on Fig. 2, and one sees in this particular example that the axiom of Transfer Principle cannot directly entail that z is preferable to z', because agent i loses a lot, and also because their labor times differ. But one can use

[3] This concept is closely related to the Equal Wage Equivalent first-best allocation rule characterized on different grounds in Fleurbaey and Maniquet (1999).

Hansson Independence and say that the other indifference curves can be anything without altering the social preferences about z and z'. Then one can focus on the case when some of these other indifference curves are like the dashed curves on Fig. 2.



- Fig. 2 -

In this particular case, one can construct intermediate allocations such as z^1, z^2, z^3, z^4, with

$$\begin{array}{rcl} & & z_i^1 \; P_0 \; z_i' \; P_0 \; z_i^3 \; P_0 \; z_i^2 \; P_0 \; z_i \; P_0 \; z_i^4, \\ & & z_j \; P_0 \; z_j^4 \; P_0 \; z_j^3 \; P_0 \; z_j^2 \; P_0 \; z_j^1 \; P_0 \; z_j', \\ c_i^2 & = & c_i^1 - \delta > c_j^1 + \delta = c_j^2, \\ c_i^4 & = & c_i^3 - \delta' > c_j^3 + \delta' = c_j^4, \\ \ell_i^1 & = & \ell_i^2 = \ell_j^1 = \ell_j^2, \\ \ell_i^3 & = & \ell_i^4 = \ell_j^3 = \ell_j^4, \end{array}$$

and rely on Transfer Principle to conclude that $z^2 \; R \; z^1$ and $z^4 \; R \; z^3$. And Weak Pareto applied to this pair of agents[4] would imply that $z \; P \; z^4$, $z^3 \; P \; z^2$, and $z^1 \; P \; z'$, so that, by transitivity, one can conclude that $z \; P \; z'$. Since this kind of construction can be done even when the gain is very small for j while i's loss is huge, one then obtains an infinite inequality aversion regarding indifference curves of agents with identical preferences.

The second central argument of the proof is the following. Consider two agents i and j and two allocations z and z' such that $z_k = z_k'$ for all $k \neq i, j$, and

$$W_i(z_i') > W_i(z_i) > W_j(z_j) > W_j(z_j').$$

[4] When the population is larger, the application of Weak Pareto is not so simple, but the thrust of the argument is the same.

Introduce two new agents, a and b, whose identical wage rate w is such that $W_i(z_i) > w > W_j(z_j)$, and whose preferences are $R_a = R_i$ and $R_b = R_j$. Let z^* denote the laisser-faire allocation for the two-agent economy formed by a and b, and (z_a, z_b) be another allocation which is feasible but inefficient in this two-agent economy, and such that

$$W_i(z_i) > W_a(z_a) > w > W_b(z_b) > W_j(z_j).$$

Let $R_{\{a,b\}}$, $R_{\{a,b,i,j\}}$ and $R_{\{i,j\}}$ denote the social preferences for the economies with population $\{a, b\}$, $\{a, b, i, j\}$ and $\{i, j\}$, respectively. By Laisser Faire and Weak Pareto, one can say that $z^* \ P_{\{a,b\}} \ (z_a, z_b)$. Therefore, by Separability, it must necessarily be the case that

$$(z^*_a, z^*_b, z_i, z_j) \ P_{\{a,b,i,j\}} \ (z_a, z_b, z_i, z_j).$$

By the above argument producing a strong inequality aversion among agents with identical preferences, we derive from

$$\begin{gathered} z'_i \ P_i \ z_i \ P_i \ z_a \ P_i \ z^*_a \\ z^*_b \ P_j \ z_b \ P_j \ z_j \ P_j \ z'_j \end{gathered}$$

that

$$(z_a, z_b, z_i, z_j) \ P_{\{a,b,i,j\}} \ (z^*_a, z^*_b, z'_i, z'_j).$$

As a consequence, one has

$$(z^*_a, z^*_b, z_i, z_j) \ P_{\{a,b,i,j\}} \ (z^*_a, z^*_b, z'_i, z'_j),$$

from which Separability entails that

$$(z_i, z_j) \ R_{\{i,j\}} \ (z'_i, z'_j).$$

It is actually easy to obtain a strict preference $(z_i, z_j) \ P_{\{i,j\}} \ (z'_i, z'_j)$ by referring, in the previous stages of this argument, to another allocation (z''_i, z''_j) Pareto-dominating z', instead of z' itself. Then, from Separability again, one can finally derive the conclusion that $z \ P \ z'$ in the initial economy.

>From this second central argument, one is not far from the conclusion of the theorem, because when two allocations z and z' are such that

$$\min_i W_i(z_i) > \min_i W_i(z'_i),$$

it is always possible to go from z' to z by a sequence of moves that either rely on Weak Pareto, or have two agents i and j for whom the inequality between the levels W_i and W_j at the contemplated intermediate allocations is reduced.

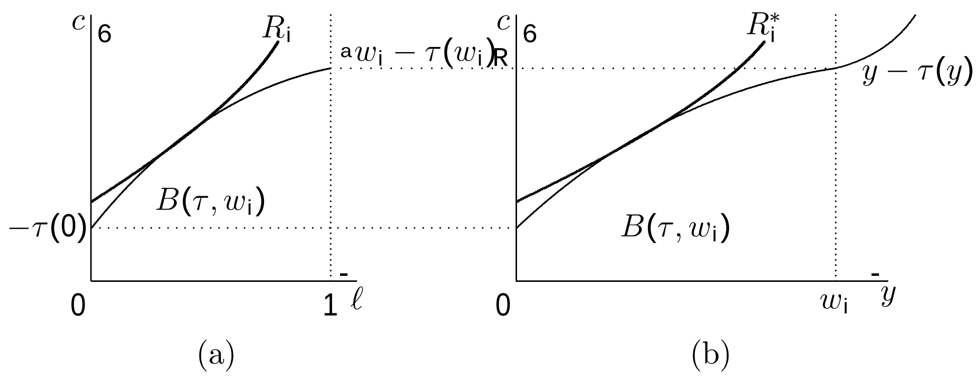
8 Tax redistribution

In this section and the following ones, we examine the issue of devising the redistribution system under incentive-compatibility constraints and with the objective of achieving foreseeable consequences that are the best according to the above social preferences. As is standard in this second-best context, we assume that only earned income $y_i = w_i \ell_i$ is observed, so that redistribution is made via a tax $\tau(y)$. This tax is actually a subsidy when $\tau(y) < 0$. Under this kind of redistribution, agent i's budget set is defined by (see Fig. 3a):

