Inequality, Lobbying and Resource Allocation

Joan Esteban
Instituto de Análisis Económico, CSIC and Universitat Pompeu Fabra
Debraj Ray
New York University and Instituto de Análisis Económico, CSIC

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Abstract

In this paper we propose a particular channel through which wealth inequality distorts the public resource allocation process. Our analysis rests on four premises: [1] governments play a role in the allocation of resources; [2] governments lack information just as private agents do regarding the identity of efficient sectors; [3] agents lobby the government for preferential treatment; and [4] a governmenteven if it honestly seeks to maximize economic efficiency may be confounded by the possibility that both high wealth and true economic desirability create loud lobbies. We construct a model to describe this scenario, and study the effects of aggregate wealth and wealth inequality on the efficiency of public allocation. Broadly speaking, both poorer economies and unequal economies display greater public inefficiency. The paper warns against the conventional wisdom that this is so because such governments are more corrupt.

1 Introduction

In this paper we propose a particular channel through which wealth inequality distorts the resource allocation process. Our analysis rests on four premises:

[1] Governments play a role in the allocation of resources. They can facilitate (or hinder) economic activity in certain geographical regions or in certain sectors by the use of subsidies, tax breaks, infrastructural allocation, preferential credit treatment, and permissions or licenses.

[2] Governments lack information — just as private agents do — regarding which sectors are worth pushing in the interests of economic efficiency.

[3] Agents lobby the government for preferential treatment. Moreover, lobbying sometimes but certainly not always does entail bribery of corrupt government bureaucrats. It involves industrial confederations, processions, demonstrations, signature campaigns, media manipulation and a host of other visible means to demonstrate that preferential treatment to some group will ultimately benefit "society" at large.

[4] A government — even if it honestly seeks to maximize economic efficiency — may be confounded by the possibility that *both* high wealth and true economic desirability create loud lobbies.

Of these four premises, it is immediately necessary to defend the premise of "efficiency maximization". We certainly do not believe that every public decision-maker is truly honest. Nor do we believe that all honest governments will seek to maximize economic efficiency before all else. We merely use this assumption as a device to understand the signal-jamming created by the interplay of wealth and true profitability. In doing so, we develop a theory of the interaction between economic inequality and imperfect resource allocation.

We argue that in a world of imperfect information, where lobbying or other forms of costly signaling play the role of providing information to policy-makers, wealth inequality may distort the signals transmitted by economic agents. The point that lobbying can be conceived as a costly device to transmit information has already made by Austen-Smith (1994) and Austen-Smith and Banks (2000, 2002), among others. We take this idea a step further, by explicitly linking the cost of lobbying to capital market imperfections, and so to wealth. We wish to capture the idea that profitable sectors have more of an incentive to lobby intensively but, at the same time, sectors dominated by wealthy interest groups find it *easier* to lobby more intensively.

Consequently, policymakers on the receiving end of such lobbies — honest though they may be — can make bad resource-allocation decisions.

Indeed, a corrupt government may not make things that much worse. There is an abundant literature arguing that high inequality is negative for growth — and especially so in developing countries — because this facilitates the buying of corrupt politicians and triggers intense rent-seeking. The point we wish to make here is that one can explain (at least partially) the observation that poor countries with high inequality appear to manage resources rather poorly without having to appeal to the *deus ex machina* of money-pocketing politicians. [We return to these matters in detail in the concluding section of the paper.]

The model we choose to make these points conceives of a government (or social planner) as an entity that publicly provides licenses, quotas, infrastructure or any essential goods to carry out productive activities. We shall call these objects *permissions*. A set of economic agents (we might think of them as individuals, production groups, or sectoral/regional interests) compete for these permissions. An agent is distinguished by two characteristics, her productivity and her wealth. This is private information: the government can observe neither wealth nor productivity.

A permission granted to an agent permits a level of output equal to the individual productivity factor. The government has a limited number of permissions and wishes to allocate them so as to maximize economic efficiency — aggregate output in this case. However, because productivity is private, the government does not know who the appropriate agents are. Instead, agents can send (costly) signals conveying implicit information regarding the returns to being awarded a permission. But the intensity of the signal emitted is conditioned both by wealth as well as by the productivity level. The government is aware of this possibility, and reacts to signals accordingly, in an effort to allocate its limited permissions as efficiently as possible. This is the model of signal-jamming we wish to explore.

We begin our paper by fully characterizing the signaling equilibria of this model, under two restrictions on beliefs and actions. The first employs a slight variation on the "intuitive criterion" (Cho and Kreps (1987)). The second demands that the policy maker treat two signals with the same information content (regarding payoffs) in an identical way. Despite the fact that our model allows for a continuum of productivities, wealths and signals, these two conditions generate a unique equilibrium (Proposition 1).

We then turn to the connections between wealth, the distribution of wealth, and resource allocation. We do this from several complementary perspectives. First, we consider a scenario in which aggregate wealth changes, keeping the distribution of that wealth constant. Then we study changes in the wealth distribution itself, with mean wealth held constant. For each of these categories of wealth change there are different ways in which we might approach the question of resource allocation. We study two in this paper: allocative efficiency pure and simple, and overall efficiency, which is just allocative efficiency *net* of lobbying costs.

The principal contribution of this paper is to lay down a methodology for studying these changes. The fact that a potentially complicated multildimensional signalling model precipitates a unique, easily computed equilibrium takes us part of the way. But the real groundwork that runs through the paper lies in Observations 2 and 3. We show there that the effects of any sort of wealth change (first- or second-order) on allocative efficiency can be understood by studying whether a particular composite distribution function (built from the primitives of the model) undergoes second-order stochastically comparable changes when the parameters of interest change.

Using this approach, we show that an increase in wealth must always raise allocative efficiency relative to maximum potential output (Proposition 2).

An equalization of the wealth distribution has more complex effects. The effects of such equalization jointly depend on existing wealth inequality, the distribution of individual productivities as well as the lobbying cost function. Redistribution relaxes the constraint in the capital market and permits relatively poor but relatively productive people to lobby harder. This is a positive effect from the allocative point of view, but the overall costs of signaling are thereby altered as well, leading to fresh entry or exit of other wealth-productivity types. Indeed, Examples 1 and 2 demonstrate that allocative efficiency may fall as a result.

At the same time, such perverse effects can only occur if the distributional changes are relatively local in nature. Proposition 3 demonstrates that if the wealth equalization is sufficiently far-reaching, allocative efficiency *must* rise. Propositions 4 and 5 extends these findings to distributional improvements that do not necessarily take us all the way to perfect equality. Because of the situations in Examples 1 and 2, one cannot hope for unambiguous results. Nevertheless, Proposition 4 establishes that *any* improvement in inequality that leads to better percentage wealth gains for the relatively poorer and induces net participation in the lobbying market must improve allocative efficiency. In a similar vein, Proposition 5 demonstrates that if one distribution function of wealth dominates another in a sense that we refer to as "strong single-crossing", then, too, allocative efficiency must improve.

Now, allocative efficiency alone neglects the costs of lobbying. Accordingly, Section 7 takes up the study of *overall* efficiency: allocative efficiency net of lobbying expenditure. It turns out that one additional condition permits us to replicate every result for allocative efficiency. This is the assumption that the hazard rate of the distribution of productivities is nondecreasing. Observations 4 and 5 show that the methodology laid down in Observations 2 and 3 extend immediately under this additional condition. Propositions 6–9 contain the counterparts of the earlier results for allocative efficiency.

Section 8 concludes and also contains fairly extensive bibliographical notes. Indeed, if there is a "bottom line" to these arguments, this section attempts to make the case for it: in many matters of public allocation, it is poverty and inequality — and not necessarily corrupt government — which may lie at the root of inefficiency This casts a different light on the inefficiency of public decision-making in developing countries, one that does not appeal to political appetite for corruption. In this sense, while we focus on an entirely different set of factors, we are in line with the work of Banerjee (1997) on "misgovernance".

2 The Licence Raj

While a development economist will immediately appreciate that government control over productive licenses and permissions often represent the rule rather than the exception, a few brief remarks may be useful for a broader audience. Governments in developing countries have long sought to control or encourage (in a particular direction) the allocation of resources. These range from the direct granting of licences or permissions to produce (which fit exactly the model of this paper), to credit or trade subsidies, and to waivers and exemptions from taxes on particular areas of economic activity.

Why these controls? A cynical answer would be that the controls exist simply so that bureaucrats and governments officials can line their pockets. A more nuanced answer would take into account the fact that for many developing countries, the path out of underdevelopment must traverse new roads, possibly in directions not taken by their established counterparts in the world economy. But new roads are typically beset by market failures, and so it is felt that government encouragement of such sectors is necessary. At the same time, and because so many more people are closer to poverty in developing countries, other key sectors (such as domestic agriculture) may also be regulated. Very often, these regulations control the freedom to carry out some economic activities or to import and/or export selected commodities by requiring that specific licences or permissions be obtained. Of course, the question of just what items are to be regulated, or encouraged, is perenially open to debate and influence.

Perhaps the classic example of government regulation has to do with the protection of certain "infant industries" and the subsidy (through tax breaks or preferential credit) of export-oriented industries. But which sectors are to be protected, and which sectors abandoned to existing foreign suppliers? Which export sectors are to be nourished, as providing an important source of future comparative advantage? Through most of the developing world (and indeed, in much of the *developed* world), these questions have come up again and again.

The title of this section comes from India (in)famous "licence raj", an era dating from post-Independence and substantially — but far from entirely — dismantled in the liberalization drive of the 1990s. Licensing requirements for the startup of any business was commonplace, as was protection of domestic sectors against imports (for an early and insightful account, see Bhagwati and Desai (1970)). Items as diverse as the environment, the art market, agriculture, telecommunications, power, banking and financial services, software are subject to significant government intervention, and the media is full of reports about how the intervention process is subject to business interests.

Similarly, the nature and scope of public intervention in China (ostensibly to shape the path of its development) is only too well known. As Ogus and Zhang (2004) point out, there are at least 146 industrial sectors in China in which special approval and registration are required. And of course, India and China are not alone in the developing world in their attempt to chart out an economic course via the regulatory process.

Such practices are also commonplace in the economically developed world. restrcting ourselves only to North America, the United States is a hotbed of licensing activity, presumably influenced by a host of business interests.[1] Indeed, every State in the United States has a "Division of Occupational Licensing" charged with the role of setting the conditions — often exams and tests — to qualify for a license. In New Jersey, for instance, there are 41 different commisions regulating different professional activities. Of course, apart from professional and business activities, we also have licences for the exploitation of resources that are under state ownership, such as mining sites, underwater resources and the radio spectrum. Similar activity and the consequent lobbying for public support is pervasive in Canada. For instance, Mork et al. (1998) refer to what they term the "Canadian disease": high inequality in inheritance confers strong lobbying power to individuals whose interests are tied to traditional production sectors. Public policies supporting these low productivity sectors slow down growth. More generally, Mork et al. (1998) find empirical evidence supporting the assertion that countries with high inheritance inequality have lower growth rates.

Similar considerations apply to the control of imports and exports in developed countries. The World Trade Organization has recently reached an agreement on import licensing procedures by the member countries in order to limit the discretionary use of licensing by governments. As we've already stated, licensing is common practice in developing countries. But even developed countries such as the European Union or the United States require licences for imports of some agricultural products. This is the case of the imports of products under quota from countries with special low tariff aggreements. Import licensing is also used for the case of industries that the government thinks should be temporarily protected. Steel, some textiles, or shoes are well-known examples.

