Sharing information in web communities

Gabrielle DEMANGE[1]

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Abstract

The paper investigates the formation of information sharing communities. The environment is characterized by the anonymity of the contributors and users, as on the Web. Furthermore information exchange is limited to simple recommendations. When preferences differ, it is argued that a community may be worth forming because it facilitates the interpretation and understanding of the posted information. The admission rule within a community and the stability of multiple communities are examined.

Keywords value of information, communities, anonymity, preference diversity

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[1]PSE, 48 Boulevard Jourdan, 75014 Paris, France. e-mail: demange@pse.ens.fr.

1 Introduction

Group structures on the Web such as peer-to-peer (P2P) systems aim at sharing various goods and disseminating information in a fully decentralized way. Quite often, information is not rivalrous and returns to scale are not decreasing. Why then do communities form with a free but restricted access? This paper argues that a basic rationale is related to the value of information. A tremendous quantity of information is posted on the Web, on blogs for instance. In some situations however, all this information is useless. As an illustration, consider a page in which an individual provides her opinion on movies. If she says that it is worth watching movie A, or that she prefers movie A to movie B, do I benefit from this knowledge? If the peer is a critic, and I am pretty aware of her tastes, her judgment may be valuable to me. If instead I have no idea at all about her preferences, I learn nothing. In other words, how useful a person's statement is much depends on whether the preferences of this person are known. As a result, when contributors are anonymous, posted information on topics on which tastes differ may have little value. Furthermore, in these situations, search engines may not be helpful either. Given a query, an engine provides a ranking based on the observed behaviors of those who have experimented the topic. The ranking is valuable to other users only if they share similar tastes and know it. This suggests that a community, by filtering the peers who contribute to a platform, may facilitate the understanding and the usefulness of the conveyed information. This paper investigates in a stylized model the criterion on which this filtering may be determined and whether information is efficiently processed.

There are individuals who regularly look for a piece of advice on a category of 'objects', on movies for instance, and who occasionally post information on a particular object, when they have seen a movie for instance. Objects differ in some characteristics and individuals differ in their tastes. More specifically, objects' characteristics and individuals' preferences are characterized by a single parameter on a circle as in Salop differentiation model (1979). Two crucial features bear on the information process. First, anonymity is preserved as often in P2P. Second, although we assume truthful behavior, language is limited. Individuals are able to describe whether they have enjoyed consuming a particular object but they are unable to describe why, in particular they are unable to describe the object's characteristic.

To analyze the value of the posted recommendations, we follow Blackwell (1953). Recommendations are valuable to a peer if they allow him to make "better" decisions, that is decisions that increase his utility. Anonymity and limited language prevent this to be generally true. Recommendations are useful for users who share similar tastes, and furthermore, posted information without control on the contributors may not only be useless but also detrimental by introducing some noise in the information relevant to other peers. Thus the admission rule in a community is essential in determining the value that each peer derives from the information provided by the community's members. This leads us to analyze preferences over admission rules. Community's members do not fully agree on an admission criterion owing to their differences in tastes, even if all of them benefit from the community. We analyze this divergence and, assuming that a leader/initiator of a community chooses the admission rule, we study how this choice is influenced by the contribution rate,[2] the cost and the probability of finding an answer to a query, and the sharing of ad revenues.

Next we study the coexistence of several communities, called a configuration. To predict which communities might form and be durable, stability concepts borrowed from cooperative game theory are useful. The threat of proposals to form a new community puts constraints on configurations. An interesting question is whether communities form so as to reach some form of efficiency. Indeed there is a trade-off between the amount of information received by peers and its relevance to them. As a community becomes less strict in its admission rule, peers are more numerous but more diverse, which makes recommendations more numerous but less informative. This trade-off is optimally solved at an efficient configuration. It is shown that stability does not imply efficiency nor does efficiency imply stability because of external effects. For instance, it may happen that at an efficient configuration each peer would like his own community to expand and to accept newcomers, which makes the configuration unstable. However expanding communities leads to the suppression of some of them, which has an overall negative effect on welfare because of the decrease in the relevant information.

Whereas the circle model is the simplest model to explore our ideas, the analysis could be extended to different or more complex settings under heterogeneity in preferences. In particular, the value for a community is explored more informally under vertical differentiation when individuals agree on ranking the objects according to an 'objective' quality but may differ in their willingness to pay. Also, in our basic model, a peer who is contemplating buying a particular object searches for a single recommendation at most. A question is whether the availability of an important number of recommendations, or of an appropriate statistic on those, reveals all the information an individual needs to make a good decision and thereby destroys the value of forming a community. The answer is negative under horizontal differentiation: even when a large number of recommendations is available, forming a community that restricts access so as to be homogeneous enough is still valuable. Due to limited language, when tastes really differ as in a horizontal differentiation model, information exchange is only valuable between similar individuals. This is not true under

[2] There is some evidence of "free-riding" in P2P systems. A main question is whether the generated difficulties are severe enough to call for the implementation of incentive schemes (see Feldman et al. 2004 and Ng, Chiu, and Liu 2005 for example). Bramoulle and Kranton (2007) and Corbo et al. (2007) provide theoretical analysis of free-riding in a network context.

vertical differentiation.

This paper is related to the growing literature on the search for relevant information on the Internet. The ranking provided by a search engine can be seen as an aggregator of preferences that determines 'objectively' the relative importance of Web pages. As such, it is more valuable under common preferences. This explains why search engine rankings share some similar features as the methods used for measuring 'quality'. Indeed, as acknowledged by Page et al. (1998), the PageRank method used initially by Google has been inspired by the citation analysis techniques (see Slutzki and Volijn 2006 who provide axiomatizations of some methods). Also, Dwork et al. (2001) explicitly assume a common underlying ranking and borrow tools from social choice theory in order to study aggregation over several engines. In a circle model as considered here, any anonymous aggregation of preferences over the whole population yields a completely flat ranking, that is the society is indifferent between any two objects, and distinct rankings provided by search engines can only be attributed to chance or to bias. In some sense, a community forms with members whose preferences are homogeneous enough to allow for a useful aggregation.

The formation of communities and the diversity of preferences are recognized as an important feature of the Web environment. Various algorithms have been proposed for detecting communities through link structure, as surveyed in Newman (2001). Recent empirical studies analyze the growth of communities as a function of the structure of an underlying social network (Backstrom at al. 2006 for instance). This paper investigates, in a specific context, why and how a community forms in the first place (without any prior network). Also, the diversity of preferences is at the root of 'collaborative filtering'. Collaborative filtering is a system that aims at giving tailored recommendations to a user on the basis of his past behaviors and a collection of 'similar' user profiles (Hofmann and Puzicha 1999). Typically, an e-commerce retailer recommends to a user to buy a particular item because another user who has behaved similarly in the past has bought that item. Hence, instead of a recommendation sent by a peer who has experienced an object, the purchase is itself interpreted as a good signal even though the buyer may have disliked it. More importantly, the system is centralized in contrast with P2P. Our model accounts both for the decentralization and anonymity aspects endemic to P2P systems. An individual voluntarily chooses a community and all peers within a community have access to the same recommendations, which allows to keep anonymity and privacy.

Finally, the paper is related to the literature on belief formation or social learning, in which agents choose actions over time based on information received by others, either by direct communication or by observing actions. Information bears on an unknown state of nature, and its transmission is constrained by a specific setting. For instance information is conveyed by neighbors in a given network structure or the agents whose actions are observed move in a given sequence. A main question analyzed by this literature is whether beliefs converge and whether they converge towards the 'true' state of nature (see Goyal 2005 for a survey). In our model, a 'state of nature' is the objects' characteristics. Because language is limited to simple subjective statements, and by the very fact that communities must be homogeneous enough to be valuable (in a circle model), objects' characteristics are not revealed, even with access to numerous peers' recommendations. Finally, DeMarzo et al. (2003) consider another form of limitation in communication in which information is processed through an exogeneous 'listening structure' (or network) with individuals who fail to disentangle what is truly new from the signals they receive over time. The main problem investigated here is instead to determine which listening structures might emerge in a setting in which the recommendations by some individuals are detrimental to others.

The plan of the paper is the following. Section 2 sets up the model and Section 3 studies a single community. Section 4 analyzes configurations of communities. Equilibrium individuals behavior is investigated when multiple and possibly overlapping proposals to form a community are made, and coordination failure is discussed. How proposals are made is explored and efficiency properties of configurations are investigated. Proofs are gathered in the final section.

2 The model

Consider one category of 'objects', such as movies, or books, or restaurants for instance. Individuals widely differ in their tastes. Differentiation models in which preferences are characterized by a single characteristic are a stylized and parsimonious way to model these differences. We adopt here the 'circle' model introduced by Salop (1979). Some variations and extensions are examined in section 3.4.

Individual's preferences over the objects are characterized by a single parameter, a point on a circle. An object, a movie or a restaurant for instance, is characterized by a point on the same circle. An individual 'located' at θ on the circle is called a θ-individual and similarly an object located at t is a t-object. An individual who buys an object derives a utility gain that is non-increasing in his distance to the object. Specifically the utility gain for a θ-individual who buys a t-object is given by $u(d(\theta, t))$ where $d(\theta, t)$ is the distance on the circle between θ and t. If this gain is negative, the individual is better off not buying the object. Function u is non-increasing, identical for all individuals. To fix the idea, at most half of the objects are valuable to an individual: There is a threshold value d^*, $0 < d^* < \pi/2$ for which $u(d) > 0$ for $d < d^*$ and $u(d) < 0$ for $d > d^*$. Furthermore function u is continuous and derivable except possibly at d^*. As a simple example, consider the situation in which individuals either do or do not enjoy consuming the object. It is represented by a *binary* function u:

$$\text{for some positive } g \text{ and } b, u(d) = g, d < d^*, u(d) = -b, d > d^*. \tag{1}$$

(The utility level at d^* does not matter because the probability of an object being distant of d^* to a person is null.)

The society is uniformly distributed on the circle. If the characteristic of a particular object is perfectly known, the set of individuals who benefit from buying it is given by those located at a distance smaller than d^*. Thus, under perfect information on an object's location, a proportion p of the people buy it, where $p = d^*/\pi$, independently of the location t. For new objects however, their characteristics are not known. This is the situation we are interested in. New objects are assumed to be *a priori* uniformly distributed on the circle. Under imperfect information on objects' characteristics, an individual forms some assessment on the location and decides whether to buy a particular object by comparing the expected utility gain from buying it with 0. A weak form of risk aversion is assumed: faced with the lottery of buying two objects with equal probability, a peer prefers not to buy if the sum of the distance is $2d^*$ (or larger), i.e. $u(d) + u(2d^* - d) < 0$ for $d < d^*$. For a binary function (1) for example weak risk aversion holds for $b > g$. Weak risk aversion implies that the expected utility gain derived from buying at 'random' is negative.

Under imperfect information, there is some scope for information sharing: Individuals who have bought an object may post their opinion on it. Our aim is to study the value of a community in gathering and sharing such opinions.

2.1 Communities

In a community, the role of contributors and users can *a priori* be distinguished. Contributors add to the content by providing information on the objects they have tested while users have access to the posted information. Here the sets of contributors and users are identical. This is induced by the following assumptions. First we assume that there is no intrinsic motive to contribute such as altruism. Thus, for a community to be 'viable' as defined in the next section, contributors are also users so as to draw some benefit. Furthermore, even though there may be no direct cost (nor benefit) in allowing users not to contribute, it may be worth restricting access to contributors simply to encourage them to contribute. In that case users and contributors coincide.

Anonymity and restricted access can be implemented by a fully decentralized mechanism such as Gnutella and Freenet. These mechanisms propagate queries through a P2P network without the need of a server. A query is sent to neighbors who provide an answer if they have one or otherwise pass the query to their own neighbors and so on until an answer is reached.[3] The system can be anonymous by recognizing members by an address only. Records, which are not public, can keep track of peers' behavior. Sanctions such as exclusion are based on these records and automatic. Records on peers' contributions for instance allow the community to sustain some contribution level by excluding users who contribute too little.

