

Individual Euler Equations Rather Than Household Euler Equations

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Abstract

This paper focuses on the identification and estimation of the intertemporal and intratemporal elasticities of substitution for the wife and separately for the husband using individual Euler equations. To that end, the household is represented as a group of agents making joint decisions. By means of this framework, individual Euler equations are derived and used to identify and estimate the parameters of interest. The main advantage of this approach is that the key parameters can be identified for all household members and not only for the household as a whole. To implement this approach it is essential to deal with an important issue: individual Euler equations depend on individual consumption which is not observable. In this paper it is shown that individual Euler equations are identified when only data on household consumption, individual labor supply and individual wages are observed. The identification strategy is then used to estimate the elasticities of substitution using the Consumer Expenditure Survey.

*I would like to thank the National Science Foundation for Grant SES-0231560 and the Graduate School, University of Wisconsin-Madison, for their support. I am very grateful to Orazio Attanasio, Pierre Andre Chiappori, James Heckman, John Kennan and Annamaria Lusardi for their insight and suggestions. I would also like to thank the participants at the NBER Summer Institute 2002 for their invaluable comments and Luke Davis for excellent research assistance. Errors are mine.

1 Introduction

An extensive literature in public finance uses intertemporal models to study consumption, savings and labor supply and to evaluate alternative policies. To that end it is important to know which parameters govern individual behavior and to have reliable estimates of them. In dynamic models the two key parameters are the intertemporal and intratemporal elasticities of substitution. The focus of this paper is on the identification and estimation of these elasticities for the wife and separately for the husband using individual Euler equations.

In the Health and Retirement Study, allowing for only four categories of risk preferences, more than 50 percent of couples report that the wife's risk preferences differ from the husband's. It is well documented that wives are expected to live several years longer than their husbands.¹ Life-time variations in costs and opportunities - due to children, unemployment of the spouse and business cycle changes - differ dramatically between household members.² Consequently, to evaluate alternative policies on savings, consumption and life-cycle labor supply it is essential to model household intertemporal decisions as the joint decisions of its members. This approach is feasible only if reliable estimates of the intertemporal and intratemporal elasticities of substitution are available for the wife and separately for the husband.

As discussed in Browning, Hansen and Heckman (1999), the estimation of those two parameters is almost exclusively based on Euler equations. One of the major challenges in the estimation of Euler equations is the lack of consumption data at the individual level. The traditional solution to this problem is to assume that the household behaves as a single agent. Under this assumption, it is possible to assign a unique utility function to the whole household and derive household Euler equations, which depend only on household consumption. The results obtained with this approach are extremely important to understand consumption, savings and labor supply dynamics.³ However, two shortcomings characterize this approach. First, in Mazzocco (2002) it is shown that a household can be represented using a unique utility function if and only if the conditions for Gorman aggregation are satisfied. In a static framework, Thomas (1990), Browning et al. (1994), Lundberg et al. (1997), Browning and Chiappori (1998) and Chiappori et al. (2002) find strong evidence against this assumption. In an intertemporal environment, the standard approach is rejected in Lundberg et al. (forthcoming) and Mazzocco (2002). Second, the unitary approach generates estimates for the intertemporal and intratemporal elasticities for the household as a whole, but not for individual members.

In this paper, the household is modelled as a group of agents making joint decisions and

¹See Lundberg et al (forthcoming) for a discussion on this issue.

²For a discussion see Heckman (1978), Killingsworth (1979) and Heckman and Macurdy (1980).

³For a survey on Euler equations see Browning and Lusardi (1996).

individual Euler equations are used to identify and estimate intertemporal and intratemporal elasticities. This framework has two main advantages. First, it is not affected by an aggregation problem. Second, elasticities of substitution can be estimated separately for the wife and the husband. To implement this approach it is essential to deal with an important issue: individual Euler equations depend on individual consumption which is not observable. This paper focuses on the identification and estimation of individual Euler equations when only data on household consumption, individual labor supply and wages are available, i.e. the information available in the Consumer Expenditure Survey (CEX). In particular, the paper deals with the following two issues:

a) Suppose that husband and wife are characterized by individual preferences and cooperate. Moreover, suppose that only total consumption, individual labor supplies, wages and interest rates are observed. It is shown that individual Euler equations are identified given the limited amount of information. Specifically, if both agents work, Euler equations of husband and wife can be identified up to a constant. Consequently, the intertemporal and intratemporal elasticities of substitution can be determined for both the wife and the husband. If only one agent works, Euler equations for the spouse supplying labor can be identified up to a constant. For the spouse not working, only the consumption Euler equation can be identified.

b) Using the suggested identification strategy, individual Euler equations will be estimated. In particular, the panel structure of the CEX will be used to estimate individual intratemporal and intertemporal elasticities of substitution.