$$B(\tau, w_i) = \{(\ell, c) \in X \mid c \leq w_i \ell - \tau(w_i \ell)\}.$$

It is convenient to focus on the pre-tax income-consumption space, in which the budget is defined by (see Fig. 3b; we retain the same notation because no confusion is possible):

$$B(\tau, w_i) = \{(y, c) \in [0, w_i] \times \mathbb{R}_+ \mid c \leq y - \tau(y)\}.$$



- Figure 3 -

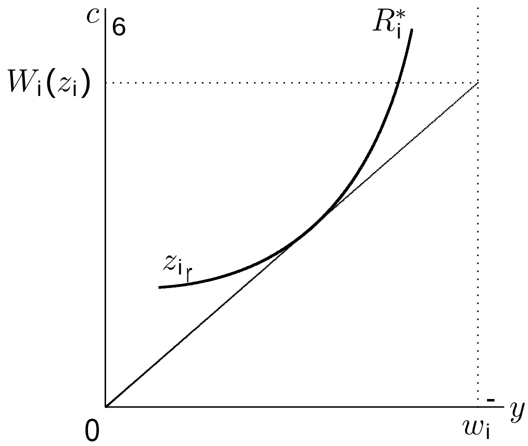
In the income-consumption space, one can define individual preferences R_i^* over income-consumption bundles, and they are derived from ordinary preferences over labor-consumption bundles by:

$$(y, c) R_i^* (c', y') \Leftrightarrow (\frac{y}{w_i}, c) R_i (\frac{y'}{w_i}, c').$$

With such preferences, the incentive-compatibility constraint can be formulated by the condition that for all agents i and j,

$$(y_i, c_i) R_i^* (y_j, c_j).$$

The way $W_i(z_i)$ is computed in the income-consumption space is illustrated in Fig. 4.



- Fig. 4 -

The budget constraint for the redistribution agency is:

$$\sum_{i=1} \tau(w_i \ell_i) \geq 0.$$

We will say that a tax function τ is *feasible* if it satisfies the above constraint when all agents choose their labor time by maximizing their satisfaction over their budget set. We restrict our attention to functions τ which are continuous and such that $y - \tau(y)$ is non-negative and non-decreasing.

The social preferences defined in the previous section are based on the maximin criterion applied to $W_i(z_i)$, for $i = 1, ..., n$. Computing the optimal tax then consists in finding the agent(s) with the lowest $W_i(z_i)$, and then maximizing the satisfaction of such agent(s).

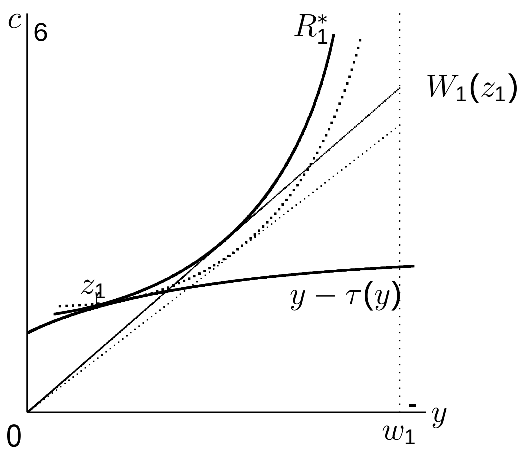
9 Optimal tax: the two-agent case

As an introductory analysis, consider the case of a two-agent population $\{1, 2\}$. Assume that $w_1 < w_2$, for instance. As a consequence, agent 2's budget set always contains agent 1's one. And if agent 1 works at the laisser-faire allocation z^*, necessarily $W_1(z_1^*) < W_2(z_2^*)$ since $W_i(z_i^*) \geq w_i$ for $i = 1, 2$, with equality $W_i(z_i^*) = w_i$ when the agent works. (If an agent is so averse to labor that $\ell_i^* = 0$, then $W_i(z_i^*)$ equals the marginal rate of substitution at $(0, 0)$, which is greater than or equal to w_i.)

If the agents have the same preferences $R_1 = R_2$, then the optimal tax is the one which maximizes the satisfaction of agent 1 (since agent 2's budget set contains agent 1's one, in the case of identical preferences one has $W_2(z_2) \geq W_1(z_1)$ in any allocation obtained via tax redistribution). This result extends immediately to a larger population: When all agents have the same preferences, an optimal tax is one which, among the feasible tax functions, maximizes the satisfaction of the agents with the lowest wage rate.

