The Stock Market and the Allocation of Capital in a Production Economy

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Abstract

We analyze the allocative role of the stock market in a multi-sector production economy. Output in each sector is determined by a Cobb-Douglas production function and subject to log-normal productivity shocks. Investors allocate capital across sectors and to an information technology that allows them to learn privately about sectoral shocks. Stock prices provide signals that guide investors in their allocation but depress their incentives to collect information. We show that wealthier economies are better informed and allocate capital more efficiently across sectors. The improved capital allocation leads to larger total factor productivity, GDP and concentration of economic activity. The real and financial sectors are positively associated. These properties are consistent with the evidence.

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1 Introduction

Economists have long argued that the stock market plays an important role in channeling capital to its best uses. Stock prices not only clear the market, they also serve as informative signals that guide investors in their decisions (Bagehot (1873), Hayek (1945)). Recent empirical studies provide systematic evidence in support for this view. For example, Wurgler (2000) documents that investments are more sensitive to value addition in countries that are more developed financially and that this sensitivity relates to the informativeness of stock prices. Yet, modeling the link from stock prices to production is difficult, precisely because of their informational role. Several authors relate managers' investment choices to the information conveyed by prices[1]. But they do not explore the macroeconomic implications. These implications are the subject of this paper.

We develop a macroeconomic model of the stock market, capital allocation and production, in which stock prices are informative. The economy is composed of many sectors subject to productivity shocks. Investors allocate capital across sectors and to an information technology that allows them to learn privately about sectoral shocks. We establish the following results.

(i) Wealthier economies invest more in the information technology.

(ii) Thanks to their superior information, they allocate capital more efficiently across sectors. That is, investments are more responsive to productivity shocks.

(iii) Total factor productivity (TFP) and GDP are larger.

(iv) Output is more concentrated across sectors.

(v) The financial sector is more developed. That is, its size, the volume of shares traded and turnover are larger.

(vi) The stock market only performs its informational role if the economy is sufficiently wealthy. Below an income threshold, no information is produced and transmitted.

(vii) The stock market exerts two conflicting effects on the efficiency of the capital allocation: on one hand, stock prices provide useful signals about productivity shocks but, on the other, they depress investors' incentives to collect costly information.

These properties are consistent with the evidence. The literature on finance and growth establishes that financial institutions contribute to growth (see Levine (1997) and (2004) for reviews). In particular, Levine and Zervos (1998), Rousseau and Wachtel (2000) and Carlin and Mayer (2003) document that output grows faster

[1] In particular, Fishman and Hagerty (1989), Holmstrom and Tirole (1993), Dow and Gorton (1997), Boot and Thakor (1997), Subrahmanyam and Titman (1999).

in countries with better-functioning stock markets. Levine and Zervos (1998) show further that TFP growth, rather than capital growth, explains most of GDP growth (Result (iii)). They also report that financial sector development as measured by size, volume of trade and turnover is positively related to output growth (Result (v)).

The importance of information is pointed out. As mentioned above, Wurgler (2000) shows that investments are more sensitive to value addition in countries that are financially more developed. He shows that this sensitivity relates to stock price synchronicity, a measure of the informativeness of stock prices introduced by Morck, Yeung, Yu (2000). In addition, Carlin and Mayer (2003) report that stock markets exert a stronger effect on innovative industries, i.e. those with high R&D investments and skilled labor (Result (ii)). Moreover, they seem more useful in wealthier economies. Carlin and Mayer (2003) again document that stock markets have a stronger growth effect in more developed countries (Result (vi)). Finally, Imbs and Wacziarg (2003) report that countries go through two stages of sectoral diversification. At first, sectoral diversification increases but, beyond a certain level of income, economic activity starts concentrating again. Our model shows that output tends to concentrate as the economy grows and more information is produced. This requires that income be above a threshold (Result (iv)).

The model belongs to the large literature on trading under asymmetric information. Output within a sector is determined by a Cobb-Douglas production function that displays decreasing returns to capital and is subject to log-normal productivity shocks. This setup, by departing from the usual Gaussian-linear structure, allows to capture the complementarity between technologies and capital that renders the allocation of capital essential, and to ensure that capital and output remain positive. The drawback, however, is that no closed-from expression is known for asset demands under the resulting log-normal payoffs. For this reason, we resort to the small-risk expansion introduced by Peress (2004).

The remaining of the paper is organized as follows. Section 2 describes the economy. Section 3 defines the equilibrium concept. Section 4 characterizes the capital allocation, output and the real sector for a given quality of information. Section 5 determines the quality of information and examines the financial sector. Section 6 concludes. Proofs are relegated to the appendix.

2 The economy

Our goal is to investigate how the stock market enhances productivity and output thanks to an efficient allocation of capital. Stock prices play a dual role: they clear the market and they convey information. Because of the latter role, closed-form solutions to models of trading with private information rarely exist. The few exceptions typically rely on assumptions that render prices linear in payoffs, signals and noise. They assume on one hand, constant absolute risk aversion (CARA) or risk neutral preferences, and on the other hand, normally distributed random variables[2]. These assumptions imply that the demand for assets is linear in their expected payoffs and that expected payoffs are linear in signals, including prices, thus making prices linear in the random variables.

These assumptions are problematic for our purpose. Stock payoffs are a fraction of firms' output, i.e. a combination of technology shocks and capital, while capital itself is a function of prices (it equals the number of shares issued multiplied by the price). Therefore, a production function linear in shocks and capital would be required to obtain expected payoffs linear in prices. Such a production function would neither capture the complementarity between technologies and capital, nor allow for returns to capital to decrease. In addition, normally distributed shocks imply that capital and output can be negative. Ideally, we would like to use a Cobb-Douglas production function subject to log-normal shocks. Unfortunately, no closed-from expression is known for asset demands under log-normal payoffs. For this reason, we resort to a small-risk expansion. Following Peress (2004), we build a sequence of economies in which fundamentals (payoffs, risks and costs) are scaled appropriately by a parameter z. The model is solved in closed-form by driving z toward zero. Peress (2004) demonstrates the convergence and the accuracy of this approximation. Throughout the paper, we assume that the scaling factor z is small enough for the approximation to be valid.

The main features of the economy are the following. The economy is composed of several sectors, each subject to a technology shock and represented by a firm. Firms raise capital in the stock market. To improve their investment decisions, agents may acquire information about the technology shocks. Their information is reflected in stock prices, but only partially because of the presence of noise. Prices in turn help all investors in their portfolio allocations. Time consists of 3 periods, a planning period ($t = 0$), an investment period ($t = 1$) and a production period ($t = 2$). The model is further defined as follows.

2.1 Technologies

2.1.1 Physical technologies

The economy is composed of M sectors. In a sector, many identical firms compete and aggregate into one representative firm so that a (representative) firm corresponds to a technology shock. Capital is deployed during the investment period ($t = 1$) and output is realized in the production period ($t = 2$). Output within a sector, Y^m, is determined by a neoclassical risky technology that displays decreasing returns to capital:

$$Y^m \equiv A^m (K^m)^\beta \qquad \text{for all } m = 1, ..., M$$

where A^m is a sector m-wide technology shock, K^m is the stock of capital allocated to sector m and β is a parameter that measures the rate of decline of the marginal product of capital ($0 < \beta < 1$).

[2]See Ausubel (1990), Rochet and Vila (1994) and Barlevy and Veronesi (2000) for alternative assumptions.

The sectoral shocks A^m ($m = 1$ to M) are assumed to be log-normally distributed and independent from one another. We write $\ln A^m \equiv a^m z$ where a^m is the growth rate of technology m and z is a scaling factor. As mentioned above, we solve the model by driving z toward zero. $a^m z$ is normally distributed with mean 0 and variance $\sigma_a^2 z$. Let $y^m z \equiv \ln Y^m$ and $k^m z \equiv \ln K^m$. Whenever we drop the m superscript from a sector-specific variable, we refer to the vector of stacked variables. For example, Y and y denote the vectors of stacked Y^m and y^m.