The preliminary results indicate that the goal of estimating elasticities of substitution for the wife as well as the husband can be reached. In particular, the identification method is evaluated using a sample of households with only one member. Since for this subset of households individual consumption is equivalent to household consumption, the intertemporal model can be estimated using both the standard method and the identification strategy. The preliminary results obtained using this sample are promising and suggest that the identification method may be a useful tool to provide reliable estimates of key parameters at the individual level.

Euler equations have been estimated for the past 20 years, as reported in the survey by Browning and Lusardi (1996). The identification and estimation approach that I propose is new, as I consider Euler equations for each household member and not for the entire household. I employ an intertemporal framework in which each spouse is represented by individual preferences, therefore generalizing the static collective model developed by Chiappori (1988, 1992). Chiappori (1988, 1992) shows that in a static framework individual preferences can be identified under some separability restrictions. Blundell, Chiappori, Magnac and Meghir (2001) extend Chiappori's results to allow for households in which only one

spouse works. While this project is concerned with the identification of preferences, the focus is on household intertemporal optimization. Specifically, the goal is to identify and estimate individual Euler equations and, as a byproduct, the intertemporal elasticities of substitution for each household member. Lundberg, Startz and Stillman (forthcoming) use a three-period collective model with limited commitment and no uncertainty to explore the retirement-consumption puzzle. Lundberg and Pollack (2001) use a non-stationary multi-stage game to analyze theoretically the location decision of a married couple. They show that marital decisions involving the future are in general not efficient.

The paper is organized as follows. In section 2, the standard approach is discussed individual. In section 3, the intertemporal collective model is introduced and individual Euler equations are derived. Section 4 outlines the identification procedure. Section 5 discusses the empirical implementation. Section 6 presents some preliminary results. Section 8 concludes the paper.

2 The Standard Approach

Consider a household composed by 2 members living for \mathcal{T} periods. In each period $t \in \{0, \dots, \mathcal{T}\}$ and state of the world $\omega \in \Omega$, member i consumes a private consumption good in quantity $c^i(t, \omega)$ and supplies labor in quantity $h^i(t, \omega)$.⁴ Denote with $l^i = T - h^i$ leisure of member i , where T is the time available to each spouse in each period. At each (t, ω) , member i is endowed with an exogenous stochastic income, $y^i(t, \omega)$. For any given (t, ω) , the household can either consume or save in a risk-free asset. Let $b(t, \omega)$ and $R(t)$ denote respectively the amount of wealth invested in the risk-free asset at (t, ω) and the gross return on the risk-free asset.⁵ Let $Y(t, \omega) = \sum_{i=1}^2 y^i(t, \omega)$ and $C(t, \omega) = \sum_{i=1}^2 c^i(t, \omega)$. The utility functions are assumed to be twice continuously differentiable.

The main obstacle in modelling household intertemporal behavior is that consumption is only measured at the household level. The standard solution to this problem is to assume that the household behaves as a single agent. Under this assumption a single utility function can be assigned to the entire household. Following the literature on consumption, it is assumed that preferences are defined over a composite consumption good. To allow for non-separability between consumption and leisure, preferences depend also on leisure. Specifically, let $U(C, l^1, l^2)$ be the household utility function. Then the intertemporal allo-

⁴Public goods are not modelled in this paper. This important issue is left for future research.

⁵All the results of the paper apply if a risky asset is introduced in the model.

cation is the solution of the following problem:

$$\begin{aligned} \max_{\{C_t, b_t, h_t^i\}_{i=1,2}^T, \omega \in \Omega} E_0 & \left[\sum_{t=0}^T \beta^t U(C_t, T - h_t^1, T - h_t^2) \right] \\ \text{s.t. } C_t + b_t & \leq \sum_{i=1}^2 (y_t^i + w_t^i h_t^i) + R_t b_{t-1} \quad \forall (t, \omega) \\ b_T & \geq 0 \quad \forall \omega. \end{aligned} \tag{1}$$

Using a standard argument, the following household Euler equations can be derived,

$$U_C(C_t, T - h_t^1, T - h_t^2) = \beta E_t [U_C(C_{t+1}, T - h_{t+1}^1, T - h_{t+1}^2) R_{t+1}], \tag{2}$$

$$U_{l^i}(C_t, T - h_t^1, T - h_t^2) = \beta E_t \left[U_{l^i}(C_{t+1}, T - h_{t+1}^1, T - h_{t+1}^2) \frac{R_{t+1} w_t^i}{w_{t+1}^i} \right] \quad i = 1, 2, \tag{3}$$

where U_C and U_{l^i} are the marginal utilities of household consumption and member i 's leisure.⁶ Household Euler equations can be used to test the validity of the intertemporal model and to estimate intertemporal elasticities of substitution.

