A Network Model of Public Goods: Experimentation and Social Learning

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Abstract: How does the social structure affect the search for new technologies and information? This paper introduces a new model of social learning to address this question. The model captures a realisitic environment: people experiment to find new technologies then share this information with friends and colleagues. That is, information is a public good among socially linked individuals. The paper asks: How do patterns of social links affect search efforts? How much do individuals experiment personally and how much do they rely on their friends? When do more communication links increase or decrease social welfare? The analysis yields a series of new insights including (1) the importance of experts – "mavens" in marketing parlance – who exclusively experiment and share their information with others; (2) the costs and benefits of links - a link increases information sharing but decreases the incentive to experiment; (3) contrary to prevailing wisdom, "bridges" that connect communities can harm other community members.
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I. Introduction

Before purchasing a product, investing in a technology, or launching a research project, people can investigate options themselves as well as inquire into the experiences of friends and colleagues. Many studies have argued the importance of such communication and social learning for economic activity.[1] This paper proposes a new model of social learning - a model that combines individual search for information and communication of this information through social links. We use this model to explore how the social structure affects experimentation and the search for new technologies.

Our analysis is motivated by a large literature on social learning and social structure outside economics, including Coleman, Katz, and Mentzel's (1957) classic study of the adoption of tetracyline. James Coleman and his coauthors found that social relationships influenced how physicians learned about the new antibiotic. Some physicians learned about new drugs and treatments by searching the medical literature, while others relied on their colleagues and friends for information.[2] Moreover, a physician's particular position within the professional network determined whether he did his own research or relied on the advice of colleagues:

> [...] the process of introduction for those doctors who were deeply embedded in their professional community was in fact different from the process for those who were relatively isolated from it. The highly integrated doctors seem to have learned from *one another*, while the less integrated ones, it seems, had each to learn afresh from the journals, the detail man (drug salesman), and other media of information. Coleman et. al. (1957, p. 262, emphasis in the original)

In our analysis we will see such outcomes and the influence of the social structure on the distribution of search efforts.

Our model is simple and captures the process described by Coleman and others. We begin with a fixed social structure. People derive benefits from new information and can acquire this information from two sources. The first source is their own private search. The second source is information gathered by friends and colleagues. People decide how much private search to conduct, understanding that they and their friends and colleagues will ultimately share their search results. That is, information is a public good among socially linked individuals. In this

[1] Citations are dispersed throughout the text.

[2] While specific results obtained by Coleman and his co-authors have later been questioned, his main messages and insights have been highly influential and are now central to the sociological study of innovation and learning.

environment, we ask how the social structure influences the final benefits, costs, and pattern of search.

This model captures many social learning and search environments. Among consumers, it represents the search for high quality products or low prices. Individuals search for their own purposes and share their findings with friends and neighbors [Feick and Price (1987), Gladwell (2000)].[3] In agriculture, farmers may experiment to find the best use of inputs, then communicate their findings to other farmers [Foster & Rosensweig (1995)]. In medicine, physicians try various therapies and tell their colleagues about their results, and a similar process occurs in many other technical fields [Valente (1995)].

We consider how the social structure affects search patterns and the generation of new information. Our analysis relies on the assumption that people willingly provide information to their friends and colleagues who ask them for it. Following Granovetter's (1985) description, we model a society where agents are embedded in a social network of long term relationships; these relations took time to form, embody mutual trust, and are not easily undone. When the need for new information arises, agents in our model turn to the social network which is already in place. This model also represents a formal organizational structure where communication is only possible between certain individuals and these individuals are contractually required or obliged to share information.[4] We abstract from possibilities of strategic pricing of information and strategic link formation. Such strategic dimensions could play a role in the long run, e.g. on the evolution of the network, and such possibilities are avenues for future research. The analysis we conduct here, with a static network and non-strategic communication, begins the study of experimentation and social learning in social networks.

Our analysis is divided into two parts. In the first part we study general social networks. We characterize equilibrium search patterns, their welfare properties, and the effect of new communication links on search and welfare. In the second part we construct precise models of social networks. These models capture main ideas in the sociological and other literature concerning pat-

[3] Market researchers Feick and Price (1987, p. 86) emphasize the importance of interpersonal communication in conveying marketing information: "In 30 years of research, remarkably consistent results have documented the significance of interpersonal sources, particularly in influencing marketplace choices and in diffusing information on new products. Research has demonstrated interpersonal information exchange is widespread, interpersonal communication affects preferences and choices, interpersonal sources are often the most important sources of information, and interpersonal sources are seen as more credible than nonpersonal sources." (references omitted).

[4] In this case, the analysis here could guide the design of such a structure.

terns of experimentation and communication. In each case we again characterize the equilibrium search patterns, their welfare properties, and the effect of new communication links.

In our analysis we distinguish two implications of social links. First, private search efforts are strategic substitutes between linked individuals. A person will search less when her friends and colleagues search more. To some degree, then, linked agents share the effort of gathering information. Second, links can allow a large benefit of new information. When an agent is linked to many people she receives potentially much more information than she would gather herself. We show how these information premia and shared search costs determine the welfare of different equilibrium patterns of search.

Our analysis yields several new insights:

(1) A few people, who we call local experts, may specialize in search and such specialization can maximize aggregate welfare. These "mavens," in the parlance of marketing researchers [Feick and Price (1987)], gather a great deal of information which they share with their friends and family members who do little of their own search. We distinguish between patterns where search is distributed among all individuals and those where search is concentrated among local experts. We show how certain social structures must lead to specialization.

(2) In contrast to previous models of social learning, welfare can be higher when not all agents are linked. When some agents are not connected to each other, they can become local experts. Overall welfare can be higher than in equilibria where all agents search for information.

(3) Bridges between communities have an ambiguous role. In the sociology literature, bridges between communities are beneficial because they bring new information into a community [e.g., Burt (1992)]. We show here that when an agent gains access to an outside source of information, he may reduce his own search effort. This reduction has negative externalities on members of his community. Welfare may not fall, but there are always distributional consequences;

(4) Agents who have a more central position in the network search relatively less than those situated at the periphery. Central agents have access to more sources of information, hence provide less of their own effort. This result corresponds closely to the finding described by Coleman et al. (1957) (see above quotation);

(5) Inequality in social connections generates tensions between individual behavior and social welfare. On the one hand, agents who have more links exert comparatively less effort in equilib-

rium. On the other hand, it is exactly the agents with more links whose private experimentation and search would be most beneficial to others. We will see that equilibrium profiles yielding high welfare often involve search done by well-connected individuals, but equilibria also exist where these individuals do less search.

This paper introduces experimentation and communication through social links to the theory of social learning. These elements are largely absent in current theories of social learning, which can be divided into three groups.[5] In the first group, agents periodically decide whether to adopt a particular technology after learning something about other agents' previous choices and payoffs.[6] The results show that despite limited observation, the population can ultimately converge on the efficient technology. In the second group of models, agents sequentially make permanent choices about technology and can observe the choices of those who have preceded them in the sequence. Individuals who make choices later in the sequence may find it optimal to ignore their own private signals, leading to a "herd" and adoption of inferior technologies [Banerjee (1992), Bikhchandani et. al. (1992)].[7] In neither of these two approaches do people communicate their private signals. This paper advances a new model of social learning where socially linked individuals can communicate their private information. Moreover, private signals are not free, and individuals choose how much information to gather personally. Thus this paper is perhaps closest to third group which consists of Foster & Rosenzweig's (1995) study of strategic experimentation for the best use of agricultural inputs, and Bolton and Harris' (1999) theory of strategic experimentation in a two-armed bandit problem.[8] Our innovation is the social network.

This paper complements the growing empirical interest in networks and social learning. Researchers are investigating whether people learn basic information from those to whom they are socially linked, and a new set of papers attempts to identify network effects on technology adop-

[5] For review of the social learning literature see Bikhchandani et al. (1998) and Coa and Hirshleifer (2000). Contributions beyond those cited in the text include Banerjee and Fudenberg (1999), Lee (1993), Schlag (1998), and Smith and Sorenson (2000).

[6] For example, in Ellison and Fudenberg (1993), agents observe the average payoffs accruing to those who chose a particular technology. In Ellison and Fudenberg (1995), agents learn the choices and payoffs of a random sample of agents. Bala & Goyal (1998, 2001) and Cowan & Jonard (1999) model learning through social links, where agents learn the choices and payoffs only of linked individuals.

[7] Coa and Hirshleifer (2000) combines the information assumptions of the two approaches: agents receive private signals, observe previous choices by other agents, and observe something about the payoffs of past decisions.

[8] Bolton and Harris (1999) consider a two-armed bandit problem in a multiple agent setting. All agents observe the outcome of others' experiments. They find that, while current experimentation discourages current experimentation by others, current experimentation can encourage future experimenation as players learn more about the underlying distribution of the payoffs.

tion (Bandiera and Rasul (2001), Conley and Udry (2002), and Munshi (2002)). Foster and Rosenzweig (1995) considers the strategic aspects of experimentation and social learning. They find evidence of a public goods problem; agents experiment less than is socially optimal. In industrial organization, of course, there is a large theoretical literature which studies how research findings spillover to other firms (for a recent study of geography and spillovers see Keller (2002)). Jaffee et al. (1993) provides evidence that supports our assumption that information sharing is local. We are the first, that we know of, to explore how the communication structure will affect search for new technology.[9]

Finally, this paper contributes to the nascent theoretical literature on networks in economics. Much work on networks considers the general properties of equilibrium networks.[10] Researchers have recently begun to build network models to represent different economic settings.[11] This paper introduces the first model of public goods in social networks.

Furthermore, this paper develops a new research strategy for the study of networks. We construct families of graphs to represent different social environments. Studying a family of graphs has two advantages. First, there is enough structure to obtain analytical results, which is often impossible in general graphs. Second, we are not limited to a single graph but can understand how the results extend and change within the family. We build formal models of three types of social structures: overlapping neighborhoods, isolated communities connected by bridges, and core-periphery structures. These models could be useful in a variety of other applications including informal insurance and credit and the labor market.

The paper is organized as follows. In the next section we present the general model. We formally represent a social structure and agents' strategic search decisions. In Section III, we study general communication graphs. We illustrate our results with basic graphs that represent canonical social structures. In Section IV, we develop our families of graphs, and for each family we consider the set of equilibria and how adding communication links affects welfare and the

[9] Research on technology diffusion and adoption includes Allen (1982a, 1982b) and Besley & Case (1993).

[10] Contributions include Myerson (1977), Aumann and Myerson (1988), Jackson and Wolinsky (1996), Bala and Goyal (2000), Jackson and Watts (1998). For a comprensive collection see Dutta and Jackson (2003) and their (2003) review.

[11] Young (1999) and Morris (2000) examine coordination games played in social networks. Bramoullé (2002a, 2002b) examines anti-coordination games. Boorman (1975), Tassier (2000), Calvo-Armengol (2003), and Calvo-Armengol and Jackson (2002) study job contact networks. Kranton and Minehart (2001) build a theory of networks of buyers and sellers. Chwe (2000) studies coordination games and communication networks. Bala and Goyal (1998, 2001) also consider communication graphs. Hendriks et. al. (1999) studies airline networks.

pattern of search. Section V concludes.

II. The Model

A. Private Search and Social Learning

In this section, we develop a model that combines private search for information and information sharing through social links. There are n agents, and the set of agents is denoted by $N = \{1, ..., n\}$. We assume that agents acquire information in two phases. First, people engage in private search or experimentation. Then people learn the results of others' private search.

As for private search, we denote by e_i the private search effort of agent i, and $\mathbf{e} = (e_1, ..., e_n)$ denotes a search profile of agents. We equate the level of information obtained through private search with the private search effort. That is, e_i directly measures how much private information has been acquired by agent i through his own efforts.

Agents learn the results of others' private search through social links, whose collection we represent as a graph $\mathbf{G}$, where $g_{ij} = 1$ if agent i communicates the results of his search to agent j, and $g_{ij} = 0$ otherwise. We assume that communication flows both ways so that $g_{ij} = g_{ji}$. Since agent i knows the results of his own search efforts we set $g_{ii} = 1$. We denote by N_i the set of agents with whom agent i communicates, called i's *neighbors*. Formally

$$N_i = \{j \in N/i : g_{ij} = 1\}$$

As in reality, agents might be highly heterogenous with respect to the number and identity of their neighbors. Let k_i denote the number of agent i's neighbors; that is, $k_i \equiv |N_i|$. Agent i's *neighborhood* is defined as himself and his set of neighbors; i.e., $i + N_i$.

We make two important simplifying assumptions concerning the diffusion of information along the social links. First, we assume agents communicate their private information to their neighbors, who do not transmit this information to their own neighbors. That is, information diffuses one step and then stops. This assumption is consistent with the intuition that first-hand information carries more weight than second or third-hand information.[12] Second, agents are interested in the same kind of information, and the information obtained from one's neighbor is the same quality

[12] The methods we use here can easily be extended to information diffusion of more than one step.

as one's own. Under these assumptions, an agent would have the benefits of $e_i + \overline{e}_i$ information where $\overline{e}_i$ denotes the amount gathered by i's neighbors; that is,

$$e_i + \overline{e}_i = e_i + \sum_{j \in N_i} e_j \tag{1}$$

This expression directly models the positive externalities of social learning. The information an agent obtains from her own search efforts benefits all her neighbors. Equation (1) indicates that social structure is likely to be important for social learning and search. For example, if every agent does the same amount of private search e, agent i has the benefits of $k_i e + e$ information, and each agent benefits from an amount of information that is linear in the number of her neighbors. Agents who are more connected would have access to more information than agents who are less connected.