We do not specify how technologies are discovered, that is what determines the a^ms. We simply assume that every period, M groups of agents (entrepreneurs) are endowed with a concept which they bring to the market through an initial public offering (IPO)[3]. Firms are assumed to have no other capital than that raised through the stock market in the investment period. This assumption allows to focus on young rather than mature firms. It is well known that the former, because they have little retained earnings, are the most dependent on external financing[4].

2.1.2 Information technology

The technology shocks, a, are not observed at the time capital is deployed but an information technology allows to learn about these shocks. An agent l may acquire a private signal s_l^m about the shock in sector m, a^m:

$$s_l^m = a^m + \varepsilon_l^m$$

where ε_l^m is an error term independent of a^m and across agents. We assume that ε_l^m is normally distributed with mean 0 and precision $x_l^m z$ (variance $1/(x_l^m z)$). In the planning period ($t = 0$), agent l chooses the precision of her signal which she will observe in the investment period. Its cost is $C(x_l^m)z$ where C is continuous, increasing, convex and $C(0) = 0$. For example, $C(x) \equiv cx^2/2$ where $c > 0$. Our setup differs from the endogenous growth literature in that the information technology does not lead to the discovery of new physical technologies nor improve existing ones. Instead, it allows to allocate capital more efficiently to the exogenous physical technologies.

2.2 Agents

The economy is inhabited by a continuum of agents whose number is normalized to 1. They are *ex ante* identical, endowed with the same income w and utility U. A key feature of their preferences is absolute risk tolerance, $\tau(w) \equiv -U'(w)/U''(w)$, which we assume to be increasing with income. As shown by Peress (2004), this assumption

[3] Our goal is to investigate how the stock market can enhance output and aggregate productivity thanks to an efficient selection of technologies. In contrast, the endogeous growth literature treats the discovery of technologies as an endogenous process (Romer (1986, 1990), Aghion and Howitt (1992)).

[4] Several empirical studies confirm that financial development enhances output mainly through these young firms (Rajan and Zingales (1998), Kumar, Rajan and Zingales (1999), Demirgüç-Kunt and Maksimovic (1998), Beck, Demirgüç-Kunt and Maksimovic (2001), Love (2003)).

implies that wealthier agents acquire more information because they invest on a larger scale. For example, if preferences display constant relative risk aversion (CRRA), then τ is linear in w.

Agents only consume in the production period ($t = 2$) and their objective is to maximize expected utility from consumption. We do not focus on agents' saving decision but on how they allocate their exogenous income across sectors. As described below, stock m ($m = 1, ..., M$) is a claim to the output of sector-m firms. Agents may allocate resources to the information technology to learn about shocks and improve their portfolios. We write f_l^m for agent l's portfolio weights or fractions, i.e. the value of her investment in stock m divided by her wealth.

2.3 Assets

Shares of representative firms (sectors) trade on the stock market. One share of firm m has a price of P^m. The number of shares issued is irrelevant and normalized to 1. To ensure that agents have an incentive to collect costly information in an environment in which it gets revealed to competitors through prices, we need more sources of randomness than assets. There are several ways of adding randomness to the model. We follow the literature in assuming that the residual supply of shares of firm m is risky (Grossman and Stiglitz (1980)).

We assume that foreigners purchase all domestic shares except for a random number. Specifically, they invest $1 - \tau(w)\theta^m$ into sector m where θ^m is a random variable. The behavior of foreign investors is not modeled here. What matters for our purpose is that their trades are perfectly inelastic and determined by exogenous shocks such as liquidity needs, changes in investment opportunities in their home countries, currency fluctuations or fads regarding the domestic economy. Thus, $\tau(w)\theta^m$ represents the supply of shares available to domestic investors once foreign investors have placed their orders. The θ^m ($m = 1$ to M) are assumed to be *normally* distributed and independent from one another, from technology shocks and from agents' private signals. They have identical mean $E(\theta)$ and variance σ_θ^2/z.

We posit a noisy supply of this form, with a mean $E(\theta)\tau(w)$ and variance $\sigma_\theta^2\tau(w)^2/z$, to ensure that noise remains commensurate with the size of the economy and the magnitude of risk. If we did not scale noise appropriately, it would disappear in the limit when the economy is large (w large) or when risk is small (z small), leading to the Grossman-Stiglitz paradox: no agent acquires any information because it is perfectly revealed; but it is precisely when no one is informed that information is the most profitable.

A riskless asset is available on the international market. Domestic investors may borrow freely (lend) at the riskless rate by issuing bonds to (buying bonds from) foreigners. The riskless rate of return is denoted $R^f = 1 + r^f z$. Because the economy under study is assumed to be small in relation to the world economy, the accumulation of assets in that economy has a negligible impact on the path of the world interest rate r^f, which is therefore treated as exogenous.

2.4 Timing

The timeline is depicted in figure 1. Agents decisions consist of two steps. First in the planning stage ($t = 0$), they decide how much resources to devote to collecting information. Second in the investment stage ($t = 1$), markets open, agents observe their private signals (if any) and allocate their income to the different assets using private and public signals. The latter three events take place simultaneously. They consume the proceeds from their investments in the production period ($t = 2$). We turn to the definition of the equilibrium.

3 Equilibrium concept

We describe the equilibrium concept for this economy. We start from individual maximization (conditions (i) and (ii)) and proceed to market aggregation (conditions (iii) and (iv)). Agents choose how much information to collect (planning stage), and how to invest their income (investment stage). In equilibrium, stock prices contain useful information. $\mathcal{F}_l \equiv \{s_l, P\}$ denotes investor l's information set and $E_l(. \mid \mathcal{F}_l)$ and $E_l(.)$ refer respectively to her *ex post* ($t = 1$) and *ex ante* expectations ($t = 0$). The problem is simplified by noting that, by symmetry, agent l chooses the same precision across stocks, i.e. $x_l^m = x_l$ for all m. Furthermore, because agents are endowed with the same income, they choose the same precision, i.e. $x_l = x$ for all l[5].

(i) In the investment stage, agent l sets her portfolio weights f_l using both public and private signals (P and s_l with a precision x inherited from the planning stage). Because technology displays decreasing returns to capital, the return an investor expects from a sector and therefore the portfolio weight she chooses depend on the stock of capital in that sector. Agents are atomistic and take as given prices and the distribution of capital across sectors. Their problem can be expressed formally as:

$$\max_{f_l} E_l \left[U(c_l) \mid \mathcal{F}_l \right] \quad \text{subject to} \quad \begin{aligned} c_l &= \left[w - MC(x)z \right] R_l \\ R_l &= R^f + \sum_{m=1}^{M} f_l^m \left(R^m - R^f \right) \end{aligned} \tag{1}$$

where c_l, R_l and R^m denote respectively agent l's consumption, the (simple) return on her portfolio and on stock m. Sector m's output is $Y^m \equiv A^m (K^m)^\beta$ for a total investment K^m. Therefore, the return on stock m is $R^m = Y^m / K^m = \exp\left[a^m z - (1 - \beta) k^m z \right]$. $MC(x)z$ represents an agent's total information expenditure. Agents may borrow and short stocks if they wish. Call v the value function for this problem.