This approach is characterized by two shortcomings. First, by means of this approach only intertemporal elasticity of substitution for the whole household can be computed. Several policy questions require reliable estimates of individual intertemporal elasticities, i.e. one for each spouse. Second, this framework ignores that households are composed by several agents, possibly with different preferences.

3 A Collective Approach

Suppose that the households that we observe in the data satisfy the following two conditions: (i) the two spouses cooperate, i.e. any decision is on the Pareto frontier;⁷ (ii) each member is characterized by individual preferences. In particular, suppose that individual preferences are intertemporally separable, depend on a composite private consumption good and leisure and that member i 's utility function can be written in the form:

$$U^i(c^1, 1 - h^1, c^2, 1 - h^2) = u^i(c^i, 1 - h^i) + \delta_j u^j(c^j, 1 - h^j).$$

⁶It is also possible to derive cross Euler equations, i.e. Euler equations that relate consumption today with leisure tomorrow and vice versa.

⁷The idea that household members cooperate is well established in the literature, see for instance Becker (1973, 1974, 1991) and Chiappori (1992). Additionally, the general assumption of efficiency has the advantage of imposing no restriction on which point of the Pareto frontier will be chosen. Attanasio and Mazzocco (2002), Aura (2002), Lundberg, Startz and Stillman (forthcoming) and Mazzocco (2002) analyze the effect of limited commitment on household intertemporal behavior.

i.e. the two spouses are altruistic, but altruism can only be additive. The allocation of resources can then be characterized as the solution of the following Pareto problem:

$$\begin{aligned} \max_{\{c_t^i, b_t, h_t^i\}_{t \in \mathbf{T}, \omega \in \Omega}^{i=1,2}} \quad & \mu^1(\Theta) E_0 \left[\sum_{t=0}^{\mathbf{T}} \beta_1^t u^1(c_t^1, T - h_t^1) \right] + \mu^2(\Theta) E_0 \left[\sum_{t=0}^{\mathbf{T}} \beta_2^t u^2(c_t^2, T - h_t^2) \right] \quad (4) \\ & \sum_{i=1}^2 c_t^i + b_t \leq \sum_{i=1}^2 (y_t^i + w_t^i h_t^i) + R_t b_{t-1} \quad \forall (t, \omega) \\ & b_T \geq 0 \quad \forall \omega, \in \Omega \end{aligned}$$

for some pair of Pareto weights $(\mu^1(\Theta), \mu^2(\Theta))$, where Θ is the set of variables affecting the decision power of individual members.⁸

Even if preferences are heterogeneous, it is always possible to construct the representative agent corresponding to the household solving the following problem for a given level of individual leisure:

$$\begin{aligned} v(C, \{T - h^i\}, \{\mu^i(\Theta)\}) &= \max_{\{c^i\}_{i=1,2}} \mu_1(\Theta) u^1(c^1, T - h^1) + \mu_2(\Theta) u^2(c^2, T - h^2) \\ & \text{s.t.} \quad \sum_{i=1}^2 c^i = C \end{aligned}$$

However, the household aggregator v will generally depend on the distribution factors, i.e. on all the variables affecting the decision power. In Mazzocco (2002), it is shown that household preferences do not depend on the distribution factors if and only if the conditions for Gorman aggregation are satisfied. Using the aggregator v , household Euler equations can be written in the form,

$$v_C(C_t, \{T - h_t^i\}, \{\mu^i(\Theta)\}) = \beta E_t [v_C(C_{t+1}, \{T - h_{t+1}^i\}, \{\mu^i(\Theta)\}) R_{t+1}], \quad (5)$$

$$v_{l^i}(C_t, \{T - h_t^i\}, \{\mu^i(\Theta)\}) = \beta E_t \left[v_{l^i}(C_{t+1}, \{T - h_{t+1}^i\}, \{\mu^i(\Theta)\}) \frac{R_{t+1} w_t^i}{w_{t+1}^i} \right], \quad (6)$$

where v_C and v_{l^i} are the partial derivatives of v with respect to C and l^i . Consequently, household Euler equations depend on all variables affecting decision power unless Gorman aggregation applies. In Mazzocco (2002), it is tested whether household Euler equations depend on the distribution factors. The test is based on the following argument. If the standard model (1) is a complete characterization of household intertemporal optimization, the household Euler equations (2) and (3) should be satisfied for all families independently of the number of decision-makers in the household. If the collective formulation (4) is correct, household Euler equations should be satisfied for families with one decision-maker,

⁸To be precise, the weights μ are a function of the Pareto weights and of the altruism parameters.

but rejected for families with several decision-makers. Using the PSID and the CEX, after controlling for self selection, I find that Euler equations are strongly rejected for couples, but cannot be rejected for singles. This seems to indicate that it is important to find an alternative solution to the lack of individual data on consumption.

