Markets for Information: Of Inefficient Firewalls and Efficient Monopolies *

Antonio Cabrales† Piero Gottardi‡

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Abstract

In this paper we build a formal model for environments where information is costly and where it can be used by potential competitors. The model allows to understand how such a market is organized, and whether it is efficient (ex-post and ex-ante). There is an object for sale, whose type is unknown. The buyers get utility from only one variety of the object. The type of the potential buyers are chosen independently of one another, and of the object for sale. The buyers can find out the type of the object for sale by paying a cost. Each buyer has to choose first whether or not to explore the object and then, if he has chosen to explore the object, whether to sell a report on his information to the uninformed buyers, and at which price. After the information is sold and signals revealed, all the buyers participate in a second price auction for the object. We characterize the equilibria and welfare properties for a variety of setups. Information sold may be homogeneous or heterogeneous among buyers, and the seller of information may be a potential competitor, the owner of the good, or a disinterested third party. The results show that disinterested third parties (firewalls) may lead to inefficiencies and monopolies may achieve the efficient outcome.

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†Departament d'Economia i Empresa, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. Email: antonio.cabrales@upf.edu. http://www.econ.upf.edu/~cabrales

‡Dipartimento di Scienze Economiche, Universita' di Venezia, Fond.ta S. Giobbe, Cannaregio 873, 30121 Venezia, Italy. Email: gottardi@unive.it. http://venus.unive.it/gottardi/

1 Introduction

It is common to observe potential competitors in a market exchanging information about issues pertaining to that market. For example, one can see financial analysts providing news over market events, or soccer coaches pondering on the abilities of (theirs or others') team players. Similar situations arise in the housing market, where we find agents searching and then supplying information over housing properties on sale in the market; in the labor market, where managers often discuss the performance of talented employees in their sector. This is somewhat surprising since the information supplied often has a rival nature. As an example of this rivalry, taken from the private equity market, the following quote from The Economist is illuminating: "Buy-out firms complained that banks which were supposedly advising or lending to them sometimes snatched deals from under their noses. A notorious example was the battle for Warner Chilcott, a British drugmaker, in late 2004: while working with buy-out firms bidding for the company, Credit Suisse teamed up with JPMorgan Chase to launch a bid of its own."[1] This reveals a fundamental conflict arising in information markets. The financial trader mentioned above may prefer to be the first to use the discovery of a particularly important event, and the soccer coach could be a potential competitor in a bidding war for a specially talented player. As a consequence, the provider of information may not be trusted to make truthful reports over the information he acquired: the analyst can say that his sources indicate that clinical trials for the wonder drug are going well, when in fact they are flunking. There is, in fact, a serious concern by regulators about the objectivity and the conflicts of interest faced by financial analysts.[2] This concern is particularly important when analysts are not independent, but employed by brokers, who are interested in trading the underlying stocks, as in the case of sell-side analysts, or even by the companies object of study, as in the case of fee-based analysts.

At the same time, information may be quite costly to acquire. Getting to know that the clinical trial of a particularly promising drug are going well (and thus the stock price of the company doing the research) may require nontrivial effort for the analyst. Similarly, finding out that a young left-footed striker currently playing in a second division Argentinian team is likely to be a star also requires time and money. These costs, together with the fact that information is of common interest, generate a clear incentive for the formation of a market for information, where the agents who acquired information can provide reports over

[1] The Economist, October 12, 2006: "Banks and buy-outs: Follow the money".

[2] See, e.g. the recent report of the Forum Group (2003) created by the European Comission to deal with this problem.

it, possibly in exchange for the payment of a price, to the other agents. The possibility of exchanging, or selling information to other traders may in turn affect the agents' incentive to acquire information.

The purpose of this paper is to present a model where we can analyze these issues and which allows us to provide an answer to some questions. For example, we would like to know when information is acquired, and if so, whether it is transmitted to other agents, i.e. whether a market for information forms or rather breaks down because of the problems mentioned above. Also, if there is indeed a market for information, how is it organized. That is, who and how many traders sell information, who and how many traders buy it, and hence how competitive is the market, as well as which type of information is traded. As in the previous motivating examples, we will consider the case where the truthfulness of the information transmitted is not verifiable or contractible. So another natural question concerns the veracity of the information that is transmitted in the market, which is obviously a crucial condition for the market not to break down.

Efficiency issues are also important. Is the market efficient? Also, is the level of investment in information acquisition efficient? Or is there any scope for regulatory restrictions in order to improve welfare? This is important, because in the wake of recent scandals in financial markets, the regulatory bodies of many countries have strengthened requirements on information dissemination in various markets. One typical recommendation is to separate who provides information on a market from who trades in it ("firewalls").[3] We would like to provide a formal basis to analyze and assess these issues.

We consider a market where a single, indivisible unit of a commodity is up for sale. The market is organized as a (second price) auction, where several potential buyers can participate. The good comes in different possible varieties, and each buyer only likes one randomly (i.i.d.) chosen variety. In addition to buyers, there is the seller, who initially owns the object and has no utility for it, and some other agents who are not interested in trading the object. The true variety is not known ex-ante by anybody, but can be ascertained, incurring a given cost, by any market participant. Besides the market for the commodity there is another market where information in traded: any agent who acquired information can set a price at which he sells a report over his information to other potential buyers. The information transmitted, as we said, is non verifiable, thus reports are pure "cheap talk"

[3]For example, in the report of the European Comission Forum Group (2003), we read: "Conflict avoidance, prevention and management: Analysts' firms should have in place systems and controls to identify and avoid, prevent or manage personal and corporate conflicts of interest.

Disclosure: Conflicts of interest, whether corporate or personal, should be prominently disclosed."

messages.

We will provide a complete characterization of the equilibria of such game. We find that, when information costs are not too high, information is acquired in equilibrium and in that case it is also sold. That is,. the market for information is active. However, the information sold in that market can be noisy: when the provider of information is the seller of the commodity, he tends to hype the value of the good he declares for the agents who purchase information, while when he is a potential buyer, he tends to depress it.

Typically, only one trader acquires information in equilibrium, hence the market for information is a monopoly. Information is either sold at a positive price such that all the uninformed buyers except one purchase it or, when the cost of acquisition of information is low, at a zero price so that all uninformed buyers purchase it. To understand this, notice that the seller of information may benefit even by transmitting information for free as this allows him to manipulate the behavior of uninformed traders in the auction (for instance, so as to reduce the price he may have to pay to win the object in the auction) and hence to increase the amount of surplus he can appropriate in the auction.

We also show that, if information is acquired at all, the commodity is allocated efficiently in the auction, i.e. to the agent who values it most. But the level of investment in information acquisition is not efficient, in particular there is typically underinvestment. Interestingly, this inefficiency is present no matter what is the identity of the agent acquiring and selling information, i.e. not only when this is a potential buyer or the seller, but also when he is an agent not interested in buying the object. Actually, in the last case the inefficiency is more severe. Hence restricting the access to the market for selling information only to uninterested traders (as in the case "firewalls"), while improving the truthfulness of the information transmitted, makes the inefficiency of the overall market outcome worse in the set-up considered.

On the other hand, an efficient outcome can be attained if different types of reports, of different quality, can be sold in the market (or equivalently if we permit the resale of information). We show, in fact, that this allows the information provider to appropriate all the increase in traders' surplus generated by the information acquisition and dissemination. When a single type of report is traded, part of these rents accrue either to the buyer choosing to remain uninformed, or to the seller of the object, who are free riding, thus generating inefficiently low information acquisition incentives. At the same time, when differentiated information is sold in the market, entry in such market is often profitable, so that, unless entry is restricted by some regulation, we will have multiple providers, which is also inefficient because of the duplication in investment costs. Information markets are, in a sense, natural monopolies.

In most of the paper, we consider the case where the uncertainty over the characteristics of the commodity up for sale concerns the different possible varieties of the good, over which buyers have different preferences. We thus have a situation of horizontal differentiation. At the end of the paper we also introduce an element of vertical differentiation, by allowing for the possibility that the commodity can also be of high and low quality, and all buyers prefer, at least weakly, high to low quality. In that case, the degree of truthfulness of the reports transmitted deteriorates, both when the provider is also a potential buyer or when he is the seller of the commodity. Hence, if quality is sufficiently important for buyers, we could have here a case where separation between traders and providers of information would be welfare improving.

This paper is related to different strands in the literature. More obviously, it is related to the seminal work of Crawford and Sobel (1982) on strategic information transmission. The primary focus of such work and the ensuing literature is the message game and the relationship between information transmission and alignment of the preferences of sender and receiver (or the 'conflict of interest' among them). To that literature, we add a richer game structure. The amount of information and their 'owners' is endogenously determined, as a result of the information acquisition decisions of every agent. We also allow messages to be transmitted for the payment of a price, thus formalizing a market for information. And we examine the consequences of the availability and distribution of information which is acquired and transmitted in that market for the properties of the equilibria in the market for the commodity. Finally, with regard to the message (sub)game, in our set-up the degree of coincidence of the objectives of sender and receivers is not common knowledge, as it depends on the realization of the true variety of the object and of the preferred variety of each potential buyer, which is only privately known to him.

There is also a rather large empirical literature which studies the behavior of financial analysts, and in particular the presence of biases in their reports, and its effects for the performance of asset markets; e.g., Womack (1996), Michaely and Womack (1999), Barber et al. (2001), Agrawal and Chen (2006), Bradshaw, Richardson and Sloan (2003), Jegadeesh et al. (2004).

On the other hand there is much less in terms of theoretical work on markets for information. A good part of the attention has been given to the case where the quality of the information transmitted is perfectly verifiable, thus abstracting from the problem of untruthful reports as well as from the problem of information acquisition. Admati and Pfleiderer (1986, 1990) look at the case where market participants act as price takers, where the "paradox" arises that when information is too precise, asset prices are perfectly revealing, so that information is worthless. Therefore, providers need to add some noise in order to profit from information sales.[4] When traders are strategic, information tranmission may then also provide a strategic advantage to participants, as pointed out by Vives (1990) in a general oligopoly framework, and Fishman and Hagerty (1995) in the case of financial markets.

The case where the information transmitted is non verifiable, as in our set-up, has been considered by Morgan and Stocken (2003), who study the information transmitted by an analyst when his incentives may not be aligned with those of investors, as he may be either a type that enjoys higher utility when the price of the underlying asset is high, or a type that enjoys telling the truth. They find that the analyst always "hypes" the stock; see also Kartik, Ottaviani and Squintani (2006). This is in line with our results for the case in which the information provider is the seller of the object and there is also vertical differentiation of the information.

Our analysis, being cast in a static framework, abstracts from reputational concerns. These may arise in a dynamic framework, where providers of information and traders repeatedly interact, and may mitigate the tendency of providers to send untruthful reports which may damage their future reputation, as shown by Benabou and Laroque (1992) and Ottaviani and Sorensen (2006).

The paper is organized as follows. ...

2 Model

There is one object for sale. This object is of a type $v \in S = \{1, 2, ..., k\}$. Each possible type has the same ex-ante probability. There are N potential buyers. We denote buyer $i \in \{1, ..., N\}$ by B_i; such buyer only cares (has positive utility) for one particular variety in S which we denote θ_i. The variables $\{\theta_i\}_{i \in N}$ and v are all i.i.d over S, thus for all i, j, θ_i is uncorrelated with θ_j and v; all elements of S have then the same probability, $1/k$. The seller of the object has no utility for the object and this is allocated to buyers via a second price auction.

In this paper we are interested in situations where information is not verifiable, i.e. the

[4] A similar point is made in Milgrom (1981) who explores how to inform others, if providing too good information can hurt the provider.

seller of information sends a report, which is pure ‘cheap talk’, over the signal he received. Since we abstract from both signal contractibility and reputational concerns, information markets can only exist if there is an element of non-rivalry in the information transmitted. Hence our focus on a market where buyers need not be interested in the same type of the good. Alternatively, one could reinterpret the specification of the model as capturing situations where the seller of information is not always able to profit directly from the information acquired. For example, leveraging the information may require the posession of complementary assets or skills, which he may lack. While an element of horizontal differentiation is necessary for a market for information to develop, such a market is compatible with vertical differentiation as well. In section 6 we extend the model to understand the effect of vertical differentiation in our setup.

The realization of θ_i is private information of the individual i. On the other hand, the type of the object for sale is not known to traders. Before the auction takes place, anybody can acquire, by paying a cost c, a signal over the type of the object, which we assume to be perfectly informative. If a trader acquires such signal he can in turn ‘sell information’ to other traders.

The utility of buyer B_i can then be written as

$$\pi_{B_i} = I_v - cI_e - t_{B_i}$$

In this definition t_{B_i} is the sum of the net monetary payments made by B_i to the seller to get the object and/or to the other buyers to purchase or sell information. I_v is an indicator variable that takes the value 1 if B_i gains the object and $v = \theta_i$ and 0 otherwise. Finally I_e is another indicator that takes the value 1 if B_i decides to acquire the signal over the type of the object, and 0 otherwise.

We discuss first the case where information is acquired and transmitted by potential potential buyers of the object. We discuss in section 4 the case where the seller of the object and/or other traders, not interested in purchasing the object, can also enter the market for information. Also, we consider first the case where only one type of report is sold in stage 2. of the game, at a single price (and refer to such case as no differentiation of information) We discuss in section 5 the case where different buyers of information may get different kinds of reports. Thus, the timing of the game is as follows.

1. First, each potential buyer decides whether or not to acquire the signal over the type of the object. The cost of the signal is c. The decision to acquire information, but obviously not the information itself, is commonly observable by all agents.

2. Any potential buyer who has chosen to acquire information, before learning the realization of the (perfectly informative) signal over the quality of the object, can sign a contract with other buyers for the sale of information. The contract prescribes that the seller of information will send a report over the signal, after receiving it, to all the traders who purchased information, in exchange for the payment of a price, set by the seller of information.

3. Each of the buyers who did not choose to acquire information in stage 1. chooses whether or not to purchase information from any of the traders selling information (he may choose to purchase information from more than one seller). Each of these buyers has then a final chance, after the market for information closes, to acquire the signal directly at a cost of c.

4. All agents who paid the cost c of information acquisition learn the realization of the signal. Sellers of information then issue a report to buyers.

5. A second price auction takes place among all the buyers for allocating the object.

We assume that contracting on information is done before observing the signal to avoid signaling on information at the contracting stage. Also, after the final chance to acquire information at stage 3, there is no opportunity to resell that information. This reflects the fact that acquiring information requires a simpler technology than organizing a market. This availability of a direct information acquisition technology, in turn, limits the ability of the seller of information to extract surplus from other buyers.

We consider the case where the set of messages available to the seller of information is:

$$\mathcal{M} = \{1, 2, ..., k\},$$

i.e. it coincides with the set S of possible types of the object: the report sent by the seller is then given by one of the k possible types of the object.[5] Moreover, the message structure is augmented so as to involve two phases:

a. Each uninformed buyer who has chosen to purchase information sends first a report over his type to the trader who is selling the information (the set of available messages for the uninformed buyer B_i is again the set of possible types of the buyer, given by S). Such report is observed only by the seller of information, not by the other buyers.

[5]We discuss this assumption in footnote .3.

b. Subsequently, the buyer who is selling information sends a report over it to all the buyers who purchased information from him. The price paid and the report received is the same for all such buyers.

We consider the case where the number k of possible types of the object is strictly greater than the number N of potential buyers:

$$k > N,$$

or the competition among buyers for the object is not very intense. In section 1 we discuss the main difference between the equilibria for $k > N$ and $k \leq N$.[6]

3 Equilibrium and welfare

Since information transmission need not be truthful, the game (and in particular the information subgame 2.) has many equilibria, as is common in other kinds of "cheap talk" games (see e.g. Crawford and Sobel 1982). We focus our attention on the equilibria where the degree of truthfulness in agents' reporting - and hence the revenue of the seller of information - is maximal. It should be clear from the analysis which follows that, given our focus on equilibria with maximal degree of truthfulness, the above message structure allows to enhance the revenue of the potential buyer who is selling information and, moreover, there is no essential loss of generality in restricting attention to direct messages. We will show that an equilibrium always exists where uninformed buyers truthfully report their type to the seller of information and this one adopts the following reporting strategy (both in and out of equilibrium):

$$m_i = \begin{cases} v, \text{ if } v \neq \theta_i \\ y, \text{ with probability } \frac{1}{k-N(B_i)}, \\ \text{for all } y \neq \theta_j \; \forall \, B_j \in \mathcal{N}(B_i), \text{ if } v = \theta_i \end{cases} \tag{1}$$

where B_i denotes the buyer selling information, m_i is the report issued by him, $\mathcal{N}(B_i)$ the set given by B_i plus all the buyers purchasing information from B_i and $N(B_i)$ the number of distinct realizations of θ_j across all buyers $B_j \in \mathcal{N}(B_i)$. Therefore, trader B_i tells the truth about the quality of the object as long as the true quality of the object does not coincide with his own type (i.e. with the type he likes). On the other hand, when the two coincide, there is a conflict of interest. Thus, the seller of information, B_i will not tell the

[6]The equilibrium for the case where $k \leq N$ is characterized in Appendix B.

truth. He will send a message which is a randomization over all the types which are different from the type of the seller as well as of all the buyers of information. One could interpret this message as telling the buyers of information that the object is not appropriate for any of them. An alternative form of deception in this event would entail sending a complete uninformative message, i.e. one that does not change the priors of the buyers. This could be done through a randomization among all the messages, if the set of available messages is $\mathcal{M}$. Alternatively, we could have enlarged the set of possible messages to allow also for the possibility of sending an empty message, a blank report. It can be shown, though, that in all equilibria with these alternative forms of deception, the utility of the seller of information is lower than in the one we consider (with reporting strategy 1). Notice that the form of deception we consider requires our phase a. of the message game to acquire information about the buyers' preferences. One could argue that, in many situations, the advisors indeed have information of that kind and use it in their reports.

It can be verified, on the basis of the following analysis, that such reporting strategy entails the maximal degree of information transmission at an equilibrium with maximal degree of truthfulness.

[7]

We will characterize the perfect bayesian equilibria of the game described above and evaluate their welfare properties for different parameter configurations (in particular, for different levels of the cost of information acquisition, c). As already said, we will focus our attention on the equilibria with maximal degree of truthfulness, where the buyers of information report truthfully their type and sellers of information adopt the message strategy in (1). An additional source of multiplicity of equilibria comes from traders' behavior in the auction, in the final stage of the game. There is always an equilibrium of the auction where the bid of each trader equals his expected value of the object, conditional on winning the auction. We will focus our attention on such equilibrium ('with truthful bidding'). Other equilibria may exist, but are typically non robust to trembles and we will ignore them in what follows.

Given the selection of equilibria in the message and auction subgames described above

[7]We could have enlarged the set of possible messages available to the seller of information to allow also for the possibility of sending an empty message, a blank report. In that case the degree of information transmission which obtains is the same as when the seller sends the empty message if he likes the object, instead of sending a random message as in (1). The situation is then analogous to the one we would have if phase a. of the message game described above did not exist. It can be shown that the utility of the seller of information is lower in that case than when the message strategy is given by (1).