(ii) In the planning stage, the agent selects the precision of her signals. How much information she acquires depends on how much information is collected in aggregate and revealed through prices. Let X denote *the average precision chosen by investors.* Agent l selects the precision of her signals, $x = x(w, X)$, in order to maximize her

[5] These properties are derived in the appendix. Note that agents do not receive identical signals though they receive signals of identical precisions.

expected utility, taking as given the average precision in the economy X, and averaging over all the possible realizations of s_l, P and K:

$$\max_{x \geq 0} E_l[v(s_l, x, w, P, X, K)] \tag{2}$$

(iii) In equilibrium, prices clear the market for stocks. Individuals' investments, when aggregated, coincide with the (assumed) capital stock. These two requirements are equivalent given that capital consists exclusively of newly raised funds. Furthermore, the capital stock in any sector equals the number of shares issued, 1, times the price in that sector. Formally,

$$\int_l \frac{w}{P} f(s_l, x, w, P, X, K) = \tau(w)\theta \qquad \text{and} \qquad K = P$$

(iv) The average precision X, which was taken as given in step (ii), equals the precision chosen by investors (recall that investors are *ex ante* identical and choose the same precision):

$$X = x(w, X)$$

4 Capital allocation, production and the real sector

We determine the equilibrium allocation of capital and derive output. Throughout this section, we take as given the precision of investors' signals, which we endogenize in the next section. We guess that capital and prices are approximately, i.e. at the order z, log-linear functions of technology and supply shocks, solve for portfolios, derive the equilibrium capital allocation and prices, and check that the guess is valid. We begin with a description of two benchmark economies, one in which nothing is known about the technology shocks and another in which they are observed perfectly. They will serve as reference points when we discuss the role of information.

4.1 Benchmark economies

Theorem 1 *(Benchmark economies)*

- *No-information economy*
  - *The allocation of capital to sector m equals $K^{m\mathcal{N}} = \exp(k^{m\mathcal{N}}z)$ where*

$$k^{m\mathcal{N}} \equiv k^{m\mathcal{N}}(\theta^m) = \frac{1}{1-\beta}\left(\sigma_a^2(\frac{1}{2} - \theta^m) - r^f\right)$$

  - *Output in sector m equals $Y^{m\mathcal{N}} = \exp(y^{m\mathcal{N}}z)$ where*

$$y^{m\mathcal{N}} = a^m + \beta k^{m\mathcal{N}} = \frac{\beta}{1-\beta}\left(\sigma_a^2(\frac{1}{2} - \theta^m) - r^f\right) + a^m \tag{3}$$

- *Perfect-information economy*
  - *The allocation of capital to sector m equals $K^{m\mathcal{P}} = \exp(k^{m\mathcal{P}}z)$ where*

$$k^{m\mathcal{P}} \equiv k^{m\mathcal{P}}(a^m) = \frac{1}{1-\beta}(a^m - r^f)$$

– *Output in sector m equals $Y^{m\mathcal{P}} = \exp(y^{m\mathcal{P}}z)$ where*

$$y^{m\mathcal{P}} = a^m + \beta k^{m\mathcal{P}} = \frac{1}{1-\beta}\left(a^m - \beta r^f\right) \tag{4}$$

We comment first on the behavior of capital. When nothing is known about the technology shocks, prices and capital do not relate to technology shocks, but depend negatively on supply shocks. Sectors hit by a low (high) foreign demand, offer a large (small) residual supply to domestic investors who set a low (high) price, which in turn reduces the stock of capital. When the technology shocks are known, prices and capital are perfectly correlated to technology shocks but independent from supply shocks. The stocks are riskless and domestic capital flows to equalize expected returns to the riskfree rate in all sectors.

To understand the behavior of output, note that shocks to technology and capital are beneficial because output is a convex function of these shocks, so good shocks more than compensate for bad ones[6]. Formally, $Y^m = \exp(a^m z + \beta k^m z)$ so $E(Y^m) = \exp\{E(a^m z + \beta k^m z) + Var(a^m z + \beta k^m z)/2\}$ which increases with $Var(a^m z + \beta k^m z)$. Furthermore, $Var(a^m z + \beta k^m z) = \sigma_a^2 z + 2\beta Cov(a^m z, k^m z) + \beta^2 Var(k^m z)$ is larger the more capital and technology shocks comove and the more capital varies across sectors. In the no-information economy, capital is not related to technology shocks but it does fluctuate across sectors because of the supply shocks. In the perfect-information economy, capital fluctuates though it is immune from supply shocks because it tracks technology shocks perfectly.

Two further remarks will set the stage for the analysis in the next sections. First, output is on average larger in the perfect-information than in the no-information economy, reflecting the usefulness of information

. Second, both equilibria are invariant to changes in investors' income. The price of a stock is obtained by equating domestic investors' demand to the residual supply, i.e. to the number of shares outstanding net of foreigners' demand. Portfolio weights are (approximately) proportional to relative risk tolerance so domestic investors' demand grows in proportion to absolute risk tolerance. So does the residual supply by assumption. Therefore, the same prices and capital allocation obtain for all levels risk tolerance. We shall see that this implication no longer holds when information can be acquired. We are now ready for the analysis of an imperfect-information economy. We start with the allocation of capital.

4.2 Capital allocation

Theorem 2 *(Capital allocation)*

Assume the information decisions have been made, i.e. the average precision in the economy, X, is given. There exists a log-linear rational expectations equilibrium.

- *The allocation of capital to sector m equals $K^m = \exp(k^m z)$ where*

$$k^m \equiv k^m(X; a^m, \theta^m) = k_0(X) + k_a(X)\left(a^m - \frac{\theta^m}{X}\right) \tag{5}$$

[6] Harberger (1998) documents that the dispersion in productivity at the industry level in the U.S. is large. A small number of high-productivity industries compensates for many low-productivity ones and accounts for the bulk of aggregate TFP.

$$k_0(X) \equiv \frac{1}{1-\beta}\left\{\frac{1}{h(X)}\left(\frac{XE(\theta)}{\sigma_\theta^2}+\frac{1}{2}\right)-r^f\right\} \tag{6}$$

$$h(X) \equiv \frac{1}{\sigma_a^2}+\frac{X^2}{\sigma_\theta^2}+X \qquad \text{and} \qquad k_a(X) \equiv \frac{1}{1-\beta}\left(1-\frac{1}{\sigma_a^2 h(X)}\right) \geq 0 \tag{7}$$

- *Capital is distributed across sectors with mean and variance given by*

$$E(K^m) = \exp\left(E(k^m z)+\frac{1}{2}Var(k^m z)\right) \qquad Var(K^m) = Var(k^m z) = k_a(X)^2\left(\sigma_a^2+\frac{\sigma_\theta^2}{X^2}\right)z \tag{8}$$

$$\text{and} \qquad E(k^m z) = \frac{1}{1-\beta}\left\{\frac{1}{h(X)}\left(\frac{1}{2}-E(\theta)\right)-r^f\right\}z \tag{9}$$

- *The price of stock m equals the amount of capital invested in that sector, $P^m = K^m$.*
- *Investor l who receives a signal s_l^m allocates a fraction of her wealth to stock m such that*

$$f_l^m = \frac{\tau(w)}{w}X\left(s_l^m - \frac{k^m - k_0(X)}{k_a(X)}\right) = \frac{\tau(w)}{w}(X\varepsilon_l^m + \theta^m) \tag{10}$$

The theorem confirms our initial guess that capital and prices are log-linear functions of technology and supply shocks. a^m appears directly in the price function though it is not known by any agent, because individual signals s_l^m, are aggregated and collapse to their mean, a^m. θ^m enters the price equation for stock m although it is independent of a^m because it determines the number of stocks to be held and hence the total risk domestic investors have to bear in equilibrium. P^m and K^m reveal $a^m - \theta^m/X$, a noisy signal for a^m, with error θ^m/X. Thus, $Var[\theta^m/X]z = \sigma_\theta^2/X^2$ measures the noisiness of the price system and its inverse, X^2/σ_θ^2, its informativeness. $h(X) = z/Var_l(a^m z|\, \mathcal{F}_l)$ measures the total precision of an investor's signals. She uses information from three sources: her priors (the $1/\sigma_a^2$ term), her private signal (the X term) and the price (the X^2/σ_θ^2 term), and their precisions simply add up (equation 7). Thus, the equilibrium prices and capital allocation result from the combination of signal extraction and compensation for risk.