Independently of the number of household members, under the assumption of efficiency, individual Euler equations should always be satisfied.⁹ In particular, individual consumption and leisure should satisfy the following intertemporal optimality conditions:¹⁰

$$u_c^i(c_t^i, T - h_t^i) = \beta_i E_t [u_c^i(c_{t+1}^i, T - h_{t+1}^i) R_{t+1}], \quad (7)$$

$$u_l^i(c_t^i, T - h_t^i) = \beta_i E_t \left[u_l^i(c_{t+1}^i, T - h_{t+1}^i) \frac{R_{t+1} w_t^i}{w_{t+1}^i} \right], \quad (8)$$

Consequently, if individual consumption and individual labor supply were observed, it would be possible to test the intertemporal model of household behavior, and more important to estimate individual elasticities of substitutions. Unfortunately, consumption is only measured at the household level. The remaining sections discuss the identification and estimation of individual Euler equations if total consumption, individual labor supplies, wages and interest rates are observed, but individual consumption is not.

4 M-consumption Functions and Identification of Individual Euler Equations

Consider a household characterized by an arbitrary pair of individual utility functions u^1, u^2 which depend on a private composite good and leisure. To be able to test the model and identify the intertemporal elasticities it is important to answer the following question. Which variables do we observe? Micro datasets contain at best information on total household consumption, individual labor supplies and wages. With the exception of clothing no survey contains data on individual consumption. However, according to the theory, there should be a precise relationship between individual consumption on one side and labor supply and individual wages on the other. In this section, this link between unobservable and observable variables is used to show that individual Euler equations can be identified.

Assumption 1 *In each period, at least one member is working.*

This assumption is crucial for the identification method. In the CEX, 98 percent of households satisfy this condition. If in each period at least one of the two members is working,

⁹In this paper I abstract from the important issue of liquidity constraints.

¹⁰Individual Euler equations relating consumption today with leisure tomorrow and vice versa can also be derived.

the marginal rate of substitution between leisure and individual consumption must be equal to the wage rate divided by the price of consumption. Without loss of generality suppose that in period t member 1 satisfies assumption 1. Then the first order conditions of the Intertemporal Collective Model (4) imply,¹¹

$$\frac{u_l^1(c_t^1, T - h_t^1)}{u_c^1(c_t^1, T - h_t^1)} = q^1(c_t^1, h_t^1) = w_t^1. \quad (9)$$

If the function $q^1(c, h)$ is invertible, it is possible to determine individual consumption as a function of individual labor supply and wage rate, i.e. as a function of observable variables:

$$c_t^1 = g^1(w_t^1, h_t^1). \quad (10)$$

The function $g^1(w_t^1, h_t^1)$ corresponds to the m-consumption function introduced by Browning (1999). The following proposition establishes the condition under which $q^1(c, h)$ is invertible.

Proposition 1 *The m-consumption function $g^1(w^1, h^1)$ is well-defined if*

$$u_{lc}^1(c^1, T - h^1) u_c^1(c^1, T - h^1) - u_{cc}^1(c^1, T - h^1) u_l^1(c^1, T - h^1) \neq 0 \quad (11)$$

for any c^1 and h^1 that satisfy (9) for some feasible w^1 .

Proof. For any c^1, h^1, w^1 satisfying (9) define,

$$d^1(c^1, h^1, w^1) = q^1(c_t^1, h_t^1) - w_t^1 = 0.$$

By the implicit function theorem, $g^1(w^1, h^1)$ is well-defined if $\frac{\partial d^1}{\partial c^1} \neq 0$. Which implies condition (11). ■

Consequently, even if individual consumption is not observed, it is possible to derive a function that relates it to observable variables. Total household consumption C is also observed and this information has not been used so far. Since by assumption households are composed by 2 members, member 2's consumption can be calculated as the difference between total consumption and consumption of member 1,¹²

$$c_t^2 = C_t - c_t^1 = C_t - g^1(w_t^1, h_t^1).$$

By means of these results, individual Euler equations of member 1 can be characterized as a function of her labor supply and wage rate, by substituting the m-consumption function

¹¹The assumption that household members can choose freely labor supply is implicit in the intra-period optimality condition.

¹²Since a large fraction of couples have children it is crucial to extend the model to include them. The last section deals with this issue.