In the general case when the agents may have the same or different preferences (assuming that agent 1 works at the laisser-faire allocation), then either the optimal tax achieves an allocation such that $W_1(z_1) = W_2(z_2)$, or it maximizes the satisfaction of agent 1 over the set of feasible taxes. The argument for this fact is the following. Starting from the laisser-faire z^* where $W_1(z_1^*) < W_2(z_2^*)$, one redistributes from agent 2 to agent 1, and this increases $W_1(z_1)$ and decreases $W_2(z_2)$, following the second-best Pareto frontier. When one reaches the equality $W_1(z_1) = W_2(z_2)$, redistribution has to stop, since, by Pareto-efficiency, there is no other allocation with a greater $\min_i W_i(z_i)$. But an alternative possibility is that the incentive-compatibility constraint $(y_2, c_2) R_2^* (y_1, c_1)$ puts a limit on redistribution, which occurs when the point maximizing agent 1's satisfaction is reached. Then, the inequality $W_1(z_1) < W_2(z_2)$ remains at the optimal tax.

The important lesson to be drawn from the two above facts is that it is not always the case that the optimal tax maximizes the satisfaction of the low-skilled agent. This happens when the agents have identical preferences, but not in general. One may then ask what conditions about preferences make the optimal tax more or less redistributive. A very instructive fact is the following. Suppose that the preferences of low-skilled agent 1 become less averse to labor, in the sense that at every labor-consumption bundle, the marginal rate of substitution for R_1 decreases. Then the satisfaction of agent 2 at the new optimal tax does not increase and may decrease, which is a good indication that the tax becomes more redistributive. The proof of this claim, whose details we omit here, relies on the fact that such a change of preferences makes agent 1 move to a new bundle in his initial budget set (i.e. the budget set with the initial optimal tax) in such a way that $W_1(z_1)$ cannot increase and often decreases, while the incentive compatibility constraint $(y_2, c_2) R_2^* (y_1, c_1)$ is necessarily preserved. This decrease in $W_1(z_1)$ is illustrated on Fig. 5.



- Fig. 5 -

Therefore the new optimal tax may increase the transfer from 2 to 1, thereby decreasing agent 2's satisfaction. This fact intuitively shows that the social preferences obtained in Theorem 1 tend to reward the hardworking poor especially. This lesson will carry over to the general case.

10 Optimal tax: the general case

Let us now turn to the case of a larger population. The computation of the optimal tax is quite complex in general, in particular because the population is heterogenous in two dimensions, preferences and earning ability. We will, however, be able to derive some conclusions about, first, the part of the tax schedule which should be the focus of the social planner and, second, some features of the optimal tax.

The first kind of result has to do with translating the abstract objective of maximizing $\min_i W_i(z_i)$ into a more concrete objective about the part of the agents' budget set which should be maximized. We will restrict our attention to economies in which the distribution of characteristics in the population is rich enough so that there are no significant gaps in the distribution of wages or in the distribution of agents over the budget sets. In other words, we rely on the following assumption. Let $w_m = \min_i w_i$ and $w_M = \max_i w_i$.

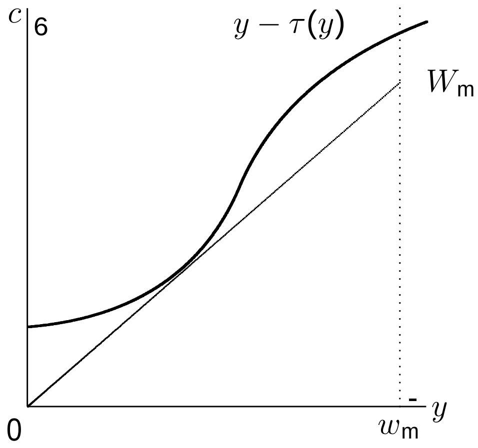
Assumption (No Gap): *It is a good approximation to consider that there are agents with wage rate equal to w for all w in $[w_m, w_M]$, and that, for all relevant tax functions τ, one finds agents at all levels of income $y \in (0, w)$ for every subgroup of agents with a same wage rate w.*

Equipped with this assumption, we can take an arbitrary feasible tax function τ, and look for the agents who have the minimum $W_i(z_i)$. The agents with lowest wage rate w_m necessarily have the smallest budget set in the labor-consumption space.

Consider first the case when $w_m > 0$. By the No Gap assumption, one finds agents with w_m essentially all over the interval $y \in (0, w_m)$, and this implies that their minimum $W_i(z_i)$ is approximately equal to

$$W_m = w_m \times \min \left\{ \frac{y - \tau(y)}{y} \mid y \in [0, w_m] \right\},$$

as shown on Fig. 6.



- Fig. 6 -

Maximizing $\min_i W_i(z_i)$ then boils down to maximizing W_m, a very concrete objective.

Now, we can say more about the optimal tax. First, it is always possible to construct a feasible tax with $W_m \geq w_m$, by taking the laisser-faire $\tau \equiv 0$. Since the objective is to maximize W_m, we therefore restrict our attention to the case $W_m \geq w_m$. We now construct a new tax by defining

$$\hat{\tau}(y) = \max\{\tau(y), w_m - W_m\},$$

that is, by cutting all subsidies which are greater than $W_m - w_m$. If τ is feasible, then so is $\hat{\tau}$, because all agents now have a smaller budget set, so that the agents who paid a tax or received a smaller subsidy than $W_m - w_m$ will stay at the same bundle, while those who received more than $W_m - w_m$ are now bound to get less. More interestingly, the value of $\min_i W_i(z_i)$ is still the same level of W_m under $\hat{\tau}$, and is equal to

$$w_m \times \frac{w_m - \hat{\tau}(w_m)}{w_m} = w_m - \hat{\tau}(w_m),$$

that is, the net income of the less skilled agents who work $\ell = 1$.

As a consequence, one can look for the optimal tax by maximizing the net income of the hardworking poor, $w_m - \tau(w_m)$, under the constraint that they have the minimum $W_i(z_i)$, that is, under the constraint that for all $y \in [0, w_m]$,

$$\frac{y - \tau(y)}{y} \geq \frac{w_m - \tau(w_m)}{w_m},$$

or equivalently,

$$\frac{\tau(y)}{y} \leq \frac{\tau(w_m)}{w_m}.$$

Since the tax will actually be negative at these levels of income, this condition means that those with a lower income than w_m must have a subsidy per income that is at least as great as for the hardworking poor.