The theorem illustrates the roles of information and the stock market. Concerning information, equation 5 shows that capital and technology shocks are positively correlated when $X > 0$. They key parameter is k_a which measures *the elasticity of investments to productivity shocks*, $\partial(\ln K^m)/\partial(\ln A^m)$. A strictly positive k_a means that funds tend to flow to the most productive sectors. Informed investors want to hold more shares of a firm with a high technology shock a^m, which pushes its price and capital stock up relative to a stock with the same supply shock θ^m but lower technology shock. Furthermore, the elasticity increases with the level of information (k_a increases with X). That is, more (less) capital is allocated to high (low) technology-shock sectors in better-informed economies. In particular, the capital allocation corresponds to those indicated in theorem 1 when there is no information ($X = 0$ and $k_a = 0$ so sectors receive capital whatever their technology shock) or when information is perfect ($X = \infty$ and $k_a = 1$ so the capital stock in a sector is proportional to its technology shock). Thus, *better-informed economies allocate capital more efficiently.*

The theorem also highlights the informational function performed by the stock market. This can best be understood by comparison to an economy in which prices do not convey any information. In such an economy, investors' total precision is reduced to $1/\sigma_a^2 + X < h(X)$ and the elasticity of investments to productivity shocks to $[1 - 1/(1 + \sigma_a^2 X)]/(1 - \beta) < k_a(X)$. The allocation of capital is not as efficient though the same private signals were produced. Thanks to the stock market, private signals do not only serve the investors who observe them but benefit all others through prices. Investors who collect private signals of precision X actually receive signals of precision $X + X^2/\sigma_\theta^2$. Thus, *the stock market plays a key informational role by allowing investors to share their information in a credible manner.*

Portfolio weights can be expressed as a weighted average of priors (the $k_0(X)$ term), prices (the k^m term) and private signals (the s_l^m term), or alternatively, as the supply per unit of risk tolerance θ^m, tilted by private-signal errors ε_l^m. Investors can go long ($f_l^m > 0$) or short stocks ($f_l^m < 0$). As in the benchmark economies, portfolio weights do not depend on absolute risk tolerance, given the average precision. But they do depend on the average precision. We shall see in section 5 that richer economies acquire more information so income indirectly affects portfolio weights.

The proof of the theorem proceeds in four steps. First, guess that capital, k^m, is linear in a^m and θ^m. Second, relate the stock return, $\ln R^m$, to a^m and the capital stock in sector m. Because the capital invested equals the value of the single share ($K^m = P^m$), $\ln R^m$ is linear in k^m and a^m. Third, solve the portfolio problem for an investor who observes k^m and s_l^m. The normality assumption leads to a return estimate, $E_l(\ln R^m \mid \mathcal{F}_l)$, linear in k^m and s_l^m and a precision, $h = z/Var_l(\ln R^m \mid \mathcal{F}_l)$, independent of k^m and s_l^m. In addition, the demand for a stock is approximately (at the order 0 in z) equal to $[E_l(\ln R^m \mid \mathcal{F}_l) + Var_l(\ln R^m \mid \mathcal{F}_l)/2 - r^f z]/Var_l(\ln R^m \mid \mathcal{F}_l)$, a linear in $E_l(\ln R^m \mid \mathcal{F}_l)$. Peress (2004) demonstrates the convergence and the accuracy of the approximation. Fourth, the law of large numbers implies that the individual signals add up to their conditional mean, a^m. Therefore, the aggregate demand is linear in a^m and k^m and, equating it to the residual supply $\tau(w)\theta^m$, yields an equilibrium capital allocation linear in a^m and θ^m as guessed. The next theorem describes production.

4.3 Production

Theorem 3 *(Production)*

Assume the information decisions have been made, i.e. the average precision in the economy, X, is given.

- *Output in sector m equals $Y^m = \exp(y^m z)$ where*

$$y^m \equiv y^m(X; a^m, \theta^m) = a^m + \beta k^m = y_0(X) + y_a(X)a^m - y_\theta(X)\theta^m \tag{11}$$

$$y_0(X) \equiv \beta k_0(X) \qquad y_a(X) \equiv 1 + \beta k_a(X) \qquad \textit{and} \qquad y_\theta(X) \equiv \frac{\beta k_a(X)}{X} \tag{12}$$

- *Output is distributed across sectors with mean and variance given by*

$$E(Y^m) = \exp\left(E(y^m z) + \frac{1}{2}Var(y^m z)\right) \qquad Var(Y^m) = Var(y^m z) = y_a(X)^2\sigma_a^2 z + y_\theta(X)^2\sigma_\theta^2 z \tag{13}$$

$$\textit{and} \qquad E(y^m z) = \beta E(k^m z) \tag{14}$$

In the previous section, we emphasized the stock market's role in raising funds and directing them to the best sectors. The theorem shows how investments translate into greater aggregate output. Output in sector m, $Y^m = \exp(y^m z)$, is given by equations 11 and 12. It derives from the production function, $y^m = \beta k^m + a^m$, in which we plug the formula for k^m (equation 5).

There are two channels through which information enhances output. First and most important, a given stock of capital is more efficiently distributed across sectors. Second, more capital is invested in risky technologies overall, at the expense of the riskless asset. To isolate the first channel which is the focus of the paper, we can estimate an economy-wide production function (which is more accurate when the number of sectors is large):

$$E(Y^m) = E(A^m)[E(K^m)]^\beta \exp\{\beta[Cov(a^m z, k^m z) - (1-\beta)Var(k^m z)/2]\}. \tag{15}$$

The term $E(A^m)\exp\{\beta[Cov(a^m z, k^m z) - (1-\beta)Var(k^m z)/2]\}$ is known in the growth literature as the "Solow residual" or "total factor productivity" (TFP). It encompasses any factor, beyond labor and capital, that contributes to output. We focus here on its endogenous component, $Cov(a^m z, k^m z) - (1-\beta)Var(k^m z)/2$. The first term, $Cov(a^m z, k^m z) = k_a(X)\sigma_a^2 z \geq 0$ (equation 5), reflects the more efficient distribution of capital, which better-informed economies can achieve. It increases with the average precision X, starting from 0 in the no-information economy (capital is distributed independently from technology shocks) and rising up to $\sigma_a^2 z/(1-\beta)^2$ in the perfect-information economy (capital is perfectly correlated to technology shocks).

The second term, $(1-\beta)Var(k^m z)/2$, reflects the tempering role played by the declining marginal product of capital ($\beta < 1$). This role can best be illustrated in the perfect-information economy. In this economy, it would not be optimal to assign the entire capital stock to the best sector as the last units of capital would be more productive in the second-best sector. Thus, efficiency requires a balance between favoring the most productive sectors and investing evenly across sectors. We show that $Cov(a^m z, k^m z) - (1-\beta)Var(k^m z)/2$ increases with the average precision X, from $-(\sigma_a^2)^2\sigma_\theta^2 z/2$ in the no-information economy to $\sigma_a^2 z/[2(1-\beta)]$ in the perfect-information economy. Hence, *the availability of information expands the production possibility set. The more informed the economy (greater X), the further away the production possibility set is expanded. This expansion occurs thanks to a more efficient allocation of capital but is limited by its declining marginal product.* The next section describes the behavior of the real economy as information improves.

4.4 The real sector

We relate several characteristics of the real sector to the average precision (we will examine the financial sector in the next section).

Theorem 4 *(The impact of information on the real sector)*

Suppose information improves (X increases).

(i) Investments are more sensitive to productivity shocks ($dk_a/dX \geq 0$).