In our model, a proposal to form a community is represented by an arc. It will be assumed that the system may detect whether the recommendations given by the members are indeed compatible with characteristics in the selected arc. By convention, an arc $[\theta, \theta']$ designates the arc from θ to θ' going clockwise. The *center* of community $[\theta, \theta']$ is $(\theta + \theta')/2$ and its *size* is defined by $(\theta' - \theta)/2$. We start by considering a single arc and identify a community with the set of all individuals whose characteristics belong to that arc. (Section 4 introduces the distinction between a proposal to form a community (an arc) and the set of individuals who effectively join.)

The technology is characterized by two data: a probability of 'success', which is the probability of finding a recommendation for a particular object in reasonable time and an individual participation cost. In line with decentralized behavior, the size of a community determines the probability of success, which is denoted by $P(\alpha)$ for a community of size α. P is assumed to be increasing and concave. For example, it can be described by a Poisson process $P(\alpha) = 1 - e^{-\lambda\alpha}$ for some positive parameter λ which reflects the contribution rate of the community members and the efficiency of the search mechanism. The participation cost includes the cost for searching and contributing. Normalizing by the average number of requests, it is denoted by c. When it is small, as is likely, it does not play an essential role in the analysis. The probability P and the cost c are first assumed to be given. However contributions have a public good aspect, and a minimal amount of contributions can be asked for to increase the success probability P. Section 3.3 investigates this point more closely.

2.2 Signals

Consider a particular object, recognized by a title for a book or a movie. Opinions on that object are described by signals. Stating a detailed judgment is difficult. To account for this, signals are assumed to be limited. A signal s on an object takes two values, *yes* or *no*, which are interpreted as a recommendation to buy or not to buy. Since there is no benefit from sending a false signal, signals are assumed to be truthful: a θ-individual having bought a t-object sends *yes* if $u(d(\theta, t)) \geq 0$ and

[3] See for example Kleinberg and Raghavan (2005) for a description of decentralized mechanisms and an analysis of the incentive to pass the information.

no if the inequality is reversed.[4] As will be clear later on, the limitation in communication is not due to the use of a binary signal. What matters is that a signal pertains to feelings (or utility levels) rather than directly on objects' characteristics.

Let us consider a signal on a particular object from a member of community $[\theta, \theta']$. It can be seen as a 'random' recommendation $\tilde{s}$ since the sender is considered as drawn at random from the community. In the sequel $\tilde{s} \in [\theta, \theta']$ refers to a signal sent by a member of community $[\theta, \theta']$.

The value of a signal can be analyzed from the viewpoint of Blackwell (1953). Signal $\tilde{s}$ is valuable to an individual if it enables him to make 'better' decisions in the sense that his expected payoff is increased. More precisely, the signal is used as follows. The joint distribution of $(\tilde{t}, \tilde{s})$ for a signal $\tilde{s}$ sent by a member of community $[\theta, \theta']$ can be computed. After learning the realized value of a signal, peers revise their prior on the characteristic t according to Bayes' formula and decide to buy or not. Clearly a signal that does not change the prior on the object's location is useless. The ignorance of the sender's location in the community has the following consequences.

- (i) A signal $\tilde{s}$ from the whole society is useless.
- (ii) A signal from a community smaller than the whole society may be useful : it changes the prior.
- (iii) Two simultaneous signals may convey less information than each one.

Point (i) is straightforward. A signal sent by an individual chosen at random in the whole group does not modify the prior, hence is not informative.

Point (ii) is also clear. Figure 1 illustrates the case of a community $[-\alpha, \alpha]$ and $d^* = \pi/2$. Each peer sends *no* for an object located in $[\alpha + \pi/2, -\alpha - \pi/2]$. Thus the posterior density conditional on the signal being *yes* is null on that arc: the posterior clearly differs from the prior density.[5]

To show Point (iii), consider a signal from a community reduced to a point, say 0, so that the sender's preferences are known. The signal is informative from point (ii) or, more directly, because a *yes* indicates that the object is $[-d^*, d^*]$ and a *no* in the complement $[d^*, -d^*]$. Add a signal sent from $[\pi]$. The important point is that on the receipt of the two signals, it is not known

[4]Malicious individuals can be incorporated. Under some expectation on their distribution, their presence introduces additional noise in the information inferred by a signal. The same argument applies if individuals make error in their judgment.

[5]More generally, consider the probability of a positive signal from community $[-\alpha, \alpha]$ for a t-object $g^{\alpha}(t) = P(yes \in [-\alpha, \alpha]|t)$. Each member in the community sends a positive signal *yes* for $t \in [\alpha - d^*, -\alpha + d^*]$, which we call the *acquiescence zone*, ($g^{\alpha}(t) = 1$) and each one sends *no* on $[\alpha + d^*, -\alpha - d^*]$, which we call the *refusal zone* $g^{\alpha}(t) = 0$. Outside these two zones there is *disagreement* and the probability is linear in between. By Bayes'rule, the posterior density conditional on a *yes* $\in [-\alpha, \alpha]$ is equal to $g^{\alpha}(t)/(2d^*)$.

which peer has sent which signal. The two signals, which are always opposite to each other, give no information because the prior is not changed. In this simple example, the new signal not only adds no information but also destroys the information conveyed by the first signal. The reason is that adding a signal introduces an additional source of randomness due to the anonymity of the sender. In contrast, in the standard framework, adding a signal is never harmful because it can simply be ignored.

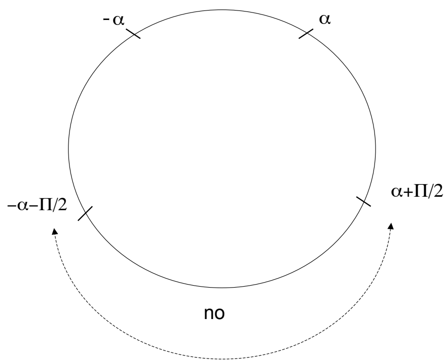

Figure 1: $d^* = \pi/2$

An informative signal has some value to a person if its receipt affects this person's optimal decision. Here, there are three options: to follow the recommendations (that is to buy the object on the receipt of a positive signal and not to buy it in the opposite case), or not to listen to them (which gives 0), or to take always the opposite action to the recommendation. Providing that tastes are restrictive enough, for d^* smaller than $\pi/4$ for example, the third case never occurs.[6]We shall focus on that case (another justification is that if a peer had interest to take the opposite action to the recommendations, another community, closer to the peer's tastes, is likely to form. Multiple communities are considered in Section 4). Thus a signal can be (strictly) beneficial to a person only by following the recommendations. Furthermore, the benefit depends on his/her location and

[6]The person who could draw the largest benefits from a strategy of buying upon a negative signal only is located at the opposite of the leader, at π. In that case, objects that are bought are in the refusal zone and in the disagreement zone (with some probability). The refusal zone $[\alpha + d^*, -\alpha - d^*]$ is an arc centered at π with size $\pi - \alpha - d^*$ which is larger than $\pi - 2d^*$. By risk aversion the expected utility for a peer from buying the objects distant of at most δ where $\delta \leq 2d^*$ is negative. Hence, when $2d^* < \pi - 2d^*$ the expected utility for the π-peer from buying the objects in the refusal zone is negative. For such a value of d^* all objects in the disagreement zone provide a negative payoff.

on the size of the community. To analyze this further, consider a community of size α and an individual whose distance to the center is θ. Let $U(\theta, \alpha)$ denote the expected utility conditional on having an answer to a query and on following the recommendation. The *a priori* probability for a positive signal is equal to $p = d^*/\pi$ since, given an object at random, each individual says *yes* with probability p. Up to a rotation, the arc may be assumed to be centered at zero. Using that there is no purchase on the reception of a negative signal gives

$$U(\theta, \alpha) = pE[u(d(\theta, \tilde{t}))|yes \in [-\alpha, \alpha])]. \tag{2}$$

The following proposition states how this value is affected by the individual position in the community and the size of the community.

Proposition 1 *Let $U(\theta, \alpha)$ be the expected value from following a recommendation of a community with size α for an individual whose distance is θ from the center and $V(\alpha) = U(\alpha, \alpha)$ the value for a peer located at an extreme point.*

- *(i) Given α, $\alpha \leq d^*$, utility $U(\theta, \alpha)$ decreases with the distance θ to the center, $\theta \leq d^*$.*
- *(ii) Given θ, $\theta \leq d^*$, utility $U(\theta, \alpha)$ decreases with α on $[0, d^*]$.*
- *(iii) there is a value α^{max} smaller than d^* such that $V(\alpha)$ is positive for $\alpha \leq \alpha^{max}$ and negative otherwise.*

The properties hold for all individuals located at a distance smaller than d^* from the center, hence for some outsiders to the community. Point (*i*) is natural given the symmetry. It says that the expected benefits derived from following a signal decrease with the distance to the center. According to point (*ii*), the expected value per signal is greater the smaller the community, that is the less uncertain the sender. This is easy to understand for a peer located at the center ($\theta = 0$). As the size α increases, the objects that he dislikes (distant of more than d^*) are more likely to be recommended and those he likes get less recommended. For an individual who is not at the center, the distribution of signals is 'biased' with respect to his own preferences and as α increases some objects that he likes get more recommended. Increasing α is however still harmful because the distribution of the distance to a peer of the recommended objects becomes riskier in the sense of first order stochastic dominance as shown in the proof. An implication of property (*ii*) is the superiority of an expert 'everything else being equal'. More precisely, a system in which an expert sends as many signals as the communities' members at the same total cost makes every peer better off. Point (*iii*) is important for the sequel. It follows from weak risk aversion which implies that two persons distant of more than $2d^*$ cannot both benefit from following the same recommendations.[7] The value α^{max} depends on the function u and is related to risk aversion. For a binary function for example easy computation gives $U(\theta, \alpha) = p[g - \frac{(g+b)}{4d^*}(\alpha + \theta^2/\alpha)]$ and $\alpha^{max} = \frac{2gd^*}{(g+b)}$ decreases from d^* to 0 when b/g ranges from 1 to ∞.

3 Community Choice

The purpose of this section is to analyze which community might form. A first requirement is that all community members benefit from it, a property that we call viability. There are many different viable communities, provided the search cost c is not too large. Which community will form is unclear because there is no unanimous criterion on the community's boundaries (as would be the case if transfers were available or if a firm was organizing the community so as to maximize profit). Community's members may have conflicting views about its scope, i.e., about the membership rule. We shall analyze the choice of a 'leader'.

3.1 Viable community

Anybody is free not to join a community. A community is said to be *viable* if each of its members benefits from it, accounting for the failure of search and the participation cost. Since, in the absence of information, an individual does not buy, reservation values are null. Hence, an individual distant of θ from the center of a community of size α is indeed willing to participate if $P(\alpha)U(\theta, \alpha) - c \geq 0$. To cover the cost, the utility from following a signal, U, must be positive. From point (i) of Proposition 1, the peers who achieve the lowest benefit are located at the extreme points of the community. The viability condition can be written simply as

$$P(\alpha)V(\alpha) \geq c \text{ where } V(\alpha) = U(\alpha, \alpha). \tag{3}$$

From (iii) of Proposition 1, viable communities are of size smaller than d^*, and under a small enough cost, the set of viable sizes is non-empty. Property (ii) points out a trade-off faced by peers: increasing the size increases the probability of getting an answer but decreases the value of an answer. To analyze this trade-off, we make the following assumptions throughout the paper. The size α is restricted to be smaller than α^{max} so that utility levels are all positive.