In the following analysis, we explore how social structure influences the search for information.

B. Strategic Interaction

We specify the following simple game. Given a social structure $\mathbf{G}$, agents simultaneously choose search effort levels. We restrict attention to pure strategies, where a strategy profile is a vector of search efforts $\mathbf{e}$. To focus on the effects of social structure, we make simple assumptions regarding payoffs. Each agent receives payoffs from information according to a (twice-differentiable) benefit function $b(e)$ where $b(0) = 0$, $b' > 0$ and $b'' < 0$; that is, more information is better, and the benefits of additional information decrease with the level of information. The individual marginal cost of search is constant and equal to c.[13] Since an agent knows his own search results and those of his neighbors, an agent i's payoff from search profile $\mathbf{e}$ in graph $\mathbf{G}$ is

$$U(\mathbf{e}; \mathbf{G}) = b\left(e_i + \sum_{j \in N_i} e_j\right) - ce_i$$

To fix ideas for these payoffs, consider the following discrete example. Suppose each agent wants to acquire a unit of a good. The good is provided by many suppliers, yielding a distribution

[13] Convex costs of search would induce *a priori* more effort sharing, while concave cost of search would induce *a priori* more specialization. The assumption of linear search cost allows us to isolate the effects of the social structure on the distribution of search efforts in the population. In fact, an important message of our analysis is that specialization can emerge for purely structural reasons.

of prices. Each agent can obtain draws from this distribution at cost c per draw (by visiting a store, say),[14] and e_i is the number of i's draws. Agents then share information, and i's final information set consists of $e_i + \overline{e}_i$ draws. Each agent then obtains the good from the supplier with the lowest price in her set. Under the assumption that all draws are independently distributed, let the expected benefits be $b(e) = B - \mu(e)$ where $\mu(e)$ is the expectation of the lowest order statistic of e draws. The benefit function is then increasing and concave in e (see Appendix). In another interpretation, each agent tries (or experiments with) a number of technologies to determine their efficacy, at cost c per trial. The expected benefits are directly related to $\sigma(e)$ where $\sigma(e)$ is the expectation of the highest efficacy of e trials.

III. Equilibrium Search in General Graphs

A. The Shape of Equilibrium Search Profiles

In this section, we analyze the shape and properties of the (pure-strategy) Nash equilibria of the game and how they depend on social structure. We first derive an abstract characterization of Nash equilibria and use it to show that there always exist a Nash equilibrium. We then distinguish between equilibria where all agents search and where only a few agents search. We call these latter agents *local experts*, since they are the only agent in their neighborhood who search. We show how equilibria involving local experts can yield higher aggregate welfare than other equilibria. We illustrate our results in basic graphs, with four agents. In the next section we model and study equilibria of more complex social structures.

We characterize Nash equilibria of the search game as follows. Let e^* denote the level of search at which, to an individual agent, the marginal benefit of search is equal to its marginal cost. Formally, $b'(e^*) = c$.[15] Thus, e^* is a decreasing function of the search cost c. It is easy to see that:

Proposition 1. *A profile* $\mathbf{e}$ *is a Nash equilibrium if and only if for every agent i either (1) $\overline{e}_i \geq e^*$ and $e_i = 0$ or (2) $\overline{e}_i \leq e$ and $e_i = e^* - \overline{e}_i$.*

The intuition is straightforward. Agents want to search as long as their total information is less

[14] For simplicity, the agent is making these draws with replacement.

[15] Given $b()$ is strictly concave, a level of search $e^* > 0$ exists and is well-defined as long as $b'(0) > c$.

than e^*. Thus, if the information they acquire from their neighbors is more than e^*, they do no search. If this information is less than e^*, they search up to the point where their total information is exactly equal to e^*.

Proposition 1 tells us that search efforts are strategic substitutes. In equilibrium, the more an agent searches, the less his neighbors search. If agent i searches an amount $e_i > 0$, the sum of the efforts of his neighbors is $e^* - e_i$.

The simple characterization in Proposition 1 allows us to distinguish different possible equilibrium search patterns and distribution of search among agents. In general, the distribution of search among agents could vary among two extremes. At one extreme are profiles in which all agents are either local experts or do no search. At the other extreme are profiles in which all agents search for information. We call an agent who searches the maximum amount, e^*, a *local expert*, since in his neighborhood he is the only agent who conducts any private search. (Because of strategic substitutability, neighbors of agents who search e^* do no search themselves.) We say a profile $\mathbf{e}$ is a *distributed profile* when all agents search for information; for all agents $i \in N$, $e_i > 0$. We say a profile $\mathbf{e}$ is *specialized* when some agents do not search; i.e., for at least one agent i, $e_i = 0$. Among specialized profiles, we will pay particular attention to *expert* profiles, where all agents who search are local experts; for all agents i either $e_i = 0$ or $e_i = e^*$. We make this distinction because the descriptive literature on social learning and experimentation often emphasizes the information leaders and "mavens" who conduct a great deal of private search and share their information with others. We will see that certain social structures exclusively lead to expert equilibria.

We next illustrate Proposition 1 and these different types of equilibria using three basic graphs, each with four agents. The graphs we consider shown in Figure 1 - the complete graph, the star, and the circle - represent canonical social structures and are building blocks for the families of graphs we consider in Section IV below.

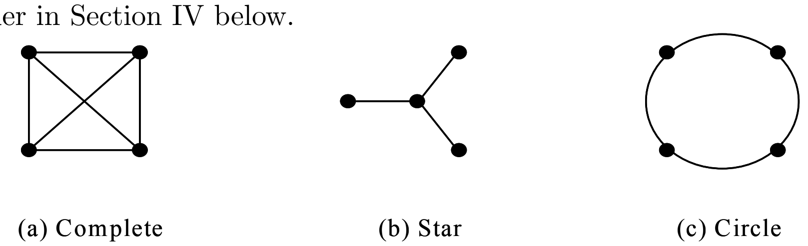

Figure 1. Basic Graphs with Four Agents

The complete graph represents a densely connected society; all agents know each other. The star graph represents a hierarchical society; only one agent knows everyone else in the society. The circle gives an intermediate case of an equal society, but where each agent only knows a subset of the population.

Example 1. *Expert and Distributed Equilibria. Consider first the complete graph. Since information is essentially public, the Nash equilibria are all the profiles such that the amount of total search is equal to* e^**. That is, the search of* e^* *can be split in any way among the agents. E.g., search could be equally distributed among agents, so that each agent searches* $\frac{1}{4}e^*$*, or one agent could be an expert, as shown in panel (a) Figure 2.*[16] *On the star, results are quite different. Only expert profiles are equilibria. Thus the star graph provides a simple example of social structure leading to specialization. There are only two Nash equilibria: Either the center is an expert, or the three agents at the periphery are experts, as shown in Figure 2 panel (b). Finally, on the circle, there are both expert and distributed equilibria. The search can be distributed among the agents, or concentrated among experts, as shown in Figure 2 panel (c).*

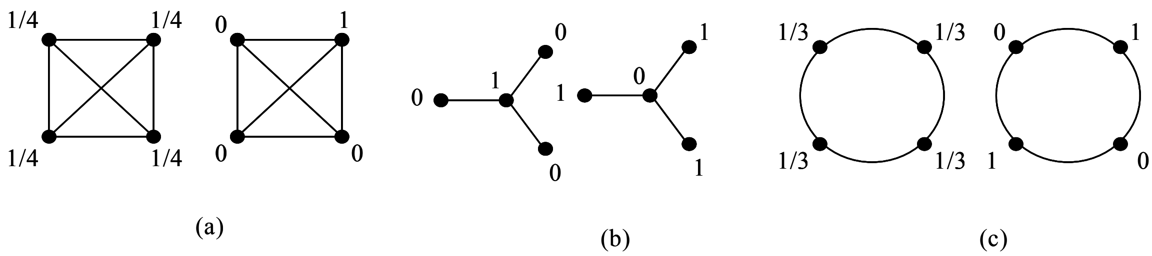

Figure 2. Equilibria in Basic Graphs with Four Agents

The results in Example 1 generalize to complete graphs, stars, and circles for n agents:

Corollary 1. *When the graph* $\mathbf{G}$ *is complete, a profile* $\mathbf{e}$ *is a Nash equilibrium if and only if* $\sum_i e_i = e^*$.

[16] *Note that in all Figures we suppress the term* e^* *for ease of exposition.*

Corollary 2. *On the star graph, a profile* **e** *is a Nash equilibrium if and only if: either (1) the agent in the center plays* e^* *and all the other agents play* 0*; or (2) the agent in the center plays* 0 *and all the other agents play* e^**.*

We defer the Corollary for the circle to Section IV below.

B. The Existence of Nash Equilibria: Expert Profiles and Independent Sets

We next show that for any social structure, there exists a Nash equilibrium. Moreover, this equilibrium is an expert equilibrium.

We find it useful to employ the following elementary notions of graph theory. An *independent set* I of a graph $\mathbf{G}$ is a set of agents such that no two agents who belong to I are linked; i.e., $\forall i, j \in I$ such that $i \neq j$, $g_{ij} = 0$. An independent set is *maximal* when it is not a proper subset of any other independent set. Any maximal independent set I has the property that every agent either belongs to it or is connected to an agent who belongs to it.[17] For any agent i, there exists a maximal independent set I of the graph $\mathbf{G}$ such that i belongs to I.[18] This notably implies that any graph possesses at least one maximal independent set.

Maximal independent sets are a natural notion in our context because of the strategic substitutability of agents' search efforts. Because search efforts are substitutes, in equilibrium no two experts can be linked. Hence, expert equilibria are characterized by this simple structural property of a graph:

Proposition 2. *An expert profile is a Nash equilibrium if and only if its set of experts is a maximal independent set of the social structure* $\mathbf{G}$*. Since for every* $\mathbf{G}$ *there exists a maximal independent set, there always exists an expert Nash equilibrium.*

Corollary 3. *For any graph* $\mathbf{G}$ *and any agent* i*, there exists an expert Nash equilibrium in which* i *is a local expert.*

The next example illustrates the concept of maximal independent sets and the relationship to expert equilibria using the three graphs in Figure 1.

[17] To see this, suppose not. Let I be a maximal independent set, and let i be an agent who does not belong to I and is not connected to any agent who belongs to I. Then the set $I \cup \{i\}$ is an independent set, and hence I is not maximal.

[18] To see this, note that i itself is an independent set. To build a maximal independent set, begin with i and successively add agents not linked to i, then agents not linked to those agents, etc.

Example 2. *Expert Equilibria and Maximal Independent Sets. Consider first the complete graph. Since all agents are linked, an independent set can include at most one agent. Hence, each agent constitutes a maximal independent set. There are then four expert equilibria, corresponding to each agent. On the star, there are two maximal independent sets. The agent at the center is one set. The second set contains the three agents in the periphery. These two sets correspond to the two expert equilibrium profiles for the star (as seen in Figure 2 (b)). In the circle, there are two maximal independent sets. Each set contains two agents on opposite sides of the circle. Again, these two sets correspond to the expert equilibria for the circle (as seen in Figure 2 (c)).*

C. Welfare Analysis

We now consider the welfare of different Nash equilibria. In general there are multiple Nash equilibria, with widely different welfare properties. Equilibria involve different distributions of individual search efforts and benefits as well as different aggregate search costs and benefits. To gain a basic understanding of the differences among search patterns, we take a standard utilitarian approach to welfare. We specify social welfare associated with a search profile $\mathbf{e}$ for a graph $\mathbf{G}$ as the sum of the payoffs of the agents:

$$W(\mathbf{e};\mathbf{G}) = \sum_{i\in N} b\left(e_i + \overline{e}_i\right) - c\sum_{i\in N} e_i$$

where recall $\overline{e}_i$ is the sum of the search efforts of i's neighbors. The first term is the sum of the benefits accruing to the agents, and the second term is the total cost of private search efforts.

Even with this basic notion of welfare, the question of which type of equilibrium leads to higher welfare is not trivial. Are specialized equilibria preferred to distributed equilibria? Among expert equilibria, is it better if there are few experts or many experts? How should the experts be located in the graph? We discuss here general elements of answers to these questions, and explore these questions further in our subsequent analysis of families of graphs. We will see the important role of experts in gathering and disseminating information, as in much of the descriptive literature. We will further see how the structure of the graph determines whether expert or distributed equilibria lead to higher aggregate welfare.

Our first welfare result confirms the public goods problem depicted in our model of search and information sharing. We say a profile $\mathbf{e}$ is *efficient* for a given structure $\mathbf{G}$ if and only if there is no other profile $\mathbf{e}'$ such that $W(\mathbf{e}';\mathbf{G}) > W(\mathbf{e};\mathbf{G})$. It is straightforward that, for any non-empty graph, no Nash equilibrium search profile is efficient. This result is due to the externalities of search efforts. Agents do not consider the benefits that accrue to their neighbors when they make their private search decisions:

Proposition 3. *When at least one pair of agents is linked, no Nash equilibrium search profile is efficient.*

We therefore take a second-best approach. Among the equilibrium profiles for a given social structure $\mathbf{G}$, we ask which yield the highest welfare. We say that an equilibrium profile $\widetilde{\mathbf{e}}$ is *second-best* for a given structure $\mathbf{G}$ if and only if there is no other equilibrium profile $\widetilde{\mathbf{e}}'$ such that $W(\widetilde{\mathbf{e}}';\mathbf{G}) > W(\widetilde{\mathbf{e}};\mathbf{G})$. We denote the welfare associated with a second-best profile as $\widetilde{W}(\mathbf{G})$.