The signaling model described in the Introduction applies at two levels. First, there is the question of allocation *within* a particular regulated sector. Consider, for instance, the licensing on the imports of dairy products to the US by the USDA. In order to qualify for an import license firms have to satisfy costly conditions related to minimum production level or minimum imports of other non-licensed products, besides paying a flat fee. If the number of firms emitting the threshold signal or higher exceeds the available number of licences, these are randomly allocated among qualifying applicants.

Second — and far more important — the *set* of regulated activities is itself subject to intense debate. Should a developing country encourage particular kinds of software? Should it encourage on other avenues of outsourced business? Should it permit foreign firms to have a 51participation in telecom? Should it subsidize pharmaceutical research on traditional herbal cures? Should there be protection of domestic steel manufacture? These are critical questions for economic direction, but the informational basis for answering such questions is dispersed. More often than not, it is the firms in these very sectors who have the best idea of how productive they can be. However, while the government may be interested in the best directions overall, these sectors want valuable subsidies. So they will lobby to argue that theirs is indeed the best direction for the country.

For instance, import licensing activities are often the object of lobbying. The basic argument (provided, of course, by the potential beneficiaries) is that the domestic industry has to be protected from foreign competition. An efficiency seeking government has to evaluate whether the cost of imposing import quotas and licenses can be justified by the gains in helping the local industry to regain momentum. However, the true potential for future competitiveness is not directly known by the government, and the intensity of lobbying will reveal something about it.

Other sectors, such as the Pharmaceutical Research and Manufacturers Association and the International Federation of Pharmaceutical Manufacturers Associations actively lobby the United States and European Union trade officials in the opposite direction, seeking their active support for international treaties and policies that would ban or restrict the use of compulsory licensing for medicines in the importing countries — in turn imposed in order to protect the domestic industry. [The case of AIDS drugs has been recently received significant attention.]

In all of these cases, the better-heeled the sectoral interests the harder it is to "interpret" the lobby. Are lobby dollars (pesos, rupees) being burnt because much is genuinely at stake, or is it because the marginal cost of burning money is low? It is in this sense that wealth inequality coupled with capital market imperfections may corrupt costly signals and hence reduce the efficiency of public resource allocation.

3 Allocating Permissions: A Model

A government must allocate a fixed stock of a publicly provided input to facilitate production among a unit measure of economic agents. The input must be provided as a single indivisible unit to each agent, if it is provided at all. Think of it as a *permission* to engage in economic activity. The important restriction is that the number of permissions α is limited: $\alpha \in (0, 1)$.

Agents are distributed on $[0, 1]$, and they are endowed with two privately observed characteristics, *productivity* (λ) and *wealth* (w). The restriction that wealth is privately observable becomes more compelling provided we agree that we are dealing with a set of agents who are all in an economic position to make large investments. While wealth differences across entrepreneurs and workers may be easier to observe publicly, such differences within the class of entrepreneurs will be less visible.

An agent with wealth w who has expended resources r in lobbying is assumed to incur a cost $c(w, r)$ in the process. If she has productivity λ and is awarded a permission with probability p, her overall expected return is given by

$$p\lambda - c(w, r). \tag{1}$$

3.1 Assumptions

We will suppose that wealth and productivity are uncorrelated. Nothing of qualititative substance hinges on this unless wealth and productivity are positively *and* closely correlated. But even the polar assumption of independence is not indefensible. Wealth is a proxy for past successes, but if an economy is undergoing rapid change (and this is precisely the sort of economy for which this model has greatest relevance anyway), past successes may be a poor predictor for what will work in the present. Formally, we assume

[A.1] Productivity and wealth are independent draws from distributions F and G respectively. F and G have supports $[0, \infty)$ and $[\underline{w}, \infty)$ respectively (with strictly positive densities in the interior).

[A.1] is stronger than what we need, but it will save on expositional resources if we don't have to worry about mass points, gaps in the support, etc.[2]

Our next two assumptions concern the cost function $c(w, r)$:

[A.2] The cost function is smooth, positive and strictly increasing in lobbying expenditure r when r is positive ($r > 0$), and strictly decreasing in wealth (whenever $r > 0$), perhaps with some positive asymptote, with $\lim_{r \to 0} c(w, r) = 0$ and $\lim_{r \to \infty} c(w, r) = \infty$.[3]

The assumption that c decreases in w implies that access to capital markets improves with wealth (see Section 3.2 for a detailed discussion).

The next assumption states that the ability of increased wealth to create a given relative reduction in lobbying expenditure weakens as wealth increases. For any (w, r) with $r > 0$, define the *elasticity of cost with respect to wealth* (or the *wealth elasticity* in short) as

$$\epsilon(w, r) \equiv -\frac{wc_w(w, r)}{c(w, r)},$$

where we include the negative sign to capture the absolute value. To be sure, this elasticity will depend on both wealth w and the required lobbying expenditure r. We assume

[A.3] If w and r increase in a way that leaves total cost unaffected, the wealth elasticity declines.

How reasonable is [A.3]? At least for high wealths (relative to c), it is easy enough to defend a declining wealth elasticity: the efficacy of increased wealth in achieving a given percentage reduction in expenditure has to die out. The assumption requires closer examination for lower levels of wealth: after all, wealth might have to cross some minimum threshold to obtain access to the credit market, so that substantial wealth effects only kick in after that threshold. At the same time, as we have stated before, we are studying not global wealth distributions but across the entrepreneurial class, among entities who are in a reasonable position to start major investment projects in the first place. We return to this discussion below in Section 3.2, in which we explictly consider a credit market model.

The reader might wish to refer to a simple family of functional forms that satisfy [A.3]. Suppose that the cost function is of the form

(2) $$c(w, r) = \hat{c}(r)\left[\frac{1}{w^\theta} + a\right],$$

where $\hat{c}(r)$ is some increasing function, $\theta > 0$, and $a > 0$. Under this specification, higher wealth lowers costs, but ultimately the marginal cost always asymptotes to some fixed limit $a\hat{c}'(r)$. It is easy to check that [A.3] is satisfied in this case, though, as the discussion to follow reveals, the separable structure assumed in (2) is by no means necessary for our restrictions to apply.

3.2 An Example: Imperfect Credit Markets

The cost function $c(w, r)$ is a bit abstract, and at one level we are happy to keep it that way. There is even no reason to insist that w is wealth: it may be some surrogate for access to contacts, or even political power. At the same time, it is important to show that fairly standard models of wealth and capital do fit our framework. To this end we interpret our model using a simple credit market story.

Suppose that r is drawn from the capital market, and that lobbying is one of many (productive or unproductive) uses that r can be put to. Assume banks cannot control this outcome and have the following overall view of the situation: r generates a stochastic nonnegative unit return y (so that *total* output is ry), distributed according to some smooth increasing cdf M.

Suppose that the bank observes borrower wealth w and lends out r under a simple debt contract that asks for a repayment of $c = c(w, r)$ (this is the function we want to solve out for). There is limited liability, though: if $ry + w \geq c$, c is repaid; otherwise only $ry + w$ can be seized. Consequently, expected repayment is given by

$$\int_{\frac{c-w}{r}}^{\infty} c dM(y) + \int_{0}^{\frac{c-w}{r}} (ry + w) dM(y) = c - r \int_{0}^{\frac{c-w}{r}} dM(y).$$

If there is competition among lenders and the best alternative return on r is $r\sigma$ for some $\sigma > 0$, then

$$c - r \int_{0}^{\frac{c-w}{r}} M(y) dy = \sigma r. \tag{3}$$

This arbitrage equilibrium equation implicitly characterizes the cost of credit c in terms of wealth w and the amount borrowed r.

If wealth exceeds σr (minus the lowest possible return on r) there are no wealth effects, of course, and c simply equals σr. For w smaller, simple differentiation shows that the resulting wealth elasticity is given by

$$\epsilon(w, r) = -\frac{w c_w(w, r)}{c(w, r)} = \frac{M(\theta) w}{[1 - M(\theta)] c(w, r)}. \tag{4}$$

where for notational compactness we write $\theta = (c - w)/r$.

To examine [A.3], we consider all changes dw and dr such that $c(w, r)$ is unchanged: $c_w(w, r) dw + c_r(w, r) dr = 0$, so that $dr/dw = -c_w(w, r)/c_r(w, r)$. Using (3) to solve this, we have

$$\frac{dr}{dw} = \frac{M(\theta)}{\frac{c}{r} - \theta M(\theta)}.$$

Using this restriction, we differentiate the elasticity $\epsilon(w, r)$ with respect to w and r. Doing so, and after some computational steps, we conclude that [A.3] holds whenever

$$\theta h(\theta) > (k - 1) M(\theta) - \frac{(k - 1)^2}{k} M(\theta)^2, \tag{5}$$

where $h(\theta) \equiv M'(\theta)/[1 - M(\theta)]$ is the hazard rate of M, and $k \equiv c/w$.

Condition (5) helps us describe cases in which [A.3] holds (under some restrictions on minimum wealth). For instance:

Observation **1** *If the distribution of unit returns is given by a Pareto distribution with finite mean, then* [A.3] *holds whenever individual wealth w is no smaller than one-fourth of the total outlay $c(w, r)$.*

It is easy enough to prove this observation. For a Pareto distribution with exponent δ, we know that $\theta h(\theta) = \delta$. Furthermore, the right hand side of (5) is bounded above by $k/4$ for all $k \geq 1$. If mean is finite, then $\delta \geq 1$. Combining these observations, (5) holds as long as k is no larger than 4, or equivalently, the ratio of w to total lobbying outlay is no smaller than $1/4$.

Thus if there are no individuals who are "poor enough" in the sense of the ratio described above, [A.3] will be satisfied for this credit markets example. Of course, (5) can be easily applied to derive wealth-cost bounds for other distributions, such as the exponential. Our example therefore reinforces the intuitive discussion earlier that [A.3] is more likely to be met for relatively high wealths, where "relatively high" does not appear to imply a particularly large ratio of wealth to lobbying expenditure.[4]

This example has the virtue of making an explicit connection between a model of imperfect credit markets — with the imperfection ameliorated by borrower wealth — and the more abstract formulation of the paper, to which we now return.

3.3 Equilibrium

The government cares about efficiency alone: it would like to single out the most productive types and give them permissions to produce. [In a later section, we remark on an extension with multidimensional government objectives.] Ideally, the government would like to award licenses to all who have productivity exceeding $\lambda(\alpha)$, where this threshold is defined by the condition $1 - F(\lambda(\alpha)) = \alpha$. On the other hand, individuals would like to be identified as productive types and thus qualify for a permission. To this end, they engage in lobbying in an attempt to persuade the government that they value a permission very highly. Thus, we view lobbying as a potential device to solve the informational problem.

We may think of an equilibrium of this game as consisting of three objects:

[1] A map from types into lobbying expenditure r. This map is a best response in that for each type (w, λ), the announcement r is optimal given the government's allocation rule (see (3) below).

[2] A map from all conceivable expenditures r (not just the equilibrium ones) to posterior beliefs held by the planner regarding the distribution of productivities associated with r. Of course, for values of r in the support of the map in [1], we require that posterior beliefs must be obtainable using Bayes' Rule. We will impose extra off-equilibrium restrictions on beliefs; see below.