And, since $W_m = w_m - \hat{\tau}(w_m)$, a remarkable feature of $\hat{\tau}$ is that

$$\hat{\tau}(y) \geq w_m - W_m = \hat{\tau}(w_m)$$

for all y. This means that the optimal tax can be found among the class of tax functions τ such that $\tau(y) \geq \tau(w_m)$ for all y.

Let us now turn to the case when $w_m = 0$. By the No Gap assumption, there are agents with arbitrarily low but positive w. For such agents the No Gap assumption again implies that they can be found at all levels of labor $\ell \in (0, 1)$. If those agents had the minimum $W_i(z_i)$, the above reasoning would entail that the optimal tax maximizes $w - \tau(w)$ under the constraint that $\tau(y)/y \leq \tau(w)/w$ for all $y \in [0, w]$. Since this reasoning can be applied for an arbitrarily low w, the conclusion is that the optimal tax must actually maximize the minimum income $-\tau(0)$.[5] These results are summarized in the following theorem.

Theorem 2 *If* $w_m = 0$, *the optimal tax maximizes the minimum income* $-\tau(0)$. *If* $w_m > 0$, *then the optimal tax maximizes the net income of the hardworking poor,* $w_m - \tau(w_m)$, *under the constraint that*

$$\frac{\tau(y)}{y} \leq \frac{\tau(w_m)}{w_m}$$

for all $y \in [0, w_m]$. *The optimal tax can always be made to satisfy the property that for all* y,

$$\tau(y) \geq \tau(w_m).$$

This result nicely shows how the social preferences contemplated here lead to focusing on the hardworking poor, who should get, as stated in the last part of the theorem, the greatest *absolute amount* of subsidy. However, those with a lower income than w_m are not forgotten, as they must obtain at least as great a *rate of subsidy* as the hardworking poor.

When there are agents with zero earning ability, however, these results boil down to a simple maximization of the minimum income, in a more traditional fashion. The case of a zero w_m can be related to physical disabilities but also to unemployment. Since unemployment may be viewed as nullifying the agents' earning ability, this result should best be interpreted as suggesting that the focus of redistributive policies should shift from the hardworking poor to the low-income households when the extent of unemployment is large, and especially when long term unemployment is a significant phenomenon.

[5] The same conclusion would also be reached under the simple assumption that there is at least one individual with wage rate $w_m = 0$ who is indifferent to labor, that is, whose preferences are defined by $(`, c)R_i(`', c') \Leftrightarrow c \geq c'$.

On the other hand, physical disabilities and unemployment are more or less observable characteristics, which may elicit special policies toward those affected by such conditions, as can be witnessed in many countries. If this is the case, then the above result should apply to the rest of the population, and the relevant value of w_{m} is then likely to be the minimum legal hourly wage.

11 Are there undeserving poor?

The results of the previous sections are well in tune with the current mood in many Western political parties of the whole spectrum, which are mostly concerned with the fate of the "deserving poor". But, as emphasized above, the results have the nice feature that they avoid condemning the non hardworking population to a harsh treatment, by guaranteeing a generous rate of subsidy to all low incomes. This particular result can be traced back to the Transfer Principle axiom, which aptly applies to all levels of work.

Nevertheless, one may still be worried about the way in which the above social preferences favor the hardworking. After all, people who are averse to labor may suffer from consequences of their upbringing or from other problems and constraints that excuse their aversion. Let us focus, for simplicity, on the extreme case of what we will call *nonworking* agents. A nonworking agent is characterized by the property that his marginal rate of substitution at any bundle $(0, c)$ is greater than or equal to w_{i}. Any agent with such preferences never works unless his wage rate is subsidized. A nonworking agent may either be an agent with reasonably hardworking preferences but a low w_{i}, or someone with high skill but preferences which are strongly averse to labor.

The idea that deserving poor should be more favorably treated would lead to make distinctions between these two kinds of nonworking individuals. And this is indeed what the social preferences defined above actually do. One striking implication of such social preferences is that they consider it a social improvement when equality of consumption between nonworking agents is broken so as to give a smaller consumption to those who have a stronger aversion to labor, at the benefit of those with a smaller aversion to labor. This is because when they have equal consumption (and do not work), the nonworking with the less labor-averse preferences have a strictly lower $W_{\mathsf{i}}(z_{\mathsf{i}})$ than the nonworking with more labor-averse preferences.

This kind of social preferences may therefore seem somewhat severe with those who, for any reason, turn out to display a strong aversion to labor. Fortunately, the constraints of the second best force the tax to give the same consumption $-\tau(0)$ to all nonworking agents. But it remains worrisome that this absence of discrimination is obtained only because of second-best constraints, and goes against the underlying social preferences.

At this point, it is natural to ask what kind of social preferences would be obtained if a requirement of non-discrimination among the nonworking agents was built in the social preferences themselves, via an appropriate axiom. Would we then obtain different preferences, and different conclusions about the optimal tax?

The following requirement excludes any kind of discrimination between the various kinds of nonworking agents, by applying the Pigou-Dalton transfer principle to such agents.

No Undeserving Poor: If z and z' are two allocations, and i and j are two nonworking agents, such that $\ell_i = \ell_j = \ell'_i = \ell'_j = 0$, and for some $\delta > 0$,

$$c'_i - \delta = c_i > c_j = c'_j + \delta,$$

whereas for all other agents k, $z_k = z'_k$, then $z \; R \; z'$.

This axiom is again compatible with an arbitrarily low degree of aversion to inequality. It is also compatible with Laisser-Faire.[6] It must be emphasized that this axiom does not require redistribution to be made from working agents to nonworking agents. But combining it with Transfer Principle, in replacement of Laisser-Faire, yields a new kind of social preferences.