(quite natural, we would like to argue, given our purposes), we will show that the overall equilibrium is, for almost all parameter values, unique:

THEOREM **1** *There is a unique perfect bayesian equilibrium of the game with no differentiation of the quality of information sold where sellers of information adopt the truthful reporting strategy in (1), buyers of information truthfully report their type to sellers, and participants in the auction adopt a truthful bidding strategy:*

1. *If* $c \geq \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} = \overline{\overline{c}}$, *no buyer chooses to acquire information; the object is then gained by one, randomly chosen, buyer, at a price* $1/k$.

2. *If* $\overline{\overline{c}} \geq c \geq \frac{1}{k^2}\left(\frac{1}{N-2}\right) = \overline{c}$, *one buyer acquires information and sells a report over it to all the other buyers except one, for a price* $p = \min\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}, c\right\}$; *the object is then always gained by a buyer who likes it, if such a buyer exists, at a price* $1/k$, *or* 1 *(if at least two buyers of information like the object and the seller of information does not like it).*

   In addition, in the subset of this region where $\overline{\overline{c}}_D = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq c \geq \overline{c}$ *another equilibrium exists, where two buyers acquire information and each of them sells a report over it to all other buyers, at a price* $p = 0$*; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price of zero if nobody else likes it, and one otherwise.*

3. *If* $\overline{c} \geq c$, *one buyer acquires information and sells a report over it to all the other buyers, for a price* $p = 0$*; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to zero when the seller of information likes (and gets) the object as well as when any other trader is the only one to like the object, and one otherwise.*

Let us summarize the content of the proposition. When information costs are low enough, information is acquired in equilibrium. Whenever it is acquired, information is transmitted via a report that in some events is not truthful. Information is sold for a low enough price so that all buyers, or all buyers except one, acquire it. The market for information is typically a monopoly. Figure 1 summarizes the result.

COROLLARY **1** 1. *If* $c \geq \overline{\overline{c}}$, *the price in the auction is equal to* $1/k$.

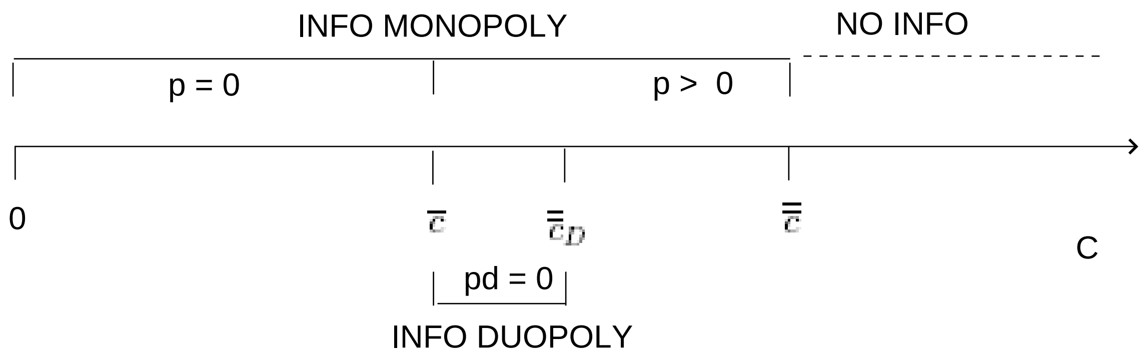

Figure 1: Equilibrium with homogeneous messages

2. *if $\overline{\overline{c}} \geq c \geq \overline{c}$, the price in the auction is equal to* 1, *when at least two indirectly informed buyers like the object; the price in the auction is equal to* $1/k$, *when either the directly informed buyer likes the object, or when the directly informed buyer does not like the object and exactly one indirectly informed buyer likes it; the price of the object is zero in all other cases.*

3. *if $\overline{c} \geq c$, the price in the auction is equal to* 1, *when at least two indirectly informed buyers like the object; the price of the object is zero in all other cases.*

3.1 Sketch of the proof and intuitions for Theorem 1

We provide a constructive proof of the proposition in Appendix A. Here we first discuss the consistency of beliefs associated with the signal/message structure in equation (1). Then we proceed to completely describe the actions of agents in all stages of the game, and check their optimality given the beliefs. In the following subsections we provide a sketch of the crucial parts of the argument with the intuitions associated to them.

Beliefs With the message structure in (1), there are no out-of-equilibrium messages. Thus, we can find the beliefs for an uninformed buyer, say buyer B_j, who receives a report from a single informed buyer, say buyer B_i, using Bayes' rule in all cases:

When buyer B_j receives from B_i a message $m_i = \theta_j$ he knows for sure that he likes the object (the message is truthful). That is, $\Pr(v = \theta_j | m_i = \theta_j) = 1$. On the other hand, when buyer B_j receives a message different from his type, this may happen for two reasons: either the message is truthful, and then B_j does not like the object, or it is not truthful, as the sender likes the object and is randomizing over types absent from the population, in which case there is a positive probability j may like the object. As we will see later, the precise value of beliefs in this case (which we compute in Appendix A for completeness) does not affect the outcome of the auction. Finally, the buyers who do not purchase information, nor acquire it directly, have beliefs equal to their prior beliefs. That is, $\Pr(v = \theta_j) = 1/k, \quad \Pr(v \neq \theta_j) = (k-1)/k$. The beliefs of an uninformed buyer who is receiving two (or more) distinct reports from two (or more) informed buyers are similar.

Behavior in the auction Given the beliefs of the buyers who purchased information from a single informed buyer we can show that a 'truthful bidding strategy' (where a trader's bid equals his expected value of the object, conditional on winning the auction) is always optimal if all other traders adopt such strategy:

When buyer receives a message indicating that the objects is of the type he likes, he knows for sure he likes the object. His optimal bid when the other bidders adopt 'truthful bidding strategies' is then equal to 1, i.e. to his posterior beliefs about the valuation of the object, and is then also truthful.

On the other hand, when a buyer receives a message different from the type he likes, he may have received this message for two reasons. First, it may be that he likes the object, but so does the sender of the message. In that case he cannot win the object in the auction with positive surplus as the seller of information, is better informed and (if he adopts a truthful bidding strategy) will make a higher bid, equal to 1. In this case, it is optimal to bid 0. Thus, when receiving a message different from his type, the buyer of information optimally bids zero.

Notice that even though we are in a second price auction, the bidder does not bid his expected valuation in all cases. This is due to the affiliation of the information of traders that is induced by the sender's reporting strategy. Once the information conveyed by winning the auction is taken into account, we see that the receiver's optimal bid when is zero after receiving a message different from his type, which is equal to the value of the object conditional on winning the auction (thus again a truthful bidding strategy), but distinct from his posterior belief over the value of the object.

In contrast, the buyers who do not purchase any information do not suffer from an affiliation problem (there is no relevant information in the event of winning the auction). Thus, the optimal bid in their case is: $\Pr(v = \theta_j) = 1/k$.

How important is the restriction to 'truthful bidding strategies'? We claim such restriction only bites when there is a single trader who is not purchasing information from other buyers. In this case any strictly positive bid lower than $1/k$ gives the same payoff as a bid of

$1/k$ to the uninformed buyer, hence is also optimal for the uninformed buyer and this may in turn affect the other traders' optimal bidding strategies generating other equilibria. On the other hand, when there is more than one trader who is not purchasing information, or all traders are purchasing information the only equilibrium in the auction game is the one with truthful bidding strategies. We can argue then that equilibria where not all bidders follow a truthful bidding strategy, if they exist, are non robust to trembles concerning the decision of purchasing information.

The bidding behavior of an uninformed buyer receiving more than one report from different informed buyers is analogous to the one described above. When his posterior belief over the value of the object equals one, he will bid one, and will bid zero otherwise (when the messages received are not fully informative).

Behavior in the message game In this stage we first check that the reporting strategy we postulated for the seller of information is indeed optimal for such trader. We then verify that also buyers of information are willing to report truthfully their types to the agent from whom they purchase the information.

First of all, notice that by changing the message strategy the seller or the buyer of information can only affect the outcome of the auction.

The seller:

There are two possible cases to check optimality for the message of the buyer.

When the seller of information likes the object, he may deviate by announcing a type corresponding to one of the agents purchasing information from him. In this case, the bid of that agent will be equal to 1, and the seller will end up paying more for the object than if he followed the equilibrium message, so he would never want to deviate.

When the seller of information does not like the object, he may deviate by announcing a type different from the true type. But that only changes the outcome in the auction, in which he is not interested as the object yields no utility to him. So he can never gain with such deviation.

The buyer:

A deviation by a buyer of information consists in reporting something different from his type. We also have to divide this discussion in two cases.

When the seller of information does not like the object, the seller of information reports the truth, whatever the reports received from the buyers of information. Hence the utility of any buyer of information is not affected by a change in his report in this case.

If the seller of information likes the object, a buyer of information, by changing his report, may affect the report sent by the seller of information. However, in this case the seller of information always bids 1, whatever the buyers' reports, and never truthfully reports the type of the object, so there is no possibility for a buyer of information to obtain any extra surplus by misreporting his type.

Stage 3 Purchase of information.

In this stage each uninformed buyer has to choose whether to purchase information from any of the informed buyers selling information, at the price posted by them, or alternatively to acquire the information directly (at the cost c), or do nothing of the two. Obviously, the agent will choose to acquire the information directly when the payoff from doing so is higher than otherwise. As we will see, the seller of information always chooses to set its price at a level such that no trader wishes to acquire information directly; on the other hand, he may find optimal to set the price of information at a level such that some of the traders prefer to remain uninformed.

Stage 2 Information sale.

This is the key stage of the game. We determine here the optimal choice of sellers of information when the market for information opens. Each seller sets a price at which he is willing to sell information to any buyer who wishes to purchase it. The price will be set at a level such as to maximize the utility of the seller of information, equal to his expected payoff in the auction plus the revenue from the sale of information, taking as given the strategies of the other sellers and buyers' responses to the prices posted in the next stage. The price for information set by sellers in this stage will determine how many uninformed buyers choose to purchase information in the subsequent stage and then the outcome of the auction.

To find the optimal pricing strategy of a seller of information, we need to consider first the various possible situations concerning the numbers of sellers of information and of buyers of information (we call each of them a "configuration"). For every configuration, when there is a single seller of information we determine the optimal price level, i.e. the one which maximizes the payoff of the seller and induces the given configuration. When there is more than a seller of information, we look for a set of prices, posted by the sellers of information, such that the price posted by each seller is the best reply to the price posted by the other sellers, and the prices induce the given configuration.

Once the characterization of each possible configuration is completed, it remains to find the configuration that obtains in equilibrium, given the past decisions of information acquisition by traders. For this we need to find the optimal choice of the sellers of information concerning the number of buyers they wish to sell information to, as well as the optimal choice of the agents who are not buying information in any given configuration to verify they indeed do not want to do it (on the other hand, the optimality of the choice of buyers of information for any given configuration is ensured by the price level set by sellers).

A complete characterization of payoffs and prices, together with actions, is given in the Appendix A. Here, we concentrate on a few important characteristics.

The **auction surplus for indirectly informed buyers** is determined by how many other buyers are directly or indirectly informed. In other words, the probability of obtaining the object with positive surplus with m directly or indirectly informed buyers is $(k-1)^{m-1}/k^m$. The surplus itself depends on the price in the auction, $1/k$ if there are uninformed buyers, and 0 otherwise.

The **price of information with a monopolist seller** extracts all surplus of the buyers up to the outside option. This outside option is the maximum between the payoff for being uninformed or obtaining directly the information in the last stage (thus being unable to resell it). Thus, the **payoff for indirectly informed buyers** in this case is simply the outside option just described.

The **auction surplus (thus the payoff) for indirectly informed buyers** when he is alone is the probability that no one else likes the good (in which case he gets it for free). When there is more than one uninformed buyer the surplus is zero, as those uninformed buyers bid away their surplus.

This leads us to the **crucial tradeoff for the monopolist seller**. By raising the price of information, he can select how many other buyers, purchase information. There are two qualitatively different classes of situations.

First, with strictly positive number of uninformed buyers. The price the seller of information pays in the auction does not change for any strictly positive number of buyers. To choose among all these cases, the only consideration is the loss in revenue from lowering the price from existing buyers, against the extra buyer of information. We show that, in fact, when $k > N$ it is always profitable to attract an extra buyer, and it is optimal to have at most one uninformed buyer. This is the main (only) change for the case with $k \leq N$, where it may be optimal to leave more than one buyer uninformed The intuition is that $k > N$, each additional buyer of information leads to relatively small competition for the good, whereas for $k \leq N$ competition among buyers is more intense.

A second situation is when there are no uninformed buyers. This happens if information is sold at zero price. On the one hand the price in the auction is lower (0 vs. $1/k$). On the other hand, all revenue from information is lost. It turns out that for low enough c, it is optimal to give away the information. The reason is that, for low c, the price that can be charged for information by the monopolist is quite low, as the outside option of obtaining information in the last stage is binding (it is cheap to get the information directly). Thus the price charged to other buyers is so low, that it is actually profitable to give the information away to guarantee that at least the auction price for the seller of information is low.

The **price of information with more than one seller is zero.** Note first that, if not all the other buyers have purchased information from at least one of the two sellers of information, then purchasing information from one of the two sellers has positive value for a buyer, as there is one less competitor in the auction. However, the additional benefit of purchasing a second report from the other seller is always zero. This follows from the fact that purchasing information from one seller allows to gain a positive surplus in the auction only when the buyer likes the object and nobody else who is informed, or purchased information, likes the object. Purchasing information also from the other seller allows the buyer to have more precise information in the event in which one of the two sellers of information likes the object (since the other tells the truth), however in such event no positive surplus can be gained since the seller of information who likes the object bids one.

On the other hand, if all the other buyers have purchased information from at least one of the two sellers of information, an uninformed buyer is not willing to pay a positive price to buy information from any of the two sellers. This follows from the fact that the agent's expected payoff in the auction depends on two factors: the probability of obtaining the object with a positive surplus, and the price he pays in the auction. The first factor only depends on how many traders bought at least one report, not on the number of reports they purchased. This is because if either of the sellers of information likes the object, such trader bids 1 for the object, so no positive surplus can be gained in that case. If neither of the sellers likes the object, they both tell the truth to all the traders buying information from them, so that if at least one of them likes the object he would bid 1, so no positive surplus can be gained in this case either. Therefore a positive surplus can only be obtained when the trader likes the object and nobody else likes it, and this occurs whether or not the trader purchases information, which has then no value to the trader in this case.

Given that each buyer wants to pay a positive price only for one signal, and only if not all other buyers purchase information, the only possible equilibrium with positive prices would entail a split of the buyers of information between the two providers of information, with at least one buyer not purchasing information. But then, each of the two sellers has an incentive to undercut the posted price. For the reasons explained before, a trader who has chosen to purchase information is always indifferent between buying from one or the other seller. Thus, a seller of information by lowering his price retains all those already buying from him and manages to steal buyers from the other seller of information, which entails a discrete jump in his surplus. This follows from the fact that the seller of information gains a positive surplus in the auction (equal to $1 - 1/k$ if at least one trader is not purchasing information) when he likes the object and neither the other seller of information *nor any other buyer that is purchasing information from the other seller*, likes the object. Hence the probability of having a positive surplus increases with the number of buyers of information purchasing information only from him. Since such incentive to undercut persists as long as the posted prices for information are positive, the only possible equilibrium is the one where the two sellers post a zero price of information, and all other buyers then buy information from both sellers, as stated..

Payoffs for information oligopolists. Since the price of information is zero, and all players are informed, the buyers of information have the same payoff as all other buyers, minus the cost of information, which explains why it is so hard to obtain equilibria with more than one informed buyer. In fact, the only reason why they exist is that the payoffs for informed oligopolists are the same as those of indirectly informed buyers with a monopolist and low cost of information.

Armed with these intuitions, we can now provide an explanation for the outcome of the information acquisition stage and thus for Theorem 1.

Stage 1 Information acquisition

High costs ($c \geq (k-1)^2/k + (N-2)(k-1)^{N-1}/k^N$), in equilibrium no buyer becomes informed. In this case not even a monopolist finds it profitable to become informed. The condition simply says that the payoffs of a monopolist (his auction surplus $(k-1)^2/k$ plus proceeds from information sale $(N-2)(k-1)^{N-1}/k^N$) are negative.

Intermediate costs ($(k-1)^2/k + (N-2)(k-1)^{N-1}/k^N \geq c \geq 1/(k^2(N-2))$),, the cost is low enough that it pays to be a monopolist, but not low enough that the monopolist is willing to forgo revenues (give away the information to all other buyers rather than selling it to all but one) to lower the auction price to zero. In part of this range $(k-1)^{N-1}/k^N \geq c \geq 1/(k^2(N-2))$, the buyers are indifferent between getting the information directly or indirectly. As a result there is another equilibrium with two informed agents.

Low costs $(1/(k^2(N-2)) \geq c)$, the cost is low enough that the monopolist is willing to forgo revenues (give away the information to all other buyers) to lower the auction price to zero.

3.2 Welfare

We now discuss the welfare properties of the equilibria described in the previous section. Given the assumptions made on traders' utilities, welfare can be simply evaluated by considering the total sum of agents' utilities, or traders' total surplus. In particular we are interested in comparing equilibria to the allocations which are Pareto efficient, and their associated welfare level, i.e. to the allocations which could be attained by a planner who is also uninformed about the type of the object and may acquire information, at the same cost c, over it.

As already noticed, if information is acquired by some trader, the resulting equilibrium allocation is always ex post efficient. Hence the only possible source of inefficiency may concern the information acquisition decision: is that also efficient at equilibrium, or rather is there overinvestment, or underinvestment in information? Evidently, the equilibrium with two buyers acquiring information, which obtains when $(k-1)^2/k+(N-2)(k-1)^{N-1}/k^N \geq c \geq 1/(k^2(N-2))$, is always inefficient as the duplication of the investment in information acquisition is always wasteful. On the other hand, we showed that when $(k-1)^2/k + (N-2)(k-1)^{N-1}/k^N \geq c$ there is always an equilibrium where only one buyer acquires information, hence without any wasteful duplication, while when $c \geq (k-1)^2/k + (N-2)(k-1)^{N-1}/k^N$, there is no information acquisition.