[7] The sum of the distance of an object to two individuals θ, θ' is at least the distance between $d(\theta, \theta')$. Hence if they are distant by more than $2d^*$, we have $u(d(\theta, t)) + u(d(\theta', t)) < 0$ for any t. If the two individuals follow the same recommendations, taking expectation over objects, the sum of their utility levels is negative, in contradiction with viability (even for a null cost c). Observe that this argument holds more generally when peers search for more than one recommendation or when they receive a statistic of recommendations, as will be investigated in section 3.4.

Assumption A0 (concavity assumption) The functions $logU(\theta, \alpha)$ and $logV(\alpha)$ are concave with respect to α in $]0, \alpha^{max}[$ for each $\theta \leq \alpha^{max}$.

Assumption A1 (elasticity assumption)
$\frac{-U'_\alpha}{U}(\theta, \alpha)$ decreases with θ and $\frac{-U'_\alpha}{U}(0, \alpha) \leq \frac{-V'}{V}(\alpha)$ for α, θ in $]0, \alpha^{max}[$

Under *A0*, PV is log-concave as a product of log-concave functions. Hence the set of viable sizes is a non-empty interval $[\underline{\alpha}, \overline{\alpha}]$ for a low enough cost c. Under the elasticity assumption *A1* the relative loss incurred by a peer due to an increase in the size is larger for the center. Furthermore these relative losses are all smaller than the relative decrease in the utility of an individual located at an extreme. In particular $\frac{-U'_\alpha}{U}(\alpha, \alpha) < \frac{-U'_\alpha}{U}(0, \alpha) \leq \frac{-V'}{V}(\alpha)$. These inequalities are compatible because $V'(\alpha)$ includes not only the variation due to the size but also the one due to the position, which is negative ($V'(\alpha) = [U'_\alpha + U'_\theta](\alpha, \alpha)$ and U'_θ is negative because U decreases with the distance to the center from (i) of Proposition 1). For example, assumptions *A0* and *A1* hold for a binary function.

3.2 Leaders' Choice

In practice, a 'leader' initiates a community and possibly defines criteria for accepting peers. The leader's optimal community size is given by the value α^0 that maximizes the payoff $P(\alpha)U(0, \alpha)$. This will be the leader's choice provided it is viable. Similarly let α^θ denote the value that maximizes the payoff $P(\alpha)U(\theta, \alpha)$, that is the preferred size of a θ-peer in a community centered at zero.

Proposition 2 *The leader's optimal choice is less than the peers' optimal one: $\alpha^0 \leq \alpha^\theta$. The leader's choice is*

1. *either the leader's optimum α^0 if it is viable, i.e. if $P(\alpha^0)V(\alpha^0) \geq c$, which occurs for a small enough cost; Peers who are not at the center would prefer a larger size and the community is closed to outsiders.*

2. *or the maximal viable size $\overline{\alpha}$. In that case all peers would prefer a larger size.*

The first point makes precise the direction of possible disagreements with the leader: when the leader is unconstrained and can choose his preferred size as in case *1*, peers all prefer a larger size than the leader. Except in the boundary case where the community is just viable, we have $P(\alpha^0)V(\alpha^0) > c$. Thus, close enough outsiders would achieve a positive payoff by joining (by continuity of the payoffs), but they are not allowed to do so: the community is closed. Otherwise, in case *2* when the leader's optimum is not viable, it is because it is too large. In that case all peers, including the leader, would benefit from an increase in the community size up to α^0. However no outsiders want to join and peers at the extreme of the community just cover their cost.

To illustrate Proposition 2, consider a Poisson process $P(\alpha) = 1 - e^{-\lambda\alpha}$ and a binary function. We first consider the impact of the probability of success and then discuss the impact that advertisements may have.

The impact of the probability of success. An improvement in the technology or an increase in the number of members due to an increase in Internet users results in an exogenous increase in the probability of success (other things being equal), here an increase in the parameter λ. Figure 2 depicts the maximal viable size $\overline{\alpha}$ (the increasing line) and the leader's optimum α^0 (the decreasing line) as a function of λ. Since the leader's choice is the minimum of these two values, increasing the population has different effects on the leader's choice depending on whether this choice is constrained or not. The following configurations are obtained as λ increases. First, for λ low enough, there is no viable community, second the leader's choice is constrained equal to the maximal viable size, and third the leader can choose his optimum value. This can be explained as follows. Increasing the population within a community makes it more attractive to outsiders. When the community is constrained by viability, for intermediate values of λ, these outsiders are welcome. As a result, the size is increased. Instead, when the community is closed, for a large enough λ, increasing the population allows the leader to choose a community restricted to peers whose tastes are more and more similar to his own: the size decreases. The impact of λ on the size directly translates into an impact on the precision of information: as λ increases, information is first made less precise (but the higher chance of getting some information compensates the loss) and then more and more precise.

Advertisements. Advertisements provide revenues that may change the leader's choice criteria. To simplify, assume that peers do not mind ads and that ads do not influence their preferences on the object on which they are searching information. Let the revenues generated by ads be proportional to the number of peers and consider two alternative ways of distributing them.

First, the leader captures all ad revenues. In that case he sets up a community that maximizes a combination of his own interests and the revenues. His choice is unchanged if the viability constraint binds. Otherwise, instead of choosing his own optimum, α^0, he chooses a larger size (between α^0 and $\overline{\alpha}$). The more he cares about revenues, the closer his choice to the maximal viable size. As a result, information is less precise. The effect can be substantial for large λ because the maximal viable size $\overline{\alpha}$ is large and α^0 is small. Whereas a community could be tailored to his specific tastes, the leader may choose a loose criteria so as to capture ad revenues.

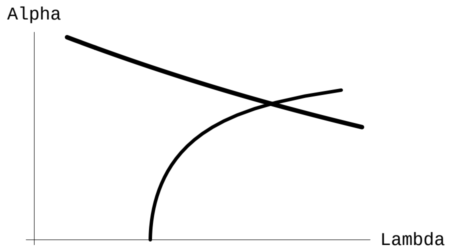

Figure 2: $c = 0.1$, $b/g = 1.5$

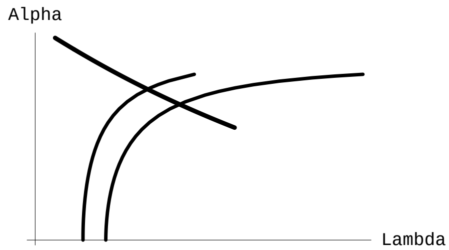

Figure 3: $c = 0.1$ and $c = 0.07$

Second, ad revenues are distributed equally among peers, which amounts to diminish cost c. This results in an increase in the maximal viable size and leaves the optimal leader's size unchanged. Hence, the leader's choice is closer and more often equal to his optimal value. In Figure 2, the maximal viable size is drawn for two distinct values of the cost (the upper increasing line corresponds to the lower value of the cost).

3.3 Enticing Contribution

Various factors influence the probability of successful search. Some result from a policy imposed to the community's members such as the enforcement of a minimal contribution rate. We assume here that P is influenced by the peers' contribution rates. A minimum rate is asked for, implemented through records on peers' contributions. The leader now chooses both the size and the minimal contribution rate.

We assume that the peer's participation cost c is an increasing function of his contributions. As a result, no peer will contribute more than the minimum required rate: his cost would increase with a null benefit since the impact of a single individual on the success probability is negligible. This is a standard effect in public good provision. Given the minimum required rate λ, let $P(\lambda, \alpha)$ and $c(\lambda)$ denote respectively the probability of success when *each* peer contributes λ and the incurred individual cost. P is non-decreasing and concave in λ and c is non-decreasing and convex.

Without constraint on viability, the leader's optimum is the value of (λ, α) that maximizes

$P(\lambda, \alpha)U(0, \alpha) - c(\lambda)$. The maximal viable size now depends on λ; it is denoted by $\overline{\alpha}(\lambda)$.

Proposition 3 *The leader's choice is*

1. *either the leader's optimum community if it is viable; the community is closed to outsiders. Peers who are not at the center would prefer a larger size and a lower participation.*

2. *or a community with maximal viable size $\overline{\alpha}(\lambda)$ for the chosen rate; the choice of λ trades off the benefits from increasing contribution and the loss due to a smaller community size (i.e. $\overline{\alpha}(\lambda)$ decreases at the chosen value of λ.)*

Given the chosen contribution rate, the choice of the size is dictated by the same considerations as in the previous section and leads to two cases. As for the contribution rate, note that the marginal benefit from increasing the contribution rate, $P_{\lambda}U(\theta, \alpha) - c'$, is decreasing as U with the distance to the center. Hence, surely, at the chosen contribution rate, the leader's marginal benefit is non-negative: otherwise a Pareto improvement within the community would be found by reducing the rate. Thus, when the leader is not constrained by viability as in case *1*, all peers with different characteristics would prefer to increase the size and to decrease the contribution rate. In case *2*, the leader would benefit from an increase in the contribution rate and from an increase in size. Increasing contribution however incites some peers at the extreme to leave thereby decreasing the size, which results in a trade-off.

3.4 Extensions

We briefly examine how results are modified in alternative differentiation models and investigate next the situation where peers have access to multiple signals.

Alternative differentiation models The previous analysis builds on the trade-off between the amount of information (increasing in the scope of the community) and its relevance (decreasing in the scope). Such a trade-off naturally arises in other settings. Consider the Hotelling model (1929) and the vertical differentiation model (Shaked and Sutton 1983). In both models, preferences and objects differ through a single characteristic. Uniform distributions are assumed.

In a Hotelling differentiation model, characteristics (both for objects and individuals) are on an interval instead of a circle. Individuals still value the objects according to their distance to their own location. Most properties extend. In particular, the expected value of a signal to a peer is decreasing in the size of the community, and communities must be of limited size (smaller than d^*) to be viable. In particular, when a leader is at a distance larger than $2d^*$ from both boundaries, Proposition 2 exactly applies (because in that case, for any community the leader might choose, i.e. with size smaller than d^*, the distributions of the recommended objects are identical in the circle and the line models).

In a vertical differentiation model, objects differ in quality. Individuals all agree on the ranking of quality levels but differ in their willingness to pay. For example, individuals' preferences are represented by a utility of the form $u(q-\theta)$ in which q is the quality level of the object and θ a minimum quality that makes the object valuable to a peer (taking $u(0)=0$). Individuals' tastes do not widely differ here so that peers may have less incentive to limit the access to a community than in the circle model. Let us first investigate viability.

Let $U(\theta,[\theta_1,\theta_2])$ denote the expected utility per recommendation for a θ-individual who follows the recommendations given by the peers of community $[\theta_1,\theta_2]$. Viability takes a slightly different form than in the circle model because optimal behavior in the absence of any recommendation may differ across peers: those with a low enough θ may benefit from buying whereas those with a high enough θ will not. The reservation utility of a θ-peer is therefore the maximum of 0 and $B(\theta)$ where $B(\theta)$ denotes the expected utility per object bought at random. A peer who is in a community adopts the strategy that is optimal without information when no answer to a query is obtained. This gives that the overall utility derived from joining community $[\theta_1,\theta_2]$ is $P(\beta)U(\theta,[\theta_1,\theta_2])+(1-P(\beta))\max(B(\theta),0)-c$ where $\beta=(\theta_2-\theta_1)/2$ denotes the community's size. The peer wants to join if this value is larger than the reservation value $\max(B(\theta),0)$. These conditions simplify into

$$\begin{aligned} P(\beta)U(\theta,[\theta_1,\theta_2])-c &\geq 0 \text{ for } \theta=\theta_2 \\ P(\beta)[U(\theta,[\theta_1,\theta_2])-B(\theta)]-c &\geq 0 \text{ for } \theta=\theta_1. \end{aligned}$$

The first condition says that a peer with the largest threshold, θ_2 is at least as well off by searching for a recommendation on an object rather than never buying. If it holds true, it is also satisfied for θ smaller than θ_2. It is violated when the community accepts 'easy' individuals who inflict a loss to a θ_2-peer by inciting too much to buy, which occurs for θ_1 low enough. The second condition says that a peer with the smallest threshold is at least as well off by searching for a recommendation on an object rather than buying it systematically. If it holds true, it is also satisfied for θ larger than θ_1 (by a simple argument). It is violated when the community accepts individuals who are too strict and make a θ_1-peer lose many opportunities, which occurs for θ_2 large enough. To sum up, viability requires preferences not to be too diverse.