(10) for c_t^1 and c_{t+1}^1 ,

$$\begin{aligned} v^1(w_t^1, h_t^1) &= \beta_1 E_t [v^1(w_{t+1}^1, h_{t+1}^1) R_{t+1}], \\ f^1(w_t^1, h_t^1) &= \beta_1 E_t \left[f^1(w_{t+1}^1, h_{t+1}^1) \frac{R_{t+1} w_t^1}{w_{t+1}^1} \right], \end{aligned}$$

where,

$$v^1(w^1, h^1) = u_c^1(g^1(w^1, h^1), T - h^1), \quad f^1(w^1, h^1) = u_l^1(g^1(w^1, h^1), T - h^1). \quad (12)$$

Similarly, individual Euler equations of member 2 can be characterized as a function of total household consumption, his labor supply and the labor supply and wage rate of members 1, by substituting $C_t - g^1(w_t^1, h_t^1)$ and $C_{t+1} - g^1(w_{t+1}^1, h_{t+1}^1)$ for c_t^1 and c_{t+1}^1 ,

$$\begin{aligned} v^2(C_t, w_t^1, h_t^1, h_t^2) &= \beta_2 E_t [v^2(C_{t+1}, w_{t+1}^1, h_{t+1}^1, h_{t+1}^2) R_{t+1}], \\ f^2(C_t, w_t^1, h_t^1, h_t^2) &= \beta_2 E_t \left[f^2(C_{t+1}, w_{t+1}^1, h_{t+1}^1, h_{t+1}^2) \frac{R_{t+1} w_t^2}{w_{t+1}^2} \right], \end{aligned}$$

where,

$$v^2(C, w^1, h^1, h^2) = u_c^2(C - g^1(w^1, h^1), T - h^2), \quad (13)$$

$$f^2(C, w^1, h^1, h^2) = u_l^2(C - g^1(w^1, h^1), T - h^2). \quad (14)$$

Given that total household consumption, individual labor supplies and wage rates are observed, the functions v^1 , f^1 , v^2 and f^2 can be identified non-parametrically or parametrically. However, we are not interested in v^i and f^i , but rather in u_c^i , u_l^i and in the individual elasticities of substitution.

To identify u_c^i , u_l^i from v^i , f^i , initially suppose that husband and wife both work. Then variations in labor supply and wages are observed for both spouses and v^i and f^i , can be identified using the transformed Euler equations. From relations (13) and (14) we can deduce that,

$$v_{h^2}^2 = -u_{cl}^2, \quad f_w^2 = -u_{lc}^2 g_w^1, \quad f_{h^1}^2 = -u_{lc}^2 g_h^1.$$

Given that v^2 and f^2 are known functions, their derivatives are known as well and g_w^1 and g_h^1 can be identified,

$$g_w^1 = \frac{f_w^2}{v_{h^2}^2}, \quad g_h^1 = \frac{f_{h^1}^2}{v_{h^2}^2} \quad (15)$$

where the result follows from $u_{lc}^2 = u_{cl}^2$. Hence (15) provides a partial differential system, which can be integrated to give $g(w^1, h^1)$ up to the constant of integration. From relations (13) and (14) we can also deduce,

$$v_w^2 = -u_{cc}^2 g_w^1, \quad f_{h^2}^2 = -u_{ll}^2,$$

which imply that u_{cc}^2 , u_{ll}^2 and u_{cl}^2 can be identified,

$$u_{cc}^2 = -\frac{v_w^2 v_{h^2}^2}{f_w^2}, \quad u_{ll}^2 = -f_{h^2}^2, \quad u_{cl}^2 = -v_{h^2}^2. \quad (16)$$

The system can be solved to derive u_c^2 and u_l^2 up to a constant. From equation (12), we obtain,

$$v_w^1 = u_{cc}^1 g_w^1, \quad f_w^1 = u_{lc}^1 g_w^1, \quad f_{h^1}^1 = u_{lc}^1 g_h^1 - u_{ll}^1,$$

which imply,

$$u_{cc}^1 = \frac{v_w^1 v_{h^2}^2}{f_w^2}, \quad u_{ll}^1 = \frac{f_w^1 f_{h^1}^2}{f_w^2} - f_{h^1}^1, \quad u_{cl}^1 = \frac{f_w^1 v_{h^2}^2}{f_w^2}. \quad (17)$$

Hence u_c^1 and u_l^1 can be identified up to a constant.

Theorem 1 *Let u^1 and u^2 be von Neumann-Morgenstern utility functions. Assume that both agents work and that either u^1 or u^2 satisfies the invertibility condition (11). Then individual Euler equations are identified up to an additive constant.*

Consider a household in which only one spouse works. Without loss of generality suppose that agent 1 supplies labor. The function $g^1(w^1, h^1)$ is still well-defined and the approach outlined for households in which both members are employed can be implemented setting $h^2 = 0$. Since no variation in member 2's labor supply is observed, only the consumption Euler equation of member 2 can be identified. The following theorem summarizes the result.¹³