What can we say on the efficient (i.e., the planner's) information acquisition decision? If information is acquired, total surplus equals one if any of the traders happens to like the object, and zero otherwise, minus the cost of information; hence total welfare is:

$$W_1 = P(\exists i | v = \theta_i) - c = 1 - \left(\frac{k-1}{k}\right)^N - c.$$

On ther other hand, if information is not acquired total surplus is one only if the agent who receives the object (and this, with no information, can be only be arbitrarily, or randomly chosen) happens to like it. Thus total welfare is in this case:

$$W_0 = \frac{1}{k}.$$

By comparing W_0 and W_1 we see that it is socially efficient for information acquisition to take place if $1 - ((k-1)/k)^N - c \geq 1/k$, or:

$$\left(\frac{k-1}{k}\right)\left(1 - \left(\frac{k-1}{k}\right)^{N-1}\right) \geq c. \tag{2}$$

This threshold can then be compared to the one found in Theorem 1 for information acquisition to take place in equilibrium:

PROPOSITION **1** *In equilibrium there is a less than efficient level of investment in information. In particular, for values of c lying in the following, nonempty interval:*

$$\frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} < c \leq \left(\frac{k-1}{k}\right)\left(1 - \left(\frac{k-1}{k}\right)^{N-1}\right) \tag{3}$$

no information is acquired in equilibrium, though it would be socially efficient to acquire it.

Thus there is a range of values of c for which acquiring information is efficient but in equilibrium nobody chooses to become informed.

We now discuss the distribution of gains and losses between the situation where no information is acquired to the situation with a monopolist of information. This will additionally allow us to shed light on the inefficiency we uncover in proposition 1.

Who gains and who loses from information acquisition We now consider the effects on the distribution of welfare, i.e. who gains and who loses, from information acquisition. It is clear from our earlier analysis that when information is acquired the only buyer who is not purchasing information strictly gains (with respect to the situation where no information was acquired), as he can clearly free ride on the others' information acquisition (his payoff goes from 0 to $1/k$). The same is typically true for the buyer who is directly acquiring and selling then the information (B_1), while the remaining buyers, who purchase information from B_1, strictly gain only when c is not too close to the threshold beyond which information is not acquired in equilibrium. What about the seller of the object? We want to investigate under which conditions the seller of the object also gains (when information is acquired in equilibrium). The payoff of the seller of the object, in the region where $\bar{\bar{c}} \geq c \geq \bar{c}$ is

$$\left[\left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-2}-(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-3}\right)\right]+\frac{1}{k}\left[\frac{1}{k}+(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\right]$$
$$= \left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-2}\right)-(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}+\frac{1}{k}\frac{1}{k}$$

The first expression reflects the fact that, as discussed in Corollary 1, the price for the object is 1, $1/k$ or 0. The terms in square brackets are the probabilities of the events for which the respective prices hold. Since the price in the auction is always $1/k$ when no one is informed, the difference in revenues of the seller of the object between the case when $\bar{\bar{c}} \geq c \geq \bar{c}$ and when $c \geq \bar{\bar{c}}$ is given by:

$$\Delta\pi_S = \left(1-\frac{1}{k}\right)\left[\left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-2}-(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-3}\right)\right]-\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \tag{4}$$

We call the gains of the seller, which are in the positive first term, *rent dissipation* since those are rents that the buyer who acquires information will not appropriate, and instead go the seller of the good. Additionally, the negative second term are losses in payoffs that go from the seller of the good to the buyer who remains uninformed. The expression (4) is positive if:

$$1 > \left(\frac{k-1}{k}\right)^{N-3}\left(\frac{k+N-2}{k}\right) \tag{5}$$

As we show in section 4.2, there are values of k and N for which (5) is satisfied and some for which it is not.

The source of the inefficiency Given that the equilibrium leads always to ex-post efficient allocations, the sums of increases in traders' welfare between the situation when no buyer is informed to the one where at most one is uninformed equals the change in total welfare, $W_1 - W_0$. Since the indirectly informed buyers welfare does not change, being zero in both cases, the change in total welfare equals the change in welfare of the seller $\Delta\pi_S$ plus the payoff of the buyer who sells information and the buyer who remains uninformed (B_N). That is:

$$W_1 - W_0 = \pi_{B_1} + \pi_{B_N} + \Delta\pi_S.$$

. The analysis of the distribution of the welfare changes deriving from information acquisition allows us to gain some further understanding of the underinvestment in information result we obtained. Underinvestment obtains whenever $\pi_{B_N} + \Delta\pi_S > 0$. Note that from equation (4) plus the fact that from equation (19) in the appendix $\pi_{B_N} = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$ we have that:

$$\left(1-\frac{1}{k}\right)\left[\left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-2}-(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-3}\right)\right]>0 \qquad (6)$$

since it is equal to the positive *rent dissipation* term in equation (4). Note that π_{B_N} cancels with the second, negative, term in equation (4).We call this term *free riding* since it is a positive payoff obtained by B_N in the situation with information, for which he pays nothing. This informational *free riding*, entails a pure transfer of surplus from the seller to B_N, hence it does not undermine incentives for efficient information acquisition. What does undermine such incentives, and is reflected in equation (6) is the fact that, when more than one indirectly informed buyer gains the object, the revenue of the seller of the object goes up, from $1/k$, to 1. Therefore, some of the rents generated by information acquisition do not accrue to the agent investing in such acquisition. We will show in the section 5 that the seller of information may indeed be able to solve this problem by selling information of different quality to different buyers, thus avoiding the "ties" that generate the extra rents for the owner of the object.

But before doing that, we show that having an "independent" seller of information (i.e. somebody commonly known to have no interest in the object) is not a solution for the inefficiency we have discovered.

4 Who should sell information

4.1 An uninterested trader

A common proposal for solving inefficiencies in information gathering/transmission in markets is the separation between information providers and traders. We model this by introducing a new type of agents, who have no utility for the object and do not participate to the market where the auction is traded. We find that, when agents of this type can acquire and sell information the efficiency characteristics of equilibria do not improve. This is true whether or not disinterested sellers are the only one licensed to sell the information.

First of all, one should note that a disinterested seller is never present in equilibria where there are two or more sellers of information, as such seller only gains from the revenue obtained from the sale of information, and the price of information when there are two or more sellers, by the argument given in the previous section, is always zero.

Thus, we can have an equilibrium where information is sold by a disinterested trader only if he is the monopolist provider of information. Note furthermore that there can never be an equilibrium when the seller of information is a disinterested trader and information is sold at zero price. In that case the revenue from the sale of information is less than the cost of information acquisition.

The reporting strategy of a disinterested trader is clearly different from the one of an interested trader since he has never an interest in lying over the type of the object. Hence the truthful reporting strategy of the uninterested trader is to always tell the truth. Hence the quality of the information sold is clearly higher when the seller is a disinterested trader. Does this imply the equilibrium has better efficiency properties? We will show that the answer to such question is negative, the efficiency properties are actually worse in this case, as the incentives to information acquisition are weaker, hence the problem of the inefficient underinvestment in information acquisition is more severe.

PROPOSITION **2** *When information is sold by uninterested traders, in equilibrium there is again a less than efficient level of investment in information. Furthermore, the interval of values of c for which information is not acquired in equilibrium though it is socially efficient to acquire it is*

$$(N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} < c < \left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-1}\right)$$

larger than in the case where information is sold by interested traders.

The intuition for this result is not too hard to understand. With respect to an interested seller, the uninterested one has one more customer (the one who would be information seller in the alternative scenario). This means an extra gain of $(k-1)^{N-1}/(k)^N$ (the price of information at the relevant margin). On the other hand, an uninterested buyer does not gain any surplus in the auction, so he loses that surplus (worth $(k-1)/(k)^2$), with respect to the interested seller. Clearly, the loss is larger than the gain, which explains the greater region of inefficiency for the uninterested seller.

4.2 The owner of the good as a seller of information

We have seen that when the seller of information is one of the buyers, and he cannot discriminate the information sold, information is acquired inefficiently. It is natural to ask whether the owner of the good could do better in terms of efficiency than one of the potential buyers.

To keep the comparison clean, we still assume that the owner of information has to pay a cost c to acquire the information. Since the inefficiency arises when there is a monopolist, we will concentrate in that case, which will clearly be the equilibrium when the cost of acquisition c is high enough. Also, we assume that the monopolist does not know, and can commit not to find out, the type of the buyers. This is, in fact, the best for him, as we will show later. Since the seller does not know the types, he is assumed to tell the truth, as the uninterested trader.

Proposition **3** *When the seller of information is the owner of the good, in equilibrium there is a less than efficient level of investment in information. In particular, for values of c lying in the following, nonempty interval:*

$$\left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-2}\right) \leq c \leq \left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-1}\right) \tag{7}$$

information acquisition is socially efficient, but it is not acquired by the owner of the good.

The logic of the result is easy to understand. As in the case when information is sold by a potential buyer of the good, the equilibrium leads always to ex-post efficient allocations. Hence, the sums of increases in traders' welfare between the situation when no buyer is informed to the one where at most one is uninformed equals the change in total welfare, $W_1 - W_0$. Since the informed buyers welfare does not change, being zero in both cases, the change in total welfare equals the change in welfare of the seller $\Delta\pi_S$ plus the payoff of the buyer who sells information and the buyer who remains uninformed (B_N). That is:

$$W_1 - W_0 = \pi_{B_N} + \Delta\pi_S.$$

. Thus, underinvestment obtains whenever $\pi_{B_N} > 0$. Since $\pi_{B_N} = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$, the result follows. Notice that unlike in the case of the buyer as a seller of information the source of the inefficiency is not *rent dissipation* but rather, the informational *free riding* by the uninformed buyer.

Another natural question is when is the owner of the good more efficient than a buyer of the object in terms of information acquisition. Comparing the threshold for information acquisition to take place for the potential buyer found in theorem 1, $\bar{\bar{c}}$, with the one found in proposition 3 for the owner of the good, we obtain that the inefficiency is more severe with the owner of the good selling information when:

$$\frac{1}{k}\left(\frac{k-1}{k}\right)+(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}>\left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-2}\right)$$

Or equivalently,

$$\left(\frac{k-1}{k}\right)^{N-3}\left(\frac{k+N-2}{k}\right)>1 \tag{8}$$

We now show that there are values of k and N such that (8) is satisfied, and some where it is not.[8]

Let $x \equiv k-N$, so that the condition (8) can be equivalently written as:

$$\left(1-\frac{1}{N+x}\right)^{N-3}\left(1+\frac{N-2}{N+x}\right)>1. \tag{9}$$

Since the term on the left hand side approaches one as $x \to \infty$, if such term is strictly decreasing in x, for all x sufficiently large, then the condition (9) holds for k sufficiently large. Notice that a function is increasing if its logarithm is decreasing. Take then the logarithm of the left hand side of (9)

$$(N-3)\ln\left(1-\frac{1}{N+x}\right)+\ln\left(1+\frac{N-2}{N+x}\right)$$

and differentiate it with respect to x. We obtain:

$$(N-3)\frac{1}{(N+x)^2}\frac{1}{\left(1-\frac{1}{N+x}\right)}-\frac{(N-2)}{(N+x)^2}\frac{1}{\left(1+\frac{N-2}{N+x}\right)}=\frac{1}{(N+x)^2}\left(\frac{\frac{(N-2)^2-(N+x)}{N+x}}{\left(1-\frac{1}{N+x}\right)\left(1+\frac{N-2}{N+x}\right)}\right)$$

which is strictly negative if and only if $(N-2)^2-N<x$. This implies that $\left(1-\frac{1}{N+x}\right)^{N-3}\left(1+\frac{N-2}{N+x}\right)$ is first increasing and then decreasing in x. Since the function is 1 in the limit of x large, this implies, first, that the buyer is more efficient for x large (thus for k sufficiently larger than N). But it also means that if $\left(1-\frac{1}{N+x}\right)^{N-3}\left(1+\frac{N-2}{N+x}\right)>1$ for $x=0$, then, the buyer will be more efficient for all x.

On the other hand, let $x=0$ (i.e. $N=k$). Then, we have that condition 8 can be written:

$$2\left(\frac{k-1}{k}\right)^{k-2}>1$$

which is true for $k<6$ and false for $k\geq 6$. We gather the previous observations in the following:

[8]Note that this is simply a reverse inequality of condition (5), which describes when $\Delta\pi_S>0$. The reason for this can be seen from the discussion on the source of inefficiency for the buyer and the seller. Those discussions show that condition (8) is satisfied when π_{B_N} is bigger than $\Delta\pi_S+\pi_{B_N}$. That is, when $\Delta\pi_S<0$.

REMARK **1** *For x sufficiently large, or for $k < 6$, the buyer is more efficient than the seller as provider of information. When x is sufficiently small and $k \geq 6$, the opposite holds.*

Thus, for N sufficiently smaller than k, the buyer is more efficient than the seller. This is a consequence of the fact when N is much smaller than k, the probability that two agents wish the same object becomes small. Hence, the *rent dissipation*, the source of inefficiency for the buyer, is small.

Before concluding this section we need to establish the claim made earlier that the seller of the good is better off not knowing the types of the seller. Denote by B_i^{Win} the event in which buyer B_i wins the auction. We have that:

LEMMA **1** *In any equilibrium*

$$\pi_S = \sum_{i=1}^{N-1} E_S \left(\Pr(B_i^{Win} | v = \theta_i) \right) - c$$

The price paid in the auction disappears from the sellers' payoff, since it adds to his revenues directly, but detracts from them in the sale of information. Thus, the payoff for the seller are maximal when the equilibrium probability of buyers acquiring the good when they like it is maximal. A commitment from the seller of information to ignore the players' valuations, as we assumed from the beginning of this section, ensures that in an equilibrium with maximal truthfulness this property is indeed satisfied, since then it is always optimal for the seller to tell the truth.

On the other hand, if the seller knows the types of buyers, he may not be able to maximize the equilibrium probability of buyers acquiring the good when they like it. The reason is that if he knows the types of the buyers, the seller may sometimes misallocate the good. Suppose for example that the seller knows the types of the buyers, and that they bid higher when they think their type is more likely. Then, if $v = \theta_1$ but $\theta_2 = \theta_3$, the seller wants to announce anything that increases the conditional probability of θ_2, as ties increase the auction revenue. But if that happens, and a player with type θ_2 obtains the object, it will have been misallocated. On the other hand, if no announcement makes the players increase their prior probability that they like the object, then all players will bid the same, and hence the object will also be misallocated with positive probability.

Lying about the type of the good in order to increase the demand of the object is akin to the "hyping" by securities' analysts that so worries the authors of the report of the European Comission Forum Group (2003). Notice that this happens in spite of the fact that the information transmitted here is not about quality, but about the type of the good for sale. It will occur 'a fortiori' when information about quality is transmitted as well. See our discussion of an extension of the model with information on quality in section 6.

5 Information of endogenously heterogeneous quality

5.1 The monopolist case

We show next that a way to have efficient information gathering at equilibrium is to allow for the possibility that information of heterogeneous quality is sold. In this subsection we consider the case where informed traders may sell different kinds of reports over their information, at different prices. To see why this may be useful for the seller of information, notice that in the case where a single type of report is sold, the price a buyer is willing to pay for information depends on over how many other buyers he gains an informational advantage. This informational advantage manifests itself in a priority to obtain the good when the buyer wants it. With a single type of report, there are up to three priority levels. First, the directly informed buyer. Then, all the indirectly informed buyers (which share the same priority level). Finally, the uninformed buyers, if any. We now show that by arranging the set of indirectly informed buyers in several distinct priority levels, the seller of information can increase his revenues. This improves the incentives for efficient information acquisition, since it eliminates the *rent dissipation* which we showed in section 3.2 was at the root of inefficiency in information acquistion.

Informed traders can choose the number of types of reports and the price at which they are willing to sell them. Facing the menu of reports on offer, each buyer chooses which ones to buy. We will consider in particular the case where information can be vertically differentiated, or the different types of reports offered for sale can always be arranged in a hierarchy, of reports of decreasing quality, or informativeness. Let L denote the number of different types of reports sold. The hierarchy of the qualities of the different reports is modelled by assuming that buyers purchasing a report of type l, $l \in \{1, .., L\}$ observe all the messages m_j, $j = l+1, .., L$. The information provided by the reports has then a nested structure, in the sense that receiving report i conveys no additional information to the one in report $l > i$, while the reverse is not true. We consider again the case where the set of possible messages available to the sellers of information for any l is the set of direct messages, $m_l \in \mathcal{M} = \{1, 2, ..., k\}$.

In this subsection we look in particular at the situation where there is a single informed trader, i.e. only one buyer has chosen to pay the cost c to acquire direct information over the quality type of the object up for sale. Let us denote by B_1 the informed buyer, and by B_i, $i = 2, .., N$ all the uninformed buyers who are purchasing information from B_1. We will find his optimal choice concerning the differentiation of information sold and his maximal payoff, thus determining also the maximal level of c such that information acquisition is worthwhile. In subsection 5.2 we allow for the possibility of entry in the market for information, i.e. that more than a single buyer acquires information and sells suitably differentiated information in the market.

We will find and characterize the equilibria of the subgame starting from the node where a single buyer, B_1, has acquired information and is selling differentiated information in the market. We determine first for any given level of L the optimal choice of B_1 concerning the prices posted for the different types of reports sold, and the equilibrium strategies in the rest of the subgames (purchase of information, reporting strategies and bids in the auction). We then find the level of L which maximizes the revenue of the seller B_1. By comparing the maximal payoff which obtained for B_1 we can then see also when information acquisition is worthwhile (when entry is restricted to at most one seller of information).

As in the previous sections, there are two phases in the reporting of messages. First each buyer of information sends a report over his type to the seller of information and this one subsequently sends the messages $m_l, l = 1, .., L$. The buyers of report of type l receive then the messages $(m_L, m_{L-1}, .., m_l)$, for any $l \in \{1, .., L\}$.

We still focus our attention on the equilibria where agents' reporting is characterized by the maximal degree of truthfulness and, at the same time, is consistent with the differentiation of information in L levels (so that the revenue of the seller of information is maximal). In particular, we will show that there is always an equilibrium where the uninformed buyers always report their type and the seller of information adopts the reporting strategy described below.

To this end, it is convenient to adopt some notational conventions. Given the hierarchical structure of the information, we will sometimes refer to the buyers purchasing from B_1 a report of quality l as the buyers in layer l of the hierarchy. For any $l \geq 2$, let $\mathcal{N}_l(B_1)$ denote then the set of buyers in layer l or below (i.e. purchasing reports of type $i \geq l$) and $N_l(B_1)$ the number of different realizations of θ_i across all buyers $B_i \in \mathcal{N}_l(B_1)$; hence $\mathcal{N}_{l-1}(B_1)/\mathcal{N}_l(B_1)$ indicates the set of buyers in layer $l-1$. $\mathcal{N}_1(B_1)$ and $N_1(B_1)$ are then similarly defined except for the fact that $\mathcal{N}_1(B_1)$ is augmented by B_1, i.e. is the set of buyers who purchased any type of report from B_1, plus B_1. The reporting strategy of B_1 for the messages $m_1, .., m_L$

is then defined recursively as follows:

$$m_1 = \begin{cases} v, \text{ if } v \neq \theta_1 \\ y, \text{ with probability } 1/[k - N_1(B_1)], \\ \text{for all } y \neq \theta_j,\ \ B_j \in \mathcal{N}_1(B_1), \text{ if } v = \theta_1 \end{cases} \tag{10}$$

and, for $l = 2, .., L$

$$m_l = \begin{cases} m_{l-1}, \text{ if } m_{l-1} \neq \theta_i \text{ for all i} \in \mathcal{N}_{\mathrm{l-1}}(\mathrm{B}_1)/\mathcal{N}_{\mathrm{l}}(\mathrm{B}_1) \\ y, \text{ with probability } 1/[k - N_l(B_1)], \\ \text{for all } y \neq \theta_j,\ \ B_j \in \mathcal{N}_1(B_1), \text{ if } m_{l-1} = \theta_i \text{ for some } B_i \in \mathcal{N}_{\mathrm{l-1}}(\mathrm{B}_1)/\mathcal{N}_{\mathrm{l}}(\mathrm{B}_1) \end{cases} \tag{11}$$

Thus at each layer l the informed trader tells the truth about the quality of the object as long as the true quality of the object does not coincide with his own type or with the type of any buyer who has purchased information of higher quality. Otherwise, the informed trader randomizes over any value different from the type of any of the agents who purchased information, including his own.