Consider now the issue of choosing the scope of a community. Write a community as $[m-\beta,m+\beta]$. The value per signal can be shown to be decreasing in the 'scope', that is $U(\theta,[m-\beta,m+\beta]$ decreases[8] with β whatever θ in $[m-\beta,m+\beta]$. Hence, as in the circle model, increasing the scope

[8]Up to a normalization, take the objects' characteristics to be distributed on $[0,1]$. Utility per signal $U(\theta,[m-$

and making the senders' preferences more diverse is harmful to any peer. Taking into account the probability of finding the recommendation as a function of the size β, this defines the trade-off between receiving more useful information but less often. There is still a value for a community, possibly closed: peers may want to restrict the access to the community. There is however a main difference with horizontal differentiation models, which occurs when signals are numerous, as we now investigate.

Multiple signals So far, we have assumed that a peer searching for a recommendation stops at the first one she finds. We examine here the situation in which a peer may obtain several recommendations, as generated by the following behavior. A peer searches a given lapse of time and considers all the recommendations that she obtains during that lapse. The receipt of $\tilde{n}$ signals is described by a vector of 0 and 1, but the only relevant information is summarized in the number of positive signals, $\tilde{s}_{\tilde{n}}$. An alternative interpretation is that the community provides a statistic on the number of positive and negative recommendations. If all recommendations are recorded, this is the best statistic that can be provided in our setting. The analysis is easily adapted to less precise information (as for example, when a peer learns whether a majority of the recommendations are positive or negative). Consider a community C of size α (for the moment we do not need to specify the differentiation model). The expected utility is defined as

$$\sum_{n\geq 1} P(\alpha, n)U(\theta, C, n) \tag{4}$$

where $P(\alpha, n)$ denotes the probability of receiving n signals from a community of size α and $U(\theta, \alpha, n)$ denotes the expected utility derived by a θ-peer upon the receipt of n signals from C under optimal behavior. A natural assumption is that as α increases the chances of obtaining more signals in a stochastic dominance sense (i.e., $P(\alpha, n) - P(\alpha', n)$ increases with n for $\alpha > \alpha'$). For example, $P(\alpha, n)$ is given by a Poisson process: $P(\alpha, n) = exp^{-\lambda}\frac{\lambda^n}{n!}$.

A natural question is wether at the limit, when there are more and more recommendations, peers receive almost all the information that is necessary for them to make the right decision. To investigate this question, note that, given a t-object, each received signal is distributed according the probability of a positive signal from the community conditional on t. Let $P(yes \in C|t) = g^C(t)$ denote this probability. By the law of large numbers, the proportion of positive signals, $\tilde{s}_n/n$, converges to $g^C(t)$ when n tends to ∞. As n increases, the statistics $\tilde{s}_n$ (knowing n) are getting

$\beta, m+\beta]$ is equal to $\frac{1}{\beta}\int_{m-\beta}^{m+\beta} u(q-\theta)(q-(m-\beta))dq + \int_{m+\beta}^{1}(u(q-\theta)dq$. A marginal increase in the scope $d\beta$ leads to a marginal variation in utility equal to $\frac{d\beta}{\beta^2}\int_{m-\beta}^{m+\beta} u(q-\theta)[m-q]dq$. Since u is increasing, the inequality $[u(q-\theta)-u(m-\theta)][m-q] \leq 0$ holds for each q. We thus have $\int_{m-\beta}^{m+\beta} u(q-\theta)[m-q]dq \leq u(m-\theta)\int_{m-\beta}^{m+\beta}[m-q]dq = 0$. Thus marginal utility with respect to β is negative for each θ.

more informative in the sense of Blackwell[9] and furthermore each one is less informative than the value of $g^C(t)$. This gives that $U(\theta, C, n)$ is increasing with n and bounded above by $U(\theta, C, \infty)$, the utility derived by a θ-peer who is told the value of $g^C(t)$. Let us apply this property to the circle and the vertical differentiation models.

In the circle model, observe first that a community can be valuable to all its members only if its size is less than d^*. Because of weak risk aversion, two individuals distant of more than $2d^*$ cannot benefit both from the community whatever the number of signals.[10] Consider the value of $U(\theta, C, n)$, which is derived assuming that a θ-peer optimally behaves for each possible realization of s_n. This behavior is not as simple as following or not a recommendation as is the case of a single signal. Hence, peers may react differently on the receipt of n signals. The optimal strategy is characterized by a threshold value such that a peer buys the object if the number of positive recommendations exceeds that threshold. Similarly, the value of $U(\theta, C, \infty)$ is derived assuming that optimal behavior when a θ-peer is told the value of $g^C(t)$. Optimal behavior is easy to compute in that case as follows.

The probability of a positive signal from community of size α centered at 0 is given in footnote 5: It is null in the refusal zone where every peer says *no* (t is in $[-\alpha + d^*, \alpha - d^*]$), equal to 1 in the acquiescence zone where every peer says *yes* (t is in $[\alpha - d^*, -\alpha + d^*]$) and is linear in between. Hence a peer in the community who is told 0 or 1 knows for sure that the object is in the refusal or in the acquiescence zones, which is all the relevant information he needs to make the good decision (observe that this is not true for individuals outside the community). A value strictly between 0 and 1 indicates that the object is in the disagreement zone. Furthermore the value $g^\alpha(t)$ reveals the location t or $-t$ each with equal probability. Defining the threshold value for t (which depends on θ) by the equation $u(|\theta + t|) + u(|\theta - t|) = 0$ it is optimal for a θ-peer to buy the object if the value t exceeds that threshold and not to buy in the opposite case.[11] For peers who are located at the center, the threshold value is d^*: they achieve the same utility as under perfect information since they buy all the objects that are valuable to them and only these ones. Instead, for peers

[9] To see this, given a value s_n create a n-vector of 0 and 1 by assigning at random s_n values 1 and $n - s_n$ values 0. Taking the $n-1$ first components yields a $n-1$-vector that is distributed according to $n-1$ signals, which is informationally equivalent to the statistic s_{n-1}.

[10] Let the community be centered at 0 of size larger than d^* and choose θ with $d^* < \theta < \alpha$. We have that $u(|t + \theta|) + u(|t - \theta|) < 0$ whatever t since the sum of the distance $|t + \theta|$ and $|t - \alpha|)$ is the maximum of 2θ and $2t$, hence larger than $2d^*$. This implies that $U(\theta, C, n) + U(-\theta, C, n) < 0$ and by symmetry $U(\theta, C, n) < 0$ for all n. Hence individuals at or close to the extreme point behave as if they had no information and never buy. Whatever positive value for the cost c, they strictly prefer not to join the community.

[11] A θ-peer wants to buy if $u(|\theta + t|) + u(|\theta - t|) > 0$. The function is decreasing in t, and the inequality satisfied at $t = 0$ because the community size is smaller than d^* and the inequality is reversed at t equal d^* for $\theta \neq 0$ because of risk aversion and satisfied as an equation for $\theta = 0$.

who are not located at the center, the threshold value is strictly smaller than d^* (because of risk aversion). As a result, they lose some opportunities of valuable purchases and are strictly worse off than under complete information. Hence, since the value $U(\theta, C, \infty)$ provides an upper bound to $U(\theta, C, n)$, most peers do not reach the maximum payoff even if they have access to many recommendations.

Consider now a vertical differentiation model. In opposite to the circle model, there is no a priori upper bound on the size for a community to be viable. Let peers obtain a statistic on the recommendations from their community and consider as above the limit case when peers are told the probability for an object to be recommended (to which the frequency $\tilde{s}_n/n$ converges as n gets large). We show that this probability provides peers with all the relevant information, allowing them to behave as if they had complete information. The probability $P(yes \in [\theta_1, \theta_2]|q)$ is null for $q < \theta_1$ (refusal zone), equal to 1 for $q > \theta_1$ (acquiescence zone) and is linear in between in $[\theta_1, \theta_2]$. As in a circle model, a peer in the community who is told 0 or 1 knows for sure that the object is in the refusal or in the acquiescence zones, which is the relevant information to make the good decision (as before, this is not true for individuals outside the community since a 1 simply indicates that the quality exceeds the upper bound θ_2 and a 0 that it is less than θ_1). If the value is strictly between 0 and 1, the position q can be inferred: the probability for an object to be recommended reveals exactly its quality and again the peer can make the right decision. Hence peers buy the objects of quality larger than their reservation value θ and only those: peers get enough information (at the limit) to reach the maximum utility level possible. The basic reason for this result is that individuals' rankings are the same. Even though recommendations differ, their aggregation sends a clear message. This suggests an important difference in the rationale for forming communitites. Since all peers within a community obtain all the information they need at the limit, the largest one, without any restrictions on membership, should emerge: more recommendations are provided, useful to all. (This argument however should be checked more precisely: by arguing directly on the limit probability, we are neglecting the randomness in the (finite) statistic, the variance of which depends on the size of the community.)

4 Configurations of communities

We have so far considered a single community. This section analyzes the coexistence of several communities. Individuals are free to join whatever community open to them, or not to join any. We shall assume that each individual is willing to join one communiy at most. We first discuss equilibrium behaviors given a set of proposed communities, and next address the question of which proposals are made.

4.1 Equilibrium and Coordination issues

Given a set of proposed communities, individuals' choices are based on how much value they expect to derive from a community. Expectations on others decisions play a role and we look for an equilibrium under which these expectations are correct. As a result of expectations, we show that there may be multiple equilibria, some of which entail some form of coordination failure.

A proposal to form a community is represented by an arc. In the following discussion, it is important to distinguish between the potential members, that is the set of individuals whose characteristic belongs to the arc, and the set of individuals who join effectively, which is the community. When all potential members join, the community is said to be *full* . The probability $P(\alpha)$ should be interpreted as the success probability only for a full community of size α. If nobody joins the proposal for example, then the probability of success drops to zero. Hence, the viability condition, which is computed under the success probability P, implicitly assumes full communities.

Consider a proposed arc that intersects no other one, for example a single arc as in the previous section. Assume the arc to be viable. Since potential members face no other proposal, joining is 'rational' for them if they expect all others to join. Thus there is an equilibrium in which the full community forms. But, if nobody joins the proposal, then the probability of success drops to zero, making it rational not to join. Thus there is also an equilibrium in which no community forms. Coordination fails since everybody is worse off than at the equilibrium in which they all join. Such coordination failure does not occur under the following expectations. When a proposed arc is the only choice to its potential members, they compute the value of joining the community under maximal participation, that is by assessing the success probability by P.

When proposed arcs overlap, some individuals (in non-negligible number) have the choice between several proposals. Saying that each one picks up a preferred community among those open to him does not pinned down a well defined behavior. The reason is that individuals base their choice on the comparison between the utility levels expected from each proposal, but such levels depend in turn on the expected community memberships (in their characteristics and numbers) through the derived distributions of the signals and the impact on the success probability. Hence individuals' decisions and expectations on those interact. As a result, several configurations may be perfectly compatible with rational behavior. Let us illustrate this point with a simple example.