[3] A map from lobbying expenditures r to the probability $p(r)$ that an announcement at r will receive a permission from the planner. Given the posterior beliefs held by the planner, we require that p must be chosen in order to maximize expected productivity.

Notice how our definition posits a *reactive* government which cannot commit to a particular line of action during the lobbying process.[5] We take this approach because we believe that the no-commitment case is often a better description of reality when lobbying is involved. Numerous government officials are frequently involved in the allocation decisions, so that the reputational concerns that might underpin a commitment model (with screening) are attenuated.

3.4 Equilibrium Refinements

The game as described displays several equilibria. There are two reasons for this multiplicity. The first is standard in the signaling literature: there are equilibria which rely on "implausible" beliefs off the equilibrium path. We employ a version of the Cho-Kreps intuitive criterion to rule these out. The second source of multiplicity arises from the possibility that the planner may discriminate between signals that are identical from its perspective (i.e., signals that generate the same posterior expected profitability). We will rule this out by assumption. Now we turn to details.

3.4.1 Variation on the Intuitive Criterion

As in the standard version of the intuitive criterion, we say that a signal is *equilibrium-dominated* for some type if emitting that signal were to yield that type a lower payoff than its equilibrium payoff, even under the assumption that such a signal would receive the best possible treatment from the planner. This is standard (and among the weakest in this class of refinements), but we will impose a slightly stronger condition.

Consider an equilibrium, and any off-equilibrium lobbying expenditure r. To begin with, the planner rules out all types for whom r is equilibrium-dominated. That still possibly leaves a multiplicity of "admissible" wealth-productivity types. Say that the off-equilibrium signal *encourages productive agents* if, whenever (w, λ) gains from emitting it, and $\lambda' > \lambda$, then (w, λ') gains just as much (or more) from emitting it (relative to equilibrium payoff).

Now we impose the additional restriction. Define "baseline productivity" of the admissible types to simply be the (conditional) average productivity — using population frequencies — after eliminating those types which are equilibrium-dominated. We require that whenever an off-equilibrium signal encourages productive agents, the planner must entertain no lower an expectation of average profitabililty among the admissible types than the baseline productivity generated by using conditional population frequencies. For all other signals, no restrictions are imposed.

This condition is mild. It simply states that if higher-productivity types benefit more from emitting an off-equilibrium signal, the planner's beliefs should not "discriminate against" such types relative to their population average. The condition is satisfied, of course, if planner beliefs simply *equal* the population distribution (after removing the dominated types), but handles several other cases as well.[6]

The collective restrictions in this section will be dubbed IC.

3.4.2 Belief-Action Parity

We will assume the following: if the planner believes that the posterior expected profitability is identical across two announcements, then she will allocate permissions with equal probability to individuals making the two announcements.

Belief-action parity — hereby christened BAP — does constitute an additional restriction.

In the rest of our analysis, we impose the BAP, as well as the IC.

4 Equilibrium

In this section, we state and prove the following result:

PROPOSITION **1** *There is a unique equilibrium of the lobbying game, with two announcements,* 0 *and* $r > 0$*. The former signal receives no permissions, while everybody who emits the latter signal receives a permission for sure.*

All individuals of type (w, λ)*, with* $\lambda \geq c(w, r)$*, announce* r*, and there is a measure of precisely* α *of them. The announcement* r *thus solves the equation*

$$\alpha = \int_{\underline{w}}^{\infty} [1 - F(c(w, r))] dG(w) \tag{6}$$

It will be worth running through the proof of this proposition in the main text, because it is very intuitive. As a first step, note that BAP alone takes us quite far in narrowing the set of equilibria:

STEP 1. *Given BAP, no more than two equilibrium announcements can be serviced with strictly positive probability.*

Suppose this assertion is false. Then there must exist three (and perhaps more) equilibrium announcements r_1, r_2 and r_3. with strictly positive allocation probabilities associated with them. Moreover, the probabilities must be distinct. [For if the probabilities were not distinct, no one would would make the costlier announcement.] But then, by BAP, the government has distinct posterior profitabilities associated with each of the announcements. But then the only way in which the least profitable anouncement can be serviced with positive probability is if the other two announcements are being *fully* serviced, which contradicts distinctness.

As an aside, this observation is extremely general, and is implications for the limited number of equilibrium announcements may be worth pursuing in separate research. In the context of the current model, however, we can obtain more specific restrictions, which rely on the particular structure here and on a repeated application of IC.

STEP 2. *Under the additional imposition of IC, no more than one equilibrium announcement receives a permission with positive probability.*

Suppose not. Then exactly two equilibrium announcements r_1 and r_2, with $r_1 < r_2$, receive permissions with strictly positive probability. Once again, these probabilities must be distinct (otherwise no one would announce r_2), but by BAP this implies that $p(r_1) \equiv p \in (0, 1)$, and $p(r_2) = 1$. Notice that types in the set

$$\mathbf{T} \equiv \{(w, \lambda)|0 < \lambda - c(w, r_2) > p\lambda - c(w, r_1)\}$$

must announce r_2 and get a permission for sure. [We don't have to worry about weak vs. strict inequalities as these are of measure zero by [A.2].] Now pick any $r \in (r_1, r_2)$. This is an off-equilibrium announcements. Notice that r is equilibrium-dominated for any type with $\lambda - c(w, r) < 0$ or with $\lambda - c(w, r) < p\lambda - c(w, r_1)$. So the set of admissible types for the announcement is

$$\mathbf{T}' \equiv \{(w, \lambda)|0 < \lambda - c(w, r) > p\lambda - c(w, r_1)\}$$

(again neglecting weak inequalities). An inspection of this set immediately reveals that the announcement r encourages productive agents. By IC, our planner's beliefs must be restricted to $\mathbf{T}'$, her estimate of expected profitability must be no lower than the "baseline expected profitability" calculated by using population frequencies.

At the same time, notice that when r is close to r_2, the set $\mathbf{T}'$ is close to $\mathbf{T}$, so that the baseline expectation of types in $\mathbf{T}'$ must be close to that in $\mathbf{T}$. But (by BAP), the latter has a higher expected profitability than those types announcing r_1. Combining the information in this and the previous paragraph, we may conclude that the our planner *must* reward such an announcement with a permission for sure.[7]

But now observe that all types (w, λ) in $\mathbf{T}'$ — and there is a positive measure of such types — will want to deviate from their equilibrium announcement of r_1. This means we don't have an equilibrium to start with.

So we are down to a case in which there are at most two equilibrium announcements 0 and $r \geq 0$. In the last step, we claim that

STEP 3. $p(r) = 1$.

Suppose, on the contrary, that $p(r) \equiv p < 1$. Clearly, types in the set

$$\mathbf{T} \equiv \{(w, \lambda) | p\lambda - c(w, r) > 0\}$$

will announce r. Now pick some $r' > r$. Notice that all types *not in* the set

$$\mathbf{T}' \equiv \{(w, \lambda) | 0 < \lambda - c(w, r') > p\lambda - c(w, r)\}$$

are equilibrium-dominated. So by IC, the planner must believe that the announcement r' comes from $\mathbf{T}'$.

Now, $\mathbf{T}'$ is not directly comparable to $\mathbf{T}$ but a little reflection shows that for r' large,

$$IE(\lambda | (w, \lambda) \in \mathbf{T}') > IE(\lambda | (w, \lambda) \in \mathbf{T}), \tag{7}$$

where these expectations are computed using population distributions. Now observe that r' encourages productive types. Consequently the planner's expectation of productivity in $\mathbf{T}'$ must be no lower than the statistical baseline. Combining this observation with (7), we must conclude that an announcement of r' must be rewarded with a permission for sure. But then all types such that

$$\lambda - c(w, r') > p\lambda - c(w, r) > 0$$

— and there is a positive measure of them — will have an incentive to deviate from their equilibrium announcement of r.

These three steps show that there is a unique equilibrium: it features two announcements with a positive announcement of r earning a permission for sure, and a zero announcement receiving no permissions. The rest of the argument simply balances supply and demand. The volume of available permissions is α, and only types with $\lambda > c(w, r)$ will announce r, so that (by Step 3) r must solve the equation

$$\alpha = \int_{\underline{w}}^{\infty} [1 - F(c(w, r))] dG(w),$$

which is (6).

5 Wealth and Lobbying: General Approach

Our goal is to examine the relationship between different aspects of the wealth distribution and the efficiency of resource allocation. In particular, we study two sorts of changes: one in which all wealth is scaled up by some fixed proportion, and another in which wealth is redistributed from relatively rich to relatively poor, creating a Lorenz-improvement.

There are, of course, other exercises that could be carried out, such as a rescaling of productivity or simultaneous productivity-wealth changes. In the interests of brevity, we concentrate on wealth alone, though we shall remark on some of these extensions below.

For each type of wealth change there are two different ways in which we might approach the question of resource allocation. One might consider "ex-post" efficiency, which is simply allocative accuracy after lobbying costs are already sunk. A measure of allocative inefficiency would be the difference between maximum potential output — that achievable under perfect information — and equilibrium output. However, while of interest, allocative efficiency cannot tell the whole story, for the transmission of information through signaling is also costly to individuals. Therefore, someone interested in *overall* efficiency losses must add lobbying costs to allocative shortfalls, and study the impact on this variable.

To summarize, then, there are several combinations of potential cause-effect scenarios. Fortunately, as we shall see below, some methodological unification of the various cases can be accomplished.

6 Allocative Efficiency

6.1 Methodology

We begin the analysis with a study of allocative efficiency, which we simply equate with equilibrium output, given by

$$Y \equiv \int_{\underline{w}}^{\infty} \left[\int_{c(w,r)}^{\infty} \lambda dF(\lambda) \right] dG(w), \tag{8}$$

[We should really take the *ratio* of equilibrium output to maximal output, but as long as there are no alterations in productivity, maximal output will not change.[8]]

We describe a method for identifying the effect of distributional and scale changes. As we shall see later, this method can be applied (with some qualifications) to the study of overall efficiency as well.

The two situations we wish to compare involve different wealth distributions G and $\tilde{G}$. Let r and $\tilde{r}$ the equilibrium lobbies before and after the change. Define two variables

$$z(w) \equiv 1 - F(c(w,r)) \tag{9}$$

and

$$\tilde{z}(w) \equiv 1 - F(c(w,\tilde{r})). \tag{10}$$

These have easy interpretations as the measure of individuals who receive permissions (before and after the parametric change) at each level of wealth. We may also think of z and $\tilde{z}$ as random variables with supports on subintervals of $[0,1]$. Within the support the cdf H of z is given by

$$H(z) = G(c^{-w}(F^{-1}(1-z), r)),$$

where $c^{-w}(\lambda, r)$ stands for the inverse function of $c(w,r)$ with respect to w.[9] Similarly, the cdf $\tilde{H}$ of $\tilde{z}$ is given by

$$\tilde{H}(\tilde{z}) = \tilde{G}(c^{-w}(F^{-1}(1-z), \tilde{r}))$$

for all $\tilde{z}$ within its support.

Now the equilibrium conditions (see (6)) determining r and $\tilde{r}$ easily reduce to

$$\int_0^1 z dH(z) = \int_0^1 z d\tilde{H}(z) = \alpha, \tag{11}$$

(where we use the same integrating index z instead of z and $\tilde{z}$), which simply states that these two random variables have the same mean.