With this message structure and a monopolist, there are a couple of interesting features of the equilibrium.

PROPOSITION 4 *A monopolist which sells differentiated information under a reporting strategy as in 10 and 11 sells information to all buyers but possibly one. In both cases the optimal distribution is to create as many layers as players with whom he communicates. The condition that determines when is it optimal to leave no buyer uninformed is:*

$$c \leq \left(1 - \left(\frac{k-1}{k}\right)^{N-1}\right)\frac{1}{N-2} = \overline{c}_h \tag{12}$$

The condition that determines when is it optimal for the monopolist to acquire information is:

$$c \leq \left(\frac{k-1}{k}\right)\left(1 - \left(\frac{k-1}{k}\right)^{N-1}\right) = \overline{\overline{c}}_h \tag{13}$$

Figure 2 summarizes the result.

The result that the monopolist wishes to create as many layers as buyers with whom he communicates, is the main innovation of the equilibrium with heterogeneous information. The key insight for this result is that for a given buyer any other buyer who is **equally or better informed** than himself lowers his payoff by exactly as much. This means that the price this individual is willing to pay in the auction will not change if anybody who is

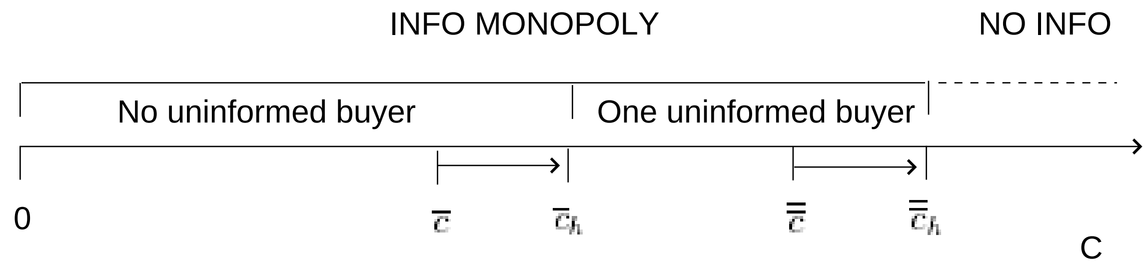

Figure 2: Equilibrium with heterogeneous messages

equally or better informed gets an even better signal. The one who gets ahead of the queue, however, does improve his payoff, and therefore is willing to pay more. The monopolist can thus increase revenue from information by improving the signal of one player (at no cost to him in the auction as long as he still gets the best information). By induction, the best is to completely rank all the buyers with whom he communicates.

Condition (12) reflects a tradeoff for the seller of information that is slightly different from the case of homogeneous information. By maintaining one uninformed buyer, the price in the auction is higher for all the informed buyers. Once this individual is informed, the price of the auction is lower for all other buyers. This benefits the buyer who acquires information directly, and also through an increase in the rents of other buyers and hence on the willingness to pay for information. But, at the same time, when all buyers are informed, the payoff of remaining uninformed (the outside option) increase, and thus reduces the maximum willingness to pay for information. The key difference with respect to the homogeneous case is that through the heterogeneous reports, the provider of information can benefit indirectly from the increased willingness to pay of indirect buyers when all are informed. Because of this, the level of costs at which all buyers become informed in equilibrium is higher with heterogeneous messages (that is, $\overline{c}_h > \overline{c}$).

5.1.1 Efficiency and equilibrium revisited

The condition for a monopolist to prefer being informed to not having information is given in equation 13. Since the condition for efficient investment in information is:

$$W_1 \geq W_0 \Longleftrightarrow \left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-1}\right) - c \geq 0.$$

Since this is equivalent to 13, we immediately have efficient information acquisition by the monopolist in equilibrium.

To summarize: we have shown that the optimal structure of reports is given by a number of reports L equal to the number $N-1$ of potential buyers of information, with prices such that each buyers chooses to purchase one report, and each type of report is purchased by only one agent. We show in particular that the optimal level of prices for such reports is such that B_1 can appropriate (via the revenue from the sale of information and the gain in B_1's surplus from the auction with respect to the case where information is not acquired) all the social surplus generated by the acquisition of information, which as shown in (2) is given by

$$W_1 - W_0 = \left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-1}\right) - c.$$

Notice the *free-riding* by the uninformed buyers still occurs in this case. The *rent dissipation*, however, now disappears. This is because the information structure generated completely ranks buyers, and thus there will be no ties in the auction. This immediately implies that there is no underinvestment in information acquisition.

5.2 The competitive case

In the previous section, we showed that a monopolist selling information of heterogeneous quality could achieve an efficient outcome in this market. In this section, we show that the possibility to sell heterogeneous information does not guarantee efficiency. In particular, it is likely that when costs of information are not too high, too much information will be acquired. More precisely, we show that more than one individual may acquire information in equilibrium, which in our setup is inefficient.

Before we do that, we should note that the outcome with a monopoly seller of heterogeneous information is always an equilibrium. The reason is that if even if entry profits are potentially very high, if any "entrant" is assumed to send uninformative signals, he may optimally send uninformative signals after entry (out of the equilibrium path). But no uninformed buyer will pay any positive price for such signals. As a consequence he will not buy information.This equilibrium is ruled out since we focus our attention on equilibria where the degree of truthfulness in agents' reporting is maximal.

Similarly, the outcome with the monopolist can always be sustained by assuming that in the case two informed buyers are present, both of them give away the signal truthfully (whenever different from their type) and without discrimination for free. We also rule out this kind of *destructive subgame outcome*. Since the strategies used in this kind subgame equilibrium are weakly dominated, they would be selected against by refinements in the spirit of *forward induction* (Van Damme 1989). Intuitively, sellers in a subgame should ask themselves: "Why would my competitor buy information just to give it away?" and conclude that another kind of equilibrium is likely in the subgame.

For these reasons, we refine the equilibrium and assume that there is no *destructive subgame outcome* and that the signaling policy of all buyers of information is like the one pursued by the monopolist in the previous section. More precisely, we assume that any seller of information B_i arranges his buyers in a set of L_i *layers (or levels),* $L_i \geq 1$. This seller of information sends a different message m_l^i to all buyers in each layer l, $l = 1, .., L_i$. Buyers in any layer l observe not only the message sent to layer l but also the messages sent to all other layers below l, i.e. observe m_j^i, $j \geq l$ thus ensuring information to be of (weakly) lower quality for buyers in lower layers.

We now describe the reporting strategy of B_i. Let layer 0 be an initial layer where only B_i lies. Also, let $N(B_i)$ denote the number of different realizations of θ_j across all buyers who buy information from B_i, $j = 1, .., N_i$.In particular we assume that all competitors are (correctly) assumed to follow the following signaling policy:[9]

$$m_1^i = \begin{cases} v, \text{ if } v \neq \theta_i \\ y, \text{ with probability } 1/[k - N_1(B_i)], \\ \text{for all } y \neq \theta_j, \ \ j = 1, .., N_i, \text{ if } v = \theta_i \end{cases} \tag{14}$$

and, for $l = 2, .., L_i$

$$m_l^i = \begin{cases} m_{l-1}^i, \text{ if } \ m_{l-1}^i \neq \theta_j \text{ for all } j \text{ corresponding to } B_j \text{ belonging to layer } l-1 \\ y, \text{ with probability } 1/[N - N_l(B_i)], \\ \text{for all } \ y \neq \theta_j, \ \ j = 1, .., N_i, \text{ if } \ m_{l-1}^i = \theta_j \text{ for some } B_j \text{ belonging to layer } l-1 \end{cases} \tag{15}$$

Proposition **5** *For sufficiently low c, in all "refined" equilibria (i.e. where the signals are as in 14 and 15), and without* destructive subgame outcome, *there are at least two sellers of information.*

Remark **2** *To understand this result we have to remember what prevented multiple entry in the game with homogeneous information (and with low costs). In that situation the (unique) signal would get its price driven to zero with multiple entrants, and thus entry would only be marginally profitable for at most one extra entrant (in the case of low costs). Now, however,*

[9]This is the same as in the monopoly case, except that some more notation is needed to distinguish the different potential senders - a superinded for messages and a subindex for M_i and L_i.

there may be two entrants or more sharing the (now higher) rents from information. A buyer of information would have to buy information from more than one provider to retain his privileged position in the hierarchy of information. That is, the (credible) threat if a buyer did not buy a (redundant) signal from a second (or third) provider is that this provider would then offer a signal of the same quality to another buyer, which would ruin the value of the signal(s) already purchased.

6 Discussion

We have built a model for information markets which allows us to study the equilibrium and welfare properties of such markets in contexts where information is vital and may be of use to potential rivals. The model is versatile, so we have been able to study a number of potential setups. Information transmitted may be homogeneous or heterogeneous, and the seller of information may be a potential buyer of the good, the seller or a "neutral" third party. We have found that in most setups the outcome is ex-post efficient. This was the result of selling through an auction and of almost everybody getting informed. As we discussed, more intense competition for the object ($N > k$) would reverse this result. Obviously, another way in which this would cease being true is if the verification technology for the type of the good was imperfect.

Ex-post efficiency is not, nevertheless, a major problem in our setup. Efficiency in information acquisition is tougher to achieve. We have seen that one potential avenue for success in this respect is for information to be transferred in different qualities. This may give a potential efficiency rationale to the behavior of the likes of Henry Blodget, the noted Merril Lynch analyst who was issuing an "accumulate" recommendation for Excite at Home while in an internal e-mail he was writing "ATHM is such a piece of crap!" This does not mean that heterogeneous information is the solution to all information acquisition problems in our setup. For low costs, it may lead to too much information acquisition as providers join in an excessive race to share the rents from the activity.

There are several extensions that could be studied within our framework. A possible extension would consider multiple units of the good for sale. Suppose in this case that the directly informed buyers had limited capacity for "enjoying" the good. This could lead to ex-post inefficiency in the case of homogeneous information. The seller of infomation will lie about the type of the object to lower the competition in the auction, as he does there is a single unit. But now, this will lead to some units sold to buyers who did not like them, and thus ex-post inefficiency. Another possible extension concerns an imperfect verification technology for the type of the object. In that case, multiple signals could improve ex-post efficiency with respect to single signals (unlike in our setup).

There is an extension which we can discuss in some more detail. We have seen that, in contrast with a potential buyer of the good, the owner of the good prefers to tell the truth about the object in all cases like the uninterested trader. This is somewhat surprising, since common sense tells us that an owner of a good (and his agents) should also be somewhat untrustworthy as a source of information, albeit for different reasons. A potential buyer of the good has an incentive to tell his competitors that the good is not "right for them." Our model captures well this phenomenon. The owner of the good should have incentives to exagerate how much the buyers like the good. However, in our context, with horizontal differentiation, there is no way to do that, unless the seller knows the types of the buyers, and can use a coincidence of wants to increase auction competition. But there is a more natural way in which the seller can exagerate, which leads to misallocation problems.

Assume that, beyond the types, the good may come in 2 quality levels, H (High) and L (Low). The buyers, as before, like only one randomly chosen variety. Formally, the good has a type $v \in S^{HL} = \left\{1^H, ..., k^H, 1^L, ..., 1^L\right\}$. Denote by $S^H = \left\{1^H, ..., k^H\right\}$ the set of High quality goods, $S^L = \left\{1^L, ..., k^L\right\}$ and the set of Low quality goods and by $i = \{i^H, i^L\}$ the set of goods of variety i, independent of the quality, for $i \in S = \{1, ..., N\}$.. In addition, the buyers are also of two types: some buyers are sensitive to quality (Se) and some of them are insensitive (In). An In consumer has a constant valuation of 1 for a good of the type he desires. An Se consumer values a good of the type he desires as V, if the good is of H quality; and he values it at 0 if it is of L quality. Let us assume for simplicity that H and L have identical probabilities for each type of good, and that consumers have identical probabilities to be of type Se and In. Also, assume that messages are structured so that the message $m \in M$ must consist of a variety $i \in S$ and a quality $J \in \{H, L\}$, so that a generic $m = (i, J)$.

Proposition **6** *For any pair of messages $m^H = (i, H)$ $m^L = (i, L)$ from an owner of the good, we must have beliefs such that* $\Pr(v \in S^H | m^H) = \Pr(v \in S^H | m^L) = \Pr(v \in S^H) = \frac{1}{2}$. *Thus, if $V > \frac{1}{2}$, for any equilibrium message m the Se type buyers bid more for the good after hearing m than the In type buyers. If $V < \frac{1}{2}$, for any m the Se type buyers bid less for the good after hearing m than the In type buyers.*

The previous proposition shows that there is an inefficient allocation of the good as long as $V \neq \frac{1}{2}$, clearly a knife-edge case. This inefficient allocation also guarantees that not all social value can be appropriated by any seller, and therefore that there will be inefficient information acquisition, at least when c is expensive enough for a "natural monopoly."

APPENDIX A

Proof of Theorem 1

The informativeness of the report sold With the message structure in (1), there are no out-of-equilibrium messages. Thus, we can find the beliefs for an uninformed buyer, say buyer B_j, who receives a report from a single informed buyer, say buyer B_i, using Bayes' rule in all cases:

$$\Pr(v = \theta_j | m_i = \theta_j) = 1 \tag{16}$$

$$\begin{aligned} \Pr(v &= \theta_j | m_i \neq \theta_j) = \frac{\Pr(v = \theta_j \cap m_i \neq \theta_j)}{\Pr(m_i \neq \theta_j)} = \frac{\Pr(v = \theta_j) - \Pr(v = \theta_j \cap m_i = \theta_j)}{\Pr(m_i \neq \theta_j)} \\ &= \frac{\frac{1}{k} - \left(\frac{k-1}{k}\right)\frac{1}{k}}{1 - \left(\frac{k-1}{k}\right)\frac{1}{k}} = \frac{1}{k(k-1)+1} \end{aligned} \tag{17}$$

Finally, the buyers who do not purchase information, nor acquire it directly, have beliefs equal to their prior beliefs:

$$\Pr(v = \theta_j) = \frac{1}{k}, \quad \Pr(v \neq \theta_j) = \frac{k-1}{k} \tag{18}$$

The comparison between the beliefs of a buyer who purchased a report from an informed buyer (in (16), (17)) and the beliefs of the buyers who did not purchase information (in (18)) provides a clear illustration of the informativeness of the report sold.

The beliefs of an uninformed buyer who is receiving two (or more) distinct reports from two (or more) informed buyers can be derived along similar lines to (16),(17).

Behavior in the auction Already fully described in the main text.

Behavior in the message game Already fully checked in the main text.

Behavior in the first three stages of the game

Stage 3: Purchase of information and message game In this stage each uninformed buyer has to choose whether to purchase information from any of the informed buyers selling information, at the price posted by them, or alternatively to acquire the information directly (at the cost c), or do nothing of the two. Obviously, the agent will choose to acquire the information directly when the payoff from doing so is higher than otherwise. As we will see, the seller of information always chooses to set its price at a level such that no trader wishes to acquire information directly; on the other hand, he may find optimal to set the price of information at a level such that some of the traders prefer to remain uninformed.

Stage 2: information sale To find the optimal pricing strategy of a seller of information, we need to consider first the various possible situations concerning the numbers of sellers of information and of buyers of information (we call each of them a "configuration"). For every configuration, when there is a single seller of information we determine the optimal price level, i.e. the one which maximizes the payoff of the seller and induces the given configuration. When there is more than a seller of information, we look for a set of prices, posted by the sellers of information, such that the price posted by each seller is the best reply to the price posted by the other sellers, and the prices induce the given configuration.

Once the characterization of each possible configuration is completed, it remains to find the configuration that obtains in equilibrium, given the past decisions of information acquisition by traders. For this we need to find the optimal choice of the sellers of information concerning the number of buyers they wish to sell information to, as well as the optimal choice of the agents who are not buying information in any given configuration to verify they indeed do not want to do it (on the other hand, the optimality of the choice of buyers of information for any given configuration is ensured by the price level set by sellers).

Optimal prices and payoffs for given numbers of sellers and buyers of information We identify each possible configuration with a number:

1. We first check the case with one seller of information (wlog let it be B_1), $N-2$ (let it be B_2 to B_{N-1}) buyers of information, and one agent not buying information at all (let it be B_N). The payoffs for the different players are:

$$
\begin{aligned}
\pi_{B_1}(1) &= \frac{1}{k}\left(1-\frac{1}{k}\right)+(N-2)p(1)-c \\
\pi_{B_i}(1) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\left(1-\frac{1}{k}\right)-p(1)=\max\{\pi_C(1),\pi_U(1)\} \text{ for } i=2,..,N-1 \\
\pi_{B_N}(1) &= \left(\frac{k-1}{k}\right)^{N-1}\frac{1}{k} \qquad (19) \\
\pi_C(1) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\left(1-\frac{1}{k}\right)-c=\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}-c \\
\pi_U(1) &= 0
\end{aligned}
$$

and the price of information is

$$p(1)=\min\left\{c,\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\}$$

(a) In this configuration, given the reporting strategy of the seller of information, given by (1), and of the buyers of information (truthtelling), the seller of information, B_1, gets the object when he likes it ($v=\theta_1$), an event which has probability $\frac{1}{k}$, and he pays in the auction the bid of B_N, which is $\frac{1}{k}$ (so the surplus is $\left(1-\frac{1}{k}\right)$ and the total expected value $\frac{1}{k}\left(1-\frac{1}{k}\right)$). In addition he obtains the price of information, paid by the $N-2$ buyers, B_2 to B_{N-1} (thus $(N-2)p(1)$ is his total revenue) and he pays the cost of information, c. We will see in a moment how is this price determined, but before doing that let us consider the expressions of the payoffs of the other traders under this configuration.

(b) The expression of the payoff for the agents B_2 to B_{N-1} is explained as follows. Each buyer B_i in this group only gets the good with positive surplus when he likes it, and neither B_1 nor any other buyer in $\mathcal{N}(B_1)$ likes it (in all the other cases there is somebody bidding 1, so no positive payoff can be obtained), which has probability $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}$. In that case, he gets the object and pays the bid of the uninformed buyer, so the surplus is $\left(1-\frac{1}{k}\right)$ and the total expected value $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\left(1-\frac{1}{k}\right)$. In addition he pays the price of this configuration, $p(1)$.