Let two viable proposals be overlapping and of identical size, $[-\alpha, \beta]$ and $[-\beta, \alpha]$ where $\alpha > \beta > 0$, as in Figure 4.1 (arcs are 'stretched'). Let arcs $[-\alpha, -\beta]$ and $[\beta, \alpha]$ be not viable. There is an equilibrium in which the full community $[-\alpha, \beta]$ forms: Assuming it forms and it is the only

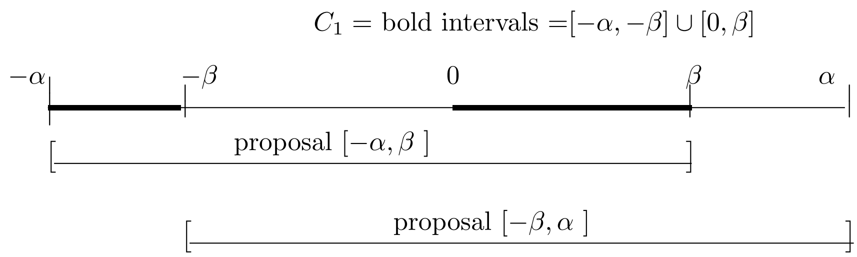

Figure 4: Coordination failure

one, all members who have the choice between the two proposals are better off by joining proposal $[-\alpha, \beta]$ (this is rational by viability), and the remaining individuals, who are in $[\beta, \alpha]$ and can only choose proposal $[-\beta, \alpha]$, do not accept it since no community included in $[\beta, \alpha]$ is viable. Similarly, there is an equilibrium in which the full community $[-\beta, \alpha]$ forms. In addition one may consider an equilibrium in which the peers who have the choice between the two proposals split themselves according to a 'cutoff' point. Here, taking 0 as the cutoff point by symmetry, the configuration with the two communities $[-\alpha, 0]$ and $[0, \alpha]$ (assumed to be viable) form an equilibrium: expecting each one forms, peers in $[-\beta, \beta]$ choose the proposal they prefer (since with equal size, the closest community is preferred, peers in $(-\beta, 0)$ prefer community $[-\alpha, 0]$ to $[0, \alpha]$ and the reverse holds for peers in $(0, \beta)$).

Rational behavior does not prevent coordination failure, as occurs when individuals split themselves in the 'wrong way'. Still in the same example, let individuals who have the choice between the two proposals, those in $[-\beta, \beta]$, split at the cutoff point 0 but make the opposite choice to the one just considered. This results in two communities, each one composed with the individuals whose characteristics belong to disjoint arcs: community C_1 with individuals' characteristics in the two arcs $[-\alpha, -\beta]$ and $[0, \beta]$, and C_2 with the two arcs $[-\beta, 0]$ and $[\beta, \alpha]$. For some parameters α and β, the two communities C_1 and C_2 form an equilibrium.[12] This occurs when individuals in $(-\beta, 0)$ prefer C_2 to C_1 and those in $(0, \beta)$ prefer C_1 to C_2: expecting individuals' choices to lead to the formation of C_1 and C_2 makes it a rational choice. For example, taking a binary function, this occurs for $\beta = 2\alpha/3$ (see computation in the appendix). There is some form of coordination failure, because everybody is better off at the equilibrium with the two communities $[-\alpha, 0]$ and $[0, \alpha]$, where they split at 0 but make the opposite choice. The argument does not depend on

[12]The expected utility derived from joining C_1 assuming it forms is

$$P(\alpha/2)[\frac{(\alpha-\beta)}{\alpha}U(\theta, [-\alpha, -\beta]) + \frac{\beta}{\alpha}U(\theta, [0, \beta])] - c$$

because the chances of obtaining the signal from each interval are proportional to their size.

symmetry and is more generally valid. Whenever an equilibrium results in two communities each one with two disjoint arcs, everybody would be better off if there was an exchange of peers of equal measure between the two communities: this exchange would keep the probability of success constant for both communities but would improve the quality of the information.

From now on, I focus on configurations without coordination failure, in which each community is composed with the set of individuals in a given arc. Such a configuration is described by a set of non-overlapping arcs. I focus on symmetric configurations. A symmetric configuration is given by n non-overlapping communities of equal size α, with n at most equal to π/α. For n strictly smaller than π/α there are gaps between any two consecutive arcs (we do not necessarily require the gaps to be of equal size). For n equal to π/α, the union of the arcs fill the entire circle and the configuration is said to be without gaps. In what follows, we address two questions. Which configurations may last? Which configurations are optimal (in a sense to be made precise)?

4.2 Stability

A configuration may last if it is stable against proposals to form new communities or to rearrange the existing ones. Stability requires that no proposal of change is accepted. Here, a proposal is said to be *accepted* if all members in the *new* community are better off than under the standing configuration. This notion is basically the blocking condition of cooperative game theory. As discussed previously, expectations matter for evaluating a proposal. We shall assume that peers evaluate a proposal under the expectation that everybody joins.

Denote by $\bar{u}(\theta)$ the utility level achieved by a θ-individual at the standing configuration, where θ is any point on the circle. For a symmetric configuration, $\bar{u}(\theta)$ is given by

$\bar{u}(\theta) = 0$ if the individual does not belong to any community,

$\bar{u}(\theta) = P(\alpha)U(d, \alpha) - c$ if he does and d is the distance to the center.

Consider a proposal to form a community. It can be written as $[m - \beta, m + \beta]$ where m is its center and β is its size. The utility for a θ-individual in $[m - \beta, m + \beta]$ who expects everybody to join the community is $P(\beta)U(|\theta - m|, \beta)$. Proposal $[m - \beta, m + \beta]$ is said to *block* a standing configuration with utility levels $\bar{u}(\theta)$ if

$$P(\beta)U(|\theta - m|, \beta) - c > \bar{u}(\theta) \text{ for each } \theta \text{ in }]m - \beta, m + \beta[. \tag{5}$$

The left hand side gives the utility evaluated by a θ-peer under the expectations that everybody joins the proposal. Condition (5) requires that such expectations are fulfilling since they make it rational for everybody to join.

I consider here proposals to form a new community that contains a community's leader, who can be assumed to be located at zero. Let us distinguish stability with respect to proposals that are larger in size than the existing community from those that are smaller. A symmetric configuration with communities' size α is *stable against enlargement* (resp. *stable against reduction*) if there is no proposal $[m-\beta, m+\beta]$ which includes zero and with size β larger (resp. smaller) than α that is accepted. Observe that although the proposals contain the leader, they are not necessarily centered at it. Hence an enlargement proposal does not necessarily contain the community $[-\alpha, \alpha]$ and a reduction proposal is not necessarily included in it. The only proposals that are excluded are those that try to attract people of two consecutive communities, less than half of each one because it contains no leader.

Stability against enlargement. Let α^{ext} be the maximum of PV, the payoff to an extreme individual. Observe that α^{ext} is less than α^0, thanks to assumptions *A0* and *A1* (the function $\log(PV)$ is concave and is decreasing at α^0 since $\frac{P'}{P}(\alpha^0) + \frac{V'}{V}(\alpha^0) \leq \frac{P'}{P}(\alpha^0) + \frac{U'_\alpha}{U}(0, \alpha^0) = 0$).

Proposition 4 *A symmetric configuration with gaps is stable against enlargement if and only the communities size is larger than the leader's choice. A symmetric configuration without gaps is stable against enlargement if the communities size is larger than α^{ext} (the maximizer of PV).*

For a configuration with gaps, it is easy to understand why stability requires communities size not to be smaller than the leader's choice. Recall that the leader's choice is $min(\alpha^0, \overline{\alpha})$. If $\alpha < min(\alpha^0, \overline{\alpha})$, consider a slight enlargement with the same leader. The new members, who do not belong to any community, are willing to join (because $\alpha < \overline{\alpha}$) and furthermore all insiders benefit from such an enlargement (because $\alpha < \alpha^0$ and using Proposition 2).

For a configuration without gaps, the stability requirement is much weaker. Since communities are adjacent, outsiders, whatever close, achieve a positive payoff at the standing configuration. Thus outsiders are more difficult to attract than in a configuration with gaps, which explains why smaller sizes than the leader's choice are compatible with stability.

Stability against reduction.

Proposition 5 *A symmetric configuration is stable against reduction if the communities' size is smaller than some value α^{int}, where $\alpha^{int} > \alpha^0$.*

The distinction between configurations with or without gaps is not relevant as far as reduction is concerned. As shown in the proof, stability against proposals that are smaller in size can be checked by considering only proposals that are included in one of the community: such a proposal excludes some of the members of an existing community and outsiders play no role.

Whereas stability to reduction requires communities not to be too large, stability to enlargement requires them to be large enough. From the previous results, both stability conditions are compatible.

Corollary. *A symmetric configuration with communities' size equal to the leader's choice is stable both against enlargement and reduction.*

A direct proof is straightforward since stability is checked against proposals that contain the leader. One cannot rely on general results to prove the existence of a stable configuration (see Demange 2005 for a survey). A first reason is that preferences are characterized by a parameter on a circle (as opposed to an interval). The second, and deeper reason, is that communities' payoffs may decrease with newcomers, as typically occurs under adverse selection. Hence even in a Hotelling or vertical differentiation models, in which the parameter belongs to an interval, known existence results cannot be applied.

In our model, whereas we have been unable to show the existence of a stable configuration, we have not found a counterexample either. Consider a configuration of size α^0 for instance. As stated previously, the excluded proposals are of size smaller than α^0 since they do not contain a leader. In fact they must be much smaller to be successful (if any). To make this precise, consider a proposal that tries to attract peers from two adjacent communities, $[-\alpha^0, \alpha^0]$ and $[\alpha^0, 3\alpha^0]$. Let it be centered at α^0 (it can be shown to be the one that have the more chances to be successful). It cannot contain $\alpha^0/2$: an $\alpha^0/2$-peer is at the same distance from the center in the standing configuration and in the proposal and prefers the size α^0 to any smaller size, hence the peer rejects the proposal. This implies that a proposal has to be of size smaller than $\alpha^0/2$ to be successful, which makes it unlikely because of the impact on the success porobability.

4.3 Efficient configurations

Our objective here is to investigate efficiency. In particular we study efficient configurations and their relationships with the stable ones. In a setup as here in which utility is not transferable, there is not a unique welfare criterion to assess efficiency. In line with our framework, we restrict ourselves to *anonymous* criteria. Such a criterion is characterized by a scalar function Φ that assigns a weight $\Phi(v)$ to a utility level v. Φ can be any increasing and continuous function, and is normalized by taking $\Phi(0) = 0$. To simplify the presentation, individual cost c is taken to be nil (which implies that the leader's choice is unconstrained). The welfare reached by a community of size α is given by

$$\int_{-\alpha}^{\alpha} \Phi[P(\alpha)U(\theta, \alpha)]d\theta. \tag{6}$$

and the total welfare at a configuration is the sum of the welfare within each community. Consider a symmetric configuration with viable community size α. Given the size, welfare is maximum at the maximal number communities. We shall neglect integer problems and take that there are $n = \pi/\alpha$ communities. Normalizing by the total size 2π for convenience, this gives the following expression for the welfare reached at a symmetric configuration with community size α

$$W_{\Phi}(\alpha) = \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} \Phi[P(\alpha)U(\theta,\alpha)]d\theta. \tag{7}$$

In words, the normalized welfare is the average weighted utility per member. A concave function Φ represents an aversion to inequality, and at the opposite a convex function favors dispersion by putting increasingly larger weight on large utility levels. This can be made precise by the following argument. Applying the intermediate value theorem to (7) gives that welfare is equal to $\Phi[P(\alpha)U(\hat{\theta}(\alpha),\alpha)]$ for some $\hat{\theta}(\alpha)$, the welfare of a 'representative individual'. The more concave the criterion is, the more distant from the center the representative individual is. Specifically, taking a criterion Φ_1 that is a concave transformation of Φ (i.e., $\Phi_1 = G(\Phi)$ for some G increasing and concave) $\hat{\theta}_1(\alpha)$ is larger than $\hat{\theta}(\alpha)$ (with obvious notation). At the limit, taking increasingly concave criteria, the representative position approaches the extreme position, ($\hat{\theta}(\alpha)$ tends to α) leading to the Rawlsian criterion, which puts weight on the minimum utility level only (a Rawlsian criterion is not associated to a function Φ and is obtained only at the limit). Similarly, an 'elitist' criterion that only values the maximum utility level within a community, here the center's utility level, is obtained as a limit of increasingly convex functions. To sum up, a criterion that is 'very' concave favors individuals with the lowest utility levels in a community, that is individuals close to the extreme, and at the opposite, a 'very' convex criterion favors the individuals with the highest utility levels in a community, those close to the center.