Moreover, a simple change-of-variables reveals directly that

$$Y = \int_{\underline{w}}^{\infty} \left[\int_{c(w,r)}^{\infty} \lambda dF(\lambda) \right] dG(w) = \int_0^1 \left[\int_{F^{-1}(1-z)}^{\infty} \lambda dF(\lambda) \right] dH(z). \tag{12}$$

and exactly the same calculation for output after the change shows that

$$\tilde{Y} = \int_0^1 \left[\int_{F^{-1}(1-z)}^{\infty} \lambda dF(\lambda) \right] d\tilde{H}(z). \tag{13}$$

Combining (12) with (13), we may conclude that

$$\tilde{Y} - Y = \int_0^1 \phi(z) d\tilde{H}(z) - \int_0^1 \phi(z) dH(z), \tag{14}$$

where

$$\phi(z) \equiv \int_{F^{-1}(1-z)}^{\infty} \lambda dF(\lambda).$$

The following observation is fundamental:

Observation **2** *$\phi(z)$ is strictly concave.*

The proof of the observation is immediate from differentiating $\phi(z)$ twice with respect to z. Its power lies in the fact that it allows us to generate a useful sufficient condition for a parametric change to result in a gain in allocative efficiency.

Observation **3** *The change from G to $\tilde{G}$ results in an increase in allocative efficiency y if the random variable $\tilde{z}$ second-order stochastically dominates the random variable z.*

This methodology is potentially useful in studying all kinds of change in aggregate wealth or its dispersion across the population. As already discussed, we will study two special cases: scalings in wealth and Lorenz improvements in the wealth distribution.

Before embarking on these specific exercises, it may be worth noting that our methodology can also incorporate changes in productivity. To be sure, maximal output will change as well, and so allocative efficiency must be judged by the ratio of equilibrium output to maximal output. See our supplementary notes — Esteban and Ray (2004) — for more details.

6.2 Across-The-Board Increases in Wealth

To apply the ideas of the previous section, consult Figure 1. The thick solid line in the diagram depicts the random variable z as a deterministic function of w (following (9)). The dotted line describes what z will become at each old wealth level when wealth is increased across the board, say proportionately. The old wealth level w now acquires the larger value w', and so — if equilibrium r remains at the old level — simply inherits the value $z(w')$. Because z is increasing in w, this means that the dotted line lies above the original. In other words, more individuals see fit to bid for permissions at every old level of wealth, at the going lobby rate. This cannot be an equilibrium state of affairs, of course, and the required lobby expenditure r must increase. This has the effect of sliding the dotted line downwards at every wealth level (see the thin solid

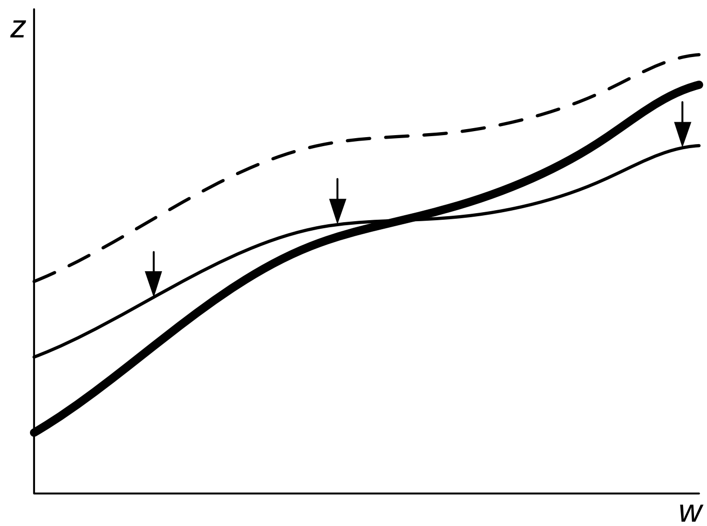

Figure 1: Effects of a Change in Wealth

line) until "demand equals supply" for permissions. This thin solid line is precisely the new random variable $\tilde{z}$, except that we express it diagrammatically as a deterministic function of the old wealth levels.

Observation 3 asserts that if $\tilde{z}$ stochastically dominates z, then allocative efficiency will have increased post-change. A sufficient condition for such domination — readily visualized in Figure 1 — is that the new $\tilde{z}$-line intersect the old z-line "from above". [Of course, we are discussing sufficient conditions and weaker restrictions will suffice, but this one is easy to grasp.] In turn, the required swivelling of the z-function may be achieved by several means, but one important requirement is that *wealth effects should diminish after some point*. This is where [A.3] plays its part; a more formal account follows.

PROPOSITION **2** *Under* [A.1]–[A.3]*, a proportional scaling-up of wealth (by a factor $\delta > 1$) cannot reduce allocative efficiency. Indeed, as long as $c(\delta w, \tilde{r}) \neq c(w, r)$ for some w, allocative efficiency must strictly increase.*

Proof. The following intermediate step will be needed:

LEMMA **1** *Assume* [A.1]–[A.3]*. Suppose that for some r, $\tilde{r}$, δ and w such that $\delta > 1$ and $\tilde{r} > r$, we have $c(\delta w, \tilde{r}) \geq c(w, r)$. Then for all $w' > w$, $c(\delta w', \tilde{r}) \geq c(w', r)$ as well.*

To prove the lemma, consider the function

$$\psi(x) \equiv \frac{c(\delta x, \tilde{r})}{c(x, r)}$$

for fixed r, $\tilde{r}$ and δ. Then, if the lemma is false, $\psi(w) \geq 1$ but $\psi(w') < 1$ for some $w' > w$. It follows that there exists $w^* > w$ such that $\psi(w^*) = 1$ and $\psi'(w^*) \leq 0$. Define $\tilde{w} \equiv \delta w^*$. Then these conditions translate into

$$\frac{\tilde{w}|c_w(\tilde{w}, \tilde{r})|}{c(\tilde{w}, \tilde{r})} \geq \frac{w|c_w(w^*, r)|}{c(w^*, r)}$$

for some $\tilde{w} > w^*$ and $\tilde{r} > r$ such that $c(w^*, r) = c(\tilde{w}, \tilde{r})$. This contradicts [A.3] and proves the lemma.

Returning to the proof of the proposition, we will now show that $\tilde{H}$ is (weakly) less risky than H in the sense of second-order stochastic dominance. Observation 3 then guarantees the first part of the proposition.

To this end use the definitions of z and $\tilde{z}$ from (9) and (10) to express both variables as functions z_1 and z_2 of the *original* wealth w (before the scaling). That is, $z_1(w) = z(w) = 1 - F(c(w, r))$ (just as in (9)), while $z_2(w) = \tilde{z}(\delta w) = 1 - F(c(\delta w, \tilde{r}))$.

It is easy to see that in the new equilibrium, $\tilde{r} > r$. Applying Lemma 1, if for some wealth w we have $c(\delta w, \tilde{r}) \geq c(w, r)$, then the same inequality is true of all $w' > w$. This proves that there exists w^* such that

(15) $$z_1(w) \leq z_2(w) \text{ for } w \leq w^* \text{ and } z_1(w) \geq z_2(w) \text{ for } w \geq w^*.$$

This implies that the random variable $\tilde{z}$ (weakly) second-order stochastically dominates z, and completes the the first part of the proof.

To establish the remainder of the proposition, simply note that if $c(\delta w, \tilde{r}) \neq c(w, r)$ for some w, then strict inequality must hold somewhere in (15). The strict concavity result established in Observation 2 then assures us that allocative efficiency must *strictly* increase. ∎

To see the role of [A.3] from an intuitive viewpoint, consider a scaling of the wealth distribution, with new equilibrium lobby $\tilde{r}$. Which wealth levels might benefit (or lose) from this change? An agent (w, λ) now has endowments $(\delta w, \lambda)$, and so is in a "worse" position than she was before if

(16) $$c(\delta w, \tilde{r}) \geq c(w, r)$$

Otherwise, she is relatively "better off". Now observe that *all* agents cannot be better off (nor can they all be worse off). For then the overall measure of active bidders would change in equilibrium, a contradiction.

So — unless there is no net change at all[10] — some agents must be worse off and some better off. For the worse off, (16) applies. But [A.3] — via Lemma 1 — tells us that (16) must then *continue* to apply for all higher wealths. This proves us that the net gainers have relatively low wealth, while the net losers have relatively high wealth.

So at relatively low wealths, there is entry, while at relatively high wealths, there are dropouts. But the marginal entrants at low wealth must have higher productivity than the marginal dropouts at high wealth. This increases allocative efficiency under the new equilibrium.

It is clear that the restriction [A.3] is important — though, of course, not logically necessary — in obtaining this result. We have already discussed this restriction in detail and there is little new to be added here. We only remark that it is implausible that [A.3] would fail over an entire range of wealths, and that if it fails at all it is likely to do so for low levels of wealth relative to lobbying cost (e.g. ratios of 0.25 or less in the credit market example of Section 3.2).

6.3 Changes in the Inequality of Wealth

The relationship between inequality and allocative efficiency (and likewise, on overall efficiency as we'll see later) is a complex one. Simple intuition would suggest that, as inequality is reduced, highly productive individuals who previously were wealth constrained could now send a signal that awards them a permission, while less productive rich individuals would cease lobbying after having lost wealth. This intuition isn't wrong, but there is much more going on, because the equilibrium lobby itself changes, with subsequent effects on allocative efficiency.

6.4 Perfect Equality

But first things first: from the characterization of Observation 3 it is clear that perfect equality of wealth (or more precisely, given our full-support assumptions, "near-perfect" equality of wealth) is conducive to perfect allocative efficiency. We record this formally as

Proposition 3 *Under* [A.1] *and* [A.2]*, consider any distribution of wealth G that causes some allocative loss in equilibrium. Then for every sequence of wealth distributions G^m with the same mean wealth as G that converge to perfect equality in the sense of weak convergence, G^m has higher allocative efficiency for all m large enough.*

This proposition has two aspects: one relatively conceptual, the other relatively technical. The conceptual part is that full equality maximizes efficiency, the technical part is that a continuity argument can be made to extend this to "near-equal" distributions, which fit the formal model. The technicalities are of little interest and are omitted (see our supplementary notes in Esteban and Ray (2004) for details), but the salubrious effects of full equality merit some discussion. In Esteban and Ray (2000), we study a signaling model restricted to full equality of wealth: it turns out that perfect equality is *not* always conducive to efficiency, especially when overall levels of wealth are low. The reason is that wealth constraints may shut down revelatory signaling altogether, especially when wealth imposes an absolute upper limit to the amount of r that can be expended. This would happen in a model of "pure" credit rationing where the costs of lobbying become infinite at finite values of r.[11] The assumptions of this paper rule this out: the cost of an incremental lobby may be large, but it is never infinite, and there is always some room to manouver.[12] One implication of such a cost function is that perfect equality guarantees allocative efficiency. However, the point is that the link between full equality and full allocative efficiency present in this model is not a mere triviality.

6.5 Complications

Matters are more complex when there are changes in wealth inequality that don't lead to full equalization of wealths. Indeed, one can construct examples in which an increase in equality *lowers* allocative efficiency. The details of such a construction are tedious and complicated. But here are the main ideas. In both the examples to follow, we begin with a equalizing Dalton transfer of wealth.