Following the descriptive literature, we explore under what conditions specialized profiles are second best. To do so, we calculate the welfare of an equilibrium profile $\mathbf{e}$ to highlight the benefits of specialized equilibria. Following Proposition 1, in any equilibrium, each agent has benefits of at least e^* information. Hence, $nb(e^*)$ is the minimum aggregate benefits in any equilibrium. In a distributed equilibrium, each agent has exactly e^* information and, hence, the $nb(e^*)$ is exactly the aggregate benefits. In specialized equilibria, however, some agents have the benefits of more than e^* information. This situation would occur, for example, when an agent is linked to more than one local expert. The benefits of such information are equal to $\sum_{j:e_j=0}[b(\bar{e}_j) - b(e^*)]$ where the summation is over all agents j who do not search and rely on others for their information. We can therefore express the welfare an equilibrium $\mathbf{e}$ as the sum of three terms:

$$W(\mathbf{e};\mathbf{G}) = nb(e^*) + \sum_{j:e_j=0} [b(\bar{e}_j) - b(e^*)] - c\sum_i e_i. \tag{2}$$

The first term, $nb(e^*)$, is the minimum aggregate benefits for any equilibrium. The second term is the *information premium* associated with specialized equilibria. The last term, $-c\sum_i e_i$ is the aggregate cost of effort.

With formula (2), we can compare welfare across equilibria for a given graph and explore for which graphs specialized equilibria are second-best. Of course, when a graph does not admit distributed equilibria, specialized equilibria are second-best by default. When a graph admits both types of equilibria, a more interesting question arises. In this case, there can be a tradeoff between the information premium from specialized equilibria and higher search cost.

For example, for a given graph, compare the welfare of an expert equilibrium $\mathbf{e}$ to the welfare of a distributed equilibrium $\mathbf{e}'$. In any distributed equilibrium, as noted above, aggregate benefits are simply $nb(e^*)$. The total search cost is at least ce^*. Hence, the highest possible welfare for a pure distributed equilibrium is $nb(e^*) - ce^*$. As for the expert equilibrium, let I be the set of experts in $\mathbf{e}$. Let s_j be the number of experts linked to agent j. Since I is a maximal independent set, the welfare of the expert equilibrium $\mathbf{e}$ is

$$nb(e^*) + \sum_{j \in N/I} [b(s_j e^*) - b(e^*)] - c\,|I|\,e^*$$

where $|I|$ is the number of experts. An expert equilibrium will therefore yield higher welfare if the information premium exceeds the additional search costs:

$$\sum_{j \in N/I} [b(s_j e^*) - b(e^*)] > (|I| - 1)\,ce^* \tag{3}$$

Clearly this inequality would be satisfied for a "sufficiently increasing" benefit function and an equilibrium with small sets of experts with many links to other agents. By "sufficiently increasing," we mean that quantities of information above e^* are sufficiently valuable (the appendix provides a precise definition).

We formalize this intuition in the following proposition, where, recall, k_i is the number of agent i's neighbors. The proposition tells us that if, in a given social structure, sufficiently many agents could be linked to local experts, then expert equilibria can yield higher welfare than distributed equilibria.

Proposition 4. *For a given graph* $\mathbf{G}$ *that admits both expert and distributed equilibria, there exists a sufficiently increasing benefit function such that an expert equilibrium yields higher welfare if there is a maximal independent set* I *in* $\mathbf{G}$ *such that the total number of links connecting agents in the set to agents outside the set is more than* $(n-1)$*; that is,*

$$\sum_{i \in I} k_i > n - 1 \tag{4}$$

This Proposition provides a method to weigh the potential benefits and costs of local expert equilibria. For the benefits, we can approximate the information premium of a local expert equilibrium by counting the number of links between local experts and others and by subtracting the number of non experts. This premium would arise when the benefit function is linear, so that each agent gains equally from any link to an expert. There is always a "sufficiently increasing benefit" function that would yield an amount arbitrarily close to this premium. We can thus represent this premium as the term $(\sum_{i\in I} k_i - |N\backslash I|)ce^*$. The cost of a local expert equilibrium, as discussed above, is the increased search costs. Since the lowest possible search costs in a distributed equilibrium is ce^*, the greatest possible cost difference with a local expert equilibrium is $(|I|-1)ce^*$, which would arise when total search in the distributed equilibrium is e^*. Thus equation (4) gives a sufficient condition under which, for a sufficiently increasing benefit function, the information premium of a local expert equilibrium would exceed any possible cost increase over a distributed equilibrium.

Equation (4) also allow us to compare the welfare of different expert equilibria. Consider two expert equilibria $\mathbf{e}$ and $\mathbf{e}'$ with local experts I and I'. If the benefit function b is sufficiently increasing, $W(\mathbf{e};\mathbf{G}) > W(\mathbf{e}';\mathbf{G})$ when $\sum_{i\in I} k_i > \sum_{i\in I'} k_i$.

We show how to apply this result using the graphs in Figure 1.

Example 3. *Welfare, Expert Equilibria, and Independent Sets. For a given graph, to determine whether a local expert equilibrium can yield higher welfare, Proposition 4 tells us to compare $(n-1)$ with the number of links between experts - that is, agents in a maximal independent set I - and non experts. Consider the graphs in Figure 1 for $n=4$. In the complete graph, each maximal independent set consists of a single agent. Hence, the inequality is not satisfied. And indeed all equilibria, expert and distributed yield the same welfare. In the star graph, there is no comparison to make, since there are no distributed equilibria. In the circle, each maximal independent set consists of two agents, and each agent in the set has two links to agent outside the set. Hence, $\sum_{i\in I} k_i = 4$. Since $(n-1)=3$, there exists a sufficiently increasing benefit function such that the expert equilibrium yields higher welfare. Calculations confirm this result. For this circle, there is a unique distributed equilibrium, where each agent searches $\frac{1}{3}e^*$;[19] this equilibrium is shown in panel (c) Figure 2. Total welfare is $4b(e^*) - \frac{4}{3}ce^*$. In any expert equilibrium, there*

[19] *A distributed equilibrium must satisfy the following system of four equations: $e_1+e_2+e_3=e^*$, $e_2+e_3+e_4=e^*$, $e_3+e_4+e_1=e^*$, $e_4+e_2+e_3=e^*$, which has the single solution of $e_1=e_2=e_3=e_4=\frac{1}{3}e^*$.*

are two experts located at opposite sides of the circle, and one such equilibrium is shown in panel (c) Figure 2. Total welfare is $2b(e^*) + 2b(2e^*) - 2ce^*$. An expert equilibrium yields higher welfare when the benefit function is sufficiently increasing so that the information premium exceeds the additional search cost; that is, when $[b(2e^*) - b(e^*)] > \frac{1}{3}ce^*$.

D. The Costs and Benefits of New Links

We finally examine the effect of adding a link on welfare. We show that opening a new channel of communication has an ambiguous effect. If private search efforts were fixed, adding a link would weakly increase welfare, since an agent would learn the results of her new neighbors search efforts. However in our model, access to a new source of information lowers the individual incentive to search. The link could lead to a loss in welfare, despite the added possibilities for learning. We explore this possibility below.

Consider a graph $\mathbf{G}$ and two agents i and j who are not linked in $\mathbf{G}$. Denote by $\mathbf{G}+ij$ the graph obtained by connecting i and j in $\mathbf{G}$. We say that the link leads to a "loss in welfare" when the second-best level of welfare for graph $\mathbf{G}+ij$ is lower than that for $\mathbf{G}$; that is, $\widetilde{W}(\mathbf{G}) > \widetilde{W}(\mathbf{G}+ij)$. To show when a link can lower welfare, let $\mathbf{e}$ be a second-best equilibrium profile for the social structure $\mathbf{G}$. There are two cases. First, in equilibrium $\mathbf{e}$, either i or j does not search. In this case, $\mathbf{e}$ is also an equilibrium for $\mathbf{G}+ij$ and hence $W(\mathbf{e};\mathbf{G}+ij) \geq W(\mathbf{e};\mathbf{G})$, and the link cannot lead to a loss in welfare.[20] Second, in equilibrium $\mathbf{e}$, both i and j search for information. In this case, $\mathbf{e}$ is not an equilibrium for the structure $\mathbf{G}+ij$. Adding a link between the two destroys the equilibrium pattern. This new link can therefore lead to a loss in welfare, especially when search by both agents is required to secure high aggregate benefits. We summarize these considerations as follows:

Proposition 5. *Consider a graph $\mathbf{G}$ and two agents i and j not linked in $\mathbf{G}$. If there exists a second best equilibrium in which either agent does no search, then $\tilde{W}(\mathbf{G}+ij) \geq \tilde{W}(\mathbf{G})$. Therefore, a necessary condition to obtain a loss in welfare from linking i and j is that both agents search in all second-best equilibrium profiles on $\mathbf{G}$.*

This result combined with expression (2) indicates that linking a pair of agents induces a loss

[20] More precisely, if $e_i = e_j = 0$, all agents obtain the same utility, while if $e_i > 0$ and $e_j = 0$, all agents except j obtain the same utility and j's utility increases.

in welfare in two settings: (1) if search done by both agents brings a high information premium that cannot be obtained otherwise, and (2) if the agents' search is needed to guarantee a low search cost.

We illustrate both positive and negative effects of a new link in the following example.

Example 4. *The Costs and Benefits of a New Link. Proposition 5 tells us to ask whether the two agents i and j search in all second-best search patterns. Consider the two stars in Figure 3 panel (a). The Figure shows the unique second-best equilibrium pattern - both centers are local experts (see Appendix for proof). Connecting a peripheral agent to the center of the other star, as shown in Figure 3 panel (b), does not disrupt the equilibrium. The link thus increases welfare. Without the link, second best welfare is $8b(e^*) - 2ce^*$ and with the link it is at least equal to $7b(e^*) + b(2e^*) - 2ce^*$. In contrast, connecting the two centers, as shown in Figure 3 panel (c), destroys the equilibrium pattern and thus can decrease welfare. With the link between the two centers, there are two second-best equilibrium profiles - the center of one star and peripheral agents of the other star are local experts. With the link, second best welfare is $7b(e^*)+b(4e^*)-4ce^*$. Welfare falls relative to the graph with no link between the centers if the increased search costs exceed the added information premium: $2ce^* > b(4e^*) - b(e^*)$.*

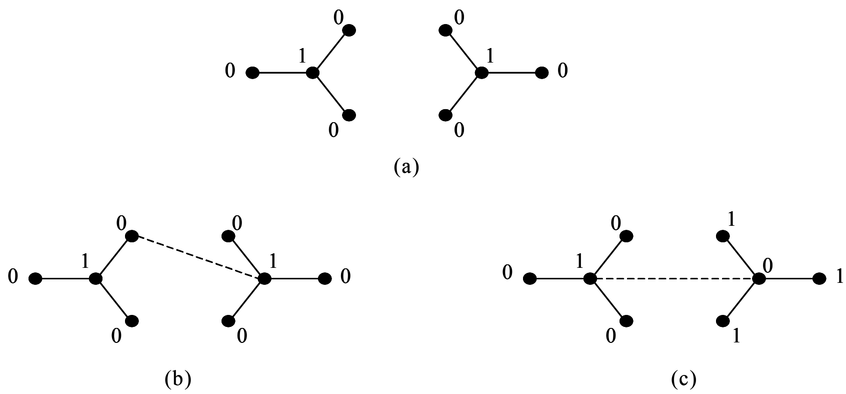

Figure 3. Costs and Benefits of a New Links

In this first part of our analysis, we explored three aspects of our model: the shape of the equilibrium profiles, their welfare, and the effect of adding links. We derived results and intuition for general communication networks, and simple examples with four agents showed that the precise architecture of the graph might affect each of these aspects. For instance, distributed equilibria are possible in the complete graph and the circle, but not the star. The extent of specialization might vary between different graphs, e.g. in an expert equilibrium, there is one local expert on the complete graph and two on the circle, as well as within the same graph, e.g. equilibria on the star have one or three local experts. Specialization yields information premia that can exceed the search costs, especially when effort is exerted by well-connected agents. Thus, on the circle expert equilibria are second-best when benefits from information are sufficiently high. On the star where all equilibria are specialized, search done by the center always yields greater welfare than peripheral search. Finally, new links have ambiguous effects. Connecting a peripheral agent of one star to the center of another star increases welfare, while connecting both centers may lower it. In the next section we turn to more complex social structures.

IV. Models of Social Networks

In this section we build stylized models of different social structures. We build three families of graphs, where each family represents a different organization of individuals in social and geographic space. Each family captures key social interactions described in sociological and other literature. With these precise models of different social structures we can gain insights on how social structures influence the the search for new information as well as the effects of new communication links. For example, we will see that equilibrium profiles yielding high welfare often involve search done by well-connected individuals, but equilibria also exist where these individuals do less search. For each section below, we first describe and model the social structure. We then solve for the Nash equilibrium search profiles, evaluate the welfare of different equilibria, and study the effect of adding links.