(c) Agent B_N only bids more than the others when he likes the object and nobody else likes it (an event with probability $\left(\frac{k-1}{k}\right)^{N-1}\frac{1}{k}$), in which case the others bid zero, so his payoff in the auction is 1. So the total expected payoff is $\left(\frac{k-1}{k}\right)^{N-1}\frac{1}{k}$.

(d) The price $p(1)$ is then set at the level which maximizes the profits of the seller of information among all prices which support the given configuration. Hence

$p(1)$ is such that any buyer B_i in $\mathcal{N}(B_1)$ is indifferent between buying the information and the best of his alternative options (acquiring information directly at the subsequent stage or remaining uninformed). That is, $p(1)$ is determined as follows $\pi_{B_i}(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\left(1-\frac{1}{k}\right) - p(1) = \max\{\pi_C(1), \pi_U(1)\}$, or, equivalently, $p(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\left(1-\frac{1}{k}\right) - \max\{\pi_C(1), \pi_U(1)\}$, which simplifies to the expression above. In this expression $\pi_C(1)$ is the payoff of buyer B_i if he chooses to acquire information directly at the subsequent stage, and $\pi_U(1)$ his payoff under the alternative action of remaining uninformed. Those payoffs are as follows:

i. $\pi_U(1) = 0$. Since there is already one agent (B_N) not buying information, if B_i were also not to purchase information and remain uninformed, the two traders are always going to bid their expected valuation $\frac{1}{k}$, so they pay the expected value for the good when they win the auction and hence have no gain.

ii. $\pi_C(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\left(1-\frac{1}{k}\right) - c$. The reason is that by acquiring information directly B_i would obtain the good with positive surplus when he likes it and none of the other informed players like the good, which has probability $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}$. In that event, B_i pays for the good what the uninformed players bid, so his surplus is $\left(1-\frac{1}{k}\right)$, for a total expected payoff from the auction of $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\left(1-\frac{1}{k}\right)$, to which the cost of information needs to be subtracted.

2. We now examine the configuration where there is again one seller of information (wlog let it be again B_1), and $N-1$ (let it be B_2 to B_N) buyers of information from him. The payoffs are now.

$$
\begin{aligned}
\pi_{B_1}(2) &= \frac{1}{k} + (N-1)p(2) - c \\
\pi_{B_i}(2) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - p(2) = \max\{\pi_C(2), \pi_U(2)\} = \pi_U(2) \text{ for } i = 2, .., N \\
\pi_C(2) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \\
\pi_U(2) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \Longrightarrow \pi_U(2) > \pi_C(2) \Longrightarrow p(2) = 0
\end{aligned}
$$

(a) In this situation, and given the message structure, the seller of information, B_1, gets the object when he likes it ($v = \theta_1$), an event with probability $\frac{1}{k}$, and he pays in the auction the bid made by all the other buyers, which is equal to zero (so his surplus is 1 and the total expected payoff $\frac{1}{k}$). In addition he obtains the payment of the price of information from the $N-1$ buyers, B_2 to B_N (thus $(N-1)p(2)$ is his total revenue) and he pays the cost of information, c. We will see in a moment how is this price determined.

(b) Consider then the payoff of all other buyers, B_2 to B_N. Each of them can only get the good with positive surplus when he likes it, and neither B_1 nor any other buyer likes it (in all the other cases there is somebody bidding 1, so no possible surplus can be obtained), which has probability $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. In that case, he gets the object and pays the bid of the other players, which is zero, so the surplus is 1 and the total expected gain from the auction is $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. In addition he pays the price of information, which for this configuration is given by $p(2)$.

(c) The price $p(2)$ is set by the monopolist seller at the level which makes the buyers indifferent between buying the information and the best of their alternative options. That is, $p(2)$ is determined so that $\pi_{B_i}(2) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - p(2) = \max\{\pi_C(2), \pi_U(2)\}$, or, in other words, $p(2) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - \max\{\pi_C(2), \pi_U(2)\}$. Those payoffs are as follows:

   i. The payoff of a buyer if he chooses the alternative action of remaining uninformed is $\pi_U(2) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. This follows from the fact that he would be the only uninformed buyer and hence get the object when he likes it and nobody else does (in all other cases somebody would bid 1) and in that case he pays the bid made by the others, which is zero. There is no other cost for him, so his expected payoff is $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$.

   ii. The payoff of a buyer under the alternative action of acquiring information at the next stage is $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c$. The reason is that he would obtain the good with positive surplus when he likes it and none of the other buyers (which are all either directly or indirectly informed via the purchase of information) likes the good, an event which has probability $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. When he gets the good, he pays the bid made by the other buyers, which is zero, for a total expected payoff from the auction of $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$, to which the cost of information needs to be subtracted.

Hence we find that the maximal price the seller can charge to buyers of information in this configuration is indeed $p(2) = 0$, i.e. information is sold for free.

3. We examine next the configuration where there are two sellers of information (wlog let them be B_1, B_2), and $N-2$ (let them be B_3 to B_N) buyers of information from

the sellers. We show that in this case: (a) each uninformed buyer purchases the signal from both informed buyers and (b) the price of information is zero for both sellers of information: $p_1(3) = p_2(3) = 0$.

(a) Note first that, if not all the other buyers have purchased information from at least one of the two sellers of information, then purchasing information from one of the two sellers has positive value for a buyer, by a similar argument to the one used in configuration 1. to show that $p(1) > 0$. However, the additional benefit of purchasing a second report from the other seller is always zero. This follows from the fact that purchasing information from one seller allows to gain a positive surplus in the auction only when the buyer likes the object and nobody else who is informed, or purchased information, likes the object. Purchasing information also from the other seller allows the buyer to have more precise information in the event in which one of the two sellers of information likes the object (since the other tells the truth), however in such event no positive surplus can be gained since the seller of information who likes the object bids one.

(b) On the other hand, if all the other buyers have purchased information from at least one of the two sellers of information, an uninformed buyer is not willing to pay a positive price to buy information from any of the two sellers. The argument is similar to the one used in configuration 2. to show that $p(2) = 0$. It follows from the fact that the agent's expected payoff in the auction depends on two factors: the probability of obtaining the object with a positive surplus, and the price he pays in the auction. The first factor only depends on how many traders bought at least one report, not on the number of reports they purchased. This is because if either of the sellers of information likes the object, such trader bids 1 for the object, so no positive surplus can be gained in that case. If neither of the sellers likes the object, they both tell the truth to all the traders buying information from them, so that if at least one of them likes the object he would bid 1, so no positive surplus can be gained in this case either. Therefore a positive surplus can only be obtained when the trader likes the object and nobody else likes it, and this occurs whether or not the trader purchases information, which has then no value to the trader in this case.

(c) Given that each buyer wants to pay a positive price only for one signal, and only if not all other buyers purchase information, the only possible equilibrium with positive prices would entail a split of the buyers of information between the two providers of information, with at least one buyer not purchasing information. Note that in this case a buyer purchasing information from one seller earns a positive surplus when he likes the object and no other buyer who is purchasing information from any of the two sellers likes it. Also, the price paid is determined by the bid of the agent not purchasing information, hence no buyer has any incentive to buy from the second seller additional information, and every buyer is indifferent between buying from one or the other seller, at the same price (the only gain from buying information is to 'jump ahead'of the -single agent- not buying info). But then, each of the two sellers has an incentive to undercut the posted price. For the reasons explained in (a), a trader who has chosen to purchase information is always indifferent between buying from one or the other seller. Thus, a seller of information by lowering his price retains all those already buying from him and manages to steal buyers from the other seller of information, which entails a discrete jump in his surplus. This follows from the fact that the seller of information gains a positive surplus in the auction (equal to $1 - 1/k$ if at least one trader is not purchasing information) when he likes the object and neither the other seller of information *nor any other buyer that is purchasing information from the other seller*, likes the object. Hence the probability of having a positive surplus increases with the number of buyers of information purchasing information only from him. Since such incentive to undercut persists as long as the posted prices for information are positive, the only possible equilibrium is the one where the two sellers post a zero price of information, and all other buyers then buy information from both sellers, as stated..

Given these observations, the payoffs are then:

$$\begin{aligned} \pi_{B_i}(3) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \quad \text{for } i = 3, .., N \\ \pi_{B_1}(3) &= \pi_{B_2}(3) = \pi_{B_i}(3) - c \end{aligned}$$

The above expressions follow from the fact that, since all uninformed buyers purchase information from both sellers, any agent gets the object with positive surplus when he likes it and nobody else does (otherwise, there is always somebody else either bidding more or bidding 1), an event which has probability $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. In such event the agent pays the highest alternative bid which is zero, so his surplus is 1, and the expected payoff in the auction is then $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. Prices of information, as we showed are zero, so the total expected payoff for buyers B_3 to B_N coincides with the expected payoff in the auction. The sellers of information have the same expected payoff in the auction, gain zero from the sale of information and in addition have to pay the cost of information, c; their utility is then strictly lower.

4. We now consider the situation where no buyer acquires information, hence there is no market for information as there is nobody selling it. In this case, the payoff of every buyer is:

$$\pi_{B_i}(4) = 0 \text{ for all } i = 1, .., N$$

Since all buyers are uninformed, they all make a bid equal to their expected valuation, $\frac{1}{k}$. The object is then randomly allocated to one buyer, who pays for it an amount equal to his expected value for the good and hence gets no surplus.

5. We examine the situation where there is one seller of information (wlog let it be B_1), $J \geq 2$ agents do not buy information (let them be $B_{N-J+1}, ..., B_N$) and the remaining $N - J - 1$ (let them be B_2 to B_{N-J}) buyers purchase information from the single seller. The payoffs for the different players are in this case:

$$\begin{aligned}
\pi_{B_N}(5) &= ... = \pi_{B_{N-J+1}}(5) = 0 \\
\pi_{B_1}(5) &= \frac{1}{k}\left(1 - \frac{1}{k}\right) + (N - (J+1))p(5) - c \\
\pi_{B_i}(5) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}\left(1 - \frac{1}{k}\right) - p(5) = \max\{\pi_C(5), \pi_U(5)\}, \text{ for } i = 2, ..., N - J \\
\pi_C(5) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}\left(1 - \frac{1}{k}\right) - c = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J} - c \\
\pi_U(5) &= 0
\end{aligned}$$

and the price of information is

$$p(5) = \min\{c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}\}$$

(a) In this configuration, given the reporting strategy of the seller of information, B_1, he gets the object when he likes it ($v = \theta_1$) - an event with probability $\frac{1}{k}$ - and pays in the auction the bid of the uninformed traders B_i, $i = N - J + 1, .., N$, which is equal to $\frac{1}{k.}$. Hence B_1's surplus is $\left(1 - \frac{1}{k}\right)$ and its total expected value $\frac{1}{k}\left(1 - \frac{1}{k}\right)$. In addition B_1 receives the payment of the price of information, paid by the $N - J - 1$ buyers, B_2 to B_{N-1} (thus $(N - J - 1)p(5)$) and he pays the cost of information, c. We will see in a moment how the price of information is determined.

(b) The expression of the payoff for buyers B_2 to B_{N-J} is explained as follows. Any of these buyers can only get the good with positive surplus when he likes it, and neither B_1 nor any other buyer B_i, $i = 2, ..., N-J$, likes it (in all other cases there is somebody else bidding 1, so no possible surplus can be obtained). In that event, which has probability $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}$, the buyer gets the object and pays in the auction the bid of the uninformed buyers, so the surplus is $\left(1-\frac{1}{k}\right)$ and its total expected value is $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}\left(1-\frac{1}{k}\right)$. In addition each of these buyers pays the price for information, $p(5)$, which needs then to be subtracted from the previous expression.

(c) Buyers B_{N-J+1} to B_N get the good when none of the other buyers (who either directly acquired the signal or purchased information) like the good. They always bid their expected valuation, $\frac{1}{k}$, so when they get the object they pay the expected value for the good and get no surplus.

(d) The price $p(5)$ is again set by the monopolist seller at the level which makes each buyer indifferent between buying the information and the best of his alternative options. That is, $p(5)$ is determined so that, for $i = 2, ..., N-J$,

$$\pi_{B_i}(5) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}\left(1-\frac{1}{k}\right) - p(5) = \max\{\pi_C(5), \pi_U(5)\},$$

or

$$p(5) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}\left(1-\frac{1}{k}\right) - \max\{\pi_C(5), \pi_U(5)\}.$$

In this expression $\pi_U(5)$ is the payoff of a buyer under the alternative action of remaining uninformed, and $\pi_C(5)$ the payoff under the alternative action of acquiring information directly in the last stage. Those payoffs are as follows:

i. $\pi_U(5) = 0$. The reason is that by remaining uninformed these agents become identical to buyers B_{N-J+1} to B_N who get zero surplus as they obtain the object at a price equal to their expected valuation for it.

ii. $\pi_C(5) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J-1}\left(1-\frac{1}{k}\right) - c$. This expression follows from the fact that by acquiring information directly a buyer obtains the good with positive surplus when he likes it and none of the other informed traders like the good, an event with probability $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J-1}$. When he gets the good, he pays what uninformed players bid, so his surplus is $\left(1-\frac{1}{k}\right)$, for a total expected payoff from the auction of $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J-1}\left(1-\frac{1}{k}\right)$, to which the cost of information has to be subtracted.

6. Consider finally the case with $J > 2$ sellers of information (wlog let them be $B_1, ..., B_J$), and $N-J$ (let them be B_{J+1} to B_N) buyers of information. In this case, and for the same reasons as in 3.: (a) each seller posts a zero price for information; (b) each uninformed buyer purchases information from all sellers. The payoffs in this case are then perfectly analogous to the ones obtained for 3.:

$$\begin{aligned}\pi_{B_i}(6) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}, \; i = J+1, .., N \\ \pi_{B_1}(6) &= ... = \pi_{B_J}(6) = \pi_{B_i}(6) - c\end{aligned}$$

Equilibria in stage 2 (for given number of sellers of information) We now check for each of the possible configurations of information sale described above, which of them can be an equilibrium in stage 2 of the game. In such stage the number of sellers of information is given, hence for each configuration, the deviations that need to be checked are: (a) whether the sellers wish to sell to the number of buyers in that configuration or to a different one; (b) whether buyers of information in any configuration indeed prefer to buy information or not to do it/acquire directly information in the subsequent stage and (c) whether the uninformed buyers prefer to stay uninformed. As we already argued, the level of prices of information found in the previous section takes care of (b), so we only need to check (a) and (c).

1. Configuration 1. is an equilibrium of stage two of the game, when there is a single seller of information, if:

$$\pi_{B_1}(1) \geq \max\{\pi_{B_1}(2), \pi_{B_1}(5)\} \tag{20}$$

$$\pi_{B_N}(1) \geq \pi_{B_N}(1) - p(1) \tag{21}$$

   Condition (20) guarantees that the seller of information does not want to raise the price, thus discouraging some buyers of information, and moving to configuration 5., nor to lower it, attracting more buyers, moving so to configuration 2.. Condition (21) guarantees that the uninformed individual prefers to stay uninformed rather than to buy information.

   The latter condition, (21), is actually trivially satisfied. Trader B_N, by purchasing the information, will get the good with a positive surplus in the same circumstances as when he does not buy information: when he likes it and nobody else does. In addition, he would have to pay for the information, so his utility would always be lower.

We now check condition (20):

(a) The only seller of information prefers configuration 1. to 2. (i.e. setting the price equal to $p(1)$ rather than $p(0)$) if

$$\pi_{B_1}(1) = \frac{1}{k}\left(1 - \frac{1}{k}\right) + (N-2)p(1) - c \geq \pi_{B_1}(2) = \frac{1}{k} - c$$

or equivalently,

$$(N-2)\min\left\{c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\} \geq \frac{1}{k} - \frac{1}{k}\left(\frac{k-1}{k}\right) = \frac{1}{k^2}$$

When $\min\left\{c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\} = c$, the above condition reduces to:

$$\begin{aligned}(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &> (N-2)c \geq \frac{1}{k^2}, \text{ or:}\\ \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &> c \geq \frac{1}{N-2}\frac{1}{k^2}\end{aligned}$$

Else, when $\min\left\{c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\} = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$ the condition is

$$c > \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq \frac{1}{N-2}\frac{1}{k^2}$$

The join of the two conditions is then:

$$c \geq \frac{1}{N-2}\frac{1}{k^2} \tag{22}$$

(b) Next, we consider the alternative choice of configuration 5.. In the discussion of 5. below we will show that configuration 5. is not admissible if $c < \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$; hence in considering a possible deviation to configuration 5. we can restrict our attention to the case:

$$c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}, \tag{23}$$

under which we have $\max\{\pi_C(1), \pi_U(1)\} = \pi_U(1)$, hence $p(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. We then show that a deviation to configuration 5. is never profitable in this case. Let us examine first the the case of 5. with $J = 2$:

$$\begin{aligned}
\pi_{B_1}(1) \quad &\geq \quad \pi_{B_1}(5) \iff \frac{1}{k}\left(1-\frac{1}{k}\right)+(N-2)p(1)-c \geq \frac{1}{k}\left(1-\frac{1}{k}\right)+(N-3)p(5)-c \\
&\iff \quad (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq (N-3)\min\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}, c\right\} \\
&= \quad (N-3)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2},
\end{aligned}$$

where the last equality follows from condition (23). The inequality condition we obtained, $(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq (N-3)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}$, can be equivalently rewritten as:

$$\frac{N-2}{N-3} \geq \frac{k}{k-1} \iff 1+\frac{1}{N-3} \geq 1+\frac{1}{k-1} \iff k-1 \geq N-3,$$

and is always satisfied since $k > N$. With $J > 2$, we have:

$$\begin{aligned}
\pi_{B_1}(1) \quad &\geq \quad \pi_{B_1}(5) \\
&\iff \quad \frac{1}{k}\left(1-\frac{1}{k}\right)+(N-2)p(1)-c \geq \frac{1}{k}\left(1-\frac{1}{k}\right)+(N-(J+1))p(5)-c \\
&\iff \quad (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq (N-(J+1))\min\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}, c\right\} \\
&= \quad (N-(J+1))\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}
\end{aligned}$$

Here we use an induction argument. We already showed the inequality above holds for $J = 2$ and we will show that for $J > 2$:

$$(N-(J+1))\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J} \geq (N-(J+2))\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}. \tag{24}$$

The above inequality can be rewritten as

$$\frac{(N-(J+1))}{(N-(J+2))} \geq \frac{k}{k-1} \iff 1+\frac{1}{(N-(J+2))} \geq 1+\frac{1}{k-1} \iff k-1 \geq N-(J+2)$$

which is always satified.