Theorem 3 *If social preferences satisfy Transfer Principle, No Undeserving Poor, Weak Pareto, Hansson Independence and Separability, then for any allocations* z, z', *one has*

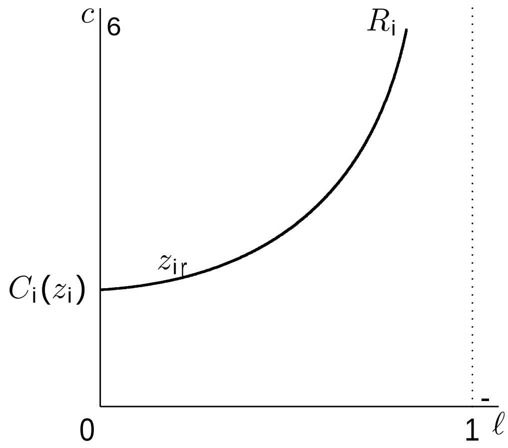
$$\min_i C_i(z_i) > \min_i C_i(z'_i) \geq 0 \Rightarrow z \; P \; z',$$

where $C_i(z_i) = \max\{c \in \mathbb{R}_+ \mid z_i \; R_i \; (0, c)\}$.

The computation of $C_i(z_i)$ is illustrated on Fig. 7.[7]

[6] A Pareto-efficient allocation can be obtained by granting a lump-sum transfer t_i to any agent i, and letting the agents choose the best bundle under the budget constraint $c_i \leq t_i + w_i \ell_i$. One can generalize and for any allocation compute the implicit t_i that would give agent i her current level of satisfaction. Now, the social preferences based on the "aggregate wealth" $\sum_i t_i$ do satisfy Laisser-Faire and No Undeserving Poor, in addition to Weak Pareto, Hansson Independence and Separability. Of course, they fail to satisfy Transfer Principle.

[7] Notice that $C_i(z_i) = -\infty$ whenever $(0, 0) \; P_i \; z_i$.



- Fig. 7 -

The surprising feature of this result is that even though No Undeserving Poor is, in terms of redistributive properties, quite weak, and only worries about inequalities among the nonworking, the combination with the other axioms gives us social preferences which seek to maximize the consumption of the nonworking (when it is the smallest).

The proof is in the appendix. The structure of the argument is similar to the proof of Theorem 1, as described in section 7.

With such social preferences, the agents with minimum $C_i(z_i)$, under the No Gap assumption, will always be those with a zero pre-tax income. The consequence is that the optimal tax for such social preferences is very easily defined.

Theorem 4 *For social preferences maximizing* $\min_i C_i(z_i)$, *the optimal tax maximizes the minimum income* $-\tau(0)$, *among the feasible tax functions.*

It seems to us that the two approaches proposed here present the decision-maker with an interesting ethical choice. The first kind of social preferences, focusing on $\min_i W_i(z_i)$, is appealing when one holds the households fully responsible for their preferences and their choice of labor time. The second kind of social preferences, focusing on $\min_i C_i(z_i)$, is preferable when one instead wants to protect the agents who would spontaneously choose a low consumption because they are averse to labor for some reason. The issue of personal responsibility for the choice of labor participation is actually hotly debated nowadays, although most of the underlying problems remain often somewhat implicit. Our analysis shows that it is indeed a central issue, and makes explicit the important consequences it has for the shape of the optimal redistribution scheme.

12 Conclusion

The main lessons to be drawn from this analysis are, in our opinion, the following. Some are methodological, others are on the substance of the issue.

Let us start with the substantial ones. A first point is that the assessment of inequalities in general, and of inequality reduction by the tax system in particular, should be made neither in terms of income (the usual measure in empirical studies) nor in terms of subjective utilities (the usual measure in optimal tax theory). This point is obviously controversial, and is conditional on ethical principles, but our system of axioms gives a clear basis on which such matters can be discussed. Notice, however, that the second kind of social preferences defined here (in the previous section) lead us to focus on the minimal income in the computation of the optimal tax, even though the general assessment of individuals' situations is made in terms of C_i, not in terms of income.

A second lesson is that the relevant measure of individual situations is W_i if one wants to respect individual choices about labor participation (as expressed in Laisser-Faire), and C_i is one wants to avoid discriminating against nonworking agents with labor-averse preferences (No Undeserving Poor). Interestingly, such measures are not very hard to put into practice in a very concrete way. Just ask people what tax-free wage rate (in the case of W_i) or leisurely consumption (in the case of C_i) would give them the same satisfaction as their current situation. Notice that in the computation of the optimal tax such information about current individual situations is not even needed. One only needs to know the set of feasible taxes, and theorems 2 and 4 then immediately pinpoint the optimal tax schedule(s).

A third lesson is that the maximin criterion can be justified in a new way, simply by combining a lenient Pigou-Dalton principle with an independence condition in the spirit of Arrow's independence (but less extreme). This result has been also noticed in different contexts by Fleurbaey (2002) and Maniquet and Sprumont (2002). This should give more respectability to the maximin criterion, which is often criticized for its extreme aversion to inequality.

As far as the optimal tax is concerned, the main lesson is that, under the fairness requirements proposed here, the best tax should either maximize the consumption of the hardworking poor (subject to the constraint that lower incomes should not have a lower rate of subsidy) and give them the greatest amount of subsidy, or simply maximize the minimum income. The choice between these two alternatives depends on the choice between W_i and C_i, or, more precisely, on the choice between the underlying axioms. The core issue about Laisser-Faire is this: Can we trust individual choices to the extent that "chosen" poverty becomes socially acceptable when good opportunities are available?