A. Overlapping Neighborhoods

We first consider an equitable social structure where each individual knows only a subset of the population, as in the circle above. We allow a person's set of neighbors to grow, and thereby capture increasing possibilities of friendship and communication across social and geographic space. This model is a stylized representation of different levels of societal integration and communication technology. At low levels of integration, people are friends only with those of a similiar ethnicity or those who live nearby. In a more integrated or technologically sophisticated society, people are friends with those of other ethnicities and in more and more distant streets and cities.

Consider the following model: agents are arranged uniformly along a circle and numbered from 1 to n. Each agent has k neighbors on the left and k neighbors on the right, so each agent has $2k$ neighbors. As k increases, an agent knows people who are further and further distant. Each person knows the agents closest to her, the size of any agent's neighborhood is symmetric, and the neighborhoods of agent i's neighbors overlap with agent i's neighborhood.[21]

We first solve for the set of Nash equilibria for this family of graphs, where the members of the family are described by the parameter k. This social structure admits each of the three possible types of equilibria: distributed, specialized, and expert. In distributed profiles, each agent takes on some share of the search effort. In expert equilibria, local experts are located at some distance apart in social space, so that each serves as an expert for his neighborhood. We have precise formulations of these two equilibrium forms:

Proposition 6. *(1) A strategy profile* $\mathbf{e}$ *is a distributed equilibrium if and only if there exists a common divisor of* n *and* $2k+1$ *- which we denote* m *- such that the sequence of the first* m *effort levels* $(e_1, ..., e_m)$ *satisfies* $\sum_{i=1}^{m} e_i = \frac{m}{2k+1}e^*$ *and the profile* $\mathbf{e}$ *is a repetition of this sequence every* m *agents. In particular, the profile where every agent searches* $\frac{1}{2k+1}e^*$ *is an equilibrium* $(m = 1)$*. (2) A strategy profile is an expert equilibrium if and only if the distance between two consecutive local experts is at least* $k+1$ *and no more than* $2k+1$*.*

Specialized equilibria also exist and are (loosely speaking) a mixture of distributed search efforts and local expertise. The sets of agents who search less than e^* are sufficiently far away from agents who are local experts.

Example 5. *Equilibria in Overlapping Neighborhood Model. We illustrate these different types of equilibria for* $n = 12$*,* $k = 1$ *and* $k = 2$ *in Figure 4. In panel (a), search is distributed among all the agents, and each agent in the same position within each group of* $2k+1$ *agents undertakes the same fraction of search effort. In panel (b), local experts conduct all the private search. Notice that the local experts are at least* $k+1$ *distance apart. In panel (c), we see a specialized, but not*

[21] The circle ($k = 1$) provides a standard way to represent local interactions, see e.g. Ellison (1993) and Eshel et. al. (1998). The family of graphs we consider in this section include the circle and also allows for expanding neighborhoods.

an expert equilibrium. Again, the local experts must be at least $k+1$ distance away from any other agent that does positive search.

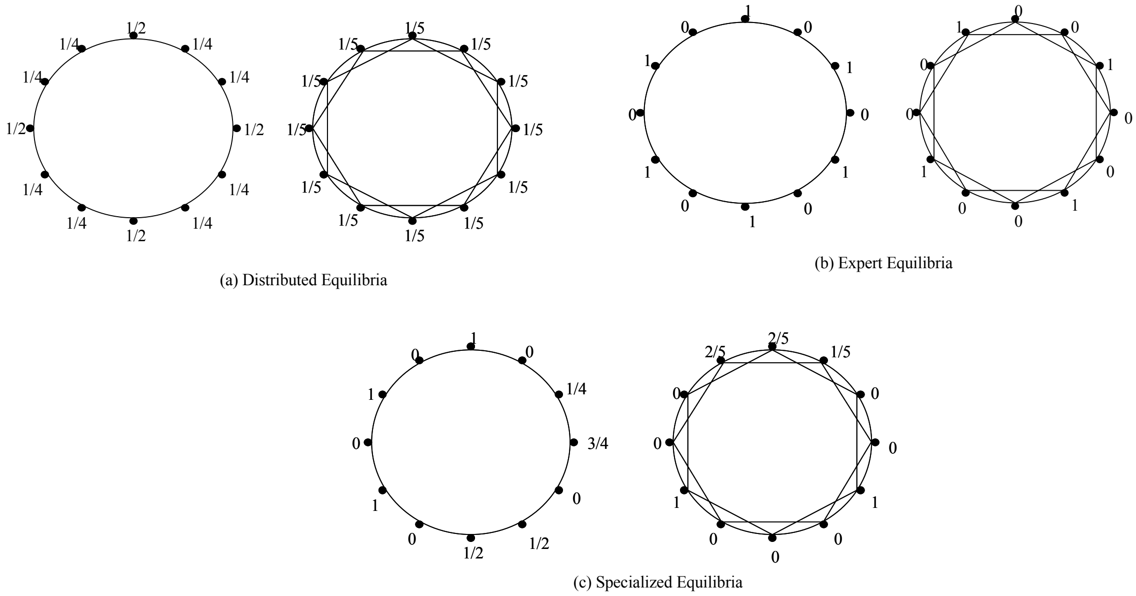

Figure 4. Equilibria in Overlapping Neighborhood Model

These different types of equilibria yield different levels of welfare, and the welfare depends on the level of societal integration k. All distributed equilibria yield the same aggregate benefits $nb(e^*)$ with total search cost $\frac{n}{2k+1}ce^*$. We show that these are the lowest search costs of any equilibrium (see Appendix). Expert equilibria yield information premia, which are highest for the expert equilibria with the greatest number of experts. Since according to Proposition 6, local experts must be at least $k+1$ distance from one another, the largest possible number of local experts is approximately $\frac{n}{k+1}$, in which case all non experts are connected to two local experts.[22] Hence the largest information premium from local expertise is $\left(n - \frac{n}{k+1}\right)[b(2e^*) - b(e^*)]$. Expert equilibria also differ in terms of their search costs, which are proportional to the number of local experts. We consider the trade-off between information premia and search costs.

We ask whether, in this social structure, specialized equilibria can yield higher welfare than distributed equilibria. We are particularly concerned with how the parameter k affects this

[22] On the other hand, since local experts must be at most $2k+1$ distance from each other, the lowest possible number of local experts is approximately $\frac{n}{2k+1}$ in which case all non experts are connected to a single expert.

comparison. We conduct this comparison for graphs that are not complete; i.e., $k < n/2 - 1$, since in a complete graph all equilibria yield the same level of welfare. We apply Proposition 4. Here, the relationship comparing specialized to distributed welfare in inequality (4) can be improved since total search in all distributed equilibria exactly equals $\frac{n}{2k+1}e^*$. Therefore, there exists a sufficiently increasing benefit function such that equilibrium with local experts I yields greater welfare if

$$\sum_{i \in I} k_i > n - \frac{n}{2k+1}$$

Since agents all have the same number of neighbors, $\sum_{i \in I} k_i = 2k|I|$, an expert equilibrium yields greater welfare when

$$|I| > \frac{n}{2k+1}$$

Hence, for b sufficiently increasing, equilibria with the largest number of local experts always yield greater welfare. Calculations confirm this result[23] and show, more precisely, that the information premium exceeds the additional search cost when

$$b(2e^*) - b(e^*) > \frac{1}{2k+1}e^*c \tag{5}$$

Thus, we observe that specialization in search is more beneficial at greater levels of societal integration. The relative benefits of expert equilibria are increasing in k. If (5) is satisfied for an integration level k' it will be also be satisfied for a more integrated society $k'' > k'$. The intuition is straightforward. As k increases, less local experts are required to reach more people. We can see this phenomenon in Figure 4 panel (b) above.

Despite the possible benefits of greater communication links, in this social structure welfare is not necessarily maximized when the graph is complete. Welfare can fall because in a complete graph there is no possibility of an information premium. When $k = n/2 - 2$, expert equilibria involve exactly two local experts and most people benefit from both sources of information. In

[23] We show in Appendix that the welfare of expert equilibrium with $|I|$ local experts is equal to

$$W(|I|) = nb(e^*) + [(2k+1)|I| - n][b(2e^*) - b(e^*)] - c|I|e^*$$

Welfare is linear in the number of experts and welfare of distributed equilibria equals $W(\frac{n}{2k+1})$. Therefore, if $b(2e^*) - b(e^*) > \frac{1}{2k+1}e^*c$, welfare is greatest when $|I|$ is greatest, while if $b(2e^*) - b(e^*) < \frac{1}{2k+1}e^*c$, welfare is greatest when $|I| = \frac{n}{2k+1}$ or for distributed equilibria.

contrast, when $k' = n/2 - 1$, communication become complete and information premium reduces to zero.In particular, we can easily show that:

Proposition 7. *Consider $k \leq n/2-2$. If b is sufficiently increasing, then for $k' = n/2-1$, welfare of the second-best profile falls.*

In this section, we studied a family of graphs where agents are symmetric; they have the same number of neighbors and identical structural positions in the graph. Consistently, we find that equitable effort sharing is sustainable in equilibrium, and distributed equilibria involve the lowest search costs of any equilibrium outcome. However, there are also specialized equilibria with local experts, characterized by heterogenous search profiles. Expert equilibria yield information premia at the expense of larger search costs. We show that these information premia can exceed the increased search costs for a sufficiently increasing benefit function when the graph is not complete. When the graph is complete, however, information premia disappear and the second-best level of welfare might fall. In the next two sections, we study families of graphs in which agents differ in terms of their number of connections and structural position. We study how the presence of better connected individuals affect equilibrium patterns and welfare.

B. Bridges between Communities

Our next model captures a social structure of communities, which are frequently observed patterns in societies.[24] People are often friends with those in the same ethnic group, or communicate and learn from those in the same village, or discuss findings with those in the same research unit within a firm. We ask how links between such communities affects the search for new information. Sociologists [e.g. Burt (1992)] have long argued that links, or bridges, between communities are critical for transmitting and spreading information and ideas. In the analysis below we will show such positive effects from social learning. We will also uncover, a new, negative, externality that comes from the search choice. When an agent gains access to a source of information outside her community, she may reduce her own search and thereby reduce the amount of information she passes on to her own community.

[24] Mapping the cohesive subgroups present in real networks is a main issue of social network analysis. Sociologists use a variety of concepts and techniques to address this task, see e.g., chapter 7 in Wasserman and Faust (1994) and Girvan and Newman (2002) for recent methodological advances.

For simplicity we consider only two communities, and we model the communities and the bridges between them as follows. We construct a social structure $\mathbf{G}$ by dividing the population into two sets, C_1 and C_2, and linking each agent in each set to every other agent in his set. That is, each community is a complete graph: $\forall i, j \in C_t, t = 1, 2, g_{ij} = 1$. While all agents know everyone in their community, only some may know agents in other communities. That is, it is possible for some $i \in C_1, j \in C_2, g_{ij} = 1$. We call such agents *bridge agents* and call the link between them a *bridge*. To maintain the idea of distinct communities, we assume throughout the analysis below that at least one agent in each community has no links to agents in the other community. Let B denote the set of bridges for a graph $\mathbf{G}$, and let β be the number of bridges.

We consider a family of such social structures, where a member of the family is characterized by the number of bridges β.

We first characterize the Nash equilibrium search patterns for this social structure. We are concerned, in particular, how the number of bridges β affects the set of equilibria. Since each community is a complete graph, in any Nash equilibrium total search effort within a community cannot be greater than e^*. Furthermore, since in both communities there is an agent who only obtains information from his own community, total search effort within each community must equal exactly e^*. Hence, the number of bridges cannot affect aggregate search efforts; in any equilibrium total search effort is $2e^*$. The number of bridges, then, can only affect the equilibrium distribution of search.

We find that the greater the number of bridges, the greater the search efforts of non-bridge agents. Because bridge agents have access to information in the other community, in equilibrium, at least one of each pair of bridge agents does no search. Hence, the greater the number of bridges, the smaller the set of equilibria, and across equilibria the average search of non-bridge agents rises. We summarize the shape of Nash equilibria in the following proposition and illustrate in Example 6.