Example 1. [The equilibrium lobby r falls, and so does allocative efficiency.] Suppose that we array individuals in order of increased wealth. Pick two nonoverlapping intervals of individuals, both at the lower end of the wealth scale, and consider a small Dalton transfer of wealth from the relatively rich interval to the relatively poor interval. This transfer will cause some dropouts in the richer region: individuals with productivities around $c(w^+, r)$, where w^+ belongs to the richer interval, will drop out. But it will also create new entrants in the poorer region: individuals with productivities around $c(w^-, r)$, where w^- belongs to the poorer interval, will enter. Because $w^- < w^+$ and c is decreasing in wealth, the types who enter are more productive than the types who drop out. *However*, if the density function for productivities falls steeply, a relatively small number of individuals will enter (compared to the dropouts), so to restore equilibrium, the equilibrium lobby r must fall.

The problem is that the fall in r encourages entry *across the board* of marginal types at *every* wealth level. So there has been an initial gain in productivity, it is true, but because of the numbers gap the resulting fall in r "contaminates" the productivity distribution by encouraging entry at all wealth levels. This contamination is larger the sharper the fall in the density function for productivity, and indeed allocative efficiency can suffer at the end of the process.

Notice that this example is contingent on the initial wealth transfer occurring at the lower end of the wealth distribution, so that the initial dropouts and entrants are *both* productive relative to the average. It is the resulting entry of average types (on average!) that worsens the outcome.

Example 2. [The equilibrium lobby rises, but allocative efficiency still falls.] A feature of Example 1 is that the equilibrium lobby r falls, so that productivity types around the average level drift into the system. Here is a variation in which this feature is reversed. Suppose once again that an equalizing wealth transfer takes place, but in the higher end of the wealth scale. Suppose, furthermore, that in the relevant region the density function for productivities is increasing. Now very low productivity types will give way to *relatively* better — but still low — productivity types. What is more, there are now more entrants than dropouts, so r must rise. This increase now has the effect of creating additional dropouts of all marginal types at all wealth levels. These types, on average, will have higher productivities than those among whom the initial transfer took place. Once again, the entire productivity mix is contaminated, and can be affected sufficiently so that allocative efficiency is degraded. In this example, allocative waste moves hand in hand with the loss due to lobbying, whereas in the previous example, the movement was in opposite directions.

The possibility that small redistributions in wealth can have effects that are different from more far-reaching interventions has not gone unnoticed in the literature. In a different exercise pertaining to societies with widespread undernutrition (but within the general framework of relating inequality to production efficiency), Dasgupta and Ray (1987) establish that small equalizations in wealth can have negative consequences for output and employment. In contrast, large changes always had salubrious effects. While the situation we study — as well as the particular chain of effects — is entirely different, the same phenomenon reappears.

6.6 Some Unambiguous Findings

The examples above provide perhaps too nihilistic a view. There are certain types of change with effects we can unambiguously describe.

6.6.1 Progressive Equalizations That Induce Participation

Consider some change in wealth distribution. A *beneficiary* is someone whose wealth is increased thereby. Say that person A *benefits at least as much as* person B if the proportionate increase in A's wealth is at least as high as that of B. Now define an equalization of wealths to be *progressive* if, whenever, some person is a beneficiary, then each individual with no higher wealth benefits at least as much.

For a given equilibrium r, say that a change in wealth distribution *induces participation* if the overall measure of types (λ, w) such that $\lambda \geq c(w, r)$ increases; i.e., "more people wish to lobby at the going lobby rate".

Proposition 4 *Under* [A.1]–[A.3]*, allocative efficiency must increase with any progressive equalization of wealths that induces participation.*

Proof. We follow the proof of Proposition 2, with some changes. Let $\phi(w)$ denote the new wealth of someone who had wealth w (the assumption of progressivity makes this a well-defined function). Recall the definitions of z and $\tilde{z}$ from (9) and (10). We may use these to express both variables as functions z_1 and z_2 of the *original* wealth w (before the change). That is, $z_1(w) = z(w) = 1 - F(c(w, r))$ (just as in (9)), while $z_2(w) = \tilde{z}(\phi(w)) = 1 - F(c(\phi(w), \tilde{r}))$.

Because the change induces participation, the new equilibrium $\tilde{r}$ exceeds r. Consequently, if, for some beneficiary with wealth w, we have $c(\phi(w), \tilde{r}) \geq c(w, r)$, then the same inequality is true of all $w' > w$. To see this, note that for every beneficiary with wealth w and for all $w' > w$, $\phi(w')/w' \leq \phi(w)/w$, and then apply Lemma 1. This proves that there exists w^* such that

$$z_1(w) \geq z_2(w) \text{ for } w \leq w^* \text{ and } z_1(w) \leq z_2(w) \text{ for } w \geq w^*. \tag{17}$$

In addition, strict inequality must hold over some intervals of wealth in (17), because there must be some nonbeneficiaries and $\tilde{r} > r$. These observations immediately imply that the random variable $\tilde{z}$ is distinct from and second-order stochastically dominates z. Observation 3 completes the proof. ∎

To reconcile this proposition with the two counterexamples — and to facilitate further discussion — note that in Example 1 the equilibrium lobby falls, something that cannot happen if the change induces participation. In our opinion, an encouragement of participation is the interesting and intuitive first-order effect of an improvement in equality: higher-wealth individuals are not too affected by their loss in wealth and so keep lobbying, while the equalizing wealth transfers enable the entry of individuals among the relatively less wealthy. It is possible to unpackage this condition further by providing assumptions on primitives which imply that participation is encouraged. The reader may consult our supplementary notes (Esteban and Ray (2004)) for more detail.

Finally, in Example 2, it is true that the improvement in equality induces participation, but the equalization isn't progressive in the sense we have defined it. In particular, the improvement in distribution takes place only at the upper end of the wealth scale. In contrast to our more cursory dismissal of Example 1, we continue to find these cases interesting, but Proposition 4 does not address them.

6.6.2 Equalizations with Strong Single-Crossing

In this section, we consider an alternative restriction on the nature of distributional equalizations. The assumption we employ is a joint restriction on the distribution of wealth and the cost function. Consider an initial distribution of wealth, G, and a new distribution of wealth $\tilde{G}$which are related in the following way:

Strong Single-Crossing (SSC). For every strictly positive (w, r) and $(\tilde{w}, \tilde{r})$ such that $c(w, r) = c(\tilde{w}, \tilde{r})$ and $G(w) = \tilde{G}(w)$,

$$\tilde{G}'(\tilde{w})|c_w(w, r)| > G'(w)|c_w(\tilde{w}, \tilde{r})|. \tag{18}$$

As an initial approach to understanding SSC, suppose that $(w,r) = (\tilde{w}, \tilde{r})$. Then SSC reduces to the condition that for every strictly positive w,

$$\tilde{G}'(w) > G'(w) \text{ whenever } \tilde{G}(w) = G(w),$$

which says that $\tilde{G}$ indeed stochastically dominates G (in a strong "single-crossing" sense). But SSC clearly implies more, for it involves conditions on the cost function as well. We comment further on this below, but first let us record what SSC does for us.

PROPOSITION **5** *Under* [A.1]–[A.3]*, allocative efficiency must increase with a fall in inequality, provided that the change in the wealth distribution satisfies SSC.*

Proof. See appendix.

Examples 1 and 2 show how the "direct" effects of an equalization may get swamped by subsequent changes in r. SSC simply does the job of guaranteeing that the indirect effects do not get in the way of the initial impact, which is always efficiency-enhancing.

SSC is a strong condition and we do not intend to push it unreservedly. At the same time, results such as this (and the preceding propositions) show that our methodology permits us to carefully pick through potential minefields, as exemplified in the counterexamples, to nevertheless reach unambiguous results in different classes of cases. As for SSC itself, we refer the reader to our supplementary notes, where discuss SSC further. In particular, we show that SSC is automatically satisfied whenever a distributional equalization occurs, provided that the cost function displays constant elasticity in wealth and the distribution of wealth is Pareto.

7 Overall Efficiency

7.1 Methodology

Allocative efficiency, while of independent interest, does not give us the full picture. A full assessment will need to take into account the fact that signaling comes at a cost. To judge overall efficiency, we need to net out this cost from any measure of allocative efficiency. Fortunately, the same methodology developed in the previous section can be applied here.

Overall efficiency is total output minus equilibrium lobbying costs, and this is given by

$$\begin{aligned} E &\equiv Y - \int_0^\infty c(w,r)[1-F(c(w,r))]dG(w) \\ &= \int_0^\infty \left[\int_{c(w,r)}^\infty \lambda dF(\lambda)\right] dG(w) - \int_0^\infty c(w,r)[1-F(c(w,r))]dG(w) \\ &= \int_0^\infty \left[\int_{c(w,r)}^\infty (\lambda - c(w,r))\, dF(\lambda)\right] dG(w). \end{aligned} \tag{19}$$

We turn now to a similar computation for the gain in overall efficiency (net of changes in lobbying costs). Use (19) and employ exactly the same change of variables as in Section 6.1 (see (9) and (10)) to see that

$$\begin{aligned} E &= \int_0^\infty \left[\int_{c(w,r)}^\infty (\lambda - c(w,r))\, dF(\lambda)\right] dG(w) \\ &= \int_0^1 \psi(z) dH(z), \end{aligned} \tag{20}$$

where

$$\psi(z) \equiv \int_{F^{-1}(1-z)}^\infty [\lambda - F^{-1}(1-z)]dF(\lambda).$$

Similarly, overall efficiency $\tilde{E}$ after the change is given by

$$\tilde{E} = \int_0^1 \psi(z) d\tilde{H}(z),$$

and combining this with (20) shows us, in a manner entirely analogous to (14), that

$$[\tilde{E} - E] = \int_0^1 \psi(z) d\tilde{H}(z) - \int_0^1 \psi(z) dH(z). \tag{21}$$

One might proceed exactly as before if ψ, like its sister function ϕ in Section 6, were to be concave. However, the analogue of Observation 2 for ψ is not automatically available. It turns out that the curvature of ψ is intimately connected to the *hazard rate* exhibited by the distribution of productivities F.

Recall that the hazard rate of F at λ is defined to be

$$\zeta(\lambda) \equiv \frac{f(\lambda)}{1 - F(\lambda)}.$$

OBSERVATION 4 *If the hazard rate ζ is nonincreasing, then $\psi(z)$ is concave (and it is strictly concave if the hazard rate is a strictly decreasing function).*

Once again, the proof follows from simple differentiation, but now it is important to focus on the assumption of a nonincreasing hazard rate. For several distribution functions this assumption is automatically satisfied, but apart from this statistical observation there may be more that can be said in favor of this assumption. Broadly speaking, the assumption of a decreasing hazard rate means that the distribution of productivity draws has a "thick tail". If such draws are viewed as the result of R&D efforts, then recent literature on the subject views the Pareto distribution of productivities as a good working approximation (see, e.g., Bental and Peled (1996), Eaton and Kortum (1999) and Kortum (1997)). The Pareto distribution does satisfy the decreasing hazard rate property.

An analogue to Observation 3 may now be established:

Observation **5** *Under the assumption that the distribution of productivities exhibits a nonincreasing hazard rate, a change from (F, G) to $(\tilde{F}, \tilde{G})$ cannot lower allocative efficiency y if $\tilde{H}$ second-order stochastically dominates H (and must strictly increase it if the hazard rate is strictly decreasing).*

7.2 Wealth, Distribution and Overall Efficiency

The striking parallel between Observations 2(4) and 3(5) guarantees that if the hazard rate of F is nonincreasing, the same results as in Section 6 can be obtained, and indeed, that *the proofs follow identical lines.* Without further ado, then, we simply record these results and comment on them very briefly.