When W_{i} is chosen, the restriction that lower incomes should not have a lower rate of subsidy than the hardworking poor is very important. It forbids policies which harshly punish the agents working part time, and give exclusive subsidies to full-time jobs. In addition, it must be stressed that when unemployment takes the form of constrained part time jobs (a less observable form than ordinary unemployment), this should best be tackled by considering it as a reduction of the agents' earning ability. An agent who only finds a half time job should be treated like an agent who works full time at a half wage rate. In our model, labor time is not observed, but this phenomenon can be taken into account by revising the distribution of earning abilities in the population, leading to a reduction of w_{m} and therefore to a more generous policy toward low incomes.

Let us now turn to methodology. The main lesson in this respect is that interpersonal comparisons of subjective utility are not needed to study redistributive issues, even in the second-best context. Of course, one has to compare *something* in order to decide what subpopulations deserve to be given priority in the distribution of resources. But comparing W_{i} (or C_{i}) between individuals has several advantages over utility comparisons. First, such comparisons are easily operationalized, in contrast with utility measurements, which still lack consensual definition and implementation method. Second, they are derived here from basic conditions of fairness. In other words, not only do our axioms enable us to justify a particular kind of social preferences (in particular, the maximin criterion), but at the same time they force us to adopt a particular measure of individual situations. This double role of the system of axioms makes our approach much more complete than the standard social choice approach, which ordinarily requires an exogenous measurement of individual well-being to be provided from outside the theory. And the discussion of the choice between W_{i} and C_{i} suggests that this does not only make our approach more complete, but that it also provides tools to discuss important ethical issues that could not be addressed in the traditional framework, and that relate the measurement of individual situations to *fairness* considerations.

A related point, which has been developed in other papers (Fleurbaey and Maniquet 1996b, 2000, 2001), is that possibility results can be obtained in the theory of social choice as soon as Arrow's independence of irrelevant alternatives is relaxed and replaced by more reasonable conditions such as Hansson Independence. And then, social choice is not only possible, but it can incorporate elaborate fairness conditions, as illustrated here.

These methodological points lead us to claim that our approach can be applied to many other issues and models. The general method is simple. First, select ethical requirements, referring to efficiency and fairness principles in particular. Second, find social preferences satisfying the requirements, or, better, derive the social preferences from the logical implications of the requirements, as illustrated by Theorems 1 and 3 above. Third, apply the social preferences in order to obtain the optimal institutions. This general method paves the way for new developments in public economics, where fairness principles would replace interpersonal utility comparisons.

Appendix: Proofs

Lemma 1 *If social preferences satisfy Transfer Principle, Weak Pareto and Hansson Independence, then for any pair of allocations z,z' and any pair of agents i, j with identical preferences R_0, such that*

$$z'_i \; P_0 \; z_i \; P_0 \; z_j \; P_0 \; z'_j \; R_0 \; (0,0)$$

and $z_k \; P_k \; z'_k$ for all $k \neq i, j$, one has $z \; P \; z'$.

Proof: Let z, z' satisfy the above conditions. By Hansson Independence, we can arbitrarily modify the preferences R_0 at bundles which are not indifferent to one of the four bundles z_i, z'_i, z_j, z'_j. Let f_i, g_i, f_j, g_j be the functions whose graphs are the indifference curves for R_0 at these four bundles, respectively. Let f_i^* be the function whose graph is the convex hull of

$$(0, f_i(0)) \cup \{(\ell, c) \mid c \geq g_i(\ell)\},$$

and f_j^* be the function whose graph is the convex hull of

$$(0, g_j(0)) \cup \{(\ell, c) \mid c \geq f_j(\ell)\}.$$

These functions are convex, and their graphs can be arbitrarily close to two indifference curves for R_0. We will indeed assume that there is an indifference curve for R_0, between f_i and g_i, arbitrarily close to the graph of f_i^*, and another one, between f_j and g_j, arbitrarily close to f_j^*.

By construction there exists ℓ_1 such that

$$g_i(\ell_1) - f_i^*(\ell_1) < f_j^*(\ell_1) - g_j(\ell_1),$$

and similarly

$$f_i^*(0) - f_i(0) = 0 < f_j(0) - f_j^*(0) = f_j(0) - g_j(0).$$

Therefore one can find $c_i^1, c_i^2, c_j^1, c_j^2$ such that

$$\begin{aligned} g_i(\ell_1) - f_i^*(\ell_1) &< c_i^1 - c_i^2 = c_j^2 - c_j^1 < f_j^*(\ell_1) - g_j(\ell_1), \\ c_i^2 &< f_i^*(\ell_1) \leq g_i(\ell_1) < c_i^1, \\ g_j(\ell_1) &< c_j^1 < c_j^2 < f_j^*(\ell_1), \end{aligned}$$

and $c_i^3, c_i^4, c_j^3, c_j^4$ such that

$$\begin{aligned} 0 &< c_i^3 - c_i^4 = c_j^4 - c_j^3 < f_j(0) - f_j^*(0), \\ c_i^4 &< f_i(0) = f_i^*(0) < c_i^3, \\ f_j^*(0) &< c_j^3 < c_j^4 < f_j(0). \end{aligned}$$

Define z^1, z^2, z^3, z^4 by

$$z_k \; P_k \; z_k^4 = z_k^3 \; P_k \; z_k^2 = z_k^1 \; P_k \; z_k'$$

for all $k \neq i, j$, and

$$z_k^1 = (\ell_1, c_k^1), \; z_k^2 = (\ell_1, c_k^2), \; z_k^3 = (0, c_k^3), \; z_k^4 = (0, c_k^4)$$

for all $k = i, j$.