Hence conditions (20) and (21) are satisfied if

$$c \geq \frac{1}{N-2}\frac{1}{k^2}.$$

2. Configuration 2. is an equilibrium, with a single seller of information, if:

$$\pi_{B_1}(2) \geq \max\{\pi_{B_1}(1), \pi_{B_1}(5)\} \tag{25}$$

Since we just showed that $\pi_{B_1}(1) \geq \pi_{B_1}(5)$, all that is needed is

$$\pi_{B_1}(2) \geq \pi_{B_1}(1).$$

Thus, reversing the inequality from the discussion of the previous case, we find that (25) is satisfied if:

$$c \leq \frac{1}{N-2}\frac{1}{k^2}.$$

3. For this configuration, with two sellers of information, to obtain the traders' payoffs in the previous section we already found the optimal level of the price set by each seller of information as a best reply to the other seller's choice: both sellers set a price $p(3) = 0$ and all other traders purchase then information from both sellers. Since there are no uninformed traders, given the number of sellers we do not need to check for further deviations in this case.

4. Here there is no sale of information, so the only possible deviation to check in this case is that an (uninformed) buyer does not wish to acquire directly information in the last stage, i.e.:

$$\pi_{B_i}(4) = 0 \geq \frac{1}{k}\left(\frac{k-1}{k}\right) - c$$

or:

$$c \geq \frac{1}{k}\left(\frac{k-1}{k}\right).$$

5. We already saw in the discussion of 1. above that this configuration is dominated by configuration 1., as long as (23) holds, or $c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$ We verify in what follows that (23) is indeed a necessary condition for configuration 5. to be an equilibrium, thus establishing that 5. can never be an equilibrium with a single seller of information.

   For configuration 5. to be an equilibrium, the following must hold:

$$\pi_{B_N}(5) = 0 \geq \max\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J+1} - p(5), \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}\left(1-\frac{1}{k}\right) - c\right\}, \tag{26}$$

   which ensures that none of the $N-J$ uninformed agents in configuration 5. wishes to deviate to either purchase information, or acquire directly information in the final stage at a cost c.

We show that condition (26) implies (23). Consider first the case where

$$\max\{\pi_C(5), \pi_U(5)\} = \pi_C(5), or \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J} \geq c,$$

and $p(5) = c$. In this case condition (26) simplifies to $0 \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J+1} - c$, or $c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J+1}$. Since

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J+1} \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1},$$

if (26) is satisfied, (23) a fortiori must hold. In the other case, where

$$\max\{\pi_C(5), \pi_U(5)\} = \pi_U(5) = 0, or \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J} \leq c,$$

since $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J} \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$ (23) must hold (this is true in fact whether or not (26) is satified).

6. For the same reasons as for configuration 3. we do not need to check for further deviations in this case.

Stage 1: information acquisition We now examine the optimal choices in the initial stage of the game, concerning the decision of each trader of whether or not to acquire directly information. This will determine the number of sellers of information at the subsequent stage, where the market for information opens and hence which of the possible configurations of information trades determined at the end of the previous section can obtain in equilibrium. To this end, for each configuration the deviations that need to be checked in the first stage are: one agent who is directly informed becomes not directly informed, and one agent who is not directly informed, becomes directly informed. We consider again in sequence the six possible configurations we found in the previous section:

1. For this configuration to obtain in an equilibrium of the overall game we need:

$$\pi_{B_1}(1) \geq \pi_{B_1}(4) = 0 \tag{27}$$

$$\pi_{B_i}(1) \geq \pi_{B_1}(3), \text{ for } i = 2, .., N-1 \tag{28}$$

$$\pi_{B_N}(1) \geq \pi_{B_1}(3) \tag{29}$$

Notice that condition (29) is always satisfied:

$$\pi_{B_N}(1) \geq \pi_{B_1}(3) \Longleftrightarrow \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c.$$

To analyze the other two conditions, consider first the case where

$$\max\{\pi_C(1), \pi_U(1)\} = \pi_C(1), or \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq c,$$

and hence $p(1) = c$. In this case condition (28) reduces to:

$$\pi_{B_i}(1) = \pi_C(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \geq \pi_{B_1}(3) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c,$$

always satisfied, while condition (27) becomes:

$$\pi_{B_1}(1) = \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)c - c \geq 0 = \pi_{B_1}(4),$$

always true.

Consider next the other case, where

$$\max\{\pi_C(1), \pi_U(1)\} = \pi_U(1) = 0$$

, or $c > \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$, and so $p(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. Condition (28) becomes:

$$\begin{aligned} \pi_{B_i}(1) \quad &= \quad \pi_U(1) = 0 \geq \pi_{B_1}(3) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \\ &\iff \quad c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}, \end{aligned}$$

which is always satisfied since in the case under consideration we have $c > \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$. Finally (27) reduces to:

$$\pi_{B_1}(1) = \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \geq 0 = \pi_{B_1}(4)$$

which is equivalent to:

$$\frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \geq 0$$

or:

$$\frac{1}{k}\left(\frac{k-1}{k}\right)\left[1 + (N-2)\left(\frac{k-1}{k}\right)^{N-2}\right] \geq c. \tag{30}$$

Therefore, conditions (27), (28) and (29) are satisfied if (30) holds. Combining this fact with the condition (22) we found in the previous section for configuration 1. to obtain as an equilibrium outcome in stage two of the game, establishes the first part of claim 2 of Theorem 1: configuration 1. obtains as an equilibrium outcome of the overall game if (it is immediate to verify that the interval below is always nonempty):

$$\frac{1}{k}\left(\frac{k-1}{k}\right)\left[1+(N-2)\left(\frac{k-1}{k}\right)^{N-2}\right] \geq c \geq \frac{1}{N-2}\frac{1}{k^2}$$

2. For such configuration to obtain in equilibrium we need:

$$\pi_{B_1}(2) \geq \pi_{B_1}(4) = 0 \tag{31}$$

and

$$\pi_{B_i}(2) \geq \pi_{B_1}(3), \text{ for } i = 2, .., N. \tag{32}$$

The first one, (31), is always satisfied under the condition we found in the previous section for configuration 2. (rather than configuration 1.) to obtain at an equilibrium in the second stage of the game, when there is a single seller: $c \leq \frac{1}{N-2}\frac{1}{k^2}$. Using such condition we obtain in fact:

$$\pi_{B_1}(2) = \frac{1}{k} - c \geq \frac{1}{k} - \frac{1}{N-2}\frac{1}{k^2} > 0$$

Turning then our attention to condition (32) we see it too is always satisfied:

$$\pi_{B_i}(2) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq \pi_{B_1}(3) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c.$$

Hence the conditions for configuration 2. to obtain as an equilibrium outcome of the game are just the conditions we found in the previous section to get 2. at an equilibrium of stage two:

$$c \leq \frac{1}{N-2}\frac{1}{k^2},$$

which establishes claim 3. of Theorem 1.

3. To get configuration 3. at an equilibrium of the overall game we need:

$$\pi_{B_1}(3) \;\geq\; \begin{cases} \pi_{B_i}(1), \text{ for } i = 2, .., N-1, \text{ if } c > \frac{1}{N-2}\frac{1}{k^2} \\ \pi_{B_i}(2), \text{ for } i = 2, .., N, \text{ if } c < \frac{1}{N-2}\frac{1}{k^2} \\ \pi_{B_i}(1) \text{ or } \pi_{B_i}(2), \text{ if } c = \frac{1}{N-2}\frac{1}{k^2} \end{cases} \tag{33}$$

$$\pi_{B_i}(3) \;\geq\; \pi_{B_1}(6), \text{ for } i = 3, .., N \tag{34}$$

When $c \geq \frac{1}{N-2}\frac{1}{k^2}$, condition (33) can be rewritten as

$$\begin{aligned}\pi_{B_1}(3) &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \geq \pi_{B_i}(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - p(1) \\ &= \max\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c, 0\right\}.\end{aligned}$$

which only holds when $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \geq 0$. On the other hand, if $c < \frac{1}{N-2}\frac{1}{k^2}$ (33) becomes:

$$\pi_{B_1}(3) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1},$$

never true.

Finally, condition (34) can be written as:

$$\pi_{B_i}(3) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c$$

always satisfied.

We conclude so that configuration 3. obtains as an equilibrium outcome of the overall game when

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \geq c \geq \frac{1}{N-2}\frac{1}{k^2},$$

which establishes the second part of claim 2 of Theorem 1.

4. For configuration 4. to obtain at an equilibrium of the overall game we need:

$$\pi_{B_i}(4) = 0 \geq \begin{cases} \pi_{B_1}(1), \text{ if } c > \frac{1}{N-2}\frac{1}{k^2} \\ \pi_{B_1}(2), \text{ if } c < \frac{1}{N-2}\frac{1}{k^2} \\ \pi_{B_1}(1) \text{ or } \pi_{B_1}(2), \text{ if } c = \frac{1}{N-2}\frac{1}{k^2} \end{cases}.$$

The last two conditions can be equivalently written as:

$$\frac{1}{N-2}\frac{1}{k^2} \geq c \geq \frac{1}{k},$$

which is never satisfied. The first condition can be rewritten as

$$\begin{aligned} c &\geq \max\left\{\frac{1}{N-2}\frac{1}{k^2}, \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\min\left[c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right]\right\} \\ &= \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\min\left[c, \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right], \end{aligned}$$

which can only be satisfied if $c > \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$, in which case the condition reduces to:

$$c \geq \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}. \tag{35}$$

Hence we conclude that when (35) holds, configuration 4. obtains at an equilibrium, which establishes claim 1 of Theorem 1.

5. We already saw that configuration 5., when admissible, is always dominated by configuration 1. and hence never obtains in equilibrium.

6. This cannot be an equilibrium because any trader $B_1 ... B_J$ can deviate to not acquiring information directly and purchasing it in the market (at a zero price); by so doing he will get a higher payoff.

This completes the analysis of the possible configurations which may arise at equilibrium and hence the proof of Theorem 1.

Proof of proposition 1 The result follows immediately by comparing the threshold for efficient information acquisition, found in (2), with the threshold found in Theorem 1 for information acquisition not to take place in equilibrium, given by (35). Hence by combining the two, we obtain that, as claimed, when (3) holds, in equilibrium no information will be gathered even though social welfare is maximized when information is acquired.

We also show that the interval of values of c identified in condition (3) is nonempty, i.e.:

$$\begin{aligned} \frac{1}{k}\left(\frac{k-1}{k}\right) + (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &< \left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-1}\right) \iff \\ \left(\frac{k-1}{k}\right)^{N-3}\left(\frac{k+N-3}{k}\right) &< 1 \end{aligned}$$

Let $x \equiv k - N$, so that the condition $\left(\frac{k-1}{k}\right)^{N-3}\left(\frac{k+N-3}{k}\right) < 1$ can be equivalently written as:

$$\left(1-\frac{1}{N+x}\right)^{N-3}\left(1+\frac{N-3}{N+x}\right) < 1. \tag{36}$$

Since the term on the left hand side approaches one as $x \to \infty$, it suffices to show that such term is strictly increasing in x. Notice that a term is increasing if its logarithm is increasing. Take then the logarithm of the left hand side of (36), we get

$$(N-3)\ln\left(1-\frac{1}{N+x}\right) + \ln\left(1+\frac{N-3}{N+x}\right);$$

differentiating it with respect to x yields:

$$\begin{aligned} (N-3)\frac{1}{(N+x)^2}\frac{1}{\left(1-\frac{1}{N+x}\right)} - \frac{(N-3)}{(N+x)^2}\frac{1}{\left(1+\frac{N-3}{N+x}\right)} &= \frac{(N-3)}{(N+x)^2}\left(\frac{1}{\left(1-\frac{1}{N+x}\right)} - \frac{1}{\left(1+\frac{N-3}{N+x}\right)}\right) \\ &= \frac{(N-3)}{(N+x)^2}\left(\frac{\frac{N-2}{N+x}}{\left(1-\frac{1}{N+x}\right)\left(1+\frac{N-3}{N+x}\right)}\right), \end{aligned}$$

which is strictly positive since $N + x > 1$, or $k > 1$.

This completes the the proof of Proposition 1.

Proof of proposition 2 The maximal price that buyers are willing to pay for information to a monopolist seller of information when a total number of $N-J$ buyers purchase information is given by $\min\left\{c,(k-1)^{N-J-1}/k^{N-J}\right\}$. By an argument similar to the one that allowed us to show the analogous result in Theorem 1, the revenue of the seller is always higher if information is sold to $N-1$ buyers than to any smaller number of buyers.

Hence, if an equilibrium exists where information is acquired and sold by an uninterested trader, his profits are:

$$\pi_{B_u}=(N-1)\min\left\{c,\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\}-c\geq 0. \tag{37}$$

It is immediate to see from (37) that the highest level of c for which an equilibrium with information acquisition exists is:

$$c=(N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}.$$

On the other hand, the threshold found in (35) for information acquisition to take place at an equilibrium when information is sold by an interested trader is

$$\frac{1}{k}\left(\frac{k-1}{k}\right)+(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$

Hence to prove the result it suffices to show that

$$\begin{aligned}\frac{1}{k}\left(\frac{k-1}{k}\right)+(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &> (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\\ &\Longleftrightarrow \frac{1}{k}\left(\frac{k-1}{k}\right)>\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\end{aligned}$$

which is always true.

This completes the the proof of Proposition 2

Proof of proposition 3 The payoffs for the seller of the good when he acquires information depend both on information and auction revenues. In terms of the revenues from information, the maximum willingness to pay for the buyers is their surplus minus their outside option (which is $\max\{\pi_C,\pi_U\}$).

Let J be the number of uninformed buyers. When $J>0$, the surplus for buyers is positive only when they obtain the good and no other informed buyer likes it. In that case the surplus is $1-1/k$ leading to an expression for the surplus:

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1-J}\left(1-\frac{1}{k}\right)=\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}$$

The outside option for $J \geq 1$, we have

$$\pi_C = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c < 0 = \pi_U.$$

Thus the price of information in this case is:

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}.$$

And therefore the revenue from information is

$$(N-J)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}. \tag{38}$$

Notice that we showed in equation (24) that this is a decreasing function of J, thus revenues from information are decreasing in J, for $J > 0$.

When $J = 0$, the surplus for buyers is positive when they obtain the good and no other buyer likes it. In that case the surplus is 1 leading to an expression for the surplus:

$$\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}.$$

The outside option for $J = 0$,

$$\pi_C = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c < \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} = \pi_U.$$

So the price of information will be zero.

The revenues at the auction when $J \geq 2$ can only be $1/k$ (if no informed buyer likes the object) or 1 (if more than one informed buyer likes the object). Thus, when $J \geq 2$ the revenues decrease with J, as the probability that more than one informed buyer likes the object is decreasing in J. Putting this together with the fact that the revenue from information is decreasing with J for $J > 0$, as we showed earlier from equations (38) and (24) tells us that the best possible option for the seller from the set $J \geq 2$ is exactly $J = 2$.

We are thus left to compare the total payoffs for the seller from $J = 0$, $J = 1$ and $J = 2$.

The payoffs when $J = 0$ are simply the revenues from the auction (which in this case are 0 if no buyer, or exactly one, likes the object, and 1 otherwise):

$$1 - \left(\frac{k-1}{k}\right)^{N} - N\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$

The payoffs when $J = 1$ are the payoffs from information sale (from equation 38) plus the revenues from the auction (which in this case are 0 if no informed buyer likes the object,

$1/k$ if exactly one informed buyer likes the object, and 1 otherwise):

$$\begin{aligned} & (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} + \left(1-\left(\frac{k-1}{k}\right)^{N-1} - (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\right) + \frac{1}{k}\Bigg((N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N} \\ = \; & (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} + 1 - \left(\frac{k-1}{k}\right)^{N-1} - (N-1)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \\ = \; & 1-\left(\frac{k-1}{k}\right)^{N-1} \end{aligned}$$

The payoffs when $J=2$ are the payoffs from information sale (from equation 38) plus the revenues from the auction (which in this case are $1/k$ if no informed buyer or exactly one likes the object, and 1 otherwise):

$$\begin{aligned} & (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2} + \left(1-\left(\frac{k-1}{k}\right)^{N-2} - (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-3}\right) + \frac{1}{k}\Bigg(\left(\frac{k-1}{k}\right)^{N-2} + (N - \\ = \; & (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2} + 1 - \left(\frac{k-1}{k}\right)^{N-1} - (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2} \\ = \; & 1-\left(\frac{k-1}{k}\right)^{N-1} \end{aligned}$$

Thus, the payoffs for the seller are maximized when $J=1$ or $J=2$, if

$$1-\left(\frac{k-1}{k}\right)^{N-1} > 1-\left(\frac{k-1}{k}\right)^{N} - N\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$

which is always true since this is equivalent to

$$N\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} > \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$

The result now follows by comparing the threshold for efficient information acquisition, found in (2), with the threshold for information acquisition not to take place in equilibrium. Thus, the threshold for information acquisition in equilibrium is when

$$1-\left(\frac{k-1}{k}\right)^{N-1} - c \geq \frac{1}{k}$$

or

$$\left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-2}\right) \geq c$$

Remember that the threshold for efficient information acquisition is:

$$1-\left(\frac{k-1}{k}\right)^{N} - c \geq \frac{1}{k}$$

or

$$\left(\frac{k-1}{k}\right)\left(1-\left(\frac{k-1}{k}\right)^{N-1}\right) \geq c$$

and the result follows.

This completes the the proof of Proposition 3

Proof of lemma 1 Denote by P the price paid in the auction and by $p_{B_i}^{Inf}$ the price payed by B_i for the information. The payoffs for the buyers of information when there are J uninformed individuals are:

$$\begin{aligned}
\pi_{B_i}^J &= \Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i) - E_i(P|B_i^{Win})\Pr(B_i^{Win}) - p_{B_i}^{Inf} \\
&= \max\{\pi_C^J, \pi_U^J\} \text{ for } i = 1,..,N-J
\end{aligned}$$

where $\Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i) = \Pr(B_i^{Win})$. Note that this is true only when $J > 0$, for $i < N$. Thus, the price of information is

$$p_{B_i}^{Inf} = \Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i) - E_i(P|B_i^{Win})\Pr(B_i^{Win}) - \max\{\pi_C^J, \pi_U^J\}$$

and i

$$\begin{aligned}
\pi_S^J &= E_S(P) \\
&\quad + \sum_{i=1}^{N-J}\left(E_S\left(\Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i)\right) - E_S\left(E_i(P|B_i^{Win})\Pr(B_i^{Win})\right)\right) - (N-J)\max\{\pi_C^J(S), \pi_U^J \\
&= \sum_{i=1}^{N-J} E_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) + \sum_{i=N-J+1}^{N} E_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) \\
&\quad + \sum_{i=1}^{N-J}\left(E_S\left(\Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i)\right) - E_S\left(E_i(P|B_i^{Win})\Pr(B_i^{Win})\right)\right) - (N-J)\max\{\pi_C^J(S), \pi_U^J \\
&= \sum_{i=N-J+1}^{N} E_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) \\
&\quad + \sum_{i=1}^{N-J} E_S\left(\Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i)\right) - (N-J)\max\{\pi_C(S), \pi_U(S)\} - c
\end{aligned}$$

This completes the the proof of Lemma 1

Proof of Proposition 6 Suppose first $V > \frac{1}{2}$. If there were a pair of messages m^H, and m^L which were sent with positive probability and $\Pr(v \in S^H|m^H) > \Pr(v \in S^H|m^L)$, then the owner of the good would induce a higher bid by from Se buyers by announcing m^H, even if the true state is such that $v \in S^L$. The reason is that the bid of In buyers does not change, as they do not care about the quality. In addition the price paid for the information is sunk at the moment when the message is sent, so there is no change in revenues from information sales by changing the announcement from m^H to m^L Thus, the owner of the good would increase profits by announcing m^H, and thus m^L would not be sent in equilibrium, which is a contradiction. If for a given type $i \in S$, only $m^H = (i, H)$ is sent in equilibrium (and not $m^L = (i, L)$), then necessarily $\Pr(v \in S^H | m^H) = \Pr(v \in S^H) = \frac{1}{2}$. Given these beliefs, it is immediate that an Se individual who likes variety i will bid more for the good for any m as their expected valuation is $\frac{V}{2}\Pr(v \in i|m)$ whereas the expected value for an In individual is $\Pr(v \in i|m)$, and since $V > \frac{1}{2}$, then $\frac{V}{2}\Pr(v \in i|m) > \Pr(v \in i|m)$. The proof for $V < \frac{1}{2}$ is similar.