In the case in which wealth is scaled up proportionately across the board, overall efficiency must increase provided that F satisfies the increasing hazard rate condition.

Proposition **6** *Assume* [A.1]–[A.3] *and suppose that F exhibits a nonincreasing hazard rate. Then a proportional scaling-up of wealth (by a factor $\delta > 1$) cannot reduce overall efficiency. Indeed, as long as the hazard rate is strictly increasing and $c(\delta w, \tilde{r}) \neq c(w, r)$ for some w, overall efficiency must strictly increase.*

This assures us that (under certain conditions) the gain in allocative efficiency recorded in Proposition 2 is not eaten away by possibly increased costs of lobbying.

The connections between inequality and overall efficiency retain the same complexity as those between inequality and allocative efficiency. However, under the hazard rate assumption, similar results may be established:

Proposition **7** *Assume* [A.1] *and* [A.2]*, and suppose that F exhibits a decreasing hazard rate. Consider any distribution of wealth G that fails to minimize equilibrium efficiency loss (over all distributions). Then for every sequence of wealth distributions G^m with the same mean wealth as G that converge to perfect equality in the sense of weak convergence, G^m has higher allocative efficiency for all m large enough.*

It should be noted that while allocations are first-best when the wealth distribution exhibits full equality, the same is not true of overall efficiency, which never attains the first-best level in any equilibrium. Nevertheless, under the decreasing hazard rate assumption, perfect equality of wealth does the best of a bad job.

The remaining positive results on inequality also survive unscathed:

Proposition **8** *Assume* [A.1]–[A.3]*, and suppose that F exhibits a decreasing hazard rate. allocative efficiency must increase with any progressive equalization of wealths that induces participation.*

Proposition **9** *Assume* [A.1]–[A.3] *and SSC and suppose that F has a nondecreasing hazard rate. Then overall efficiency cannot fall as inequality declines.*

8 Concluding Remarks

In this paper we study the effects of wealth and inequality on public resource allocation, when agents are privately informed of both their productivity and wealth. Agents require public support (in the form of "licenses", "permissions", or public infrastructure) in order to translate their potential productivity into hard economic reality. To this end they lobby the government for support. Agents with higher productivity lobby harder, but so do agents with higher wealth. The multidimensional nature of privately observed characteristics creates a signal-jamming problem.

In our model, the government seeks to maximize efficiency through its allocation of public support. Do we believe that governments really act in this way? Posed literally, the answer to this question is obviously in the negative, but this is not the interesting question. The issue is whether corruption in the public sector necessarily lies at the heart of public misallocation of resources. The model we study throws light on this question, and argues that there may be features deeper than corruption at work.

Our main results can be summarized as follows:

[A] The outcome of inefficient public decision-making is perfectly compatible with the assumption of an honest, efficiency-seeking planner. Therefore, the observation of poor performance by government agencies cannot be taken as an unambiguous indicator of more sinister motives on the part of those agencies.

[B] The extent of inefficiency in public decisions — for a given degree of inequality — depends on the aggregate level of wealth. Poor countries will tend to display higher degrees of inefficiency, both allocative and net of the lobbying cost.[13] Once again, it is tempting (though possibly erroneous) to conclude that corruption is more widespread in poorer economies.

[C] The degree of efficiency in the public allocation of resources also depends on existing inequality in the distribution of wealth. Here, the relationship is more complex. Nevertheless, in several interesting cases, a reduction in inequality improves public efficiency in decision-making. In particular, this observation is true of reductions in inequality that are sufficiently far-reaching. Thus unequal economies may appear as more "corrupt".

Our contributions are relevant to at least four strands of the literature.

In the first place, since the mid 90s, the analysis of the interaction between inequality and growth has attracted significant attention. Broadly speaking, there have been two major explanations for the negative role of inequality on growth. The first one is based on the argument that with imperfect capital markets, wealth constrained individuals will be forced to make choices which do not duly develop their abilities, so that aggregate output is below its potential level. It follows that the extent of the efficiency loss critically depends on the number of individuals for which this constraint is binding.[14] The second major type of explanation has a political-economy flavor and has been expressed in a number of ways. One line argues that high inequality will induce voters to support higher degrees of redistribution thereby inducing heavier distortions on intertemporal resource allocation, specifically dampening investment.[15] A second line of research studies the relationship between social conflict and growth.[16]

These two main approaches — the one based on missing markets, the other on political or social struggle — have been developed quite independently from each other. There is much to be gained in marrying the two. Models of imperfect capital markets, while insightful in themselves, would be enriched by taking on board the political process. The implication for the second strand of literature — at least, the part that focusses on democratic redistribution — is more damaging. As Perotti (1993, 1996), Bénabou (1996), and others have noted, initial inequalities may be related to slower growth, but evidently not through the channels proposed in this part of the theoretical literature. Unequal societies tend to under-rather than over-redistribute. Thus unequal societies may have inimical effects on growth because they *stay* unequal (and therefore suffer from one or more of the woes in the missing-markets story), and not because of some incentive-sapping drive towards equality.

A second strand of the literature to which our paper may contribute is in providing an explanation for the observed association between political power and wealth. This point goes back as far as Plato: even in a democratic society, *effective* political power is positively correlated with wealth. Rodríguez (1997) and Bénabou (2000) address this issue explictly. Specifically, Bénabou posits that the political weight of a voter depends on her rank in the wealth distribution, and then examines the implications of such a postulate. Our paper obtains this positive association between political and economic power as an equilibrium condition of a signalling game: higher wealth obscures true productive merit in the quest for public support of economic projects.

A third area of relevance is the study of lobbying as a means to transmit information to a planner. We draw on the idea that process of decision-making by governments is fraught with informational gaps. In this sense, our model shares features in common with a literature that views lobbying as a communicator of socially valuable information; see, for example, Austen-Smith (1994), Austen-Smith and Banks (2000, 2002), Austen-Smith and Wright (1992), Bennedsen and Feldman (2002), Lohman (1994) and Rasmussen (1993). Such an approach is especially relevant for societies that are undergoing rapid transformation. It may be very hard for a planner to understand and foresee the correct directions in which the economy must go.[17] In this sense, lobbying serves as a generator of possibly useful information, in contrast to the black-box models of rent-seeking analyzed in profusion in the literature.

Finally — and mentioned already at several points in this paper — the literature on corruption is certainly relevant to the issues raised here. After all, even in the most conspicuous democracies there is only a limited number of issues which are decided by majority voting. Referenda are exceptional. Thus there is always ample room for discretionary governmental decisions, which can be influenced by the citizens. It is the existence of this discretionary space that explains the development of rent-seeking, lobbying, and even corruption.[18] Indeed, almost the entire literature on lobbying or rent-seeking in developing societies explictly or implictly assumes that corruption is at the heart of the problem.[19] Agents use resources in order to induce government decisions most favorable to their interests. Whether these resources are fully wasted or used to bribe government officers does not seem to be essential to the story. Indeed, in this literature it is difficult to distinguish between a politician that is honestly impressed (in the informational sense) by the amount of lobbying done by an agent and one that is simply bribed by the agent who pays the most.

The flip side of this story is that the wealthy fuel corrupt behavior in their attempt to corner public resources. Indeed, the idea that wealthy agents may confound the resource allocation process because of their greater ability to corner resources is not new at all. The point has been made in development contexts time and again (see Bhagwati and Desai (1970) and Bardhan (1984) for insightful analyses along these lines). But the point has usually been made in the context of corrupt bureaucracies — scarce permissions or infrastructure may be bought by wealthy agents by simply bribing corrupt officials. In contrast, we attempt to argue that poor decision-making is to be expected even from honest, efficency-seeking governments. This is a case in which it does not take two to tango.

Is the distinction important? We believe it is. The emphasis on the buying of corrupt politicians (or public decision-makers) in developing countries may lead to overemphasis on the woes perpetrated by bureaucratic dishonesty, and relative neglect of the intrinsic problems of the resource-allocation process. The role of "institutional failures" in explaining the difficulties of some developing economies in taking off, despite massive foreign aid, is a standard theme. The *World Development Reports* of 1997, 2002 and 2003 make a point of this. For instance, the 2003 Report states that "high levels of corruption are associated with poverty, inequality, reduced domestic and foreign direct investment, and weak overall economic performance". The empirical evidence for this type of implied causality is rather weak. After the work of Mauro (1995) identifying a negative relationship between corruption and growth, subsequent research has substantially moderated the relevance of this effect, as in Li *et al.* (2000) and Treisman (2000).

In our paper we have argued that the causality in the opposite direction should not be overlooked. In a world of imperfect capital markets, aggregate poverty and distributional inequality pollute the transmission of information from private interests to policy-makers and end up by provoking ineffcient decisions. In fact, this reverse causality argument is alligned with the empirical findings by Treisman (2000). He finds that the log of per capita GDP can explain at least 73% of the cross-country variation in the indices of perceived corruption and that, when testing for the direction of causality, there are "strong reasons to believe that ... higher economic development does itself reduce corruption".

We do not deny the existence of corrupt practices. Our point is that the evidence of inefficient decision-making cannot be taken as a "smoking gun" for corruption. All the empirical evidence available consists of opinion indices constructed by interviewing firms, investors or plain citizens (see Treisman (2000)). The fact is that we cannot exclude the hypothesis that such opinions may reveal rationalizations of seemingly capricious public decisions rather than the direct observation of illegal money payments.

Appendix

Proof of Proposition 5. Define $z \equiv 1 - F(c(w,r))$ and $\tilde{z} \equiv 1 - F(c(w,\tilde{r}))$, just as we did before, and consider two cdfs defined over this variable:

$$H(z) = G(c^{-w}(F^{-1}(1-z), r)),$$

and

$$\tilde{H}(\tilde{z}) = \tilde{G}(c^{-w}(F^{-1}(1-z), \tilde{r})),$$

where, as before, r and $\tilde{r}$ are the lobby levels before and after the change from G to $\tilde{G}$. We will show that $\tilde{H}$ second-order stochastically dominates H. To establish this, it will suffice to prove that

$$0 < H(z) = \tilde{H}(z) < 1 \tag{22}$$

implies that

$$\tilde{H}'(z) > H'(z). \tag{23}$$

To this end, suppose that (22) holds for some z. Define Let $w = c^{-w}(F^{-1}(1-z), r))$ and $\tilde{w} = c^{-w}(F^{-1}(1-z), \tilde{r}))$. Then $c(\tilde{w}, \tilde{r}) = F^{-1}(1-z) = c(w, r)$, and $G(w) = H(z) = \tilde{H}(z) = \tilde{G}(\tilde{w})$. Also (w, r) and $(\tilde{w}, \tilde{r})$ are each strictly positive.[20] So SSC applies, and (18) holds.

Now, direct computation shows that

$$H'(z) = -G'(w)\frac{1}{c_w(w, r)}F^{-1\prime}(1-z),$$

while

$$\tilde{H}'(z) = -\tilde{G}'(w)\frac{1}{c_w(\tilde{w}, \tilde{r})}F^{-1\prime}(1-z).$$

Consequently, recalling that $c_w(w, r) < 0$ and that the density of F is strictly positive, we see that $\tilde{H}'(z) > H'(z)$ if

$$\frac{\tilde{G}'(w)}{|c_w(\tilde{w}, \tilde{r})|} > \frac{G'(w)}{|c_w(w, r)|},$$

which is assured by (18).