By Transfer Principle, one has

$$z^2 \; R \; z^1 \text{ and } z^4 \; R \; z^3.$$

By Weak Pareto (and the assumption about indifference curves close to f_i^* and f_j^*),

$$z \; P \; z^4, \; z^3 \; P \; z^2 \text{ and } z^1 \; P \; z'.$$

By transitivity, one concludes that $z \; P \; z'$. ■

Lemma 2 *If social preferences satisfy Transfer Principle, Laisser Faire, Weak Pareto, Hansson Independence and Separability, then for any pair of allocations* z,z' *and any pair of agents* i, j, *such that*

$$W_i(z_i') > W_i(z_i) > W_j(z_j) > W_j(z_j')$$

and $z_k = z_k'$ *for all* $k \neq i, j$, *one has* $z \; P \; z'$.

Proof: Let z and z' be two allocations satisfying the above conditions. Necessarily $W_i(z_i) > 0$ and $W_j(z_j) \geq 0$. Let a and b be two new agents with $w_a = w_b = w > 0$ and $W_i(z_i) > w > W_j(z_j)$, and with preferences $R_a = R_i$ and $R_b = R_j$. Let z^* be the laisser-faire allocation for the two-agent economy $\{a, b\}$, and (z_a, z_b) be another allocation such that

$$W_i(z_i) > W_a(z_a) > w > W_b(z_b) > W_j(z_j)$$

and

$$c_a + c_b < w(\ell_a + \ell_b),$$

which means that (z_a, z_b) is inefficient.

Let R_A denote the social preferences for the economy with population A. By Laisser Faire and Weak Pareto,

$$z^* \, P_{\{a,b\}} \, (z_a, z_b).$$

Therefore, by Separability,

$$(z_a^*, z_b^*, z_i, z_j) \, P_{\{a,b,i,j\}} \, (z_a, z_b, z_i, z_j).$$

We will now use the fact that

$$z_i' \, P_i \, z_i \, P_i \, z_a \, P_i \, z_a^*$$

and

$$z_b^* \, P_j \, z_b \, P_j \, z_j \, P_j \, z_j'.$$

Let $z_i^-, z_i'^+$ and z_a^- be such that

$$z_i'^+ \, P_i \, z_i' \, P_i \, z_i \, P_i \, z_i^- \, P_i \, z_a \, P_i \, z_a^- \, P_i \, z_a^*,$$

and similarly, let $z_j'^+, z_j'^{++}$ and z_b^{*+} be such that

$$z_b^{*+} \, P_j \, z_b^* \, P_j \, z_b \, P_j \, z_j \, P_j \, z_j'^{++} \, P_j \, z_j'^+ \, P_j \, z_j'.$$

Since

$$z_i'^+ \, P_i \, z_i^- \, P_i \, z_a^- \, P_i \, z_a^*$$

and $z_b^{*+} \, P_j \, z_b^*$, $z_j'^{++} \, P_j \, z_j'^+$, one can refer to Lemma 1, and conclude that

$$(z_a^-, z_b^{*+}, z_i^-, z_j'^{++}) \, P_{\{a,b,i,j\}} \, (z_a^*, z_b^*, z_i'^+, z_j'^+).$$

Similarly, since

$$z_b^{*+} \, P_j \, z_b \, P_j \, z_j \, P_j \, z_j'^{++}$$

and $z_i \, P_i \, z_i^-$, $z_a \, P_i \, z_a^-$, one obtains

$$(z_a, z_b, z_i, z_j) \, P_{\{a,b,i,j\}} \, (z_a^-, z_b^{*+}, z_i^-, z_j'^{++}).$$

By transitivity, one then has

$$(z_a, z_b, z_i, z_j) \, P_{\{a,b,i,j\}} \, (z_a^*, z_b^*, z_i'^+, z_j'^+),$$

and therefore

$$(z_a^*, z_b^*, z_i, z_j) \, P_{\{a,b,i,j\}} \, (z_a^*, z_b^*, z_i'^+, z_j'^+),$$

Separability then entails that

$$(z_i, z_j) \, R_{\{i,j\}} \, (z_i'^+, z_j'^+),$$

and by Weak Pareto one actually gets

$$(z_i, z_j)\ P_{\{i,j\}}\ (z'_i, z'_j).$$

>From Separability again, one can finally derive the conclusion that $z\ P\ z'$ in the initial economy. ■

Proof of Theorem 1. Let z and z' be two allocations such that

$$\min_i W_i(z_i) > \min_i W_i(z'_i) \geq 0.$$

Then, by monotonicity of preferences, one can find two allocations x, x' such that for all i, $z_i\ P_i\ x_i$, $x'_i\ P_i\ z'_i$, and there exists i_0 such that for all $i \neq i_0$

$$W_i(x'_i) > W_i(x_i) > W_{i_0}(x_{i_0}) > W_{i_0}(x'_{i_0}).$$

Let $(x^k)_{1\leq k\leq n+1}$ be a sequence of allocations such that for all $i \neq i_0$,

$$\begin{aligned} x_i^i &= ... = x_i^1 = x'_i, \\ z_i\ P_i\ x_i^{n+1} &= ... = x_i^{i+1} = x_i, \end{aligned}$$

while

$$x_{i_0} = x_{i_0}^{n+1}\ P_{i_0}\ x_{i_0}^{n-1}\ P_{i_0} ... P_{i_0} x_{i_0}^{i_0+1} = x_{i_0}^{i_0}\ P_{i_0} ... P_{i_0}\ x_{i_0}^1 = x'_{i_0}.$$

One sees that for all $k \neq i_0$,

$$W_k(x_k^k) > W_k(x_k^{k+1}) > W_{i_0}(x_{i_0}^{k+1}) > W_{i_0}(x_{i_0}^k),$$

while for all k, and all $i \neq i_0, k$, $x_i^{k+1} = x_i^k$. By Lemma 2, this implies that $x^{k+1}\ P\ x^k$ for all $k \neq i_0$, while $x^{i_0+1} = x^{i_0}$.