A Φ-optimal size is given by the value α^{Φ} that maximizes W_{Φ}. To understand the determinants of an optimal size, observe that there are two effects as the size of each community increases: a pure size effect within a community as if each peer could stay at the same distance to the center whereas its community expands, and a position effect, due to the rearrangement of the communities resulting from a reduction in their number. The position effect increases the distance to the center on average, which is detrimental. The size effect may or may not be beneficial to peers depending on their positions and the community's size. An optimal size balances the two effects. These position and size effects are summarized by considering the impact of the size on the 'representative' individual's utility, $P(\alpha)U(\hat{\theta}(\alpha),\alpha)$. Whereas an individual located at position $\hat{\theta}(\alpha)$ may or may not benefit from an increase in the size of the community, the representative individual's position is changing and increases as α increases, which is detrimental. At $\alpha = \alpha^0$, the size effect is beneficial so that the two effects are in opposite direction. Depending on which effect is stronger, the optimal size may be larger or smaller than α^0. In the binary case for example with Φ linear, $\Phi(v) = v$, welfare is equal to $2\pi P(\alpha)U(\alpha/\sqrt{2}, \alpha)$, that is the 'representative' individual is located at $\alpha/\sqrt{2}$. The detrimental position effect can be shown to be stronger than the size effect: α^Φ is smaller than α^0. This is more generally true under an additional assumption *A2* that we now introduce. Consider $U(k\alpha, \alpha)$, the utility per signal associated to a fixed relative position k (between 0 and 1) in the community as the size α varies. Observe that it coincides with $V(\alpha)$ for $k = 1$. $U(k\alpha, \alpha)$ decreases in α since U is decreasing in both arguments, the position and the size. The relative loss is given by $\frac{-[kU_\theta + U_\alpha]}{U}(k\alpha, \alpha)$. At $k = 0$ the position effect is null, and the relative loss coincides with $\frac{-U_\alpha}{U}(0, \alpha)$, whereas at $k = 1$, it coincides with $\frac{-V'}{V}(\alpha)$. Hence, the elasticity assumption *A1* requires the relative loss to be smaller for $k = 0$ than for $k = 1$. Assumption *A2* strengthens this condition by requiring the relative loss to be increasing in the relative position k. Writing $k\alpha = \theta$ this is stated as follows.

$$AssumptionA2 \qquad \frac{-V'}{V}(\alpha) \geq \frac{-[U_\alpha + \frac{\theta}{\alpha}U_\theta]}{U}(\theta, \alpha) \geq \frac{-U_\alpha}{U}(0, \alpha) \text{ for any } \theta \leq \alpha \leq \alpha^{max}.$$

Proposition 6 *Under A2, for any criterion Φ strictly positive, increasing and continuous, the optimal Φ-size of communities is larger than the optimal size for the Rawlsian criterion and smaller than the optimal size for the elitist criterion: $\alpha^{ext} < \alpha^\Phi < \alpha^0$.*

Recall that the preferred size of any peer is larger than the leader's one (from Proposition 2). Thus, in a configuration with communities' size smaller than α^0, every peer expects to benefit from an increase in the size of his own community. If there are gaps, the configuration is surely not efficient: expanding each community until either communities become adjacent or the size α^0 is reached leads to a Pareto improvement. Without gaps, such an expansion entails some rearrangement in the communities, which leads to an increase on average in the distance to communitites' centers. These negative external effects are not taken into account by peers as they evaluate their community. According to proposition the overall impact on welfare (as assessed by Φ) is negative when the communities' size is larger than α^Φ, which is smaller than α^0. When the size is larger than α^0, negative external effects are large so that the configuration is suboptimal whatever the criteria Φ (but there are some peers, those close to the leader, who do not want their community to expand).

It is difficult to predict which configurations will emerge, and whether some are more likely than others, without making explicit some ad-hoc formation process. However it is safe to say that at the beginning of a process only communities with size larger than α^0 will form, simply because when there are no constraints, peers unanimously reject smaller sizes. (This is also why stability requires sizes larger than α^0 when there are gaps). Hence a community with size smaller than α^0

can form only when a new proposal is made that attracts peers from two neighbored communities and results in a split. This suggests that efficiency may be rather difficult to reach.

Finally, when individuals support a cost that is small enough so as not to affect the optimal leader's choice, the efficiency analysis straightforwardly extends since the total cost paid at a configuration without gaps is constant, independent of the configuration.

5 Concluding remarks.

This paper considers a community as a cluster of individuals with similar preferences. Under anonymity and limited language, the improvement in the value of information determines the scope of a community. This has been analyzed in detail when recommendations are sufficiently rare so that a peer uses the first one obtained, if any. With multiple signals, the value of a restricted community is still positive when individuals truly differ in their tastes, as in a horizontal differentiation model. In contrast, in a vertical differentiation model, where individuals agree on ranking objects by a quality characteristic and differ only by the minimum acceptable level, the value of a community may still be positive under a limited number of signals but vanishes as the number of available recommendations per object increases. Hence, limited language and anonymity do not necessarily imply that segregation into communities is valuable. We have also investigated the formation of several communities under the assumption that an individual belongs to a single community at most and shown that stability and efficiency often enters into conflict. A natural development is to consider different behaviors for individuals or communities, and to pursue the study of aggregation of recommendations. Many questions are raised. Let us mention a few:

(i) Can we predict whether some aggregation criteria are more likely to be chosen, more apt to ensure some sort of efficiency, of stability?

(ii) How is the analysis modified if individuals benefit from joining several communities?

(iii) How would firms with profit criteria and communities interact? Can they coexist?

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6 Proofs

Proof of Proposition 1. Let $\alpha \leq d^*$. From (2), we have $U(\theta,\alpha) = p\int_{-\pi}^{\pi} f^\alpha(t)u(d(\theta,t))dt$ where f^α is the density of objects conditional on the receipt of a *yes* from $[-\alpha,\alpha]$ and $p = P(yes \in [-\alpha,\alpha]) = d^*/\pi$. We first compute f^α. The prior density of t is $f(t) = 1/(2\pi)$. The density $f^\alpha(t)$ and the conditional probability of a positive signal given t, $g^\alpha(t) = P(yes \in [-\alpha,\alpha]|t)$, are related by Bayes formula:

$$f^\alpha(t) = P(yes \in [-\alpha,\alpha]|t)\frac{f(t)}{P(yes \in [-\alpha,\alpha])} = \frac{1}{2d^*}g^\alpha(t).$$

Given t, individuals with a type θ in $[t-d^*, t+d^*]$ say *yes* and others say *no*. Hence

$$g^\alpha(t) = P(yes \in [-\alpha,\alpha]|t) = \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} 1_{[t-d^*,t+d^*]}(\theta)d(\theta)$$

In community $[-\alpha,\alpha]$, each member sends a signal *yes* for $t \in [\alpha - d^*, -\alpha + d^*]$, which we call the *acquiescence zone*, and each one sends *no* on $[\alpha + d^*, -\alpha - d^*]$, which we call the *refusal zone*. Outside these two zones there is *disagreement*. Since $\alpha \leq d^* \leq \pi/2$, all zones are non-empty. By symmetry $g^\alpha(t) = g^\alpha(-t)$. Restricting to $t \geq 0$ one has :

$$g^\alpha(t) = \begin{cases} 1 & \text{for } t \in [0, -\alpha + d^*] \text{ acquiescence zone} \\ \dfrac{\alpha + d^* - t}{2\alpha} & \text{for } t \in [-\alpha + d^*, \alpha + d^*] \text{ disagreement zone} \\ 0 & \text{for } t \in [\alpha + d^*, \pi] \text{ refusal zone} \end{cases}$$

which proves footnote 5.

To prove the monotonicity properties of U, it is convenient to rewrite it. Let $F(\theta,\alpha;.)$ denote the distribution of the distance to θ of the objects that are recommended by community $[-\alpha,\alpha]$:

$$F(\theta,\alpha;\delta) = \int_{-\pi}^{\pi} f^\alpha(t)1_{[\theta-\delta,\theta+\delta]}(t)dt. \tag{8}$$

U can be written as

$$U(\theta,\alpha) = p\int_{-\pi}^{\pi} u(\delta)dF(\theta,\alpha;\delta) \tag{9}$$

that is, $U(\theta,\alpha)$ is the expectation of $u(\delta)$ -the utility for an object distant of δ- under the distribution of the distance to θ of the objects that are recommended by community $[-\alpha,\alpha]$ time the probability of a positive recommendation. Function u is decreasing. Hence, U is decreasing with respect to θ or to α (property (i) or (ii)) if the distributions of the distance δ are increasing by first order stochastic dominance as θ or α increases. Recall that these distributions increase with θ if for any $\delta, 0 \leq \delta \leq \pi$, any θ, θ' with $\theta \leq \theta'$

$$F(\theta',\alpha;\delta) \leq F(\theta,\alpha;\delta)$$

that is the function $F(.,\alpha;\delta)$ *decreases* with respect to θ. Similarly the distributions increase with α if function F *decreases* with α.

(i) Let us show that F decreases with positive θ. From expression (8), F is derivable with respect to θ with a derivative equal to $f^\alpha(\theta+\delta)-f^\alpha(\theta-\delta)$. This term is always non-positive for θ positive because the point $\theta-\delta$ is closer to the center than $\theta+\delta$.

(ii) Let us show that F decreases with respect to α. F is derivable since function $f^\alpha(t)$ is derivable almost everywhere with respect to α. Note that f^α has a null derivative except on the disagreement zones. Hence taking the derivative under the integral (8), one has $\partial F/\partial\alpha = I+J$ where

$$I=\int_{-\alpha+d^*}^{\alpha+d^*} f^\alpha_\alpha(t)1_{[\theta-\delta,\theta+\delta]}(t)dt \text{ where } f^\alpha_\alpha(t)=-\frac{d^*-t}{8d^*\alpha^2}.$$

and J is defined similarly over the negative disagreement zone. We show $I\le 0$. Note that f^α_α is antisymmetric around d^*. Making a change of variable, I writes

$$I=\int_{d^*}^{\alpha+d^*} f^\alpha_\alpha(t)\left[1_{[\theta-\delta,\theta+\delta]}(t)-1_{[\theta-\delta,\theta+\delta]}(2d^*-t)\right]dt$$

Since f^α_α is positive on $[d^*,\alpha+d^*]$ it suffices to show that the term in brackets is non-positive, or that when $t\in[d^*,\alpha+d^*]\cap[\theta-\delta,\theta+\delta]$ then $2d^*-t\in[\theta-\delta,\theta+\delta]$. Geometrically, this is true because the middle of $[\theta-\delta,\theta+\delta]$, θ, is by assumption less than d^*, the middle of $[-\alpha+d^*,\alpha+d^*]$. More formally observe that $t\in[d^*,\alpha+d^*]\cap[\theta-\delta,\theta+\delta]$ implies $d^*\le t\le\theta+\delta$, which gives $2d^*-(\theta+\delta)\le 2d^*-t\le d^*\le\theta+\delta$. Now since θ is by assumption less than d^*, we also have $\theta-\delta\le 2d^*-(\theta+\delta)$, and finally $2d^*-t\in[\theta-\delta,\theta+\delta]$, the desired result. Hence I is surely non-positive. The same argument applies to show $J\le 0$, which proves $\partial F/\partial\alpha\le 0$.