So $\tilde{H}$ stochastically dominates H. [Notice that the dominance must be strict, because it is easy to see that H and $\tilde{H}$ must be distinct at some z.] Observations 2 and 3 now yield the desired result. ∎

9 References

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Footnotes

1. For instance, the Kentucky State Government website has a section devoted to the “One-Stop Business Licensing Program”. This section greets you thus: “Starting a new business can be a confusing process. There are over 1,800 business types and over 600 business licenses required from various agencies at the state level in Kentucky.”

2. That the distribution of productivities has positive density on $(0, \infty)$ is a simplification which can be easily dispensed with by considering additional (and tedious) special cases in the proofs. We do not impose the same restriction on w: no special cases need be considered here and besides, some of our examples make better sense when $\underline{w} > 0$.

3. Formally, we take it that $c_w(w, r) < 0$ and $c_r(w, r) > 0$, where these subscripts denote the appropriate partial derivatives.

4. If one insists further that the variance be finite in the Pareto example, the bound drops even more, to 1/8. For the exponential, the bound may be calculated to be 1/2. So while the bounds will vary depending on assumptions, none of these require an unreasonably high ratio of own wealth to lobbying expenditure for [A.3] to apply.

5. This approach can be contrasted with the *mechanism design* assumption in which the government first commits to an allocation rule and agents then react. See Banerjee (1997) for a model along these lines. For models of wealth constraints (in other contexts) that study different mechanisms, see, e.g., Fernandez and Gali (1999) and Bedard (2001) (in the context of educational attainments) and Che and Gale (in the context of auctions).

6. In particular, one can accommodate beliefs that are continuous with respect to the assessed gain that a type can make from the deviation, with marginal types being assigned zero density in the belief. Doug Bernheim encouraged us to relax our original refinement — which simply equated the population distribution over admissible types to the planner's beliefs over those types — to accommodate these cases.

7. Notice that she has permissions left over after servicing r_2, which is why $p(r_1)$ is positive to begin with.

8. Maximal output is given by $\int_{\lambda(\alpha)}^{\infty} \lambda dF(\lambda)$, where $\lambda(\alpha)$ is the value at which $F(\lambda(\alpha)) = 1 - \alpha$.

9. To specify this function in case λ lies beyond the range of $c(w, r)$ (as w varies), define $c^{-w}(\lambda, r) \equiv \underline{w}$ whenever $\lambda > c(\underline{w}, r)$ and $c^{-w}(\lambda, r) \equiv \infty$ in case $\lambda < c(\infty, r)$.

10. This is indeed possible. For instance, if $a = 0$ in the particular specification for the cost function given by (2), there will be no net change.

11. The fact that "equal poverty" can be inimical to efficiency is known in a wider context (see for example, Mirrlees (1976), Matsuyama (2002), and Ray (1998)).

12. It is easy to see that we can use our specifications to approximate, arbitrarily closely, the pure credit rationing model.

13. It should be noted that these results do depend on assumptions that may be violated in very poor economies, leading to a nonmonotonic relationship between wealth and public efficiency. See also Bardhan (1997), who presents a complementary argument for such nonmonotonicity, based on the possibility that in the early stages of growth emerging opportunities are large relative to a rudimentary public administration.

14. The role played by capital market imperfections in imposing a borrowing constraint on low wealth individuals was first examined by McKinnon (1973), Loury (1981) and more recently applied to growth models by Aghion and Bolton (1997), Banerjee and Newman (1993), Galor and Zeira (1993), Lee and Roemer (1998), Lundqvist (1993), Mani (2001), Piketty (1997) and Ray and Streufert (1993), among others.

15. This line is exemplified by the work of Alesina and Rodrik (1993) and Persson and Tabellini (1994).

16. See Bénabou (1996), Benhabib and Rustichini (1996), Saint-Paul and Verdier (1997), Chang (1998) and Tornell and Velasco (1992). Alesina, Özler, Roubini and Swagel (1996) demonstrate statistically significant associations between low growth, social polarization and political instability (see also Mauro (1995), Perotti (1996) and Svensson (1998)). Or see Olson's (1965) argument that government corruption is more likely in highly unequal societies.

17. On the possibility of degraded information in the course of development, and its implications for market functioning, see Ghosh and Ray (1996), Ray (1998, Chapters 13 and 14), and the World Development Report (1998/99).

18. On models of rent-seeking and lobbying, see, e.g., Mohtadi and Roe (1998), Rama and Tabellini (1998), Shleifer and Vishny (1993) and Verdier and Ades (1996).

19. A notable exception is Banerjee (1997), who seeks to understand bureaucratic red tape as the outcome between a welfare-maximizing government and a money-grabbing bureaucrat. Indeed, as he shows, an analysis based on the premise of a fully corrupt government can be problematic in some respects.

20. Clearly, r and $\tilde{r}$ are strictly positive. That w and $\tilde{w}$ are also strictly positive follow from the fact that $G(w) = H(z) = \tilde{H}(z) = \tilde{G}(\tilde{w}) > 0$, and the assumption that G has no atoms.

Supplementary Notes to "Inequality, Lobbying and Resource Allocation"

1. Introduction

In these notes, we supplement the paper in a number of ways:

[1] We extend the analysis to changes in productivity as well as wealth.

[2] We prove that efficiency increases for large-scale equalizations in the distribution of wealth.

[3] We provide some sufficient conditions under which an equalization in the wealth distribution encourages participation.

[4] We discuss the strong single-crossing condition and show that it is automatically satisfied for the case of a constant elasticity cost function and a Pareto distribution of wealth, whenever the distribution of wealth is equalized.

2. Productivity Changes and Efficiency

We discuss the assertion in the paper that an across-the-board increase in individual productivities will also lead to efficiency gains. This result employs the following strengthening of [A.3].

[A.3′] For every $\delta > 1$ the ratio
$$\frac{c(w, \delta r)}{c(w, r)}$$
is nondecreasing in w.

This ratio is the factor increase in the cost by an increase in the bidding expenditure by a factor of $\delta > 1$. As wealth increases both costs decrease. Our assumption simply posits that the cost at the higher level of bidding does not fall faster than the lower cost. In particular, this assumption excludes the possibility that the costs of expending δr and r tend to converge to each other as wealth becomes large. Let us just mention here that the cost function that we have proposed as an example in the main text
$$c(w, r) = \hat{c}(r)\left[\frac{1}{w^{\theta}} + a\right]$$
does satisfy [A3'].

Proposition 1. *Under* [A.1]–[A.3'], *a proportional scaling-up of productivities (by $\beta > 1$) cannot reduce allocative efficiency. Indeed, as long as $c(w, \tilde{r}) \neq \beta c(w, r)$ for some w, allocative efficiency must strictly increase.*

To establish the first part of the proposition, we will show that $\tilde{H}$ is (weakly) less risky than H in the sense of second-order stochastic dominance. Observation **??** then guarantees the result.

To this end, it will suffice to prove that

$$\text{(1)} \qquad \tilde{H}(z) > H(z) \text{ for some } z \text{ implies that } \tilde{H}(z') \geq H(z') \text{ for all } z' \geq z,$$

where
$$\tilde{H}(z) = G(c^{-w}(\tilde{F}^{-1}(1-z), \tilde{r})) = G(c^{-w}(\beta F^{-1}(1-z), \tilde{r})).$$

We first note that $\tilde{r} > r$. With the scaling of productivities, for the given r, there will be strictly more bidders at each wealth level. It follows that the equilibrium bid has to increase up to $\tilde{r}$ where the balance between demand and supply of licenses is restablished.

Secondly, we also note that, since the expected value of the distributions $\tilde{H}$ and H is the same, the two cumulative distributions must intersect at least once.

Let z^* be the smallest z at which the two distributions intersect, so that for w^* we have that

$$\frac{c(w^*, \tilde{r})}{c(w^*, r)} = \beta > 1.$$

We know there is a unique w_1 such that

$$w_1(z) = c^{-w}(F^{-1}(1-z), r)),$$

and a unique w_2 such that

$$w_2(z) = c^{-w}(\tilde{F}^{-1}(1-z), \tilde{r})).$$

Hence (1) is equivalent to $w^* \leq w_1(z) \leq w_2(z)$ for all $z \geq z^*$.

Note that

$$\frac{c(w_2(z'), \tilde{r})}{c(w_1(z'), r)} = \beta = \frac{c(w^*, \tilde{r})}{c(w^*, r)} \text{ for all } z'.$$

Suppose now that contrary to our claim there was $z' > z$ such that $w_1(z') > w_2(z')$. Then

$$\frac{c(w_1(z'), \tilde{r})}{c(w_1(z'), r)} \leq \frac{c(w_2(z'), \tilde{r})}{c(w_1(z'), r)} = \frac{c(w^*, \tilde{r})}{c(w^*, r)} = \beta.$$

But this contradicts our assumption [A.3'], so the proof of the first part is complete.

To establish the remainder of the proposition, simply note that if $c(w, \tilde{r}) \neq \beta c(w, r)$ for some w, then $\tilde{H}(z) \neq H(z)$ for *some* z . The strict concavity result established in Observation 2 then assures us that allocative efficiency must *strictly* increase. ∎

3. Large-scale reduction of inequality and efficiency

In the main text we have obtained that the effects of wealth redistribution on efficiency are rather complex and that can go either way. The sign of the effect of wealth redistribution depends on welath transfers be restricted to particular intervals of the distribution. However, if the extent of the redistribution is sufficiently large, then efficiency will go up. In the main text we estate this result as Proposition 3. We present here the proof of this proposition.

Proposition 2. *Consider any distribution of wealth G that causes some allocative loss in equilibrium. Then for every sequence of wealth distributions G^m that maintain the same mean wealth as G and converge to perfect equality, G^m has higher allocative efficiency for all m large enough.*

Proof. Let $\bar{w}$ denote the common mean wealth of G and G^m, $\bar{\lambda}$ the productivity level for which $1 - F(\bar{\lambda}) = \alpha$, and let $\bar{r}$ be such that $c(\bar{w}, \bar{r}) = \bar{\lambda}$. Denote by r^m the equilibrium value of r for each G^m.

We first claim that $r^m \to \bar{r}$ as $m \to \infty$. To establish this, let $\tilde{r}$ be any limit point of r. Observe that $\tilde{r}$ must be finite because the sequence r^m must be bounded.[1] Now notice that for all m,

$$\begin{aligned}
\alpha &= \int_0^\infty [1 - F(c(w, r^m))]\, dG^m(w) \\
&= \int_0^\infty [1 - F(c(w, \tilde{r}))]\, dG^m(w) + \int_0^\infty [F(c(w, \tilde{r})) - F(c(w, r^m))]\, dG^m(w) \\
&= \int_0^\infty [1 - F(c(w, \tilde{r}))]\, dG^m(w) + \int_{W_1}^{W_2} [F(c(w, \tilde{r})) - F(c(w, r^m))]\, dG^m(w) \\
&+ \int_0^{W_1} [F(c(w, \tilde{r})) - F(c(w, r^m))]\, dG^m(w) + \int_{W_2}^\infty [F(c(w, \tilde{r})) - F(c(w, r^m))]\, dG^m(w)
\end{aligned}$$

where W_1 and W_2 are any wealth levels such that $W_1 < \bar{w} < W_2$. The third and last terms on the RHS of the above equation must converge to zero as $m \to \infty$, because $G^m(W_1)$ and $1 - G^m(W_2)$ both converge to 0 (and the integrands in those terms are uniformly bounded). To study the second term on the RHS, observe that $F \circ c$ is a continuous function, so it is *uniformly* continuous at $\tilde{r}$ over all $w \in [W_1, W_2]$. Consequently, the second term must also go to 0 as $m \to \infty$. Finally, by weak convergence,

$$\int_0^\infty [1 - F(c(w, r^m))]\, dG^m(w) = 1 - F(c(\bar{w}, \tilde{r})).$$

It follows that $\tilde{r} = \bar{r}$, and the claim is established.