(ii) Total factor productivity increases ($d[Cov(a^m z, k^m z) - (1-\beta)Var(k^m z)/2]/dX \geq 0$).

(iii) Assume that $(1-\beta)\left(E(\theta) - 1/2\right) \geq 1/3$ *and* $\sigma_a^2 \sigma_\theta^2 \leq 5/2$*. More capital is allocated to risky technologies overall* ($dE(K^m)/dX \geq 0$).

(iv) Assume that $(1-\beta)\left(E(\theta) + 1/2\right)/\beta \geq 1/3$ *and* $\sigma_a^2 \sigma_\theta^2 \leq 5/2$*. GDP is larger on average* ($dE(Y^m)/dX \geq 0$).

(v) Assume that $\sigma_a^2 \sigma_\theta^2 \leq 5/2$ *and* $\beta \leq 3/4$*. Output is more concentrated across sectors* ($dVar(Y^m)/dX \geq 0$).

(vi) Assume that $\sigma_a^2 \sigma_\theta^2 > 1$*. The real sector's sensitivity to foreign shocks declines* ($d|\partial \ln Y^m / \partial[(1-\tau(w)\theta^m]|/dX < 0$).

Note that the conditions on the parameters are merely sufficient but not necessary for the properties to obtain. Capital is more efficiently allocated when the best (worst) sectors are more accurately identified as investors channel more (less) funds to the best (worst) sectors. Investments are more responsive to technology shocks (i). This translates into larger TFP (i) and GDP (ii). Economic activity is more concentrated as the best sectors attract a larger fraction of capital and account for a larger share of GDP (ii and iii). More resources are diverted towards risky technologies overall away from the riskless because better-informed investors face a more favorable risk-return trade-off (ii and iii).

We measure the sensitivity of output to foreign shocks as $|\partial \ln Y^m / \partial[(1-\tau(w)\theta^m]| = y_\theta(X)/\tau(w)$. It captures the percentage increase in sector-m output corresponding to a one unit increase in foreigners' demand, everything else equal.

5 Information acquisition and the financial sector

5.1 Information acquisition

Theorem 5 *(Information acquisition)*

- *Agents with wage w collect information if and only if their wage exceeds a threshold w^* where*

$$w^* \equiv 2C'(0)/\left\{\sigma_a^2 + (\sigma_a^2)^2[\sigma_\theta^2 + E(\theta)^2 - E(\theta)]\right\} \tag{16}$$

- *In that case, the equilibrium precision per stock, $X \equiv X(w)$, is such that*

$$\frac{C'(X)}{\tau(w)} = \frac{h(X) + X + [\sigma_\theta^2 + E(\theta)^2 - E(\theta)]}{2h(X)^2} \tag{17}$$

The theorem describes how much information agents collect. Its proof consists of two steps. First, given the average precision X, derive the first order condition for an investor's precision, $x(w, X)$. Second, set $X = x(w, X)$ to obtain the equilibrium precision X. The theorem implies that *more information is collected in wealthier economies.* This can be seen on two levels. First, when $w^* > 0$, agents collect information only if they are wealthy enough. Thus, *the stock market only performs its informational role if the economy is sufficiently wealthy.* $w^* > 0$ happens in particular when $E(\theta)$ is large. In that case, agents expect to hold a large number of shares and therefore find information valuable. When $w^* \leq 0$, agents collect information regardless of their income. $w^* \leq 0$ happens in particular when $C'(0) = 0$, i.e. the first piece of information is virtually free[7].

Second, when agents do collect information, the precision of their signals increases with their income. This is illustrated by figure 2. The increasing (decreasing) curve represents the left (right) hand side of equation 17. The equilibrium precision choice is located at their intersection. The picture also shows that the equilibrium precision is larger for wealthier economies (the increasing curve shifts downwards) and is confirmed by figure 3. This property obtains because the benefit from information rises with the scale of investment whereas its cost does not. Indeed, a wealthier economy is willing to bear more risk (absolute risk tolerance rises with income by assumption), and to allocate more capital to each risky technology. This induces it to acquire more information. In that sense, information generates increasing returns with respect to the scale of investment. This property holds in spite of a convex information cost which generates decreasing returns with respect to the signal precision. It is formalized in theorem 6.

We can see from equation 17 how *investors' inability to appropriate the full benefit from their private information limits the production of information.* The amount of information revealed by prices equals X^2/σ_θ^2. If there were no public revelation, this term would vanish from $h(X)$ in both the numerator and the denominator. The right hand side of the equation and hence the equilibrium precision would be larger. Thus, investors limit their collection of private information because it gets partially revealed to others through their trades. But it is precisely because prices reveal information that the stock market is useful in allocating resources. We characterize next the financial sector.

5.2 The financial sector

We consider the following financial variables. Stock-market capitalization is the value of all firms listed on the exchange and equals $E(\sum_{m=1}^{M} P^m) = ME(K^m)$ on average. The ratio of stock market capitalization to GDP,

[7] Alternatively when $C'(0) > 0$, $w^* \leq 0$ if $1/\sigma_a^2 + \sigma_\theta^2 > 1/4$ or if $1/\sigma_a^2 + \sigma_\theta^2 \leq 1/4$ and $E(\theta) \geq 1/2 + \sqrt{1/4 - 1/\sigma_a^2 - \sigma_\theta^2}$.

$(\sum_{m=1}^{M} P^m)/(\sum_{m=1}^{M} Y^m)$, can be approximated with $E(P^m)/E(Y^m)$ when the number of sectors is large. The volume of trade is the average value of shares traded and equals $\sum_{m=1}^{M}(\int_l |f_l^m w|/2 + |\tau(w)\theta^m|/2)$ where the first term captures trades by domestic investors and the second trades by foreigners. The term 1/2 avoids double counting matching buy and sell orders. We attribute by convention the initially issued shares to foreigners. Turnover can be computed indifferently as the ratio of volume of trade to GDP or to market capitalization. We measure liquidity as the resilience of stock prices to uninformative supply shocks, $|\partial \ln Y^m/\partial\theta^m| = y_\theta(X)$. It coincides with the economy's sensitivity to foreign shocks scaled by risk tolerance.

Theorem 6 *(The impact of income on the financial sector)*

Suppose income, w, rises. Then,

(i) More information is collected ($dX/dw \geq 0$, strictly if $w > w^*$).

(ii) Assume that $(1-\beta)E(\theta) \geq 2$ and $\sigma_a^2\sigma_\theta^2 \leq 5/2$. The ratio of stock market capitalization to GDP rises ($d[E(K^m)/E(Y^m)]/dX \geq 0$).

(iii) Expected stock returns and their variance decline.

(iv) The volume of trade and turnover rise.

(v) Assume that $\sigma_a^2\sigma_\theta^2 > 1$. Liquidity rises ($d|\partial \ln Y^m/\partial\theta^m|/dX < 0$).

Expected stock returns and their variance decline. The average excess return shrinks faster than the conditional standard deviation of returns, so their ratio $[E_l(R^m) - R^f]/\sqrt{Var_l(R^m \mid \mathcal{F}_l)}$, the average Sharpe ratio, also declines. Thus the unconditional mean-variance frontier contracts as income rises. Note that this effect obtains in general equilibrium only. The mean-variance frontier perceived by an investor expands as she acquires more information, holding fixed the average precision. The volume of trade and turnover increase because better-informed investors make larger trades. This effect outweighs the reduction in the dispersion of beliefs which occurs as information improves. Finally, liquidity rises, i.e. output is less sensitive to uninformative foreign supply shocks.