Theorem 2 *Let u^1 and u^2 be von Neumann-Morgenstern utility functions. Assume that only agent 1 works and that u^1 satisfies the invertibility condition (11). Then individual Euler equations of member 1 are identified up to an additive constant. Moreover, the consumption Euler equation of member 2 can be identified up to an additive constant.*

Three remarks are in order. First, the identification of the individual elasticities of substitution is unaffected by the fact that individual Euler equations are identified up to an additive constant. Second, the suggested identification strategy requires the following five assumptions: (i) individual preferences are defined over a private good and leisure; (ii) in each period, at least one agent works; (iii) altruism is additive; (iv) m-consumption functions are well defined (v) utility functions are continuous. In particular, for the identification strategy to work, no additional assumption on the functional form of u^1 and u^2 is required. Moreover, no assumption on the exogeneity of the labor force participation decision is needed.¹⁴ Third, the method proposed in this section generates a set of overidentifying restrictions, which can be employed to test the model.

¹³A formal proof of this result is not included because it is mostly a replication of the argument used for households in which both members work. The proof is available on request.

¹⁴The labor force participation decision will be important in the parametric estimation analyzed in the next section. A potential solution to the selection bias problem is also discussed.

5 Empirical Implementation

The identification strategy is implemented assuming a specific parametric formulation for individual preferences. Specifically, suppose that the one-period utility function can be written in the form,¹⁵

$$u^i(c^i, T - h^i) = \frac{\left[(c^i)^{\sigma_i} (T - h^i)^{1-\sigma_i} \right]^{1-\rho_i}}{\sigma_i (1 - \rho_i)},$$

where $0 < \sigma_i < 1$, $\rho_i > 0$. For this specification of preferences, the intra-period condition (9) becomes,

$$q^i(c^i, h^i) = \frac{1 - \sigma_i}{\sigma_i} \frac{c^i}{T - h^i} = w^i.$$

Consequently, the m-consumption function for agent 1 can be written in the form,

$$c^1 = g^1(w^1, h^1) = \frac{\sigma_1}{1 - \sigma_1} w^1 (T - h^1).$$

The functions v^1 , f^1 , v^2 and f^2 can now be computed and the following transformed Euler equations can be derived,

$$1 = \beta_1 E_t \left[\left(\frac{w_{t+1}^1}{w_t^1} \right)^{\gamma_1 - 1} \left(\frac{T - h_{t+1}^1}{T - h_t^1} \right)^{-\rho_1} R_{t+1} \right], \quad (18)$$

$$1 = \beta_2 E_t \left[\left(\frac{C_{t+1} - \phi_1 w_{t+1}^1 (T - h_{t+1}^1)}{C_t - \phi_1 w_t^1 (T - h_t^1)} \right)^{\gamma_2 - 1} \left(\frac{T - h_{t+1}^2}{T - h_t^2} \right)^{\theta_2} R_{t+1} \right], \quad (19)$$

$$1 = \beta_2 E_t \left[\left(\frac{C_{t+1} - \phi_1 w_{t+1}^1 (T - h_{t+1}^1)}{C_t - \phi_1 w_t^1 (T - h_t^1)} \right)^{\gamma_2} \left(\frac{T - h_{t+1}^2}{T - h_t^2} \right)^{\theta_2 - 1} \frac{R_{t+1} w_t^2}{w_{t+1}^2} \mid h^2 > 0 \right], \quad (20)$$

where $\gamma_i = \sigma_i(1 - \rho_i)$, $\theta_i = (1 - \sigma_i)(1 - \rho_i)$, $\phi_i = \frac{\sigma_i}{1 - \sigma_i}$. Moreover, $E_t[\cdot \mid h^i > 0]$ indicates that Euler equations containing the marginal utility of leisure are satisfied only if the spouse works. The first equation represents the consumption or leisure Euler equation of member 1.¹⁶ The second and third equations are, respectively, the consumption and leisure Euler equations of member 2.¹⁷ The coefficients γ_i , θ_i and ϕ_1 will be estimated using the Generalized Method of Moments (GMM).

¹⁵The utility function is divided by σ_i to normalize the multiplicative constant of the marginal utility of consumption to 1.

¹⁶The identification method requires the intra-period optimality condition of spouse 1. Consequently, given one of member 1's Euler equations, all the others are equivalent and redundant.

¹⁷Cross Euler equations relating consumption and leisure can also be derived.