Proposition 8. *For any community graph* $\mathbf{G}$*, the Nash equilibria are the search profiles such that: (1) total effort exerted within each community is equal to* e^**, and (2) for any two agents* i *and* j *connected by a bridge, at least one of agents* i *and* j *does no search. As a consequence, across equilibria the average search cost of non-bridge agents is higher for social structures with greater numbers of bridges* β*.*

Example 6. *Equilibrium Search Patterns for Communities connected by Bridges. Consider two communities of four agents each, pictured in Figure 6. In panel (a), there are no bridges ($\beta = 0$), and any distribution of search effort e^* in each community is a Nash equilibrium. In particular, equal distribution of search efforts is a Nash equilibrium; each agent searches $\frac{1}{4}e^*$. In panel (b), there is one bridge ($\beta = 1$). The set of equilibria is smaller. There is no equilibrium in which both bridge agents do strictly positive search. Hence, as shown in the Figure, the search of e^* is distributed among the non-bridge agents in one community, and total search of non-bridge agents increases. In panel (c), there are two bridges ($\beta = 2$). In this graph, there is another pair of agents for which it is not possible for them both to do positive search. Hence, search by non-bridge agents increases again.*

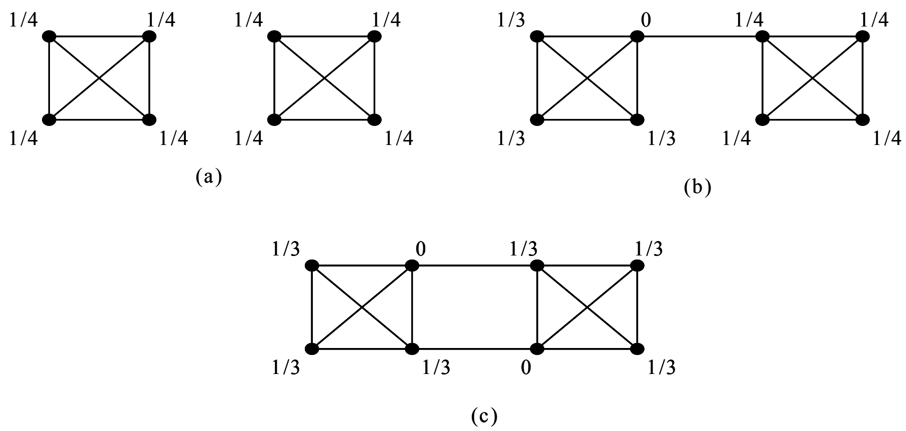

Figure 5. Equilibria in Communities Connected by Bridges

This result counters traditional sociological wisdom concerning bridges, which are a central concept of the literature on information transmission. Links between communities, it is argued, allows information to spread across social boundaries [Granovetter (1973), Burt (1992), Watts and Strogatz (1998)]. Our result highlights a different aspect of bridges. When search for information is costly and search is endogenous, bridge agents search less, which increases the burden of search on other agents. Thus, our analysis points towards a more ambiguous vision of bridges. Bridge agents are not just passive channels of information spanning across social groups. Bridge agents are also 'gatekeepers' who can use their privileged position to decrease their own effort.[25]

[25] To reconcile our result with the sociological wisdom, we also consider the set of equilibria when information

We next solve for the second-best search profiles for these social structures. We find that, while some bridge agents do no search in equilibrium, search by other bridge agents leads to the highest aggregate welfare. That is, second best profiles always involve search by bridge agents. When no bridge agents search in equilibrium, no information is shared betweem communities, and agents have the benefits of only the information generated in their own community. Welfare is simply $nb(e^*) - 2ce^*$. In contrast, in equilibria where a bridge agent does search, he shares his information in his community as well as transmits his information to all his linked partners in the other community. The agents in the other community who are linked by this bridge have an information premium. The welfare of an equilibrium profile $\mathbf{e}$ for a social structure $\mathbf{G}$ with a set of B bridges is then

$$W(\mathbf{e},\mathbf{G}){=}nb(e^*) + \left[\sum_{i\in B: e_i=0} b(e^* + \sum_{j\in N_i\cap B} e_j) - b(e^*)\right] - 2ce^* \tag{6}$$

where the second term is the information premium earned by those agents linked to bridge agents who search for information. It follows directly from this expression that the second-best equilibrium profile exclusively involves search by bridge agents in one or, often, both communities:

Proposition 9. *Consider a communication graph* $\mathbf{G}$ *with* β *bridges. When* $\beta = 0$*, all Nash equilibrium profiles are second-best profiles for* $\mathbf{G}$*. For* $\beta \geq 1$*, in second best profiles, bridge agents in one community exert all the effort. In the other community, the total effort exerted by bridge agents is either 0 or* e^**.*

Thus, our results also support sociological wisdom concerning bridges. The equilibria with highest welfare involve search by bridge agents, who share information with their own community and linked agents in the other community.

Our next set of results considers the effects of adding bridges between communities. Since communities are completely linked, this is the only way to add communication channels in this

can pass from a member of a community, through a bridge, to a member of another community. When information diffuses two steps in the graph, we obtain the following result, see Appendix. There are two types of Nash equilibria: (1) bridge agents exert a total effort of e^* and other agents do no search; (2) bridge agent do no search and in each community, non-bridge agents exert a total effort of e^*. With two step diffusion, bridge agents can either transmit information between communities (as emphasized in the sociology literature) or be gatekeepers (as we showed above for one-step diffusion). In the first case, bridge agents provide all the information, while in the second case, bridge agents take even more advantage of their position to reduce their search.

social structure. We apply Proposition 5, which tells us when adding links can lower welfare. In this analysis we must go beyond simply the number of bridges and consider the precise patterns of bridges and their effect on welfare. There are many possible patterns of bridges. An agent could be linked to more than one agent in another community, and these agents in turn could be linked to several agents in the first community. We call a bridge between two agents a *separate bridge* if and only if neither of the two agents is linked to another agent in the other community. We show that adding separate bridges between communities always increases welfare. In second-best profiles, more bridge agents search and increase the information available across community boundaries. The same is not true for bridges between two agents who are already connected to the other community. Such bridges can reduce welfare. An agent who had an incentive to search when connected to a single agent in the other community may not search when linked to two. We summarize the positive impact of separate bridges in the following Proposition. We illustrate the positive and negative effects of new bridges in Example 7.

Proposition 10. *Consider two communities connected by β separate bridges. An equilibrium profile is second-best when: (1) if $\beta = 1$, one bridge agent in one community is an expert; (2) if $\beta = 2$, one bridge agent in each community is an expert; (3) if $\beta \geq 3$ and even, $\beta/2$ bridge agents in each community share a total effort of e^* equitably; (4) if $\beta \geq 3$ and odd, $(\beta - 1)/2$ bridge agents in one community share a total effort of e^* equitably, as well as $(\beta + 1)/2$ bridge agents in the other community (not linked to those who search). As β increases, the welfare of second-best profiles increases.*

Example 7. *The Effect of New Bridges on Welfare. We first illustrate Proposition 10. Consider two communities of four agents each. In each of four panels in Figure 6, we illustrate a second best profile for $\beta = 0, 1, 2, 3$ separate bridges. The total search costs are the same in each case, and the information premium increases from 0 to $b(2e^*) - b(e^*)$ to $2\left[b(2e^*) - b(e^*)\right]$ to $\left[b(2e^*) + 2b(\frac{3}{2}e^*) - 3b(e^*)\right]$.*

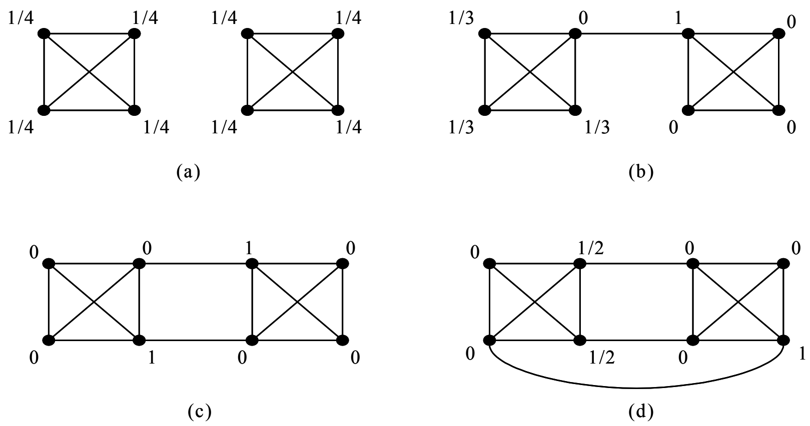

Figure 6. Second-Best Equilibria with Separate Bridges

We next show how a new bridge that is not a separate bridge can decrease welfare. Consider the two communities with four agents each in Figure 7. Panel (a) shows a social structure with three bridge agents in each community. In any second-best equilibrium for this social structure, the agents with two bridge neighbors are local experts. Hence, applying Proposition 5, a link between them can reduce welfare. Indeed, we see in panel (b) when this link is added, the second-best equilibria now involve expertise by only one of them. Calculations confirm that this link induces a fall in welfare. Without the link, the information premium in a second-best equilibrium is $4[b(2e^*) - b(e^*)]$, *and with the link it is* $3[b(2e^*) - b(e^*)]$. *Welfare is higher when the two agents who are most connected to other agents do all the search, and adding a link between these two agents leads to a fall in welfare.*

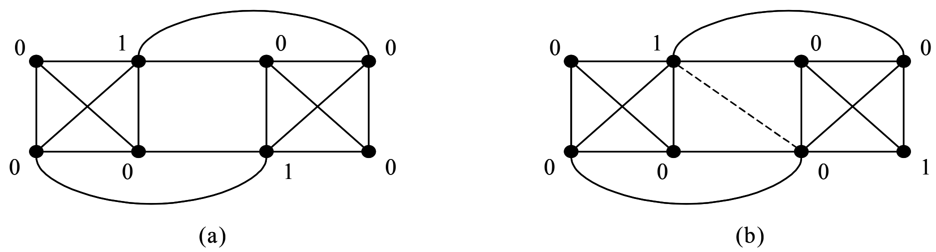

Figure 7. Negative Effect of New Non-Separate Bridge

In this section, we studied a society organized in distinct communities. We find that bridges between communities affect the distribution of search. In equilibrium, agents who gain information from the other community bear relatively less of the search costs. However, when they do search, they benefit agents in both communities, and hence bridges potentially increase aggregate benefits from information. Finally, we show that effect of new bridges between communities is ambiguous. When a new bridge links two already well-connected bridge agents, the second-best level of welfare might fall. In other cases, a new bridge is beneficial. In the next section, we develop our study of asymmetric graphs in another direction.

C. Core-Periphery Graphs

We next construct and analyze graphs that represent core-periphery social structures.[26] Such structures consist of a core - a group of densely connected agents - and a periphery - a group of agents with connections only to the core. These structures characterize many types of social and economic interactions. In economic geography, researchers have associated industrial patterns with core-periphery structures, where manufacturing activity is concentrated in one or few regions - the core - while agricultural activity is dispersed and supplies firms and people in the core [see e.g. Krugman (1991)]. Spatial organization of other activities such as trade and communication often follows a core-periphery pattern. Examples of core-periphery networks include the interlocking directorates of firms in the United States [Mintz and Shwartz (1981)], international trade [Snyder and Kick (1979)], and the social organization of academic research [Mullins et al. (1977)]. The precise structure of such networks are, of course, more complex than the stylized model we construct below. Yet, in all these examples, there is a clear distinction between a core and a periphery such that peripheral agents have few interactions - mostly with core agents - while core agents have many interactions - mostly with other core agents.

We model core-periphery structures as follows. We divide the population into two sets, the core, C, and the periphery, P, such that all agents in the core are linked to each other and no agents in the periphery are linked to each other; i.e., $\forall i, j \in C, g_{ij} = 1$ and $\forall i, j \in P$ *s.t.* $i \neq j, g_{ij} = 0$. Core agents may or may not have links to peripheral agents. We call core agents that have one or more links to peripheral agents *external core agents*, and we call all other core agents *internal core agents*. To make the analysis interesting, we assume there is at least one external core agent, so that at least one peripheral agent has a link to the core. To simplify the exposition, throughout the analysis below we assume that no external core agents have the same

[26] Bramoullé (2002a, 2002b) develops and analyzes graphs with a core-periphery structure in a study of anti-coordination games.

set of neighbors in the periphery.[27] This assumption makes every external core agent distinct in terms of its neighbors.

We consider a family of such structures, where each member of the family is distinguished by two measures. These measures capture the extent to which core agents are central sources of information for groups of agents in the periphery. For an external core agent i, consider his neighbors in the periphery, $N_i \cap P$. Some of these peripheral agents may be linked to other agents in the core. Let c_i denote the number of links between i's peripheral neighbors and core agents other than i:

$$c_i = \sum_{j \in N_i \cap P} (k_j - 1)$$

We consider the minimum and maximum such values for a graph $\mathbf{G}$: $\underline{c} = \min_{i \in EC}\{c_i\}$ and $\overline{c} = \max_{i \in EC}\{c_i\}$ where EC denotes the set of external core agents in $\mathbf{G}$. When $\underline{c}$ is small, for some agents in the periphery a single core agent is their sole source of information. When $\overline{c}$ is also small, this is true for most peripheral agents.

We first find the Nash equilibria search patterns for core-periphery graphs. For any graph, there are only two types of Nash equilibria. Both involve local experts. In the first type, external core agents are local experts. In the second type, all peripheral agents are local experts. We will see below that the welfare comparison between these different types of equilibria depends on the measures $\underline{c}$ and $\overline{c}$.

Proposition 11. *On any core-periphery graph, there are two types of Nash equilibria: (1) One external core agent is a local expert and searches e^*, all his neighbors (the rest of core and his linked agents in the periphery) do not search, and the remaining peripheral agents each search e^*. (2) All external core agents do no search, each peripheral agent searches e^*, and internal core agents collectively exert a total effort of e^*.*

In the first equilibrium, a single external core agent conducts all the search in the core. This information benefits all the other agents in the core as well as linked agents in the periphery.[28] In

[27] We have solved for all the Nash equilibrium profiles of core-periphery graphs, including those that do not satisfy this assumption. Moreover, the welfare results below hold whether or not this assumption is satisfied.

