Price Improvements in Financial Markets as a Screening Device[1]

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Abstract

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In many security markets, market-makers offer offer to trade at a discount relative to their posted bid and ask quotes. In this article we provide an explanation to this phenomenon. We show that market-makers can mitigate informational asymmetries by selectively offering price improvements to their regular clients. We study a specific type of pricing strategy which consists (a) in offering price improvements to investors who have not repeatedly inflicted trading losses to the marker-maker and (b) in temporarily suspending these discounts otherwise. We find that when a market-maker uses this pricing strategy, there are equilibria in which his clients optimally choose not to contact him when they have private information. These equilibria Pareto-dominate those which are obtained when the market-maker does not or can not make his quotes contingent on his clients' trading histories. Our model predicts that (1) market-makers should grant price improvements to their regular clients but that (2) these improvements should be temporarily suspended after sequences of purchases (sales) followed by price increases (decreases).

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1 Introduction

In most market microstructure models, traders interact anonymously. This view of the trading process is not always adequate. In many trading systems (e.g. the London Stock Exchange), traders engage in one-to-one negotiations and forge long-term trading relationships.[1] What are the benefits of these trading venues relative to more anonymous trading systems? Benveniste, Marcus and Whilhelm (1992) argue that long-term trading relationships mitigate adverse selection problems because they give dealers the leverage to sanction traders who misrepresent their trading motivation. A critical assumption is that, after a transaction, dealers are able to observe whether their clients were informed or not. This is a strong assumption. In fact Benveniste et al.(1992), p.75, point out that

> *"In reality, the specialist must decide which brokers have cheated using statistical inference from a history of trades. Inference is necessarily imperfect, and some brokers might escape detection."*

In this paper, we explore a model with long term-relationships where dealers never observe their clients' trading motivation. We show that in this case dealers can mitigate adverse selection by using pricing policies which are contingent on their past profits with a given client. We also show that these pricing policies will involve price discounts. Hence our analysis explains why dealers sometimes trade within their posted bid and ask prices. These price improvements frequently occur in dealership markets but are difficult to explain in the standard model of price formation in dealership markets (e.g. Glosten and Milgrom (1985)).[2]

We model long-term trading relationships between an investor and her 'regular' dealer as a repeated game. In each period, the investor either receives private information regarding the value of a risky security or she suffers an endowment shock that can be hedged with this security. The investor must then decide whether to trade with her regular dealer or not. If he is contacted, the dealer selects one of two prices, a 'good' price or a 'bad' price, depending on the profitability of his past transactions with the investor. The good price

[1] Other prominent examples include the Nasdaq or the upstairs market in the NYSE.

[2] Price improvements on the London Stock Exchange or on the Nasdaq have been documented by Reiss and Werner (1993) and Huang and Stoll (1996), respectively. For instance Huang and Stoll (1996) find that 45.1% of the large trades receive price improvements on Nasdaq. Price improvements have also been documented for floor markets such as the NYSE and the Frankfurt Stock Exchange (see Petersen and Fialkowski (1994) and Theissen (2000)).

improves upon the price that the investor could obtain without patronizing the dealer. The bad price does not offer such an improvement. It is charged when the dealer's cumulative losses with the investor are larger than a threshold determined by the dealer.[3]

This pricing policy confronts the investor with the following dilemma when she possesses private information. If she exploits her information by trading against her regular dealer, she earns a large return at the dealer's expense. But the dealer records a loss. Thus informed trading increases the likelihood that the investor will be denied price improvements in the future. This would be of no concern to the investor if he were *always* informed. But this is not the case. Sometimes the investor needs to hedge and risk-sharing is more efficient when the dealer offers good prices. Thus the investor impairs the value of the ongoing relationship with her regular dealer if she rips him off. If she refrains from trading, the investor loses a profitable trading opportunity but she maintains their relationship in 'good stand'. To sum up, the investor trades off the loss in the value of her relationship with the dealer with the immediate gain from informed trading.

We show that there are values of the parameters for which the investor is better off not trading with her regular dealer when she is informed. For these parameters, there exist 'cooperative' equilibria in which the dealer establishes a virtuous relationship with the investor. The latter does not trade with her relationship dealer when she has information in order to secure future price improvements. In turn, offering price improvements is a viable pricing policy for the dealer since the investor does not conduct informed trades. Furthermore, price improvements reduce the investor's hedging costs. Accordingly, the allocation of risk between the investor and her regular dealer is more efficient in cooperative equilibria. Hence the dealer's pricing policy results in a welfare gain for the two parties.

The dealer makes his prices contingent on his trading profits with a given investor because these profits are indicators of the investor's behavior. Intuitively, trading losses are more frequent when the investor exploits her private information. It is important to realize, however, that the dealer will also experience losses when the investor does not exploit private information. Thus the dealer will sometimes refuse price improvements, even if the investor refrains from trading when he is informed. This has two implications. First, in cooperative equilibria, there are *oscillations* in the price improvement received by a given trader. This is a testable implication.[4] Second, the pricing policy analyzed

[3] There are anecdotal evidences that dealers use pricing strategies which depend on the profitability of their past transactions with a given client. For instance, Smith (1985) describes the dealers' behavior in the upstairs market as follows: 'Dealers try to keep track of who bags them [...]. They try to maintain a ratio of \$6 in commissions for every \$1 in trading losses generated by a specific customer. [...] if the ratio drops to 3-to-1, someone from the firm may have to chat with the customer'.