(iii) follows: V decreases, is positive for α equal to 0 and negative for α larger than d^*. ∎

Proof of proposition 2. The optimal choice of the leader is the value of α that maximizes $P(\alpha)U(0,\alpha)$ under the viability constraint $P(\alpha)V(\alpha)\ge c$. Viability implies that α is smaller than α^{max}, the value for which $U(\alpha,\alpha)=V(\alpha)$ is null. For such sizes, U and V are positive. The objective and the constraint are log-concave so that the first order condition is sufficient (alternatively we could work with $logPU$ and $logPV$). Let μ be the multiplier associated with the constraint. The first order condition is

$$P_\alpha U(0,\alpha)+PU_\alpha(0,\alpha)+\mu[P_\alpha V+PV'](\alpha)=0. \tag{10}$$

If μ is null, the optimal choice is α^0 that maximizes $PU(0,\alpha)$, the leader's optimum. If μ is positive, the constraint binds: α^0 is not viable, i.e. outside the interval $[\underline{\alpha},\overline{\alpha}]$. Furthermore the solution solves $P(\alpha)V(\alpha)=c$, hence is either $\underline{\alpha}$ or $\overline{\alpha}$. Note that PV increases at $\underline{\alpha}$ and decreases at $\overline{\alpha}$. From (10), the derivatives of PU and PV are of opposite sign. If the latter derivative is positive, PV

increases: the solution is $\underline{\alpha}$. Since under *A1*, $P_\alpha V + PV' > 0$ implies $P_\alpha U(0,\alpha) + PU_\alpha(0,\alpha) \geq 0$, it must be that the derivative of PV is non-positive: the solution is $\overline{\alpha}$. Furthermore we surely have $P_\alpha U(0,\alpha) + PU_\alpha(0,\alpha) > 0$ at $\alpha = \overline{\alpha}$.

Let us consider now θ-peers in the community with $0 < \theta$. Under *A0*, the function $logPU$ is concave with respect to α, with a derivative given by $[\frac{P_\alpha}{P}(\alpha) + \frac{U_\alpha}{U}(\theta,\alpha)]$. From *A1* $[\frac{P_\alpha}{P}(\alpha) + \frac{U_\alpha}{U}(0,\alpha)] \geq 0$ implies $[\frac{P_\alpha}{P}(\alpha) + \frac{U_\alpha}{U}(\theta,\alpha)] > 0$. Thus, if the leader's choice is unconstrained, equal to α^0, the first term is null at α^0, which implies that $logP(\alpha^0)U(\theta,\alpha^0)$ increases for $0 < \theta$: peers not at the center prefer a larger size. If the leader's choice is constrained, we have seen that the first term is strictly positive at the leader's choice $\overline{\alpha}$, hence all peers would prefer a larger size.

Finally observe that when the cost is null, the leader is unconstrained: since the α^0-peer prefers a larger size than the leader, we surely have $U(\alpha^0,\alpha^0) > 0$. ■

Proof of proposition 3. The optimal choice of the leader is the value that maximizes $P(\lambda,\alpha)U(0,\alpha) - c(\lambda)$ over (λ,α) subject to $P(\lambda,\alpha)V(\alpha) - c(\lambda) \geq 0$. Let μ be the multiplier associated with the constraint. The first order conditions are

$$P_\alpha U(0,\alpha) + PU_\alpha(0,\alpha) + \mu[P_\alpha V + PV'](\alpha) = 0 \tag{11}$$

$$P_\lambda U(0,\alpha) - c'(\lambda) + \mu[P_\lambda V(\alpha) - c'(\lambda)] = 0 \tag{12}$$

When μ is null, the viability constraint does not bind, and the leader can choose its optimal value. The same argument as in proposition 2 yields that for the chosen value of λ other peers would like the size to increase. As for the contribution rate, since $U(\theta,\alpha) \leq U(0,\alpha)$, (12) yields $P_\lambda U(\theta,\alpha) - c'(\lambda) \leq 0$: a θ-peer would prefer a smaller contribution rate.

When μ is positive, we know that (11) and *A1* implies that α is set at the maximal viable size associated to the chosen λ, $\overline{\alpha}(\lambda)$. From the first order condition on λ (12), $P_\lambda U(0,\alpha) - c'(\lambda)$ and $P_\lambda V(\alpha) - c'(\lambda)$ are of opposite sign. Since $U(0,\alpha) > V(\alpha)$ it must be that the former is positive: the leader would prefer to increase the contribution rate. ■

Proof of Proposition 4. Consider a configuration with gaps. We proved in the text that a necessary condition for stability against enlargement is that the community size α be larger than the leader's choice, that is larger than $min(\alpha^0,\overline{\alpha})$. Conversely let α be between $min(\alpha^0,\overline{\alpha})$ and $\overline{\alpha}$. If α is the maximal viable size, an enlargement is not viable so no outsider wants to join. If the size α is between α^0 and $\overline{\alpha}$, then it is stable for another reason: the individuals at or close to the leader do not benefit from an increase in size and if the enlargement is not centered, half of them are further away from the leader, also diminishing their utility (from Proposition 2).

Consider now a configuration without gaps with community size α larger than α^{ext}. At the standing configuration, the minimum utility level is achieved by an individual located at an extreme point of a community, hence $\bar{u}(\theta) \geq (PV)(\alpha)$ for each θ. Given a proposal to an enlargement of size β larger than α, let us consider an individual located at an extreme point of the proposal. If the proposal is accepted, his expected payoff is $(PV)(\beta)$. This is strictly smaller than $(PV)(\alpha)$ since PV decreases for size larger than α^{ext}: the peer would be strictly worse off by accepting. By continuity, this is also true for individuals close to the extreme of the new community: the proposal is rejected. ∎

Proof of Proposition 5 The proof is divided into three steps.

Step 1. We first show that stability can be checked by considering proposals that are centered at the leader's position. Let $[-m-\beta, -m+\beta]$ be a reduction proposal, $\beta < \alpha$, centered at $-m$, $0 < m$. We have $m < \beta$ because the leader belongs to the proposal. We show that if it accepted, then the centered proposal $[-\beta, \beta]$ obtained by translation is also accepted,. Formally we show that

$$P(\beta)U(\theta, [-m-\beta, -m+\beta]) - c > \bar{u}(\theta) \text{ for each } \theta \in [-m-\beta, -m+\beta] \tag{13}$$

implies

$$P(\beta)U(\theta, \beta) - c > \bar{u}(\theta) \text{ for each } \theta \in [-\beta, \beta] \tag{14}$$

By symmetry it suffices to show (14) for positive θ

For θ in $[0, -m+\beta]$, this is clear since θ belongs to proposal $[-m-\beta, -m+\beta]$ and is closer to the center of $[-\beta, \beta]$ hence $U(\theta, \beta) > U(\theta, [-m-\beta, -m+\beta])$.

For θ in $[m, \beta]$, (13) holds for the translated point $\theta - m$(since it belongs to $[-m-\beta, -m+\beta]$) Furthermore $\theta - m$ is closer to 0 than θ. Hence we have $\bar{u}(\theta - m) \geq \bar{u}(\theta)$. Using $U(\theta, \beta) = U(\theta - m, [-m-\beta, -m+\beta])$ this finally yields

$$P(\beta)U(\theta, \beta) - c = P(\beta)U(\theta - m, [-m-\beta, -m+\beta]) - c > \bar{u}(\theta - m) > \bar{u}(\theta)$$ ∎

Step 2. From Step 1, stability against reduction is checked by considering proposals centered at 0, $[-\beta, \beta]$, $\beta < \alpha$. For such proposals we have $\bar{u}(\theta) = P(\alpha)U(\theta, \alpha) - c$ for each $\theta \in [-\beta, \beta]$. Hence proposal $[-\beta, \beta]$ is accepted if

$$P(\beta)U(\theta, \beta) > P(\alpha)U(\theta, \alpha), \ \theta \in]-\beta, \beta[. \tag{15}$$

By continuity, inequalities (15) imply $P(\beta)U(\beta, \beta) \geq P(\alpha)U(\beta, \alpha)$. Thus a sufficient condition for stability against reduction is

$$P(\alpha)U(\beta, \alpha) > P(\beta)U(\beta, \beta) \text{ for each } 0 < \beta < \alpha. \tag{16}$$

We show that, conversely conditions (16) are necessary for stability against reduction. By contradiction, let inequality $P(\beta^*)U(\beta^*, \beta^*) \geq P(\alpha)U(\beta^*, \alpha)$ be met for some β^* smaller than α.

Proposal $[-\beta^*, \beta^*]$ makes β^*- or $-\beta^*$-individuals at least as well off. The elasticity conditions *A1* then imply that all members in $]-\beta^*, \beta^*[$ are strictly better off. To see this, observe that the ratio $\frac{U(\theta,\alpha)}{U(\theta',\alpha)}$ decreases with α for $\theta < \theta'$ (because the derivative with respect to α of the log of the ratio is $\frac{U_\alpha(\theta,\alpha)}{U(\theta,\alpha)} - \frac{U_\alpha(\theta',\alpha)}{U(\theta',\alpha)}$ which is negative under $A1$). Taking $\theta' = \beta^*$, this gives

$$\frac{U(\theta,\alpha)}{U(\theta,\beta^*)} \leq \frac{U(\beta^*,\alpha)}{U(\beta^*,\beta^*)} \text{ for } \theta < \beta^* < \alpha.$$

Using the assumption $P(\alpha)U(\beta^*,\alpha) \leq P(\beta^*)U(\beta^*,\beta^*)$, this gives

$$P(\alpha)U(\theta,\alpha) < P(\alpha)U(\theta,\beta^*)\frac{U(\beta^*,\alpha)}{U(\beta^*,\beta^*)} \leq P(\beta^*)U(\theta,\beta^*) \text{ for each } \theta < \beta^*$$

Thus, by symmetry, (15) is satisfied at $\beta = \beta^*$: proposal $[-\beta^*, \beta^*]$ is accepted.

Step 3. Consider configurations stable against reduction. From Step 2, their sizes are described by the viable sizes that satisfy inequalities (16). Denote by A this set. We show that A is an interval of the form $]0, \alpha^{int}[$.

First the set A is non-empty: It contains $]0, \alpha^{ext}]$, where α^{ext} is the maximum of $(PV)(\alpha)$. This follows from the fact that α^{ext} is viable (otherwise there is no viable size). and the inequalities

$$P(\alpha)U(\beta,\alpha) > P(\alpha)U(\alpha,\alpha) = P(\alpha)V(\alpha) > P(\beta)V(\beta) \text{ for each } \beta,\alpha,\ 0 < \beta < \alpha < \alpha^{ext}$$

The first inequality holds because $U(\beta,\alpha)$ decreases with α, and the second one because the log-concave function PV increases on $[0, \alpha^{ext}]$.