[28] When there are no internal core agents and an external core agent has exactly one peripheral neighbor, a slight variation on the first equilibrium exists. If i is a core agent and j his unique peripheral neighbor, the profile where $e_i + e_j = e^*$, all the other core agents do no search, and their peripheral neighbors search e^* is an equilibrium.

the second equilibrium, in contrast, all peripheral agents exert an effort of e^* and search done in the core is concentrated among internal core agents - those who do not have access to peripheral information. We illustrate in the following example.

Example 8. *Equilibrium Search Patterns in Core-Periphery Graphs. Consider the graphs in Figure 8. In the graph on the left of each panel, one core agent is internal while the other three core agents are external.*[29] *In the graph on the right, all core agents are external. Panel (a) shows an equilibrium in each graph where an external core agent is a local expert. Panel (b) depicts the equilibria where all peripheral agents search. Notice the difference in panel (b) between the equilibria in the graphs on the left and right: On the left, the internal core agent searches, while on the right, all core agents are external and do no search.*

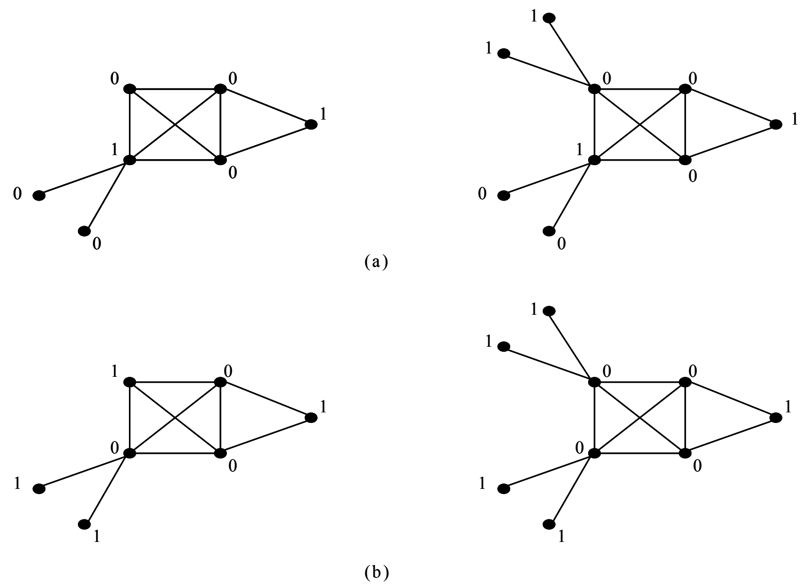

Figure 8. Equilibria in Core-Periphery Graphs

We next characterize second-best search profiles. Here we see that the pattern of links between the core and periphery, as captured in our measures $\underline{c}$ and $\overline{c}$, determines which of the two types of equilibria is second-best. Let W_e be the highest welfare of equilibria of the first type of equilibrium,

[29] *For ease of exposition the graphs in the Figure violate our assumption that no core two core agents have the same set of neighbors. All of welfare results are the same whether or not this assumption is satisfied.*

where an external core agent is a local expert, and let W_p denote the welfare of equilibria of the second type, where all peripheral agents are experts.

We start with the simplest case of sparse links between the core and periphery. Suppose that each peripheral agent has at most one link to the core; i.e., $\overline{c} = 0$.[30]

Proposition 12. *Consider any core-periphery graph* **G** *such that* $\overline{c} = 0$*. Then* $W_e > W_p$*; in every second best profile, an external core agent is a local expert and this agent is among those with the most links to the periphery.*

This case, $\overline{c} = 0$, is quite similar to the star graph, since each peripheral agent has at most one link to the core. With sparse links between the core and the periphery, information generated in the periphery does not benefit many agents in the core. The information premium is always smaller than the additional search costs, and therefore it is always better in terms of welfare for peripheral agents to search less.

As we see next, this conclusion does not necessarily hold for social structures where there are dense links between the core and periphery. Suppose now that at least one peripheral agent has more than one neighbor in the core; that is, $\overline{c} \geq 1$. In this case, it is possible that equilibria where all peripheral agents search is second-best. As we saw in section III above, second-best profiles generally trade-off search costs and information premia. Here, the only agents who might receive an information premium are external core agents, since they have access to information from neighbors in the core as well as the periphery. When $\underline{c}$ is high, many external core agents' neighbors in the periphery are well-connected to the core, and search by these peripheral agents brings more informational benefits. Hence, social welfare can be higher when search is concentrated in the periphery. To prove this result, we apply the techniques of Proposition 4. For each type of equilibrium, we identify the maximal independent set of agents that constitutes the set of local experts. We then count the links between agents in the set and agents outside the set. We then compare these numbers across equilibria, and the comparison ultimately depends on the value of $\underline{c}$.

Proposition 13. *Consider any core-periphery graph* **G** *such that* $\overline{c} \geq 1$ *: (1) If all agents in the core are linked to agents in the periphery and if* $\underline{c} < |C| - 1$*, then there exists a sufficiently*

[30] Recall we have assumed that at least one agent in the periphery has a link to an agent in the core.

increasing benefit function such that the second best equilibrium involves a core agent who is a local expert. If $\underline{c} > |C| - 1$ then there exists a sufficiently increasing benefit function such that in the second best equilibrium all peripheral agents search. (2) If there is an internal core agent and if $\underline{c} = 0$, then there exists a sufficiently increasing benefit function such that the second best equilibrium involves an external core agent who is a local expert. If $\underline{c} \geq 1$, there is a sufficiently increasing benefit function such that peripheral search is second best.

Paradoxically, the information premium associated with periphery search is higher when some core agents are not connected to the periphery. Since internal core agents have no peripheral source of information, they must search for information themselves. They then share their information with the rest of their neighbors in the core, and these external core agents have a higher information premium. Hence, if there is an internal core agent, there is a lower value of $\underline{c}$ for which peripheral search can yield higher welfare.

Example 9. *Core versus Periphery Search. Consider the graphs in Figure 8 and first compute the coefficients c_i for external core agents. We obtain $c_i = 1$ for the core agents with a unique peripheral neighbor and $c_i = 0$ for the core agents with two peripheral neighbors. Thus, $\underline{c} = 0$ and Proposition 13 applies. In both graphs, if b is sufficiently increasing, in the second-best profile a core agent with two neighbors in the periphery is a local expert. Calculations confirm this result. For instance, consider the graph on the left in panel (a). The welfare of this equilibrium is $5b(e^*)+2b(2e^*)-2ce^*$. The equilibrium where a different external core agent is a local expert yields a welfare of $6b(e^*) + b(3e^*) - 3ce^*$, which is always lower. The equilibrium where all peripheral agents are experts, shown on the left in panel (b), yields a welfare of $4b(e^*)+2b(2e^*)+b(3e^*)-4ce^*$. The difference in welfare between this equilibrium and the best equilibrium with external core agent search (shown on the left in panel (a)) is $b(3e^*) - b(e^*) - 2ce^*$, which is always negative.*

Finally, we consider the impact of adding links to this communication structure. Since agents in the core are completely connected, the only possible new links are between peripheral agents and core agents. There are many ways to think about adding links. We could begin with no links between the core and periphery, then successively connect each peripheral agent to the core. We could, of course, take any arbitrary graph and add a link between a core and peripheral agent.

It is easy to see that linking isolated peripheral agents to the core never leads to a fall in welfare. Consider a core-periphery graph where a peripheral agent is isolated, and question the effect of connecting her to the core. Let $\widetilde{\mathbf{e}}$ be a second-best equilibrium for the original graph. The isolated peripheral agent searches e^*. As for her soon-to-be new neighbors, there are two cases. Either they do no search in $\widetilde{\mathbf{e}}$, or they exert positive effort. In the first case, once connected, she might still search e^*, which benefits her new neighbors and yields greater welfare. In the second case, she can benefit from her new links to reduce her search, and this also increases welfare.

To analyze a new link in any arbitrary graph, we apply Proposition 5: a link between an agent i and an agent j will always increase welfare unless agents i and j search in every second-best search profile for the original graph. Such situations arise in core-periphery graphs.

One important case is when a core agent is a sole source of information to a set of peripheral agents. A link between this core agent and an agent in the periphery who provides information to the core can reduce welfare. Consider a social structure where some external core agents are sole sources of information to a set of peripheral agents ($\underline{c} = 0$) and other core agents are not sole information sources ($\overline{c} \geq 1$). In this case, according to Proposition 13, if there is an internal core agent, then there exists a sufficiently increasing benefit function such that the second best equilibrium involves a core agent who is a local expert. This core agent is an agent i for whom $c_i = 0$, and all peripheral agents j not linked to i are also local experts. Now consider a link between agent i and an agent j in the periphery who is connected to several other core agents. In the new set of equilibria, peripheral agent j need not search since it has another source of information. Welfare can fall, since the original core agents connected to j receive less information. The example below illustrates.

Example 10. *Effect of a New Link between the Core and the Periphery. Consider the graph in panel (a) Figure 9. We showed in the previous example that the equilibrium where the external core agent with two peripheral neighbors is a local expert is always second-best and yields a welfare of* $W = 5b(e^*) + 2b(2e^*) - 2ce^*$. *As described above, linking this agent with the third peripheral agent might lower social welfare. To see this, examine the possible equilibria on the new graph in panel (b): (1) In the equilibrium shown in the Figure, the core agent with three neighbors in the periphery is a local expert and* $W_1 = 7b(e^*) - ce^*$; *(2) Another equilibrium would be where a different external core agent is an expert yielding welfare* $W_2 = 6b(e^*) + b(3e^*) - 3ce^*$, *which is always lower than* W_1; *(3) The last equilibrium is where all peripheral agents search yielding* *welfare* $W_3 = 4b(e^*) + 2b(2e^*) + b(4e^*) - 4ce^*$. *Therefore, the second-best level of welfare falls with the new link if* $W > W_1$ *and* $W > W_3$. *That is, if* $\frac{1}{2}[b(4e^*) - b(e^*)] < ce^* < 2[b(2e^*) - b(e^*)]$.

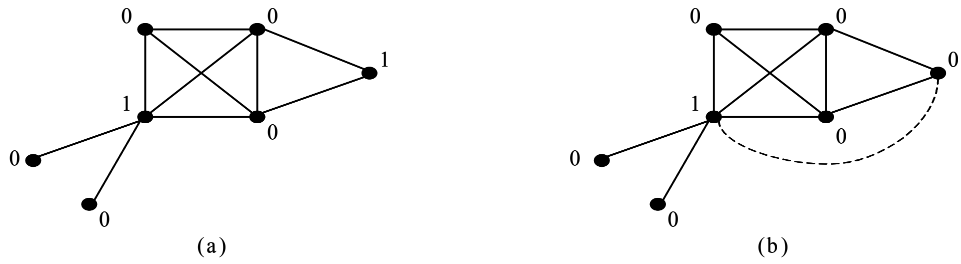

Figure 9. Effect of New Link Between the Core and Periphery

Our results for core-periphery graphs lend insights that may apply to more complex networks. Results here indicate that centrality should in general have a strong impact on the level of effort in equilibrium. Individuals who are more central (e.g. have more connections) have access to more social sources of information, hence are expected to search less in equilibrium than agents who are less central. Even when the graph does not have a core-periphery structure, it is possible to measure relative degrees of centrality of the agents [see Wasserman and Faust (1994, ch. 5)]. Our results are consistent with empirical findings concerning centrality such as (1) the discovery of major innovations often takes place at the periphery of social systems,[31] and (2) "mavens" are not at the center of social networks [Gladwell (2000)]. On the other hand, our results indicate that social welfare is generally higher when more central agents exert more effort. This outcome echoes our findings on bridges and communities and points to a general source of tension between individual rationality and social welfare when agents have unequal numbers of links and occupy different positions within a network.

V. Conclusion

This paper introduces a new model of social learning where people search for information and share their findings along social links. We explore how the social structure affects the pattern of

[31] See Kuhn (1963) and Chubin (1976) on scientific innovations, and Mansfiel (1968), Jewkes et al. (1959), and Hamberg (1963) on industrial innovations.

search. We both analyze general graphs and construct specific families of graphs to capture the features of different social structures.

In each case we first study equilibrium search patterns. We distinguish between equilibria where some agents specialize in search and equilibria where search is distributed among agents. We find several structures that admit only specialized equilibria. A star graph is an extreme example - either the center is a local expert or the agents in the periphery are local experts. In communities connected by bridges and in core-periphery graphs, in equilibrium there are always some agents who rely on others for information. In general, agents who have a more central position, or who bridge different communities, search relatively less in equilibrium. We also show that when agents have similar positions in the network, as in the graphs with overlapping neighborhoods, they can have very different effort levels in equilibrium. These different levels derive from the strategic substitutability of search efforts; when one agent searches more, his neighbors search less. We discuss below a dynamic model which can select among these equilibria.