5.1 The Data

To implement the identification procedure, the dataset must have the following two characteristics. First, information on total household consumption, individual labor supply, wages and interest rates must be available. Second, the dataset should have a panel structure to determine consumption, labor supply and wage dynamics for each household. The CEX survey satisfies these requirements. Since 1980, the CEX survey has been collecting data on household consumption, labor supply, wages and demographics. The survey is a rotating panel organized by the Bureau of Labor Statistics (BLS). Each quarter about 4500 households, representative of the US population, are interviewed: 80% are reinterviewed the following quarter, while the remaining 20% are replaced by a new randomly selected group. Each household is interviewed at most for four quarters and detailed information are elicited in regard to expenditures for each of the three months preceding the interview, and in regard to labor supply and demographics for the quarter preceding the interview.¹⁸

The data used in the estimation cover the period 1982-1998. The first two years are dropped, since the data were collected with a different methodology. As in Meghir and Weber (1996) and Attanasio and Mazzocco (2002), the rotating feature of the panel is used, i.e. household level data for the four quarters available are employed. Consequently, I drop all households that are not in the survey for all four interviews. To verify the robustness of the results to the empirical strategy I also experiment with synthetic panels.¹⁹ I exclude from the sample rural households, households living in student housing, household in which the head is younger than 21 and older than 60 and households with incomplete income responses. The identification method is useful to identify individual preferences of couples. Therefore, I concentrate on married households. Singles are used as a benchmark to evaluate the performance of the method. To implement the identification procedure, at least one household member must be employed. Hence, I exclude all households in which the husband is not working. The husband claims to be employed in all four available interviews for more than 91 percent of the households in the sample.²⁰ Finally I drop households experiencing a change in marital status.

The CEX dataset contains monthly data on consumption. However, the labor supply variables are available only every quarter. Consequently, in the estimation quarterly variables are employed. Total consumption is computed as the sum of food at home, food out,

¹⁸Each household is interviewed for five quarters, but the first interview is used to make contact and no information is publicly available.

¹⁹The synthetic panels are constructed using the year of birth of the head of the household. Intervals of five years are constructed and all households are assigned to one of them. The variables of interest are then averaged over all the households belonging to a given cohort observed in a given quarter.

²⁰Alternatively, the identification strategy can be implemented using in each period the spouse that is employed in that period. This increases the size of the sample.

tobacco, alcohol, other nondurable goods and services such as heating fuel, public and private transportation, personal care and semidurable goods which include clothing and shoes. In particular, from the definition of total consumption I exclude consumer durables, housing, education and health expenditure. Total consumption is deflated using the Consumer Price Indices published by the BLS. Specifically, the price index for the composite good is calculated as a weighted average of individual price indices, with weights equal to the expenditure share for the particular consumption good. The gross hourly wage rate is computed using three variables: the amount of the last gross pay; the time period of the last gross pay covered; the number of hours usually worked per week in the corresponding period. Since the wage rate is not directly observed, the measure used in this paper might be affected by endogeneity. In particular, the amount of the last gross pay is likely to be affected by the number of hours of work in a given period. However, this criticism applies to any work unless wage rates are directly observed. Moreover, even in this case the wage rate will depend on hours of work through the investment in human capital. To calculate the after tax wage rate, federal effective tax rates are generated using the NBER's TAXSIM model. Finally, the real after tax wage rate is determined using the individual price indices. Total time available to each household member is computed as 12 hours per day times 7 days per week times 13 weeks per quarter. This implies that $T = 1092$. Quarterly individual labor supply is obtained multiplying by 13 the number of hours usually worked per week in the corresponding period. The interest rate is the quarterly average of the 3-month Treasury bill rate preceding the interview. The real after tax interest rate is calculated by using the output of the TAXSIM model and the household price indices. Finally to account for preference shocks, the one-period utility function of each spouse is augmented to include a function of demographic variables.

Three limitations of the CEX should be discussed. First, individual labor income data are collected at the first and last interviews unless a member of the household reports changing his or her employment. In the second and third interviews the labor income data are set equal to the data reported in the first interview. Consequently, variations in gross pay check are only observed between the third and the fourth. Second, 27 percent of respondents reporting to be employed do not have data on gross pay. Therefore the sample size is smaller than predicted. Third, to estimate the model by GMM, a set of valid instruments is required. Under the assumption of rational expectations, any lagged information is a suitable instrument.²¹ The short version of the panel implies that the set of valid instruments is very small. To deal with this problem and the fact that labor income variation is only observed between the last two quarters, the third and fourth interviews are used to calculate

²¹The existence of measurement errors may introduced unexpected dependence between the Euler equation error term and lagged information. For this reason all instruments will be calculated as the first or higher lag of current variables.

consumption, leisure and wage growth, whereas the first and second interviews are used to construct the instruments. I will also experiment with a linearized version of individual Euler equations. In this case, synthetic panels can be employed. This partially solves the problems intrinsic in the short panel available in the CEX. The summary statistics of the main variables are reported in table 1.