Second, if (16) is satisfied for all viable sizes, $A =]0, \overline{\alpha}]$ and take $\alpha^{int} = \overline{\alpha}$. Otherwise, consider the smallest value, α^{int} for which (16) does not hold. We have $P(\alpha)U(\beta,\alpha) > P(\beta)U(\beta,\beta)$ for each $\alpha < \alpha^{int}$ and $\beta < \alpha$. Furthermore, since (16) does not hold at α^{int}, there is some $\alpha^* < \alpha^{int}$ with $P(\alpha^{int})U(\alpha^*, \alpha^{int}) \leq P(\alpha^*)V(\alpha^*)$. Therefore, by continuity, $P(\alpha^{int})U(\alpha^*, \alpha^{int}) = P(\alpha^*)U(\alpha^*, \alpha^*)$ and the function $\alpha \to P(\alpha)U(\alpha^*, \alpha)$ decreases at $\alpha = \alpha^{int}$. By logconcavity, it decreases for larger values than α^{int}, hence

$$P(\alpha)U(\alpha^*,\alpha) \leq P(\alpha^{int})U(\alpha^*,\alpha^{int}) = P(\alpha^*)U(\alpha^*,\alpha^*).$$

Inequality (16) does not hold for a size larger than α^{int}: the set A is the interval $]0, \alpha^{int}[$. ∎

Proof of Proposition 6. Welfare W_Φ (7) writes:

$$W_\Phi(\alpha) = \frac{1}{\alpha}\int_0^\alpha \Phi[P(\alpha)U(\theta,\alpha)]d\theta. \tag{17}$$

Take the derivative with respect to α:

$$\begin{aligned} W'_\Phi(\alpha) &= \frac{1}{\alpha}\Big\{\int_0^\alpha \Phi'[P(\alpha)U(\theta,\alpha)][PU_\alpha + P_\alpha U](\theta,\alpha)d\theta + \Phi[P(\alpha)U(\alpha,\alpha)] \\ &- \frac{1}{\alpha}\int_0^\alpha \Phi[P(\alpha)U(\theta,\alpha)]d\theta\Big\} \end{aligned} \tag{18}$$

Integration by parts of $\theta \to \theta\Phi[P(\alpha)U(\theta,\alpha)]$ over the interval $[0,\alpha]$ gives

$$\alpha\Phi[P(\alpha)U(\alpha,\alpha)] - \int_0^\alpha \Phi[P(\alpha)U(\theta,\alpha)]d\theta = \int_0^\alpha \theta\Phi'[P(\alpha)U(\theta,\alpha)]P(\alpha)U_\theta(\theta,\alpha)d\theta.$$

Hence

$$W'_\Phi(\alpha) = \frac{1}{\alpha}\int_0^\alpha \Phi'[P(\alpha)U(\theta,\alpha)][PU_\alpha + P'U + \frac{\theta}{\alpha}PU_\theta](\theta,\alpha)]d\theta. \tag{19}$$

The term $[PU_\alpha + P_\alpha U]$ represents the size effect. For a θ-peer, it depends on the community size being smaller or larger than his preferred size. In particular, for a community size strictly less than α^0, a small increase in size is beneficial to all peers. The term $[\frac{\theta}{\alpha}PU_\theta]$, which is always negative, reflects the position effect. Writing

$$\left[PU_\alpha + P'U + \frac{\theta}{\alpha}PU_\theta\right](\theta,\alpha) = (PU)(\theta,\alpha)\left[\frac{P'}{P}(\alpha) + \frac{U_\alpha + \frac{\theta}{\alpha}U_\theta}{U}(\theta,\alpha)\right]$$

and using Assumption *A2* gives

$$\left[PU_\alpha + P'U + \frac{\theta}{\alpha}PU_\theta\right](\theta,\alpha) \le (PU)(\theta,\alpha)\left[\frac{P'}{P}(\alpha) + \frac{U_\alpha}{U}(0,\alpha)\right]$$

When the size α is larger than α^0, the term on the right inside the square brackets is non-positive, giving that the function $[PU_\alpha + P'U + \frac{\theta}{\alpha}PU_\theta](\theta,\alpha)$ is negative for all positive θ. Using the expression (22) and the positivity of Φ', this implies the inequality $W'_\Phi(\alpha) < 0$ for $\alpha \ge \alpha^0$: surely $\alpha^\Phi < \alpha^0$.

A similar argument yields $\alpha^\Phi > \alpha^{ext}$ where α^{ext} is the maximum of the payoff to an extreme individual $PU(\alpha,\alpha) = (PV)(\alpha)$. For $\alpha \le \alpha^{ext}$ $\frac{P'}{P}(\alpha) + \frac{V'}{V}(\alpha) \ge 0$ (by concavity of $\log(PV)$. Under *A2*

$$[PU_\alpha + P'U + \frac{\theta}{\alpha}PU_\theta](\theta,\alpha) \ge (PU)(\theta,\alpha)\left[\frac{P'(\alpha)}{P(\alpha)} + \frac{V'}{V}\right] \ge 0$$

which gives $W'_\Phi(\alpha) > 0$ for $\alpha \le \alpha^{ext}$, and finally implies $\alpha^{ext} < \alpha^\Phi$. ■

Appendix: binary function. Given a binary function, set $k = \frac{(b+g)}{4gd^*}$. As computed below the utility levels for individuals inside a community of size $\alpha \le d^*$ is given by

$$U(\theta,\alpha) = pg[1 - k\alpha(1 + \frac{\theta^2}{\alpha^2})] \text{ for } \theta \in [-\alpha,\alpha] \tag{20}$$

and for those outside the community at a distance less than $\theta \in [-d^*, d^*]$ is

$$U(\theta,\alpha) = pg[1 - 2k\theta] \text{ for } \alpha < |\theta| \le d^*. \tag{21}$$

Let us check that assumptions *A0*, *A1*, and *A2* are satisfied. Equation (20) gives $V(\alpha) = pg(1-2k\alpha)$ which gives the maximum viable size $\alpha^{max} = 1/2k$ and

$$-\frac{V'}{V}(\alpha) = \frac{2k}{1-2k\alpha} \text{ and } -\frac{U_\alpha}{U}(\theta,\alpha) = k(1 - \frac{\theta^2}{\alpha^2})/[1 - k(\alpha + \frac{\theta^2}{\alpha})].$$

A0 is met since U and V are concave in α. *A1.* Since $2V(\alpha) = g(1 - 2k\alpha)$, we have

$$-\frac{V'}{V}(\alpha) = \frac{2k}{1-2k\alpha} > \frac{k}{1-k\alpha} = -\frac{U_\alpha}{U}(0,\alpha)$$

Function $-\frac{U_\alpha}{U}(\theta, \alpha)$ is decreasing in θ^2 if $\frac{k}{\alpha} < \frac{1}{\alpha^2}[1 - k\alpha]$ or equivalently if $1 - 2k\alpha > 0$, that is $\alpha \leq \alpha^{max} = 1/2k$. Consider now *A2.* One has:

$$U(\theta, \alpha) = U(0, \alpha) - pgk\frac{\theta^2}{\alpha},\ U_\alpha(\theta, \alpha) = U_\alpha(0, \alpha) + pgk\frac{\theta^2}{\alpha^2},\ U_\theta(\theta, \alpha) = -2pgk\frac{\theta}{\alpha}.$$

This gives $[U_\alpha + \frac{\theta}{\alpha^0} U_\theta] = U_\alpha(0, \alpha) - pgk\frac{\theta^2}{\alpha^2}$, which is decreasing with θ in $[0, \alpha]$.

Computation of (20). We shall use repeatedly that for t_1, t_2 in the (positive) disagreement zone one has

$$\int_{t_1}^{t_2} f^\alpha(t)dt = \frac{-1}{8d^*\alpha}(\alpha + d^* - t)^2|_{t_1}^{t_2} = \frac{1}{8d^*\alpha}(t_2 - t_1)(2\alpha + 2d^* - (t_2 + t_1)) \tag{22}$$

Let us compute the utility derived from buying conditional on a *yes*, for a θ-individual in the community, i.e., U divided by p. For objects $t \geq 0$, he achieves:

in the acquiescence zone $[0, -\alpha + d^*]$: g,

in the disagreement zone $[-\alpha + d^*, \alpha + d^*]$: g for any object in $[-\alpha + d^*, \theta + d^*]$, and $-b$ on $[\theta + d^*, \alpha + d^*]$. No object in the refusal zone receives a *yes*. This gives using (22)

$$g\frac{d^* - \alpha}{2d^*} + \frac{1}{8d^*\alpha}[g(\alpha + \theta)(3\alpha - \theta) - b(\alpha - \theta)^2].$$

By symmetry, the utility for a θ-individual on negative t is equal to that of a (-θ)-individual on positive t. Thus the utility for t-objects with $t \leq 0$ is equal to:

$$g\frac{d^* - \alpha}{2d^*} + \frac{1}{8d^*\alpha}[g(\alpha - \theta)(3\alpha + \theta) - b(\alpha + \theta)^2]$$

Collecting terms gives that the overall utility of a θ-peer receiving a *yes* is equal to:

$$g(\frac{d^* - \alpha}{d^*}) + \frac{1}{4d^*\alpha}[g(3\alpha^2 - \theta^2) - b(\alpha^2 + \theta^2)]$$

Rearranging and multiplying by p gives (20). ■

Consider now an individual outside the community whose distance to the center is less than d^* and let us prove (21). We compute the utility derived on objects with positive t for θ-individual and distinguish positive and negative θ.

A θ-individual with $\alpha < \theta < d^*$, gets the benefit g for all positive objects that he buys: the individual likes all t-objects with $0 \leq t \leq \theta + d^*$ and a t-object can be recommended only if $t < \alpha + d^*$ which is smaller than $\theta + d^*$. Thus he achieves $g/2$ (1/2 is the probability of an object being in the positive zone knowing that it is recommended). An individual located at

$-\theta$, $\alpha < \theta < d^*$, achieves in the acquiescence zone g for objects with $0 \leq t \leq -\theta + d^*$ and $-b$ for $-\theta + d^* \leq t \leq -\alpha + d^*$. In the disagreement zone he dislikes all objects hence achieves $-b$. This gives a total of $g\frac{d^*-\theta}{2d^*} - b\frac{\theta-\alpha}{2d^*} - b\frac{\alpha}{2d^*}$, or $g/2 - (b+g)\frac{\theta}{2d^*} = g(1/2 - 2k\theta)$. Using the symmetry property again, collecting terms and multiplying by p gives (21). The utility is positive for $\theta \leq d^*(2g/(b+g)) \leq d^*$, the last inequality because $g \leq b$. ■

Coordination failure example. Given proposals $[-\alpha, \beta]$ and $[-\beta, \alpha]$, let C_1 be the individuals whose characteristics belong to $(-\alpha, -\beta) \cup (0, \beta)$, and C_2 to $(-\beta, 0) \cup (\beta, \alpha)$. We claim that for $\beta = 2\alpha/3$ the two communities C_1 and C_2 form an equilibrium. The expected utility levels derived from choosing C_1 and C_2 under the expectations that they form are given respectively by (see footnote 12)

$$P(\tfrac{\alpha}{2})[\tfrac{(\alpha-\beta)}{\alpha}U(\theta, [-\alpha, -\beta]) + \tfrac{\beta}{\alpha}U(\theta, [0, \beta])] \text{ and } P(\tfrac{\alpha}{2})[\tfrac{(\alpha-\beta)}{\alpha}U(\theta, [\beta, \alpha]) + \tfrac{\beta}{\alpha}U(\theta, [-\beta, 0])].$$

We shall use the following property. From the expressions (20) and (21) the utility derived from an arc is concave with respect to the distance ($|\theta|$) to the center for the peers inside the arc and it is linear in that distance for the peers outside. Overall the function is concave since it is differentiable at the boundary point when $\theta = \alpha$.

We first show that peers at the boundaries, $-\beta$, 0, and β are indifferent between C_1 and C_2. Given the symmetry, it is obvious for 0 and it suffices to show it for $-\beta$ for instance. Observe that $-\beta$ is outside or at the extreme of any of the arcs considered. Hence the utility from either community is derived by computing the weighted distance of $-\beta$ to the center of each arc in the community. One checks that for $\beta = 2\alpha/3$, the weighted distance to the arcs in C_2 is $\frac{2}{3}\frac{\beta}{2} + \frac{1}{3}(\frac{\alpha+3\beta}{2})$, and that to C_1 is $\frac{2}{3}\frac{3\beta}{2} + \frac{1}{3}(\frac{\alpha-\beta}{2})$ which are both equal to $\frac{\alpha+5\beta}{6}$: the $-\beta$-peer is indifferent between the two communities.

It remains to show that the peers in the interior of $(-\beta, 0)$ or (β, α) prefer C_2 to C_1. As a function of θ in $(-\beta, 0)$, the utility for C_2 is concave and the utility for C_1 is linear (since such a θ is outside the intervals forming C_1). Hence the difference in the utility levels is a concave function of θ in $(-\beta, 0)$ and is null at the extreme $-\beta$ and 0: all peers in $(-\beta, 0)$ prefer C_2 to C_1. The same result holds for θ in the arc (β, α) since a β-peer is indifferent between C_2 to C_1 and an α-peer clearly prefers C_2 to C_1. ■