This completes the the proof of Proposition 6

Proof of Proposition 4 Most of this proof (in Appendix B) involves routine computations similar in nature to those of Proposition 1. We simply report here the proof of a crucial lemma that is clearly different from the previous material.

LEMMA **2** *The optimal distribution is to create as many layers as remaining players.*

Proof of lemma 2 To see this notice

1. The price paid at the auction does not change by increasing the number of layers.

2. If an old layer l is split in two l' and l'', then the willingness to pay of individuals in old layers $l + k$ does not change as they only care about the number of people in their layer or above, not their distribution.

3. If an old layer l is split in two l' and l'', then the willingness to pay of individuals in old layers $l - k$ does not change as they only care about the number of people in their layer or above, and this has not changed.

4. If an old layer l is split in two l' and l'', then the willingness to pay of individuals in old layer l who is now in the new lower layer l'' does not change as they only care about the number of people in their layer or above, and this has not changed.

5. If an old layer l is split in two l' and l'', then the willingness to pay of individuals in old layer l who is now in the new upper layer l' strictly increases as they only care about the number of people in their layer or above, and this is now strictly lower.

This completes the proof of Lemma 2 and Proposition 4

Proof of proposition 5 To show this, we only need to establish that profits from acquiring information are always strictly positive. This is because we showed in the previous section that for low c the monopolist drives surplus to zero for indirectly informed players. More precisely, by equation (44), the condition for there being no unconnected players, and thus $\pi_{B_i} = 0$ for an indirectly connected player, is that c is low. Thus, as long as profits from buying information are positive, there will be incentives to purchase the information.

Suppose first that in the subgame with two informed sellers there is one (call him B_{i_1}, and call the other seller B_{i_2}) not offering the complete hierarchy. There are two cases to consider:

A If B_{i_1} is not extracting full rents from the indirectly informed players (that is if there is a B_i purchasing information from B_{i_1} such that $\pi_{B_i} > \max\{\pi_u, \pi_c\}$), then B_{i_2} can just replicate the signaling structure of $B_{i_1}$and get non zero profit by charging a positive price to B_i (and zero, or even some negative $-\varepsilon$ to all other buyers of information, to be sure they accept the signal). Such B_i would lose his position in the hierarchy when not buying that extra information from B_{i_2}. This is so because, if B_i does not buy, there is always at least one individual who used to get the distorted information from B_{i_1} and who now knows the truth when B_i likes the good. Thus B_i will get only $\max\{\pi_u, \pi_c\}$ as payoff.

B If B_{i_1} is extracting full rents (conditional on $L_{i_1} < N-1$), then if B_{i_2} offers the complete hierarchy, $L_{i_2} = N-1$, and prices which leave $\pi_{B_i} > \max\{\pi_u, \pi_c\}$ to all the indirectly informed, they will all buy only from B_{i_2} and stop buying the signal from B_{i_1}, and thus give him strictly positive profits.

The only alternative is that in the subgame with two informed sellers both B_{i_1} and B_{i_2}offer the full hierarchy, so that $L_{i_1} = L_{i_2} = N-1$. Here we also have two cases:

C If B_{i_1} is extracting full rents, then if B_{i_2} offers the complete hierarchy, $L_{i_2} = N-1$, and prices which leave $\pi_{B_i} > \max\{\pi_u, \pi_c\}$ to all the indirectly informed, they will all buy only from B_{i_2} and stop buying the signal from B_{i_1}, and thus give him strictly positive profits.

D If B_{i_1} is not extracting full rents, that is if there is a B_i purchasing information from B_{i_1} such that $\pi_{B_i} > \max\{\pi_u, \pi_c\}$), then B_{i_2} can get positive surplus by replicating the hierarchy and asking $p_{l_{i_2}} = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N_l} - \max\{\pi_u, \pi_c\} - p_{l_{i_1}}$, that is, the monopoly prices we computed in the last section minus what B_{i_1}asks. The indirectly informed buyers B_i have to buy the signal, or risk their own position in the hierarchy, as we argued in case A above.

This completes the proof of Proposition 5

APPENDIX B

Proof of Proposition 4 With the message structure, there are again no out-of-equilibrium messages. Thus, we can find the beliefs of the uninformed buyers using Bayes' rule in all cases. Let n_l be the number of buyers in layer l and N_l the total number of buyers in layers 1 through l plus the seller of information $B_1 : N_l = \sum_{j=0}^{l} n_j$, where we adopt the convention $n_0 = 1$.

The beliefs of buyer B_j purchasing a report of type l are:

1. When B_j receives a message $m_l = \theta_j$ he knows for sure he likes the object. That is[10]

$$\Pr(v = \theta_j | m_l = \theta_j) = 1$$

2. On the other hand, when B_j receives a message $m_l \neq \theta_j$, this may happen either because $v = m_l$, or because the sender or somebody in an earlier layer $t < l$ likes the object, in which case the sender does not tell the truth and randomizes over the types which are absent from the population of buyers of information, including B_1. Given this:

$$\Pr(m_l = \theta_j) = \left(\frac{k-1}{k}\right)^{N_{l-1}} \frac{1}{k}, \quad \Pr(m_l \neq \theta_j) = 1 - \left(\frac{k-1}{k}\right)^{N_{l-1}} \frac{1}{k}$$

and

$$\begin{aligned}\Pr(v &= \theta_j | m_l \neq \theta_j) = \frac{\Pr(v = \theta_j \cap m_l \neq \theta_j)}{\Pr(m_l \neq \theta_j)} = \frac{\Pr(v = \theta_j) - \Pr(v = \theta_j \cap m_l = \theta_j)}{\Pr(m_l \neq \theta_i)} \\ &= \frac{\frac{1}{k} - \left(\frac{k-1}{k}\right)^{N_{l-1}} \frac{1}{k}}{1 - \left(\frac{k-1}{k}\right)^{N_{l-1}} \frac{1}{k}} = \frac{1 - \left(\frac{k-1}{k}\right)^{N_{l-1}}}{k - \left(\frac{k-1}{k}\right)^{N_{l-1}}}\end{aligned}$$

The beliefs of buyers who do not purchase information nor acquire it directly are then unchanged:

$$\Pr(v = \theta_j) = \frac{1}{k}, \quad \Pr(v \neq \theta_j) = \frac{k-1}{k}$$

Behavior in the auction We begin again by the last stage of the subgame, by characterizing the agents' behavior in the auction (still under the restriction to "truthful bidding strategies").

[10] Even though a buyer receiving message m_l also receives messages $m_j, j > l$, given the nested structure of the information these reports have no additional informational value and can be ignored in the derivation of the conditional expectations.

1. When an agent B_j receives a message $m_l = \theta_j$ it is clear that the optimal bid is equal to his posterior beliefs about the valuation of the object. That is his bid is equal to 1

2. When an agent B_j receives a message $m_l \neq \theta_j$, the optimal bid is equal to $\Pr(v = \theta_j | m_i \neq \theta_j)$. An agent may receive this signal for two reasons. Either $v = \theta_i$ for some B_i in an earlier layer, in which case he cannot win the object in the auction as agent B_i, who is better informed will make a higher bid, or $v \neq \theta_i$ for all B_i in earlier layers and $v \neq \theta_j$, in which case he may indeed win the auction by bidding a positive price. But that will entail a negative surplus. This implies that the optimal bid is zero when $m_l \neq \theta_j$, once the information conveyed by winning the auction is taken into account (affiliation).

3. Finally, the buyers who do not listen to the reports of the directly informed seller do not suffer from an affiliation problem. Thus, the optimal bid in that case is: $\Pr(v = \theta_j) = \frac{1}{k}$.

Behavior in the message game Next we consider agents' behavior in the message game. Given the reporting strategy described in equations (10) and (11) of the seller of information, we now show that the optimal reporting strategy of every buyer who is purchasing the information is to truthfully report his type. This can be shown in an almost identical way as in the case of homogeneous quality of information. Next we show that the reporting strategy of the seller of information is also optimal for this agent.

Optimality of truthful reporting for the buyers of information First of all, notice that a change in the message strategy of a buyer of information can only change the outcome of the auction, not the price paid for information. A deviation by the buyer of information consists in reporting anything other than his type. We divide this discussion in two cases.

1. Let the buyer of information be agent B_j, and suppose he purchased report l. If $v = \theta_1$ or $v = \theta_i$, for some buyer of information B_i in some layer $t < l$, either B_1 or B_i will bid 1 in the auction, no matter what is the report of B_j. Hence there is no possibility for B_j to obtain any extra surplus by misreporting his type.

2. If $v \neq \theta_1$ and $v \neq \theta_i$ for all buyers of information B_i in any layer $t < l$, then (11) prescribes that $m_t = v$ for all $t = 1, .., l$, no matter what is the report sent by agents who purchase report l. The reports of agents in layer l only affect the reports sent by B_1 to agents in layers $t' > l$. Under truthful reporting by B_j all buyers in layers

$t > l$ receive a message inducing them to bid zero when $v = \theta_j$. The effect of B_j's misreporting his type is that buyers in layers $t > l$ will receive a message which will induce them either to bid zero, or a positive amount, thus lowering, at least weakly B_j expected gains from the auction. So misreporting is not optimal for B_j.

Optimality of the message for the seller of information As for the buyers of information, a change in the seller's reporting strategy has no effect on the revenue from the sale of information, only on the outcome of the auction.

1. When $v = \theta_1$, i.e. the seller of information likes the object, he can deviate and send, for some $l \in \{1, .., L\}$ a message $m_l = \theta_k$ for some buyer B_k purchasing a report of some type. In this case, the bid of B_k will equal 1, and the seller of information has to pay more for the object than if he had followed the reporting strategy (10) and (11), so the deviation is clearly not optimal.

2. When $v \neq \theta_1$, the seller is not interested in the object. Since any deviation from the messages prescrobed by his reporting (10) and (11) only changes the outcome in the auction, in which he is not interested, the seller can never gain from such deviation.

Sale of information

Payoffs for the monopolist with no individual without information The single informed trader acts as a monopolist in the market for information. The maximal rent he can extract from the $N - 1$ uninformed buyers purchasing information from him, for any given layer structure, is determined by comparing the payoff a buyer can get by acquiring the information of the quality associated to the layer he is in with the alternative payoff he could get by not purchasing the information.

To evaluate the payoff of a buyer in layer l we need to distinguish the case (i) where in layer l there is a single buyer from the case (ii) where in that layer there is more than a single buyer. In case (i) the buyer in layer l will always get the commodity when he likes it and no other buyer in the layers above likes it (an event with probability $\left(\frac{k-1}{k}\right)^{N_l - 1} \frac{1}{k}$), and the price he will pay will be the second highest bid after his, the bid made by the bidders with less information[11], i.e. in the lowest layer, given by 0. On the other hand in case (ii) the same is true when the buyer likes the object and no other buyer in the layers above *as well as in his own layer* l likes it (an event with probability $\left(\frac{k-1}{k}\right)^{N_l - 1} \frac{1}{k}$). When some other

[11]Note that in this case the buyer will be the only one receiving a message equal to his own type.

buyer in layer l likes the object, the buyer will get the object with some probability but will pay an amount equal to his valuation so that his payoff will be zero. In case (i) we have $N_l = N_{l-1} + 1$; hence in both cases the payoff in the auction for a buyer in layer l is given by the following expression:

$$\left(\frac{k-1}{k}\right)^{N_l - 1} \frac{1}{k}$$

If we add to this the price paid to acquire the information, we obtain the expression of the total payoff to a buyer B_i of acquiring information in layer l of a chain of length L (at a price p_l), when all the N players are connected to a unique buyer, is

$$\pi_{B_i} = \left(\frac{k-1}{k}\right)^{N_l - 1} \frac{1}{k} - p_l.$$

The buyer's alternative payoff if he chooses not to buy any information and hence remain unconnected, is given by $\pi_u = \left(\frac{k-1}{k}\right)^{N-1} \frac{1}{k}$. Alternatively, the buyer could also choose to acquire directly the information, in the third and last stage, as a cost c, in which case his payoff would be $\pi_c = \left(\frac{k-1}{k}\right)^{N-1} \frac{1}{k} - c$. Note that both π_u and π_c are independent of the number L of layers. Notice that $\max\{\pi_u, \pi_c\} = \pi_u = \left(\frac{k-1}{k}\right)^{N-1} \frac{1}{k}$

The maximal rent the monopolist selling the information can extract from buyer B_i, and hence the maximal value of the price he can charge, is thus given by

$$p_l = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N_l - 1} - \max\{\pi_u, \pi_c\} = \frac{1}{k}\left(\left(\frac{k-1}{k}\right)^{N_l - 1} - \left(\frac{k-1}{k}\right)^{N-1}\right) \qquad (39)$$

The total payoff of B_1, the informed buyer, from acquiring the commodity when $v = \theta_1$ and from selling the information to the other buyers, is thus given by:

$$\pi_{B_1} = \frac{1}{k} + \sum_{l=1}^{L} n_l p_l - c$$

where p_l is as in expression (39). Note that the second term depends on the distribution of buyers across layers, i.e. on all the values $n_l, l = 0, 1, .., L$.

To obtain the optimal distribution of buyers across the layers we need then consider only its effects on the revenue from the sale of information.

$$\begin{aligned} \pi_{B_1} &= \frac{1}{k} + \sum_{l=1}^{L} n_l \frac{1}{k}\left(\left(\frac{k-1}{k}\right)^{N_l - 1} - \left(\frac{k-1}{k}\right)^{N-1}\right) - c = \\ &= \frac{1}{k}\left(1 + \sum_{l=1}^{L} n_l \left(\left(\frac{k-1}{k}\right)^{N_l - 1} - \left(\frac{k-1}{k}\right)^{N-1}\right)\right) - c \qquad (40) \end{aligned}$$

Lemma 3 *The optimal distribution is to create as many layers as remaining players.*

Proof 1 *Same as Lemma 2*

We denote then the payoff of the first buyer $\pi_{B_1}(2)$ to allude to this possible equilibrium configuration. Notice that we are thus assuming $n_l = 1$ for all l. So from (40) we have

$$\begin{aligned}\pi_{B_1}(2) &= \frac{1}{k}\left(1+\sum_{l=1}^{L}\left(\left(\frac{k-1}{k}\right)^{N_l-1}-\left(\frac{k-1}{k}\right)^{N-1}\right)\right)-c\\ &= 1-\left(\frac{k-1}{k}\right)^{N-1}-(N-2)\,\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}-c\end{aligned}$$

Payoffs for the monopolist with one individual without information In this case, again, the maximal rent the monopolist can extract from the $N-2$ buyers purchasing information from him, for any given layer structure, is determined by comparing the payoff a buyer can get by acquiring the information of the quality associated to the layer he is in with the alternative payoff he could get by not purchasing the information.

To evaluate then the payoff of a buyer in layer l we need to distinguish the case (i) where in layer l there is a single buyer from the case (ii) where in that layer there is more than a single buyer. In case (i) the buyer in layer l will always get the commodity when he likes it and no other buyer in the layers above likes it (an event, as we said, with probability $\left(\frac{k-1}{k}\right)^{N_l-1}\frac{1}{k}$), and the price he will pay will be the second highest bid after his, the bid made by the bidders with less information, i.e. the uninformed buyer, given by $\frac{1}{k}$. On the other hand in case (ii) the same is true when the buyer likes the object and no other buyer in the layers above *as well as in his own layer* l likes it (an event with probability $\left(\frac{k-1}{k}\right)^{N_l-1}\frac{1}{k}$). When some other buyer in layer l likes the object the buyer will get the object with some probability but will pay am amount equal to his valuation so that his payoff will be zero. In case (i) we have $N_l = N_{l-1}+1$; hence in both cases the payoff in the auction for a buyer in layer l is given by the following expression:

$$\left(\frac{k-1}{k}\right)^{N_l-1}\frac{1}{k}\left(1-\frac{1}{k}\right)=\left(\frac{k-1}{k}\right)^{N_l}\frac{1}{k}$$

If we add to this the price paid to acquire the information, we obtain the expression of the total payoff to a buyer B_i of acquiring information in layer l of a chain of length L (at a price p_l), when all the N players are connected to a unique buyer, is

$$\pi_{B_i}=\left(\frac{k-1}{k}\right)^{N_l}\frac{1}{k}-p_l.$$

The buyer's alternative payoff if he chooses not to buy any information and hence remain unconnected, is given by $\pi_u = 0$. Alternatively, the buyer could also choose to acquire directly the information, in the third and last stage, as a cost c, in which case his payoff would be $\pi_c = \left(\frac{k-1}{k}\right)^{N-2} \frac{1}{k}\left(1 - \frac{1}{k}\right) - c = \left(\frac{k-1}{k}\right)^{N-1} \frac{1}{k} - c$. Note that both π_u and π_c are independent of the number L of layers and that $\max\{\pi_u, \pi_c\} = \max\left\{\left(\frac{k-1}{k}\right)^{N-1} \frac{1}{k} - c, 0\right\}$

The maximal rent the monopolist selling the information can extract from buyer B_i, and hence the maximal value of the price he can charge, is thus given by

$$\begin{aligned} p_l &= \frac{1}{k}\left(\frac{k-1}{k}\right)^{N_l} - \max\{\pi_u, \pi_c\} && (41) \\ &= \min \frac{1}{k}\left(\left(\frac{k-1}{k}\right)^{N_l}, \left(\frac{k-1}{k}\right)^{N_l} - \left(\frac{k-1}{k}\right)^{N-1} + c\right) && (42) \end{aligned}$$

The total payoff of B_1, the informed buyer, from acquiring the commodity when $v = \theta_1$ and from selling the information to the other buyers, is thus given by:

$$\pi_{B_1}(1) = \frac{1}{k}\left(1 - \frac{1}{k}\right) + \sum_{l=1}^{L} n_l p_l - c,$$

where p_l is as in expression (41). Note that the second term also depends on the distribution of buyers across layers, i.e. on all the values $n_l, l = 0, 1, .., L$.