Second, for both general graphs and our families of graphs, we study the aggregate welfare of equilibrium profiles. Because of strategic substituability, equilibrium search patterns are never efficient. Hence, we adopt a second-best approach. We compare the aggregate costs and benefits of equilibrium search patterns. We find that social welfare is determined by a trade-off between information premia and search costs. Which factor dominates depends on the graph and on the shape of benefit function. We obtain a useful method to determine when, for a sufficiently increasing benefit function, specialized equilibria yield highest welfare. Using the concept of maximal independent sets from graph theory, we count the number of links between experts and non-experts. The formula tells us which equilibria with local experts yields highest welfare and when such equilibria yield greater welfare than distributed equilibria. When agents have different numbers of neighbors - as in the communities-bridges and core-periphery graphs - we find a conflict between individual behavior and aggregate social welfare. Agents who have more links exert comparatively less effort in many equilibria. But second-best profiles involve search by these well-connected agents whose private search efforts would lead to the highest social benefits.

Third, the paper explores how new links affects the pattern of search and welfare. There are two effects of new links. On the one hand, a link allows more sharing of information, leading to a potential gain from social learning. On the other hand, a link allows an agent access to more information which could lead her to reduce her own search effort. We find this negative effect has distributional consequences. For instance, contrary to the prevailing wisdom, we show that bridges between communities can be detrimental to others. Some agents who gain outside sources of information must reduce their own search efforts, which lowers the amount of information they have to share with their community. Furthermore, we show how new links might lower overall welfare. When two agents are in critical positions in a network such that their search efforts benefit many others, a link between them can reduce aggregate welfare.

Our analysis highlights the importance of agents who specialize in gathering information, who we call local experts. While our static model admits both specialized and distributed equilibria, a simple dynamic model indicates we are likely to see local experts in most social networks. Suppose individuals' needs for a new technology (such as tetracycline) do not appear simultaneously, but are random, depending on individuals' different circumstances. At time $t = 0$, people have no information about the innovation. At each later date, one agent, picked at random, has a need for a new technology. She first turns to her friends and colleagues for information. If the information obtained through social learning is not sufficient, she complements through her own costly search. This process converges to an equilibrium with local experts, where agents who are the first in their neighborhood to need the information become the local experts.

Beyond the analysis here, our results concerning local experts may shed light on the role of informed consumers in the economy. Uninformed consumers, i.e. those who lack good first-hand knowledge of the market, might rely on the suggestions of informed friends. Heterogeneity in information acquisition and the important role played by informed consumers have long been recognized in industrial organization. Even a few informed consumers can have a positive effect on efficiency, through the direct pressure they exert on producers [Wilde and Shwartz (1979), Salop and Stiglitz (1977)]. Our analysis points to potentially greater channel through which informed consumers affect the market. Social learning helps the diffusion of information gathered by informed consumers and greatly amplifies their pressure on the market. Recognizing this, many communication campaigns target informed consumers [Feick and Price (1986), Gladwell (2000)].

The model may also shed new light on the well-studied phenomenon of conformity and imitation. An individual who has been particularly successful in his search for information will be imitated by his social neighbors. Here, conformity arises because agents gather different information and communicate with each other. Conley and Udry (2002) find robust empirical evidence for this kind of "informational imitation." Informational imitation differs from other more studied sources of conformity, such as conformistic preferences [Akerlof (1997)], signaling [Bernheim (1994)], and herding due to informational cascades [Banerjee (1992), Bikhchandani et. al. (1992)].

The analysis we conduct here is a first step in the study of experimentation and social learning in social networks, and more generally the study of public goods in social networks. In our static model, three simplifying assumptions could be relaxed in future research. First, one could study heterogeneity in individual characteristics, such as search costs or benefits from information. Equilibrium profiles would then reflect such differences. Second, one could complicate the process of information diffusion. For instance, information could diffuse more than one step with, possibly, some loss of quality at each step. Notice that our model can easily accomodate information diffusion without loss of quality of several steps.[32] Third, information could have more than one dimension and people might be interested in different types of information. We believe that our central insights would extend to these cases, with some adaptation to the more complex environments. As long as substitution between private and others' efforts is present, heterogeneity in individual efforts would emerge in equilibrium and should reflect differences in network characteristics. Individual and social gains from learning should still be mainly determined by the interplay between costs savings and information premia.

A greater departure from our current modeling would allow agents to price or strategically exchange information. In our current model, welfare would be maximized if people internalized the effect their search decisions have on others. This objective could be achieved through altruism or by attaching a price to information transmission. This altruism or pricing may occur in reality – not as an exchange of money per se, but as a social exchange [Blau (1986)]. An individual who relies on another person for information may "pay him back" by offering other services or with social deference. A potentially interesting question is how experts might "compete" to provide information to their neighbors. As noted earlier, strategic pricing and strategic link formation could play a role in the long run, e.g. on the evolution of the network.

[32] k-step information diffusion on the graph $\mathbf{G}$ is formally equivalent to 1-step information diffusion on the graph $\mathbf{G}^k$ defined as follows: $\mathbf{G}^k_{ij} = 1$ if i and j are less than k steps away in $\mathbf{G}$. Formally, $\mathbf{G}^k_{ij} = 1$ if either $\mathbf{G}_{ij} = 1$ or there exist $t \leq k$ agents $i_1, ..., i_t$ such that $\mathbf{G}_{ii_1} = \mathbf{G}_{i_1 i_2} = ... = \mathbf{G}_{i_t j} = 1$.

Appendix

The following standard property will be used in several places. Consider a benefit function b that is increasing, concave, and such that $b'(e^*) = c$. Then, for any real number s greater than 1, the quantity $b(se^*) - b(e^*) - (s-1)ce^*$ is negative and decreases when s increases.

For any benefit function b such that $b'(e^*) = c$, define the indice σ as follows

$$\sigma = \frac{b(ne^*) - b(e^*)}{c(n-1)e^*}$$

This indice σ is between 0 and 1. In the text, a benefit function is called "sufficiently increasing" if σ is sufficiently close to 1.

Properties of μ in discrete illustration.

Suppose that the distribution of prices has cumulative distribution function $F(p)$. The quantity $\mu(e)$ is the expectation of the minimum of e draws that are independent and identically distributed from this distribution. Formally, $\mu(e) = \int_0^\infty pdG(e;p)$ where $G(e;p)$ denotes the cumulative distribution of the minimum, and is given by $G(e;p) = 1 - [1 - F(p)]^e$. This yields, through integration by parts,

$$\mu(e) = \int_0^\infty [1 - F(p)]^e dp$$

Since $0 \leq 1 - F(p) \leq 1$, $\mu(e)$ is decreasing in e. In addition, $\mu(e) - \mu(e+1) = \int_0^\infty [1-F(p)]^e F(p)dp$ which shows that $\mu(e) - \mu(e+1)$ is decreasing in e as well and μ is convex in e.

Proof of Corollary 1.

Let i be an agent who exerts positive effort $e_i > 0$. Then, $e_i + \bar{e}_i = e^*$. Since i is connected to all other individuals, it means that $\bar{e}_i = \sum_{j \neq i} e_j$, and $\sum_j e_j = e^*$. Reciprocally, these profiles are obviously Nash equilibria.

Proof of Corollary 2.

We first show that each profile is a Nash equilibrium. (1) Suppose the center agent i exerts e^*. All other agents in the graph then receive e^* information from i and their best responses are to do no search. Agent i receives no information from his neighbors, hence e^* effort is i's best response. (2) Suppose the center agent i exerts 0 search. All other agents in the graph then receive no information from any neighbor, and their best responses are to exert e^*. When all other agents exert e^*, the center agent i receives $(n-1)e^*$ from his neighbors, and hence his best response is to do no search.

We now show these profiles are the only Nash equilibria. Suppose the center agent i exerts a search level $0 < e' < e^*$. In equilibrium, each of his neighbor's must search $e^* - e'$. Suppose now each of agent i's neighbors search $e^* - e'$. Agent i's information will then equal $(n-1)(e^* - e') + e'$ which is strictly greater than e^* for $n > 2$. Hence, agent i would have an incentive to deviate and lower her search effort.

Proof of Proposition 3.

Let $\mathbf{e}$ be an equilibrium and i an agent such that $e_i > 0$ and for some $j \neq i, g_{ij} = 1$. From Proposition 1, we know that $\bar{e}_i + e_i = e^*$, which implies $b'(\bar{e}_i + e_i) = c$. Examine the partial derivative of the welfare function with respect to e_i.

$$\frac{\partial W}{\partial e_i} = \sum_{j \in i \cup N_i} b'(\sum_{k \in j \cup N_j} e_k) - c = \sum_{j \in N_i} b'(\sum_{k \in j \cup N_j} e_k)$$

When b is strictly concave, this derivation implies $\frac{\partial W}{\partial e_i} > 0$, and welfare could be strictly improved by increasing e_i. Agents do not internalize the benefits their search efforts have for others.

Proof of Proposition 4.

Since b is increasing and concave, for any real number s between 1 and n we have $(s-1)\sigma ce^* \leq b(se^*) - b(e^*) \leq (s-1)ce^*$. This yields

$$\sum_{j \in N/I} [b(s_j e^*) - b(e^*)] \geq \sigma \sum_{j \in N/I} (s_j - 1)ce^*$$

Thus, equation (3) is satisfied if σ is sufficiently high and if $\sum_{j \in N/I}(s_j - 1)ce^* > (|I| - 1)\, ce^*$. The last inequality simplifies to $\sum_{j \in N/I} s_j > n-1$. The proof is completed with following lemma:

Lemma A1. For any maximal independent set I on any graph $\mathbf{G}$, $\sum_{j \in N/I} s_j = \sum_{i \in I} k_i$.

Proof: We count the number of links between agents in I and agents outside I from two perspectives. Let $x_i = 1$ if $i \in I$ and $x_i = 0$ otherwise. Then,

$$\sum_{j \in N/I} s_j = \sum_{j \in N/I} \sum_{i \in N} g_{ji} x_i = \sum_{j \in N} \sum_{i \in N} g_{ji} x_i - |I|$$

Switching the double summation yields

$$\sum_{j\in N/I} s_j = \sum_{i\in N}\sum_{j\in N} g_{ji}x_i - |I| = \sum_{i\in N} x_i(\sum_{j\in N} g_{ji}) - |I| = \sum_{i\in I}(k_i+1) - |I|$$

Calculations for Example 4.

For a star with n agents, the welfare of where the center searches is $W^1 = nb(e^*) - ce^*$ while the welfare of where peripheral agents search is $W^2 = (n-1)[b(e^*) - ce^*] + b((n-1)e^*)$. Their difference is equal to $W^1 - W^2 = (n-2)ce^* - [b((n-1)e^*) - b(e^*)]$ which is positive.

Proof of Proposition 6.

Consider first a distributed equilibrium such that for every i, $0 < e_i < e^*$. It follows from Proposition 1 that for every i, $\sum_{j=i-k}^{i+k} e_j = e^*$. Subtracting the equation for i and the equation for $i+1$ yields $e_{i-k} - e_{i+k+1} = 0$. Thus for every i, $e_i = e_{i+2k+1}$. Recall that when the indice becomes greater than n, one simply substracts n from it. This shows that, more generally, $e_i = e_j$ as soon as there exists two integers t, t' such that $i - j = (2k+1)t - nt'$. Define m as the greatest common divisor of $2k+1$ and n. A standard result of arithmetics is that there always exist two integers t and t' such that $m = (2k+1)t - nt'$. This means that for every i, $e_i = e_{i+m}$ and the whole effort profile is generated by the first m individual efforts $e_1, ..., e_m$.

Consider, next, an expert equilibrium. Let I be the set of agents that exert e^*. This set must be non-empty (otherwise, some agent would increase his search). By Proposition 1, in a Nash equilibrium, no agent $j \in I$ can be linked to any other agent in I. Hence, the distance between each agent $j \in I$ must be at least $k+1$. The distance can also not be more than $2k+1$. If it were, then the agent $k+1$ away would be learning no information from her neighbors and hence would have an incentive to deviate and exert positive effort. It is evident that these strategies constitute a Nash equilibrium.

Search costs and welfare computations for equilibria in overlapping neighborhoods graphs.

Let $\mathbf{e}$ be an equilibrium. An argument similar to Lemma A1 yields $\sum_{i\in N}(e_i + \sum_{j\in N_i} e_j) = \sum_{j\in N}(k_j+1)e_j$. This equality evaluates from two points of view the total level of information that benefits the agents. On the left hand side, any agent i receives the benefits of $e_i + \sum_{j\in N_i} e_j$ information. On the right hand side, when agent j searches e_j, his k_j neighbors and himself benefit from it. Then, by Proposition 1, $\forall i, e_i + \sum_{j\in N_i} e_j \geq e^*$. Since on overlapping neighborhoods,

$\forall j, k_j = 2k$, we have $\sum_{j \in N} e_j \geq \frac{n}{2k+1} e^*$ which shows that distributed equilibria yield lowest search costs.

Consider an expert equilibrium with $|I|$ experts. No agent can be connected to three experts, since in this graph, it would mean that two experts communicate with each other. Denote by n_1 the number of agents connected to a single expert and by n_2 the number of agents connected to two experts. With these notations, welfare can be expressed as $W = nb(e^*) + n_2[b(2e^*) - b(e^*)] - c|I|e^*$. We then compute n_2. Clearly, $|I| + n_1 + n_2 = n$. We apply Lemma A1. Here, we have $\sum_{i \in I} k_i = 2k|I|$ and $\sum_{j \in N/I} s_j = n_1 + 2n_2$, which together yield $n_1 + 2n_2 = 2k|I|$. Hence, $n_2 = (2k+1)|I| - n$. Welfare only depends on the number of local experts in the expert equilibrium.

Proof of Proposition 8.