6 Conclusion

We analyze the allocative role of the stock market in a multi-sector production economy. Output in each sector is determined by a Cobb-Douglas production function and subject to log-normal productivity shocks. Investors allocate capital across sectors and to an information technology that allows them to learn privately about sectoral shocks. Stock prices provide signals that guide investors in their allocation but depress their incentives to collect information. We show that wealthier economies are better informed and allocate capital more efficiently across sectors. The improved capital allocation leads to larger total factor productivity, GDP and concentration of economic activity. The real and financial sectors are positively associated. These properties are consistent with the evidence.

A Proof of theorem 1 (Benchmark economies)

See the proof of theorem 2 and set $X = 0$ for the no-information economy and $X = \infty$ for the perfect-information economy.

B Proof of theorem 2 (Capital allocation)

The proof of theorem 1 builds on Peress (2004). We guess that the equilibrium capital allocation is given by equations 5 to 7 and solve for an investor's optimal portfolio by driving z toward zero (recall that precision choices are taken as given at this stage). The first step is to relate stock payoffs to the technology shocks and capital.

- Stock payoffs

The single share of the firm entitles stockholders to its entire output, $Y^m = A^m (K^m)^\beta$ where capital equals the value of the single share issued, i.e. $K^m = P^m$. Substituting into the production function and taking logs leads to the payoff from holding stock m, $V^m = e^{v^m z}$ where $v^m \equiv a^m + \beta k^m$. The return on stock m, $r^m z = \ln \frac{V^m}{P^m} = v^m z - p^m z = v^m z - k^m z$ where $p^m z \equiv \ln P^m$, is normally distributed. The next step is to estimate the mean and variance of stock returns using the equilibrium prices (or equivalently $\xi^m \equiv a^m - \theta^m / X$) and private signals s_l^m.

- Signal extraction

The average precision in the economy is X and the precision of agent l's signal is x_l^m. We show in the proof of theorem 6 below that chosen precisions are identical across stocks and investors so we write x for x_l^m. For the capital allocation given in equation 5 (k^m is linear in a^m and θ^m), the conditional mean and variance of $a^m z$ are for agent l:

$$Var_l(a^m z \mid \mathcal{F}_l) = \frac{z}{\widehat{h}} \quad \text{and} \quad E_l(a^m z \mid \mathcal{F}_l) = (a_{0l} + a_{\xi l}\xi^m + a_{sl} s_l^m) z$$

$$\text{where} \quad h_0(X) \equiv \frac{1}{\sigma_a^2} + \frac{X^2}{\sigma_\theta^2} \quad \widehat{h} \equiv \widehat{h}(X, x) \equiv h_0(X) + x \quad a_{0l}\widehat{h} \equiv \frac{X E(\theta)}{\sigma_\theta^2} \quad a_{\xi l}\widehat{h} \equiv \frac{X^2}{\sigma_\theta^2} \quad \text{and} \quad a_{s_l}\widehat{h} \equiv x$$

$Var_l(a^m z \mid \mathcal{F}_l)$ falls as the precision of the private signal x (the public signal X^2/σ_θ^2) increase. $E_l(a^m z \mid \mathcal{F}_l)$ is a weighted average of priors, public and private signals where the weight on the private signal (the public signal) is increasing in x (in X). If the investor does not acquire information, set $x = 0$ and s_l^m vanishes from the equations. The conditional mean and variance of stock excess returns follow:

$$E_l(r^m z \mid \mathcal{F}_l) = E_l(a^m z \mid \mathcal{F}_l) - (1 - \beta) k^m z \quad \text{and} \quad Var_l(r^m z \mid \mathcal{F}_l) = Var_l(a^m z \mid \mathcal{F}_l)$$

- Individual portfolio choice

We turn to the portfolio choice. Agent l is endowed with a wage w and chooses her portfolio allocations f_l^m to maximize $E_l\left[U(c_l) \mid \mathcal{F}_l\right]$ subject to $c_l = \left[w - MC(x)z\right] R_l$ where $R_l = R^f + \sum_{m=1}^{M} f_l^m \left(R^m - R^f\right)$ is the return on her portfolio. The first-order conditions for this problem are $E_l\left\{U'(c_l)\left(R^m - R^f\right) \mid \mathcal{F}_l\right\} = 0$ for $m = 1...M$.

Expanding $U'(c_l)$ around w yields $U'(c_l) \approx U'(w) + U''(w)\delta w_l + o(z)$ where $\delta w_l \equiv \left[w - MC(x)z\right] R_l - w = \left[w - MC(x)z\right]\left[R^f + \sum_{m=1}^{M} f_l^m \left(R^m - R^f\right)\right] - w$. $o(z)$ captures terms of order larger than z. Note that $E_l[R^m - R^f \mid \mathcal{F}_l] = E_l[\exp(r^m z) \mid \mathcal{F}_l] - R^f = \exp[E_l(r^m z \mid \mathcal{F}_l) + Var_l(r^m z \mid \mathcal{F}_l)/2] - R^f = E_l(r^m z \mid \mathcal{F}_l) + Var_l(r^m z \mid \mathcal{F}_l)/2 - r^f z + o(z)$ and $E_l\left[\left(R^m - R^f\right)^2 \mid \mathcal{F}_l\right] = Var_l(r^m z \mid \mathcal{F}_l) + o(z)$. Substituting back into the first-order condition, noting that $E_l\left[\left(R^m - R^f\right)\left(R^{m'} - R^f\right) \mid \mathcal{F}_l\right] = 0$ if $m \neq m'$ leads to $U'(w)[E_l(r^m z \mid \mathcal{F}_l) + Var_l(r^m z \mid$

$\mathcal{F}_l)/2 - r^f z] + U''(w) w f_l^m Var_l(r^m z \mid \mathcal{F}_l) = 0 + o(z)$. We solve for f_l^m and insert $\tau(w) \equiv -U'(w)/U''(w)$ to obtain the fraction of wealth allocated to stock m (at the order 0 in z):

$$f_l^m = \frac{\tau(w)}{w} \frac{E_l(r^m z \mid \mathcal{F}_l) - r^f z + Var_l(r^m z \mid \mathcal{F}_l)/2}{Var_l(r^m z \mid \mathcal{F}_l)} \tag{18}$$

Substituting the above expression for $E_l(r^m z \mid \mathcal{F}_l)$ and $Var_l(r^m z \mid \mathcal{F}_l)$ yields:

$$f_l^m = \frac{\tau(w)}{w} \left\{ \frac{X - x}{h(X)} \left(\frac{XE(\theta)}{\sigma_\theta^2} + \frac{1}{2} \right) - \left(\frac{X - x}{\sigma_a^2 h(X)} + x \right) \left(a^m - \frac{\theta^m}{X} \right) + x s_l^m \right\}$$

Setting $x = X$ leads to equation 10. The final step consists in aggregating stock demands and clearing the market.

- Market clearing

We multiply equation 10 by investors' income w and aggregate over all investors to obtain the aggregate demand for stock m at the order 0 in z:

$$\begin{aligned}\int_l \frac{f_l^m w}{P} &= \int_l \tau(w) \left\{ \frac{X - x}{h(X)} \left(\frac{XE(\theta)}{\sigma_\theta^2} + \frac{1}{2} \right) - \left(\frac{X - x}{\sigma_a^2 h(X)} + x \right) \xi^m + x s_l^m \right\} \\ &= \tau(w) \left\{ -X\xi^m + X a^m + \int_l x \varepsilon_l^m \right\}\end{aligned}$$

since $\int_l x_l^m = X$ and $\int_l x_l a^m = a^m \int_l x_l^m = a^m X$. Applying the law of large numbers to the sequence $\{x_l^m \varepsilon_l^m\}$ of independent random variables with the same mean 0 (conditional on a^m) leads to $\int_l x_l^m \varepsilon_l^m = 0$ (see He and Wang (1995) for more details). Finally, the market clearing condition for stock m is $\int_l f_l^m w + [1 - \tau(w)\theta^m] = 1$ where the left hand side is the aggregate demand for the stock (the term in parenthesis is the foreign demand) and the right hand side is the aggregate supply (recall that the number of shares issued is normalized to 1). Plugging in the expression for the domestic demand yields the equilibrium prices and capital allocation given by theorem 2. They are linear in a^m and θ^m as guessed.