To obtain the optimal distribution of buyers across the layers we need then consider only its effects on the revenue from the sale of information.

$$\begin{aligned} \pi_{B_1}(1) &= \frac{1}{k}\left(1 - \frac{1}{k}\right) + \sum_{l=1}^{L} n_l \min \frac{1}{k}\left(\left(\frac{k-1}{k}\right)^{N_l}, \left(\frac{k-1}{k}\right)^{N_l} - \left(\frac{k-1}{k}\right)^{N-1} + c\right) - c = \\ &= \frac{1}{k}\left(1 - \frac{1}{k} + \sum_{l=1}^{L} n_l \min\left(\left(\frac{k-1}{k}\right)^{N_l}, \left(\frac{k-1}{k}\right)^{N_l} - \left(\frac{k-1}{k}\right)^{N-1} + c\right)\right) - c \end{aligned}$$

Lemma **4** *The optimal distribution is to create as many layers as remaining players.*

Proof 2 *Same as Lemma 2*

When does the price change from $\left(\frac{k-1}{k}\right)^{N_l}$ **to** $\left(\frac{k-1}{k}\right)^{N_l} - \left(\frac{k-1}{k}\right)^{N-1} + c$ $\quad \left(\frac{k-1}{k}\right)^{N_l-1} \geq \left(\frac{k-1}{k}\right)^{N_l-1} - \left(\frac{k-1}{k}\right)^{N-1} + c$ if and only if

$$\left(\frac{k-1}{k}\right)^{N-1} \geq c$$

When does the monopolist prefer to have no unconnected players?

1. Assume first that
$$\left(\frac{k-1}{k}\right)^{N-1} \geq c$$
Then the payoff under 1 is
$$\pi_{B_1}(1) = \frac{1}{k}\left(1 - \frac{1}{k} + \sum_{l=1}^{L} n_l \left(\left(\frac{k-1}{k}\right)^{N_l} - \left(\frac{k-1}{k}\right)^{N-1} + c\right)\right) - c$$
$$= \left(\frac{k-1}{k}\right) - \left(\frac{k-1}{k}\right)^{N} - (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} + (N-2)\frac{c}{k} - c$$
We also know from above that
$$\pi_{B_1}(2) = 1 - \left(\frac{k-1}{k}\right)^{N-1} - (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c$$
So $\pi_{B_1}(2) \geq \pi_{B_1}(1)$ iff
$$1 - \left(\frac{k-1}{k}\right)^{N-1} \geq \left(\frac{k-1}{k}\right) - \left(\frac{k-1}{k}\right)^{N} + (N-2)\frac{c}{k}$$
So in fact the monopolist prefers to have no unconnected players provided c is low enough so that
$$\frac{1}{k}\left(1 - \left(\frac{k-1}{k}\right)^{N-1}\right) \geq (N-2)\frac{c}{k}$$
$$1 - \left(\frac{k-1}{k}\right)^{N-1} \geq (N-2)c$$

2. For
$$\left(\frac{k-1}{k}\right)^{N-1} < c$$
We are in the regime where
$$\pi_{B_1}(1) = \frac{1}{k}\left(1 - \frac{1}{k} + \sum_{l=1}^{N-2}\left(\frac{k-1}{k}\right)^{l+1}\right) - c$$
$$= \left(\frac{k-1}{k}\right)\left(\frac{1}{k} + \left(\frac{k-1}{k}\right) - \left(\frac{k-1}{k}\right)^{N-1}\right) - c$$
Thus
$$\pi_{B_1}(1) = \left(\frac{k-1}{k}\right)\left(1 - \left(\frac{k-1}{k}\right)^{N-1}\right) - c \tag{43}$$

and since

$$\pi_{B_1}(2) = 1 - \left(\frac{k-1}{k}\right)^{N-1} - (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c$$

So $\pi_{B_1}(1) \geq \pi_{B_1}(2)$ iff

$$\left(\frac{k-1}{k}\right)\left(1 - \left(\frac{k-1}{k}\right)^{N-1}\right) \geq 1 - \left(\frac{k-1}{k}\right)^{N-1} - (N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$

$$(N-1) \geq \left(\frac{k}{k-1}\right)^{N-1} = \left(1 + \frac{1}{k-1}\right)^{N-1}$$

but notice that

$$\begin{aligned}\left(1 + \frac{1}{k-1}\right)^{N-1} &= \sum_{t=0}^{N-1}\binom{N-1}{t}\frac{1}{(k-1)^t} \\ &= 1 + \sum_{t=1}^{N-1}\frac{(N-1)(N-2)...(N-t)}{(1 \cdot 2 \cdot ... \cdot t)\,(k-1)^t} \\ &\leq 1 + \frac{(N-1)}{(k-1)} + \sum_{t=2}^{N-1}\frac{1}{2}\frac{(N-1)^t}{(k-1)^t} \\ &\leq 2 + \frac{(N-2)}{2} = \frac{(N+2)}{2}\end{aligned}$$

but since

$$N \geq 4 \Longleftrightarrow (N-1) \geq \frac{(N+2)}{2}, \text{and } \frac{(N+2)}{2} \geq \left(\frac{k}{k-1}\right)^{N-1}$$

then $\pi_{B_1}(1) \geq \pi_{B_1}(2)$. So in this regime, 1 always dominates 2.

The previous considerations show that both 1 and 2 can be preferred for different values of c, and overall the condition that determines when 2 is preferred by the monopolist is:

$$1 - \left(\frac{k-1}{k}\right)^{N-1} \geq (N-2)c \tag{44}$$

This completes the proof of Proposition 4

The case $N \geq k$ In this case the message policy cannot be like the one mentioned in the text, so we need to modify it.

Signaling of sellers of information As before suppose there are J informed traders, denote by B_1 through B_J.

We also assume that the set of available messages is the set of types, S, plus an additional message $m_{0,}$ which we call the *blank* message. Let $\mathcal{N}(B_i)$ the set of buyers who buy from B_i, and $N(B_i)$ be the number of different realizations of θ_i across all buyers $i \in \mathcal{N}(B_i)$.

$$m_i = \begin{cases} v, \text{ if } v \neq \theta_i \\ y, \text{ with probability } \frac{1}{k-N(B_i)}, \\ \text{for all } y \neq \theta_j, \ \forall j \in \mathcal{N}(B_i), \text{ if } v = \theta_1 \text{ and } k > N(B_i) \\ m_0 \text{ if } v = \theta_i \text{ and } k \leq N(B_i) \end{cases} \tag{45}$$

Therefore, the informed trader tells the truth about the quality of the object as long as the true quality of the object does not coincide with his own type. Otherwise, the informed trader randomizes over any value different from the type of any of the buyers when $k > N(B_i)$ and he uses the blank message m_0 if $v = \theta_i$ and $k \leq N(B_i)$

With this message structure, there are no out-of-equilibrium messages. Thus, beliefs for an agent informed through a single directly informed buyer are determined using Bayes' rule in all cases.

Behavior in the auction

1. When an agent j receives a message $m_i = \theta_j$ it is clear that the optimal bid is equal to his posterior beliefs about the valuation of the object. That is his bid is equal to 1

2. When an agent j receives a message $m_i \neq \theta_j$, the optimal bid is not necessarily always equal to $\Pr(v = \theta_j | m_i \neq \theta_j)$. An agent may receive this signal for two reasons. Either $v = \theta_i$ and $v = \theta_j$, in which case she cannot win the object in the auction as agent i, who is better informed will make a higher bid, or $v \neq \theta_i$ and $v \neq \theta_j$, in which case she may indeed win the auction by bidding a positive price. But that will entail a negative surplus. This implies that the optimal bid is zero when $m_i \neq \theta_j$, once the information conveyed by winning the auction is taken into account (affiliation). Notice that this reasoning is independent on how many signals this individuals gets.

3. Finally, the buyers who do not listen to the reports of the directly informed seller, or the ones who listen and receive the message m_0 do not suffer from an affiliation problem. Thus, the optimal bid in ther case is: $\Pr(v = \theta_j) = \frac{1}{k}$.

Behavior in the first three stages of the game We will focus only in those cases where there are any changes with respect to the discussion in the main text.

Stage 2: information sale

Optimality of the message for the seller of information First of all, notice that by changing the message policy he cannot change the price paid for information, as that is based on expectations of the message policy. So any change in messages can only lead to changes in the outcome at the auction.

1. Let the seller of information be agent B_j. When $v = \theta_j$, he can deviate by announcing $m_j = \theta_k$ for some B_k. In this case, the bid of agent B_k will be equal to 1, and the seller has to pay more for the object than with the equilibrium message, which is not optimal. He can also deviate by announcing m_0 when $k > N(B_i)$. But that will change the bid of the buyers of information from 0 to $1/k$, which is not optimal.

2. Let the seller of information be agent B_j. When $v \neq \theta_j$, agent B_j can deviate by announcing $m_j \neq v$. But that only changes the outcome in the auction, in which he is not interested as the object produces no utility to him. So this is not optimal.

Optimality of reporting for the buyer of information Same as before.

Optimal prices and payoffs for (fixed) numbers of direct and indirect buyers of information We will discuss only those cases where there is any change in payoffs. For the rest we merely report again the payoffs. To understand why there are so few changes, notice that the only important change is the fact that when m_0 is reported the bid paid by B_1 the informant (and winner of the auction) is $\frac{1}{k}$, not zero as before. Thus, only that payoff can change. And it will only change in those cases where he was not already paying $\frac{1}{k}$.

1. We first check the case with one seller of information (wlog let it be B_1), $N-2$ (let it be B_2 to B_{N-1}) buyers of information from the single seller, and one agent not buying information at all (let it be B_N). The payoffs for the different players are:

$$\pi_{B_1}(1) = \frac{1}{k}\left(1 - \frac{1}{k}\right) + (N-2)p(1) - c$$

$$\pi_{B_i}(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\left(1 - \frac{1}{k}\right) - p(1) = \max\{\pi_C(1), \pi_U(1)\}$$

$$\pi_{B_N}(1) = \left(\frac{k-1}{k}\right)^{N-1}\frac{1}{k}$$

$$\pi_C(1) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-2}\left(1 - \frac{1}{k}\right) - c = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c$$

$$\pi_U(1) = 0$$

2. We now check the case with one seller of information (wlog let it be B_1), and $N-1$ (let it be B_2 to B_N) buyers of information from the single seller. The payoffs are now.

$$\pi_{B_1}(2) = \frac{1}{k}\left(1 - \frac{1}{k}\Pr(k \leq N(B_i)\right) + (N-1)p(2) - c$$
$$\pi_{B_i}(2) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - p(2) = \max\{\pi_C(2), \pi_U(2)\} = \pi_U(2)$$
$$\pi_C(2) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} - c$$
$$\pi_U(2) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} \Longrightarrow \pi_U(2) > \pi_C(2) \Longrightarrow p(2) = 0$$

3. We now check the case with two sellers of information (wlog let them be B_1, B_2), and $N-2$ (let them be B_3 to B_N) buyers of information from the sellers.

   In this case, and for the same reasons as before: (a) each unlinked buyer purchases the signal from both directly linked buyers and (b) under this configuration, the price is zero for both sellers of information.

   Given these observations, the payoffs are then:

$$\pi_{B_i}(3) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$
$$\pi_{B_1}(3) = \pi_{B_2}(3) = \pi_{B_i}(3) - c$$

4. We now check the case when no player obtains the information from the seller of the object. In this case, the payoffs are:

$$\pi_{B_i}(4) = 0$$

   In this case al buyers are uninformed, and they all bid their expected valuation $\frac{1}{k}$, so they pay the expected value for the good and get no surplus.

5. We now check the case with one seller of information (wlog let it be B_1), $J \geq 2$ agents not buying information at all (let them be $B_{N-J+1}, ..., B_N$) and $N-J-1$ (let them be B_2 to B_{N-J}) buyers of information from the single seller. The payoffs for the different players are:

$$\pi_{B_N}(5) = ... = \pi_{B_{N-J+1}}(5) = 0$$
$$\pi_{B_1}(5) = \frac{1}{k}\left(1 - \frac{1}{k}\right) + (N - (J+1))p(5) - c$$
$$\pi_{B_i}(5) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}\left(1 - \frac{1}{k}\right) - p(5) = \max\{\pi_C(5), \pi_U(5)\}$$
$$\pi_C(5) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-(J+1)}\left(1 - \frac{1}{k}\right) - c = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J} - c$$
$$\pi_U(5) = 0$$

6. We now check the case with J sellers of information (wlog let them be $B_1, ..., B_J$), and $N - J$ (let them be B_{J+1} to B_N) buyers of information from the sellers. In this case, and for the same reasons as in (3): (a) each unlinked buyer purchases the signal from both directly linked buyers and (b) under this configuration, the price is zero for both sellers of information. The payoffs in this case are:
$$\pi_{B_i}(3) = \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$$
$$\pi_{B_1}(3) = ... = \pi_{B_J}(3) = \pi_{B_i}(3) - c$$

Equilibrium conditions for each configuration of direct and indirect buyers of information .We now check for each possible configuration of the information sale, which of them can be an equilibrium. For each configuration, the deviations that need to be checked are: (a) whether the sellers wish to sell to the number of buyers in that configuration (b) whether the buyers prefer to buy information or not to do it/ or become directly informed in the third stage and (c) whether the uninformed prefer to stay uninformed.

Notice that the prices where chosen to take care of (b), so we only need to check (a) and (c).

1. The equilibrium conditions are:
$$\pi_{B_1}(1) \geq \max\{\pi_{B_1}(2), \pi_{B_1}(5)\} \tag{46}$$
$$\pi_{B_N}(1) \geq \max\{\pi_{B_N}(1) - p(1)\}^* \tag{47}$$

Equilibrium condition (46) guarantees that the seller of information does not want to raise the price to discourage some buyers of information (thus going to configuration 5) or lower it (thus going to configuration 2). Equilibrium condition (47) guarantees that the uninformed individual prefers to stay uninformed rather than buy information. This latter condition (47) is actually trivially satisfied. The uninformed individual, by purchasing the information will get the good for a positive surplus in the same circumstances as by obtaining the information (when he likes it and nobody else does) and he would have to pay for that information.

We now check condition (46).

(a) When there is a choice between equilibrium 1 and 2, the monopolist chooses 1 if

$$\begin{aligned}\pi_{B_1}(1) &= \frac{1}{k}\left(1-\frac{1}{k}\right)+(N-2)p(1)-c\\ &= \frac{1}{k}\left(\frac{k-1}{k}\right)+(N-2)\min\left\{c,\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\}-c\\ &\geq \frac{1}{k}\left(1-\frac{1}{k}\Pr(k\leq N(B_i)\right)-c=\pi_{B_1}(2)\\ (N-2)\min\left\{c,\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\} &\geq \frac{1}{k}\left(1-\frac{1}{k}\Pr(k\leq N(B_i)\right)-\frac{1}{k}\left(\frac{k-1}{k}\right)\\ &= \frac{1}{k^2}\left(1-\Pr(k\leq N(B_i)\right)\end{aligned}$$

Suppose that $\min\left\{c,\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\}=c$, then

$$\begin{aligned}(N-2)\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &> (N-2)c\geq\frac{1}{k^2}\left(1-\Pr(k\leq N(B_i)\right)\\ \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1} &> c\geq\frac{1}{N-2}\frac{1}{k^2}\left(1-\Pr(k\leq N(B_i)\right)\end{aligned}$$

Else, if $\min\left\{c,\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\right\}=\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$ the condition is

$$c>\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}\geq\frac{1}{N-2}\frac{1}{k^2}\left(1-\Pr(k\leq N(B_i)\right)$$

The join of the two conditions is

$$c\geq\frac{1}{N-2}\frac{1}{k^2}\left(1-\Pr(k\leq N(B_i)\right) \tag{48}$$

(b) Now we show when there is a “choice” of configurations 5 and 1, when does the monopolist prefer 1.

$$\pi_{B_1}(1) \geq \pi_{B_1}(5) \Longleftrightarrow \frac{1}{k}\left(1-\frac{1}{k}\right) + (N-2)p(1) - c$$
$$\geq \frac{1}{k}\left(1-\frac{1}{k}\right) + (N-(J+1))p(5) - c$$
$$(N-2)p(1) \geq (N-(J+1))p(5)$$
$$(N-2)\min\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}, c\right\} \geq (N-(J+1))\min\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}, c\right\}$$

So essentially the problem reduces to finding the maximum of

$$(N-(J+1))\min\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}, c\right\}$$

over $J \in \{1, 2, ..., N\}$. If the maximum is at 1, as we showed has to happen when $N < k$, then configuration 5 cannot be an equilibrium. Otherwise, it will be 5. One useful observation, though, is that when $c < \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$, then $J = 1$ is optimal in this set.

2. The equilibrium conditions are:

$$\pi_{B_1}(2) \geq \max\left\{\pi_{B_1}(1), \pi_{B_1}(5)\right\}$$

For

$$\pi_{B_1}(2) \geq \pi_{B_1}(1)$$

reversing the inequality from the discussion of configuration 1, we get then:

$$c < \frac{1}{N-2}\frac{1}{k^2}\left(1 - \Pr(k \leq N(B_i)\right)$$

But since $c < \frac{1}{N-2}\frac{1}{k^2}\left(1 - \Pr(k \leq N(B_i)\right)$ also implies $c < \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-1}$, then at that level of c, $\pi_{B_1}(1) \geq \pi_{B_1}(5)$.

3. For this configuration we already discussed the optimality of the number of individuals who obtain the information from each agent when computing the payoffs and the price $p(3) = 0$. In addition there is no uninformed agent.

4. Here there is no sale of information, so there is nothing to check in this stage.

5. For equilibrium we need:

$$\pi_{B_N}(5) \geq \max\left\{\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J+1} - p(5), \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}\left(1-\frac{1}{k}\right) - c\right\} \tag{49}$$

First let us look at the case where $\max\{\pi_C(5), \pi_U(5)\} = \pi_C(5)$. Thus, $\frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J} \geq c$, and $p(5) = c$. We just need to check when is $\pi_{B_N}(5) \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J+1} - c$. Or, in other words,

$$\begin{aligned} 0 &\geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J+1} - c \\ c &\geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J+1} \end{aligned}$$

Now let us look at the case where $\max\{\pi_C(5), \pi_U(5)\} = \pi_U(5) = 0$. Thus,

$$c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}$$

So the join of the two conditions is

$$c \geq \frac{1}{k}\left(\frac{k-1}{k}\right)^{N-J}$$

6. Same as configuration 3.

Stage 1: information acquisition There are no changes in the discussion at this stage, except insofar as to state that configuration 5 can be an equilibrium.

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