By Weak Pareto, $x^1\ P\ z'$, and $z\ P\ x^{n+1}$. By transitivity, $z\ P\ z'$. ■

Lemma 3 *If social preferences satisfy Transfer Principle, Safety Net, Weak Pareto, Hansson Independence and Separability, then for any pair of allocations z,z' and any pair of agents i, j, such that*

$$C_i(z'_i) > C_i(z_i) > C_j(z_j) > C_j(z'_j)$$

and $z_k\ P_k\ z'_k$ for all $k \neq i, j$, one has $z\ P\ z'$.

Proof: The proof is similar to that of Lemma 2. We just highlight its main structure. It consists in introducing two agents a and b, with $w_a = w_b = 0$ and $R_a = R_i$, $R_b =$

R_j. By monotonicity of preferences they have nonworking preferences. We consider two allocations $((0,c_a),(0,c_b))$, $((0,c'_a),(0,c'_b))$ such that

$$C_i(z'_i) > C_i(z_i) > c'_a > c_a > c_b > c'_b > C_j(z_j) > C_j(z'_j)$$

and $c'_a - c_a = c_b - c'_b$. By Safety Net,

$$((0,c_a),(0,c_b))\ R_{\{a,b\}}\ ((0,c'_a),(0,c'_b))$$

and by a slight perturbation of $((0,c'_a),(0,c'_b))$ one could have a strict social preference. By separability, one deduces that

$$((0,c_a),(0,c_b),z_i,z_j)\ P_{\{a,b,i,j\}}\ ((0,c'_a),(0,c'_b),z_i,z_j)$$

and by applications of Weak Pareto and the Lemma 1, one can obtain (with the help of intermediate allocations as in Lemma 2, but these details are omitted here)

$$((0,c'_a),(0,c'_b),z_i,z_j)\ R_{\{a,b,i,j\}}\ ((0,c_a),(0,c_b),z'_i,z'_j),$$

so that by transitivity

$$((0,c_a),(0,c_b),z_i,z_j)\ P_{\{a,b,i,j\}}\ ((0,c_a),(0,c_b),z'_i,z'_j)$$

and by Separability

$$(z_i,z_j)\ R_{\{i,j\}}\ (z'_i,z'_j).$$

By a slight perturbation of (z'_i,z'_j) one could get a strict social preference, so that by Separability one obtains $z\ P\ z'$ in the initial economy. ■

The proof of Theorem 3 mimics that of Theorem 1, by substituting C_i for W_i, and relying on Lemma 3 instead of Lemma 2.

References

Arrow K. J. 1951, *Social Choice and Individual Values*, New York: Wiley.

Atkinson A. B. 1973, "How progressive should income tax be?", in M. Parkin and A. R. Nobay (eds.), *Essays in Modern Economics*, London: Longmans.

Atkinson A. B. 1995, *Public Economics in Action*, Oxford: Clarendon Press.

Bossert W., M. Fleurbaey, D. Van de gaer 1999, "Responsibility, Talent, and Compensation: A Second-Best Analysis", *Review of Economic Design* 4: 35-55.

Diamond P. 1998, "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates", *American Economic Review* 88 (1): 83-95.

Dworkin R. 1981, "What is Equality? Part 2: Equality of Resources", *Philosophy and Public Affairs* 10: 283-345.

Ebert U. 1992, "A reexamination of the optimal nonlinear income tax", *Journal of Public Economics* 49: 47-73.

Fleurbaey M. 2001, "The Pazner-Schmeidler social ordering: A defense", IMW Bielefeld.

Fleurbaey M., F. Maniquet 1996a, "Fair allocation with unequal production skills: The no-envy approach to compensation", *Mathematical Social Sciences* 32: 71-93.

Fleurbaey M., F. Maniquet 1996b, "Utilitarianism versus fairness in welfare economics", forthcoming in M. Salles and J. A. Weymark (Eds), *Justice, Political Liberalism and Utilitarianism: Themes from Harsanyi and Rawls*, Cambridge : Cambridge University Press.

Fleurbaey M., F. Maniquet 1999, "Fair allocation with unequal production skills: The solidarity approach to compensation", *Social Choice and Welfare* 16: 569-583.

Fleurbaey M., F. Maniquet 2000, "Fair orderings with unequal production skills", THEMA 2000-17.

Fleurbaey M., F. Maniquet 2001, "Fair social orderings", mimeo, U. of Pau and U. of Namur.

Fleurbaey M., K. Suzumura, K. Tadenuma 2000, "Informational requirements for social choice in economic environments", mimeo, U. of Pau and Hitotsubashi U.

Fleurbaey M., A. Trannoy 2000, "The Impossibility of A Paretian Egalitarian", THEMA 2000-26.

Hansson B. 1973, "The Independence Condition in the Theory of Social Choice", *Theory and Decision* 4: 25-49.

Maniquet F., Y. Sprumont 2002, "Fair production and allocation of an excludable nonrival good", IAS Princeton.

Mirrlees J. 1971, "An Exploration in the Theory of Optimum Income Taxation", *Review of Economic Studies* 38: 175-208.

Pazner E. 1979, "Equity, nonfeasible alternatives and social choice: A reconsideration of the concept of social welfare", in J.J. Laffont ed., *Aggregation and Revelation of Preferences*, Amsterdam: North-Holland.

Sadka E. 1976, "On income distribution incentive effects and optimal income taxation", *Review of Economics Studies* 43: 261-268.

Seade J. K. 1977, "On the shape of the optimal tax schedules", Journal of Public Economics 7: 203-236.

Tuomala M. 1990, *Optimal Income Tax and Redistribution*, Oxford: Oxford University

Press.