To complete the proof, let total output produced under G^m be denoted by Y^m. Then

$$\begin{aligned}
Y^m &= \int_0^\infty \left[\int_{c(w,r^m)}^\infty \lambda dF(\lambda)\right] dG^m(w) \\
&= \int_0^\infty \left[\int_{c(w,\bar{r})}^\infty \lambda dF(\lambda)\right] dG^m(w) + \int_0^\infty \left[\int_{c(w,r^m)}^{c(w,\bar{r})} \lambda dF(\lambda)\right] dG^m(w) \\
&= \int_0^\infty \left[\int_{c(w,\bar{r})}^\infty \lambda dF(\lambda)\right] dG^m(w) + \int_{W_1}^{W_2} \left[\int_{c(w,r^m)}^{c(w,\bar{r})} \lambda dF(\lambda)\right] dG^m(w) \\
&+ \int_0^{W_1} \left[\int_{c(w,r^m)}^{c(w,\bar{r})} \lambda dF(\lambda)\right] dG^m(w) + \int_{W_2}^\infty \left[\int_{c(w,r^m)}^{c(w,\bar{r})} \lambda dF(\lambda)\right] dG^m(w).
\end{aligned}$$

Once again, the third and last terms go to zero as $m \to \infty$, while uniform continuity can be applied just as before to show that the second term also goes to zero. Finally, by the property of weak convergence,

$$\int_0^\infty \left[\int_{c(w,\bar{r})}^\infty \lambda dF(\lambda)\right] dG^m(w) \to \int_{c(\bar{w},\bar{r})}^\infty \lambda dF(\lambda)$$

[1] It is easy to check that if r^m is unbounded, the equilibrium condition (2) must fail for some m, given the assumptions on G^m.

which simply means that the equilibria under the sequence G^m asymptotically display full allocative efficiency. This implies the proposition. ∎

4. Participation encouraging wealth redistributions

Proposition 4 in the main paper asserts that a progressive redistribution of wealths will increase efficiency provided that it induces a higher participation of bidders at the old bid r.

We shall now examine the conditions under which a Dalton progressive redistribution of wealth does encourage participation (for constant bid r).

The equilibrium announcement r solves the equation

$$1-\alpha=\int_0^\infty F(c(w,r))dG(w). \tag{2}$$

The RHS is strictly increasing in r. Hence, any new distribution of wealth $\tilde{G}$ such that holding r constant satifies that

$$1-\alpha=\int_0^\infty F(c(w,r))dG(w)\geq\int_0^\infty F(c(w,r))d\tilde{G}(w)$$

will have an equilibrium $\tilde{r}\geq r$ and hence will *encourage participation*.

Using the well-known Jensen's inequality we shall have that, if $\tilde{G}(w)$ has the same expected wealth and Lorenz dominates $G(w)$, $\tilde{r}\geq r$ if $F(c(w,r))$ is convex in w.

Upon differentiation, we find that

$$\frac{\partial F}{\partial w}=f(c(w,r))c_w(w,r)\text{ and, after some manipulation,}$$

$$\frac{\partial^2 F}{\partial w^2}=\frac{f(c(w,r))c_w(w,r)^2}{c(w,r)}\left[\frac{f'(c(w,r))c(w,r)}{f(c(w,r))}+\frac{c(w,r)c_{ww}(w,r)}{c_w(w,r)^2}\right].$$

Since $c_{ww}(w,r)\geq 0$, $f'(\lambda)\geq 0$ is sufficient for the convexity of F with respect to w. The convexity of F can be more problematic at the upper tail of the distribution as f' turns negative. Indeed, if the density falls too sharply, the first term in the braces (negative) might dominate the second term. However, this needs not be the case for distributions such as the Pareto –characterized by thick upper tails– or those who assymptotically approach the Pareto distribution.

This property of the behavior of the upper tails of distributions can be analyzed by means of the "income-share elasticity" of a distribution, $\pi(x)$, as introduced in Esteban (1986). Formally,

$$\pi(x)\equiv 1+\frac{f'(x)x}{f(x)}.$$

The income-share elasticity is falling with x for all distributions. For many distributions this fall is too fast and $\pi\to-\infty$ as $x\to\infty$. In contrast, the distinctive feature of the Pareto distribution is that $\pi(x)=-\alpha$. This property makes the Pareto distribution particularly suitable to describe many interesting distributions in Economics characterized by tails that are fatter than the ones predicted by the Normal, LogNormal or Exponential distributions. The family of distributions assymptotically "behaving like" a Pareto distribution was first analyzed by Lévy (1927) and later by Mandelbrot (1960). They proposed the "Weak Pareto Law": the ratio of the cumulative distribution function to a Pareto distribution tends to unity as the variable tends to infinity. Esteban (1986) proposed instead the weaker "Weak Weak Pareto Law" (WWPL): the income-share elasticity of a distribution decreases and tends to $-\alpha$ as the variable tends to infinity. Notice that the income-share elasticity of the distributions satisfying the WWPL is always larger than $-\alpha$. Therefore, for distributions satisfying the WWPL[2]

$$\frac{c(w,r)c_{ww}(w,r)}{c_w(w,r)^2} \geq 1+\alpha$$

is sufficient for the convexity of F.

What this restriction demands?

Let us now slightly strengthen Assumption [A3]. This is,

Assumption [A3"] The wealth elasticity of the cost function $\epsilon(w,r)$ is non-increasing in w.

We know that

$$\epsilon(w,r) \equiv -\frac{wc_w(w,r)}{c(w,r)}.$$

Concerning the wealth elasticity of the cost function, it seems realistic to assume that the proportional impact on the cost of r from a wealth increase is well below unity

Differentiating we obtain

$$\begin{aligned} \frac{\partial \epsilon(w,r)}{\partial w} &= -\frac{c(w,r)c_w(w,r) + wc(w,r)c_{ww}(w,r) - wc_w(w,r)^2}{c(w,r)^2} = \\ &= -\frac{wc_w(w,r)^2}{c(w,r)^2}\left[\frac{c(w,r)c_{ww}(w,r)}{c_w(w,r)^2} - 1 - \frac{1}{\epsilon(w,r)}\right]. \end{aligned}$$

Hence, [A3"] implies that

$$\frac{c(w,r)c_{ww}(w,r)}{c_w(w,r)^2} \geq \frac{1}{\epsilon(w,r)} + 1.$$

We can conclude that, under [A3"],

$$\frac{1}{\epsilon(w,r)} \geq \alpha,$$

is a sufficient condition for the convexity of F and hence for a progressive redistribution to be participation encouraging.

[2]The WWPL is satisfied by the Pareto distributions of the second and third kind, among many others. By way of illustration, the three-parameter family defined by

$$\pi(x) = -\alpha + \beta x^{-\epsilon}, \text{with } \alpha, \beta, \epsilon > 0$$

satisfies the WWPL and generates the well-known Generalized Gamma distribution, see Esteban (1986).

5. The Strong Single Crossing SSC condition

Proposition 5 establishes that a redistribution of wealth satisfying the *Strong Single Crossing Condition* will increase efficiency. We examine now the conditions under which the SSC property is satisfied.

We start we recalling the SSC condition [A4].

[A.4] For every strictly positive (w, r) and $(\tilde{w}, \tilde{r})$ such that $c(w, r) = c(\tilde{w}, \tilde{r})$ and $G(w) = \tilde{G}(w)$,

$$\tilde{G}'(\tilde{w})|c_w(w, r)| > G'(w)|c_w(\tilde{w}, \tilde{r})|. \tag{3}$$

How strong is [A.4]? Consider a family of cost functions in which wealth has a constant-elasticity impact:

$$c(w, r) = \hat{c}(r)/w^\theta,$$

where $\hat{c}(r)$ is some increasing function and $\theta > 0$.[3] Then it is easy to see that

$$|c_w(w, r)| = \frac{\theta\hat{c}(r)}{w^{\theta+1}} = \frac{\theta c(w, r)}{w}.$$

Consequently,

$$\frac{G'(w)}{|c_w(w, r)|} = \frac{G'(w)w}{\theta c(w, r)},$$

so that (3) reduces to

$$\tilde{G}'(\tilde{w})\tilde{w} > G'(w)w \tag{4}$$

for every strictly positive pair $(w, \tilde{w})$ such that $G(w) = \tilde{G}(\tilde{w})$. Notice again that this is a specially strong form of second-order stochastic dominance (take $w = \tilde{w}$ and reinspect (4)). However, for several families of distribution functions, the stricter condition is *automatically* satisfied whenever a reduction in inequality (in the sense of Lorenz dominance) takes place.

One such class is the Pareto distribution of the second kind on w. Let

$$G(w) = 1 - m^\delta(w + m)^{-\delta},$$

where $m > 0$ and $\delta > 1$ (to ensure that a mean is well-defined). It is easy to check that w has atomless support on $[0, \infty)$, and that the mean of G is given by $m/(\delta - 1)$. Routine computation also establishes that as m and δ simultaneously increase (say, to $\tilde{m}$ and $\tilde{\delta}$), holding overall mean constant, the distribution becomes progressively more equal in the sense of Lorenz (or equivalently, second-order dominance). The question is: does (4) automatically hold when this change takes place?

To answer this, observe that the restriction $G(w) = \tilde{G}(\tilde{w})$ simply means that

$$m^\delta(w + m)^{-\delta} = \tilde{m}^{\tilde{\delta}}(\tilde{w} + \tilde{m})^{-\tilde{\delta}}, \tag{5}$$

so that

$$\frac{m}{w + m} = \left(\frac{\tilde{m}}{\tilde{w} + \tilde{m}}\right)^k,$$

[3] The reader will notice that this family is a subclass of the family introduced in (**??**).

where $k \equiv \tilde{\delta}/\delta$. Now it is easy to see that $kx < 1 - (1-x)^k$ whenever $x \in (0,1)$. Using this, we conclude that

$$\frac{w}{w+m} = 1 - \frac{m}{w+m} = 1 - \left(1 - \frac{\tilde{w}}{\tilde{w}+\tilde{m}}\right)^k < k\frac{\tilde{w}}{\tilde{w}+\tilde{m}}. \tag{6}$$

Now $G'(w)w = \delta m^{\delta}(w+m)^{-(1+\delta)}w$ and $\tilde{G}'(\tilde{w})\tilde{w} = \tilde{\delta}\tilde{m}^{\tilde{\delta}}(\tilde{w}+\tilde{m})^{-(1+\tilde{\delta})}\tilde{w}$, so that — using (5) — (4) will hold if

$$\tilde{\delta}\frac{\tilde{w}}{\tilde{w}+\tilde{m}} > \delta\frac{w}{w+m},$$

which is precisely guaranteed by (6).

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