Table 1: Summary Statistics

Independent Variable	Mean	Variance
Real Consumption per Quarter	2625.3	1430.2
Husband's Labor Supply per Week	41.1	14.6
Wife's Labor Supply per Week	25.6	18.5
Husband's Before Tax Wage per Hour	13.38	11.1
Wife's Before Tax Wage per Hour	8.55	7.8
Real After Tax Interest Rate	5.9	1.58
Number of Observations		23195
Number of Families		9214

5.2 Econometric Issues

To identify the individual preference parameters the following Euler equations will be employed: the consumption Euler equation of member 1 and 2; the leisure Euler equation of member 2. The Inada condition guarantees that the consumption Euler equations are always satisfied. The leisure Euler equations are satisfied only if the corresponding agent supplies a positive amount of hours on the labor market in period t as well as in period $t + 1$. If only a fraction of individual members works in both periods, the estimation results may be affected by a self-selection bias. Since around 91 percent of males between the ages of 21 and 60 with all four interviews supply labor in all four quarters of the survey, the assumption that the husband is always employed is not very restrictive and self-selection should not be too important. The use of member 2's leisure Euler equations is more problematic. In the sample, around 70 percent of wives work in all four quarters. Consequently the self-selection bias could be an issue. In the estimation of the non-linear Euler equations I will make the identifying assumption that the error term of the individual Euler equations is not correlated with the labor force decision. This is clearly a strong assumption. Hence, to verify the empirical relevance of self selection into the labor force for females, the identification strategy will also be implemented using a log-linearized version of individual Euler equations after

controlling for self-selection using standard methods.²²

The Euler equations will be estimated using the Generalized Method of Moments (GMM), because this approach is general enough to estimate the non-linear as well as the linear version of the model. However, the GMM is not free of problems. As for any estimator, the GMM estimator is consistent only if measurement errors are not an issue. If the GMM is used to estimate non-linear equations, the measurement error problem is exacerbated. The quality of the consumption data in the CEX is very good. However, a good look at the labor supply data suggests that they may be affected by measurement errors. Using a Montecarlo simulation, in his insightful paper, Carroll (2001) finds that measurement errors should bias the estimates of intertemporal substitution downward. To verify the impact of measurement errors on the estimation, a linearized version of the model is also used. The GMM estimation has also an important advantage: it does not require the log-linearization of the Euler equations. Carroll (2001) and Ludvigson and Paxon (2001) find that the approximation method may introduce a substantial bias in the estimation of the preference parameter. On the other hand, Attanasio and Low (2000) show that using long panels it is possible to estimate consistently log-linearized Euler equations.

A well known result is that, to estimate consistently Euler equations, a relatively large number of time periods is needed, but not necessarily on the same household. The sample used in this project covers 17 years. It is therefore likely that the aggregate shocks will average out. To control for observable heterogeneity the utility function of each household member is multiplied by a heterogeneity term, $\exp\left(\sum_{j=1}^m \xi_j z^j\right)$, where z is a vector of demographics including age and the number of children. In this paper I abstract from the important issue of liquidity constraints. If household members are restricted in their ability to borrow, Euler equations are replaced by inequalities as shown in Zeldes (1989).

6 Preliminary Results

The identification procedure developed in the previous sections relies heavily on the theoretical structure of the model. It is therefore important to evaluate the performance of the identification method. One simple way to do this is to apply the identification method to households with only one member. In this case total and individual consumption are identical and both observable. It is therefore possible to estimate individual Euler equations using both the standard method and the identification procedure. Specifically, in the identification method, the Euler equation (18) is used. In the standard method, the individual

²²Given the parametric assumption on preferences, the log-linearized individual Euler equations are linear in individual consumption. This implies that member 2's Euler equations will be linear in $\log(C - \phi_1 w^1 (T - h^1))$.

Euler equations (7) and (8) are employed, which under the assumption on preferences can be written in the form,

$$E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{\gamma-1} \left(\frac{T-h_{t+1}}{T-h_t} \right)^{\theta} \beta R_{t+1} - 1 \right] = 0,$$

$$E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{\gamma} \left(\frac{T-h_{t+1}}{T-h_t} \right)^{\theta-1} \frac{w_t}{w_{t+1}} \beta R_{t+1} - 1 \right] = 0.$$

where $\gamma = \sigma(1 - \rho)$ and $\theta = (1 - \sigma)(1 - \rho)$. By means of this approach it is also possible to establish the impact of the non-linearities of the model on the coefficient estimates. In this regard, two versions of individual Euler equations are estimated: the non-linear Euler equations discussed in the previous sections and their log-linearized version.

RESULTS TO BE ADDED.

7 Conclusions

In this paper, the identification and estimation of individual Euler equations is analyzed. It is shown that individual Euler equations can be identified parametrically and non-parametrically observing only data on household total consumption, individual labor supply and wages, i.e. with the limited information available in the CEX. Moreover, assuming a specific utility function for each household member, I estimate them by means of the identification procedure developed in this paper.

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