C Proof of theorem 3 (Production)

The proof is a straightforward consequence of theorem 2 since $y^m = a^m + \beta k^m$.

D Proof of theorem 4 (The impact of information on the real sector)

(i) Investment elasticity: $k_a \equiv \frac{1}{1-\beta}(1 - \dfrac{1}{\sigma_a^2 h(X)})$ and $Cov(a^m, k^m) = k_a \sigma_a^2$. Therefore, $dk_a/dX > 0$ and $dCov(a^m, k^m)/dX = \frac{1}{1-\beta}(1 + \frac{2X}{\sigma_\theta^2})/h(X)^2 > 0$.

(ii) TFP: $Var(k^m z) = k_a^2(\sigma_a^2 + \frac{\sigma_\theta^2}{X^2})z$ and TFP is proportional to $Cov(a^m z, k^m z) - (1-\beta)Var(k^m z)/2$. Differentiating these expressions yields $dVar(k^m z)/dX = z2(1/\sigma_a^2 - X)(1 + X/\sigma_\theta^2)/[h(X)^3(1-\beta)^2]$ and $d[Cov(a^m z, k^m z) - (1-\beta)Var(k^m z)/2]/dX = z\frac{\beta}{1-\beta}[h(X)(1 + \frac{2X}{\sigma_\theta^2}) - (1/\sigma_a^2 - X)(1 + X/\sigma_\theta^2)]/h(X)^3 = z\frac{\beta}{1-\beta}[h(X)\frac{X}{\sigma_\theta^2} + (2X + \frac{\sigma_\theta^2}{X^2})(1 + \frac{X}{\sigma_\theta^2})]/h(X)^3 > 0$.

(iii) Average capital stock: $E(k^m z) = z[1/2 - E(\theta)]/h(X) - r^f z$ and $E(K^m) = \exp\left(E(k^m z) + \frac{1}{2}Var(k^m z)\right)$. Hence, $dE(K^m)/dX = E(K^m)\exp(u_k z)$ where $u_k z \equiv dE(k^m z)/dX + \frac{1}{2}dVar(k^m z)/dX = z[(1-\beta)(E(\theta) - 1/2)(1 + 2X/\sigma_\theta^2)h(X) + (1/\sigma_a^2 - X)(1 + X/\sigma_\theta^2)]/[h(X)^3(1-\beta)^2]$. The numerator can be written as a polynomial in X whose coefficients are positive if $(1-\beta)(E(\theta) - 1/2) \geq 1/3$ and $\sigma_a^2 \sigma_\theta^2 \leq 5/2$.

(iv) and (v) Variance and average of output: $E(Y^m) = \exp\left(E(y^m z) + \frac{1}{2}Var(y^m z)\right)$ where $E(y^m z) = \beta E(k^m z)$, $Var(y^m z) = \sigma_a^2 z + \beta^2 Var(k^m z) + 2\beta Cov(a^m z, k^m z)$ and $dVar(y^m z)/dX = 2z(\frac{\beta}{1-\beta})^2[(1/\sigma_a^2 - X)(1 + X/\sigma_\theta^2) + \frac{\beta}{1-\beta}h(X)(1 + 2X/\sigma_\theta^2)]/h(X)^3$. The numerator can be written as a polynomial in X whose coefficients are positive if $\sigma_a^2\sigma_\theta^2 \leq 5/2$ and $\beta \leq 3/4$. As for output, $dE(Y^m)/dX = E(Y^m)\exp(u_y z)$ where $u_y z \equiv dE(y^m z)/dX + \frac{1}{2}dVar(y^m z)/dX = z(\frac{\beta}{1-\beta})^2[\frac{1-\beta}{\beta}(E(\theta) + 1/2)(1 + 2X/\sigma_\theta^2)h(X) + (1/\sigma_a^2 - X)(1 + X/\sigma_\theta^2)]/h(X)^3$. The numerator can be written as a polynomial in X whose coefficients are positive if $(1-\beta)\left(E(\theta) + 1/2\right)/\beta \geq 1/3$ and $\sigma_a^2\sigma_\theta^2 \leq 5/2$.

(vi) Sensitivity of investments to foreign shocks: $y_\theta(X)/\tau(w) = \beta k_a(X)/X/\tau(w)$ and $d[y_\theta(X)]/dX = [\frac{1}{\sigma_a^2\sigma_\theta^2} - 1 - \frac{2X}{\sigma_\theta^2} - (\frac{X}{\sigma_\theta^2})^2]/h(X)/(1 + \frac{X}{\sigma_\theta^2})$ which is negative if $\sigma_a^2\sigma_\theta^2 > 1$.

E Proof of theorem 5 (Information acquisition)

We proceed in two steps. First, we find an investor's optimal precision $x = x(X)$ given the average precision in the economy X. Then we solve for the equilibrium precision by equating the two, $X = x(X)$

- Investors' demand for information

To solve the information acquisition problem faced by an investor, we approximate the expected utility of an investor who chooses a signal of precision x_l : $E_l\left[U(c_l) \mid \mathcal{F}_l\right] = U(w) + U'(w)E_l\left[\delta w_l \mid \mathcal{F}_l\right] + U''(w)E_l\left[\delta w_l^2/2 \mid \mathcal{F}_l\right] + o(z)$. Plugging equation 18 into this the expression for δw_l leads to $E_l\left[\delta w_l \mid \mathcal{F}_l\right] = wr^f z - \sum_{m=1}^{M} C(x_l^m)z + \tau(w)\sum_{m=1}^{M}(\lambda_l^m)^2 z + o(z)$ and $E_l\left[\delta w_l^2 \mid \mathcal{F}_l\right] = \tau(w)^2\sum_{m=1}^{M}(\lambda_l^m)^2 z + o(z)$ where

$$\lambda_l^m\sqrt{z} \equiv [E_l(R^m \mid \mathcal{F}_l) - R^f]/\sqrt{Var_l(R^m \mid \mathcal{F}_l)} = f_l^m\sqrt{Var_l(r^m z \mid \mathcal{F}_l)}$$

is investor l's Sharpe ratio on stock m, a function of s_l^m and k^m (and x_l^m). We substitute back these expressions and obtain $E_l\left[U(c_l) \mid \mathcal{F}_l\right] = U(w) + U'(w)[wr^f z - \sum_{m=1}^{M} C(x_l^m)z + \tau(w)\sum_{m=1}^{M}(\lambda_l^m)^2 z/2] + o(z)$. We integrate over all possible values of a^m and θ^m and find the utility an investor expects in the planning stage to achieve in the investment stage:

$$E_l\left[U(c_l)\right] = U(w) + U'(w)\left\{wr^f z - \sum_{m=1}^{M} C(x_l^m)z + \tau(w)\sum_{m=1}^{M} E[(\lambda_l^m)^2]z/2\right\} + o(z)$$

In this expression, $E_l(\lambda_l^{m2})$ is the contribution to expected utility from investing in stock m. It no longer depends on s_l^m nor k^m but it is still a function of the x_l^m:

$$E_l(\lambda_l^{m2}) = \left[h_0(X) + x_l\right]A + \frac{1}{4(h_0(X) + x_l)} + q - 1$$

$$\text{where} \qquad A(X) \equiv \frac{h(X) + X + \sigma_\theta^2}{h(X)^2} + q(X)^2 \qquad \text{and} \quad q(X) \equiv \frac{E(\theta) - 1/2}{h(X)} \tag{19}$$

By symmetry, A and q are identical across stocks so investors choose the same precision across stocks, i.e. $x_l^m = x_l$ for all m. Maximizing $E_l[U(c_l)]$ with respect to x_l taking X (hence A and q) as given, leads to the first order condition:

$$\frac{C'(x_l)}{\tau(w)} = \frac{1}{2}\left[A(X) - \frac{1}{4[h_0(X) + x_l]^2}\right]$$

This condition yields an optimal precision $x = x(X, w)$, identical across investors since they have the same income. We turn to the equilibrium determination of X.