Our proof relies on the following lemma.

Lemma A2. Let $\mathbf{e}$ be an equilibrium for a graph $\mathbf{G}$. Consider two agents i and j such that $i \cup N_i \subset j \cup N_j$. If $e_j > 0$, then for every agent $k \neq i$ who belongs to $N_j \backslash N_i$, $e_k = 0$.

Proof: When $e_j > 0$, by Proposition 1, $e_j + \sum_{k \in N_j} e_k = e^*$. The information obtained by j is equal to the sum of the information obtained by i and the information produced by the neighbors of j who do not communicate with i

$$e_j + \sum_{k \in N_j} e_k = e_i + \sum_{k \in N_i} e_k + \sum_{k \neq i \in N_j \backslash N_i} e_k$$

The Nash condition on i implies that $e_i + \sum_{k \in N_i} e_k \geq e^*$ which means that $\forall k \neq i \in N_j \backslash N_i, e_k = 0$ and $e_i + \sum_{k \in N_i} e_k = e^*$.

Now, consider a bridge connecting i and j and denote by i_0 an agent in i's community who is not a bridge agent. Since $i_0 \cup N_{i_0} \subset i \cup N_i$, we can apply Lemma A2. Either $e_i = 0$ or all the agents to whom i is connected who are not neighbors of i_0 do no search. This means that if $e_i > 0$, all the bridge agents in the other community who are connected to i do no search. Reciprocally, the profiles described in the Proposition are evidently Nash equilibria.

Equilibria for Two-Step Diffusion.

Bridge agents are connected to every agent in the graph $\mathbf{G}^2$. Suppose that one bridge agent j exerts strictly positive effort $e_j > 0$ and consider i_1 a non-bridge agent in one community and i_2 a non-bridge agent in the other community. Since $i_1 \cup N_{i_1} \subset j \cup N_j$ and $i_2 \cup N_{i_2} \subset j \cup N_j$, we can apply Lemma A2 to $i_1$and j and i_2 and j. For every s belonging to $N_j \backslash N_{i_1}$ or to $N_j \backslash N_{i2}$, $e_s = 0$, which implies that all non-bridge agents do no search. Therefore, bridge agents must exert a total effort of e^*, and any such profile is an equilibrium. In contrast, if bridge agent do no search, non-bridge agents in each community do not have access to information outside their community, hence the Nash equilibria are the profiles such that non-bridge agents in each community exert a total effort of e^*.

Proof of Proposition 9.

Consider an equilibrium profile $\mathbf{e}$. If $\forall i \in B, e_i = 0$, the information premium equals zero, hence welfare is strictly greater when some bridge agent exerts positive effort. Hence, in any second-best profile $\tilde{\mathbf{e}}$ there exists an agent $i \in B$ such that $\tilde{e}_i > 0$. Now suppose that in $\tilde{\mathbf{e}}$ some search is done by non-bridge agents in i's community. That is, $\exists j \notin B$ such that $g_{ij} = 1$ and $e_j > 0$. Consider the profile $\mathbf{e}'$ defined as follows: $e'_j = 0$, $e'_i = e_i + e_j$ and $e'_k = e_k$ for every $k \neq i, j$. The search done by j in $\tilde{\mathbf{e}}$ is done by i in $\mathbf{e}'$. Then, $\mathbf{e}'$ is an equilibrium profile in which bridge neighbors of i receive greater informational benefits than in $\tilde{\mathbf{e}}$. Thus $W(\mathbf{e}'; \mathbf{G}) > W(\tilde{\mathbf{e}}; \mathbf{G})$ which is contradictory. This shows that in second-best profiles, as soon as some bridge agents do some search in one community, they do all the search.

Proof of Proposition 10.

When $\beta = 1$, the result is a direct consequence of Proposition 9. Suppose that $\beta \geq 2$. Our proof proceeds in two steps: (1) We determine the equilibria that yield greatest welfare, conditional on the hypothesis that t bridge agents in one community and $\beta - t$ bridge agents in the other community exert positive effort; and (2) We use the previous calculations to determine the optimal t. First, suppose that t bridge agents in one community exert a total effort of e^*. These agents are denoted by $1, ..., t$. How should we allocate the effort to maximize the information premium received by their neighbors in the other community? This optimization problem can be expressed as follows:

$$\max_{e_1 + ... + e_t = e^*} \sum_{i=1}^{t} b(e_i + e^*) - b(e^*)$$

Since b is concave, the solution to this problem is standard and involves equal sharing of effort. That is, an optimal solution is such that $\forall i, e_i = \frac{1}{t} e^*$. Next, compute the optimal t. For clarity, denote by $\pi(t) = b((1 + \frac{1}{t})e^*) - b(e^*)$. The greatest welfare in equilibrium with t is $W(t, \beta) = t\pi(t) + (\beta - t)\pi(\beta - t)$ if $0 < t < \beta$ and $W(0, \beta) = \beta\pi(\beta)$. This last case can easily be eliminated.

Since b is increasing, we have $\pi(\beta) < \pi(\beta - 1)$ and $\pi(\beta) < \pi(1)$, which means that $\beta\pi(\beta) < (\beta - 1)\pi(\beta - 1) + \pi(1) = W(1, \beta)$. Therefore, in any second-best profile, $0 < t < \beta$ and the optimization problem reduces to

$$\max_{t \in \{1,2,\ldots,\beta-1\}} t\pi(t) + (\beta - t)\pi(\beta - t)$$

One can check that the function $t \mapsto t\pi(t)$ is increasing and concave,[33] which implies that the optimal t is an integer closest to $\beta/2$.

Calculations for Example 7.

We characterize the second-best profiles for the graph of Example 7 (see Figure 7). First, consider the graph without the link. The profile depicted in figure 7 yields an information premium of $4[b(2e^*) - b(e^*)]$. Following Proposition 9, search is exclusively done by bridge agents in either one community, or both communities. When bridge agents in a single community search, the three bridge agents in the other community benefit from their information, hence the information premium cannot be greater than $3[b(2e^*) - b(e^*)]$ and it is not second-best. When a bridge agent with a single bridge searches, his neighbor in the other community does not search, hence an agent with single bridge in the other community searches as well. Hence, the two bridge agents with two bridges do no search, and the information premium equals $2[b(2e^*) - b(e^*)]$. Second, consider the graph with the link. Either a bridge agent i with three bridges search, and it is best when $e_i = e^*$, which yields an information premium of $3[b(2e^*) - b(e^*)]$. Or only bridge agents with single bridges search and the information premium is $2[b(2e^*) - b(e^*)]$.

Proof of Proposition 11.

Consider an equilibrium profile $\mathbf{e}$. Clearly, total search done on the core $\sum_{i \in C} e_i$ cannot be greater than e^*. Suppose that there is an external core agent i who exerts positive effort, but who is not a local expert: $0 < e_i < e^*$. Recall, $N_i \cap P$ denotes the set of peripheral neighbors of i. We distinguish two cases: $|N_i \cap P| = 1$ and $|N_i \cap P| \geq 2$.

(1) Suppose that i has a unique peripheral neighbor j. Since $j \cup N_j \subset i \cup N_i$, Lemma A2 tells us that agents in $C \backslash N_j$ do no search. N_j is the set of core agents connected to j, to which i belongs. There are two cases. (a) $e_j = 0$. In this case, i receives all her information from

[33] The first derivative equals $b((1 + \frac{1}{t})e^*) - b(e^*) - \frac{1}{t}e^* b'((1 + \frac{1}{t})e^*)$ and is positive because b is concave, while the second derivative equals $\frac{1}{t^3}(e^*)^2 b''((1 + \frac{1}{t})e^*)$ and is negative.

agents in N_j, and hence the total search done on N_j is exactly equal to e^*. This is possible only when agents of N_j who search do not have other peripheral neighbors than i. (Since total search done on N_j is e^*, these other peripheral neighbors could not search, hence would receive less than e^*).All core searchers have the same, unique peripheral neighbor. (b) $e_j > 0$. In this case, $e_j + e_i + \sum_{N_j \backslash \{i\}} e_k = e^*$. If j has other core neighbors than i, j must be their unique peripheral neighbor. Again, all core searchers have the same, unique peripheral neighbor.

(2) Suppose that $|N_i \cap P| \geq 2$. Take $j \in N_i \cap P$. Since $j \cup N_j \subset i \cup N_i$, Lemma A2 yields $\forall k \in N_i \backslash N_j, e_k = 0$. Thus, all other peripheral neighbors of i do no search. The same argument applied to another peripheral neighbor of i implies that $\forall k \in N_i \cap P, e_k = 0$ and all peripheral neighbors of i do no search. Thus, for any j peripheral neighbor of i, the total search done by her neighbors other than i must be at least equal to $e^* - e_i$. Since these agents all belong to the core, it means that their total search is exactly equal to $e^* - e_i$. Thus, any core searcher must be connected to all the neighbors of i and cannot have peripheral neighbors not connected to i. That is, all core searchers must have the same peripheral neighbors.

When core agents have distinct neighbors, the only case where an external core agent exerts positive effort but is not a local expert is when this agent has a unique peripheral neighbor, who does not have other neighbors, and there is no internal core agents (as described in the Footnote). In general, we showed that there are only two additional possibilities with respect to the set of equilibria described in Proposition 11: (1) When several core agents have a unique and common peripheral neighbor and when there are no internal core agents, any profile such that e^* is allocated in any way among these core agents and their peripheral neighbor, other core agents do zero and other peripheral agents are experts is an equilibrium; and (2) When several core agents have the same peripheral neighbors, any profile where these core agents exert a total effort of e^*, their common peripheral neighbors do no search, other core agents do no search, and other peripheral agents are experts is an equilibrium.

Proof of Proposition 12.

For each agent j who belongs to EC, denote by $k_{j,P}$ the number of peripheral neighbors of j. That is, $k_{j,P} = k_j - (|C| - 1)$. We examine two cases. First, suppose that there exists an internal core agent. Consider an equilibrium where internal core agents exert an effort of e^*. All peripheral agents earn $b(e^*) - ce^*$, while the external core agent j earns $b[(k_{j,P} + 1)e^*]$. Welfare in equilibrium equals

$$W_0 = nb(e^*) - ce^* + \sum_{j \in EC} \{b[(k_{j,P}+1)e^*] - b(e^*) - k_{j,P}ce^*\}$$

Consider next the equilibrium where i is the external core expert. Internal core agents earn $b(e^*)$ and do not search. External core agent j who does not search earns $b[(k_{j,P}+1)e^*]$, while the payoff of i is $b(e^*) - ce^*$. Peripheral agents not connected to i earn $b(e^*) - ce^*$, while peripheral neighbors of i earn $b(e^*)$. This leads to

$$W_i = nb(e^*) - ce^* + \sum_{\substack{j \in EC \\ j \neq i}} \{b[(k_{j,P}+1)e^*] - b(e^*) - k_{j,P}ce^*\}$$

Substracting both formulas yields $W_i - W_0 = -\{b[(k_{i,P}+1)e^*] - b(e^*) - k_{i,P}ce^*\}$ which is positive and increasing with $k_{i,P}$ since b is concave and $b'(e^*) = c$. Second, suppose that there is no internal core agent. The expression for W_i does not change, hence is increasing in $k_{i,P}$. Welfare of an equilibrium where core agents do no search is $W_0 = nb(e^*) + \sum_{j \in EC}\{b[k_{j,P}e^*] - b(e^*) - k_{j,P}ce^*\}$ since total search is equal to zero on the core. Again, substracting both expressions yields

$$W_i - W_0 = -\{b(k_{i,P}e^*) - b(e^*) - (k_{i,P}-1)ce^*\} + \sum_{\substack{j \in EC \\ j \neq i}} \{b[(k_{j,P}+1)e^*] - b(k_{j,P}e^*)\}$$

which is positive.

Proof of Proposition 13.

First, note that any equilibrium has a welfare less than or equal to the welfare of an expert equilibrium (even when several core agents have the same set of neighbors). To determine second-best profiles, we only need to determine which expert equilibrium yields greatest welfare. To do this, we compute $\sum_{i \in I} k_i$ for all expert equilibria where, recall, I is the maximal independent set of experts. Consider first the equilibrium where the external core agent j is an expert. Here, the set of experts is composed of j and all peripheral agents not connected to j. Since $k_j = |C| - 1 + |N_j \cap P|$, we obtain $\sum_{i \in I} k_i = |C| - 1 + |N_j \cap P| + \sum_{i \in P, i \notin N_j} k_i$. Using the parameter c_j, this expression can be simplified to $\sum_{i \in I} k_i = |C| - 1 + \sum_{i \in P} k_i - c_j$. Next, consider the equilibrium where all peripheral agents search. Here we have to distinguish two cases: (1) If the internal core is not empty, there is an internal core expert and $\sum_{i \in I} k_i = |C| - 1 + \sum_{i \in P} k_i$; (2) If the internal core is empty, $\sum_{i \in I} k_i = \sum_{i \in P} k_i$. Combining these expressions and the techniques of Proposition 4 induces the results described in the Proposition, except for the case when $\underline{c} = 0$ and there is no internal core agent. In this last case, direct examination of welfare yields the result. Some core agents are such that all their peripheral neighbors are only connected to them. Search by one of these core agents with most neighbors yields greatest welfare. (Computations are similar to the case of the star graph.)

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