- Equilibrium precision

In equilibrium, $x = X$. We solve for X and show that it increases with wealth. Let $D(X,w) \equiv \frac{C'(X)}{\tau(w)} - \frac{1}{2}\left[A(X) - \frac{1}{4h(X)^2}\right]$. For a given level of wealth w, the equilibrium precision is the root of D. Replacing A with its expression yields $D(X,w) \equiv \frac{C'(X)}{\tau(w)} - [h(X)+X+\sigma_\theta^2+E(\theta)^2-E(\theta)]/[2h(X)^2]$. $D(X,w)$ increases with X (holding w constant) since the first term increases while the second decreases with X. Furthermore, $\lim_{X\longrightarrow\infty} D(X,w) > 0$ because C' is positive and increasing so $\lim_{X\longrightarrow\infty} C'(X) > 0$. Finally, $D(0,w) = C'(0)/\tau(w) - (\sigma_a^2)^2/[1/\sigma_a^2+\sigma_\theta^2+E(\theta)^2 - E(\theta)]/2$. It follows that D admits a root if and only if $D(0,w) \leq 0$ and that it is unique. This happens if $w < w^x$ defined in equation 16. If $C'(0) = 0$, then an interior solution exists whatever the value of w. Finally, a larger w shifts D upwards, leading to a higher root X

F Proof of theorem 6 (The impact of income on the financial sector)

(i) Average precision: see the proof of theorem 5.

(ii) Ratio of stock market capitalization to GDP: the ratio equals $(\sum_{m=1}^{M} P^m)/(\sum_{m=1}^{M} Y^m) \approx E(P^m)/E(Y^m) = \exp\left(E(k^m)z + \frac{1}{2}Var(k^m)z - E(y^m)z - \frac{1}{2}Var(y^m)z\right)$ when the number of sectors is large. Differentiating this expressions yields $\frac{1}{1-\beta}[(1+\beta)(1/\sigma_a^2-X)(1+X/\sigma_\theta^2)+((1-\beta)(E(\theta)+1/2)-1/2+\beta/2)(1+2X/\sigma_\theta^2)h(X)]/h(X)^3$. The numerator can be written as a polynomial in X whose coefficients are positive if $(1-\beta)E(\theta) \geq 2$ and $\sigma_a^2\sigma_\theta^2 \leq 5/2$.

(iii) Stock returns:

(iv) Volume of trade and turnover: the volume of trade equals

$$\sum_{m=1}^{M}\left(\int_l |f_l^m w|/2 + |\tau(w)\theta^m|/2\right) = \frac{Mw}{2\sqrt{2\pi}}\left(\sqrt{\sigma_\theta^2 + X} + \sqrt{\sigma_\theta^2}\right)$$

which increases with X. It is computed at the order 0 in z. GDP and market capitalization are both independent of X at the order 0 so turnover is equal to the volume of trade at the order 0.

(v) Liquidity: see the proof of theorem 4.

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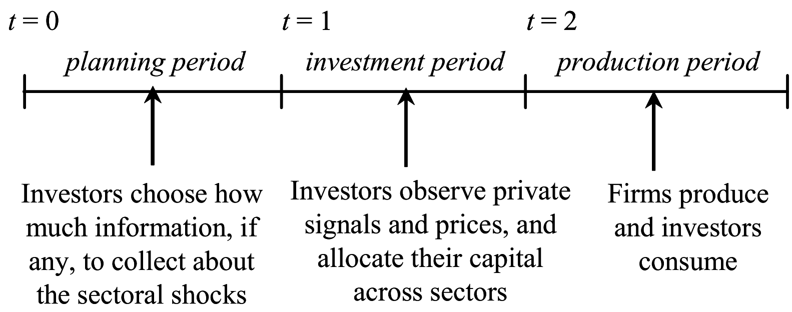

Figure 1: Timing.

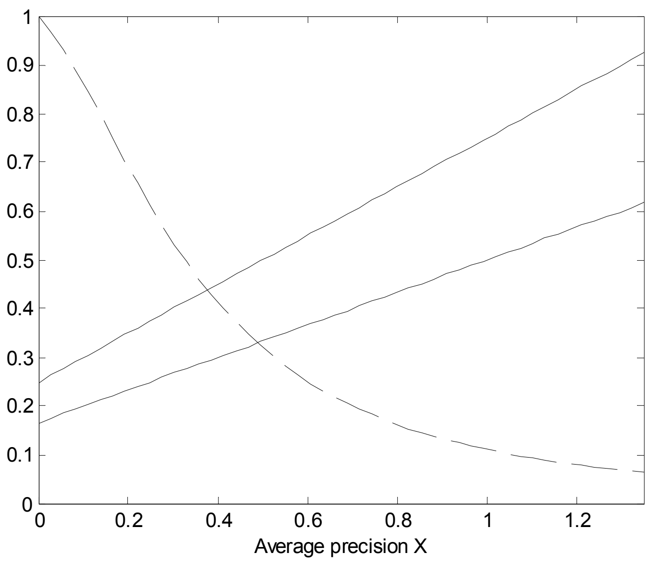

Figure 2: The average precision in equilibrium. The solid curves represent the marginal cost of information (the left hand side of equation 17) and the dashed curve its marginal benefit (the right hand side of the equation). The average precision in equilibrium is located at the intersection of a solid and dashed curve. The top solid curve corresponds to income $w = 2$ and the bottom one to income $w = 3$. If income is below a threshold $w^* = 0.5$ then there is no intersection and the average precision is 0. The other parameters are $\beta = 0.5$, $\sigma_a^2 = 1$, $\sigma_\theta^2 = 1$, $E(\theta) = 1$, $r^f = 0.02$ and $M = 20$.

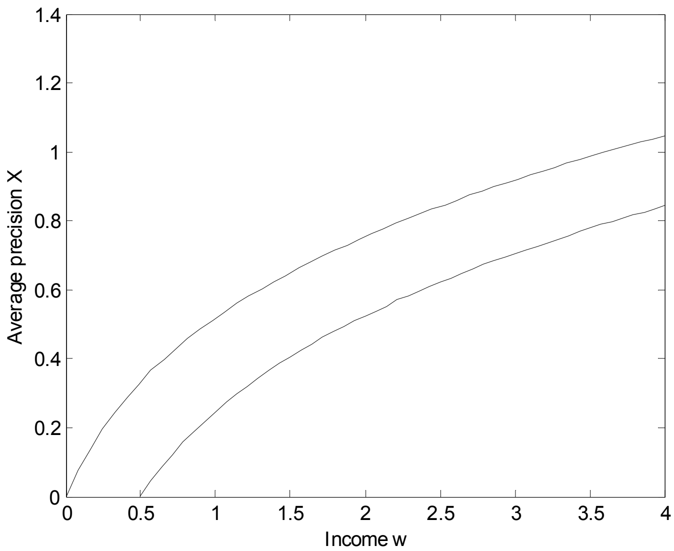

Figure 3: The average precision as a function of income. The top curve corresponds to $C(x) = x^2$ and the bottom curve to $C(x) = x^2 + 0.5x$. In the latter case, no information is collected if income is below a threshold $w^* = 0.5$. The other parameters are $\beta = 0.5$, $\sigma_a^2 = 1$, $\sigma_\theta^2 = 1$, $E(\theta) = 1$, $r^f = 0.02$ and $M = 20$.