[4] According to the model, the switch from 'good' to 'bad' prices should occur when the dealer has lost a large amount of money with the investor.

in this paper does not always discipline the investor. Actually, if penalty phases occur too frequently, the value of maintaining the relationship in good stand becomes small compared to the one shot gain an informed investor can obtain by ripping her dealer off. In such cases, the investor cannot be deterred from exploiting her private information. This problem is more likely to occur for traders with a small probability of being informed. Actually, in this case, the probability distributions of the dealer's profits are not markedly different when the investor exploits her private information and when she does not. This means that past profits are poor indicators of the investor's unobserved behavior.

As usual in models of adverse selection, our model features market breakdowns (no trading in equilibrium). We show that the set of parameters for which a market breakdown occurs is smaller when the dealer uses a profit-contingent pricing strategy than when he does not. This vindicates the view that trading venues in which investors can develop long-term relationships facilitate trades that could not otherwise occur.[5]

Two recent papers (Rhodes-Kropf (1998) and Bernhardt et al.(1999)) have also provided theories which explain price improvements. Rhodes-Kropf (1998) considers a static model in which dealers post non-competitive prices. Hence they can offer price improvements, without making losses, to those traders who negotiate trades. The fraction of investors with the ability to negotiate trades and their bargaining power are assumed to be exogenous. Bernhardt et al.(1999) endogenizes these assumptions by considering repeated relationships between a dealer and his clients. Their central result is that more regular traders obtain larger price improvements. Asymmetric information is a crucial ingredient of the present article whereas it is not in Rhodes-Kropf (1998) and Bernhardt et al.(1999).[6] Price improvements are part of the incentive mechanism which deters investors from using their private information in our model. A main prediction of Bernhardt et al.(1999) is that price improvements should be positively correlated with trade sizes.[7] In our setting, the investor trades in larger size when she obtains a price improvement. Thus, our model would also predict a positive correlation between trade sizes and price improvements. The prediction that price improvements, for a given trader, should oscillate over time is unique to our model, however. A test of this prediction could therefore be used to distinguish empirically our explanation for price improvements from previous explanations.

[5] For instance, Madhavan (2000), page 31, notices that 'The upstairs market major role may be to enable transactions that would not otherwise occur in the downstairs market'.

[6] In Bernhard et al. (1999), investors and dealers have symmetric information. Rhodes-Kropf (1998) considers the case of asymmetric information but he assumes that informed investors can not negotiate price improvements.

[7] This positive correlation is indeed observed on the London Stock Exchange. See Reiss and Werner (1993).

Finally our analysis is related to models of repeated games in which players' actions are imperfectly observed (see e.g. Green and Porter (1984), Radner (1985) and Abreu et al. (1990)). To our knowledge, this literature has not addressed yet the case in which the repetition of the interactions between two players can be used to solve an adverse selection problem, as it is the case in this paper.

The paper is organized as follows. In the next section, we describe the model. Section 3 analyzes the benchmark case of short-term (or anonymous) trading relationships. Section 4 considers the case in which the dealer and the investor are engaged in long-term trading relationships. Section 5 discusses competition among relationship dealers. Section 6 concludes. All the proofs are in the Appendix.

2 The Model

The investor

We consider a risk averse investor and a risk neutral dealer (henceforth the relationship or regular dealer) who are engaged in long-term trading relationships. The investor can be thought as an institution. On the Nasdaq for instance, institutional investors directly engage in long-term relationships with dealers and in fact receive the bulk of price improvements for large orders (see Huang and Stoll (1996) and Smith et al. (1998)).

The investor contacts her relationship dealer when she has an opportunity to trade a risky security, either because she has private information on the value of the security or because she needs to hedge. These trading opportunities arise at dates $0, 1, 2, ...t, ..., ad \quad inf$. We call a 'period', the interval of time between two dates. At the end of each period, the risky security is worth $\tilde{v}_1 = v_0 + \tilde{\epsilon}$ where $\epsilon = +1$ or $\epsilon = -1$ with probabilities $\lambda_U = \lambda_D = \frac{\mu}{2} > 0$ or $\epsilon = 0$ with probability $\lambda_0 = 1 - \mu$ (so that $E(v_1) = v_0$). The parameter μ measures the asset volatility since $Var(\tilde{v}_1) = \mu$. The investor can also invest in a riskless asset. The (intra period) risk free rate is set to zero, for simplicity.

At the beginning of each period, with probability $\alpha > 0$, the investor receives perfect information on the final value of the security. With probability $(1 - \alpha)$, the investor has no private information but she receives a risky endowment which has a payoff $\tilde{z}$, at the end of the period. The security can be used to hedge the risky endowment since $\tilde{z} = \tilde{h}\tilde{\epsilon}$ where $\tilde{h} = +Q$ or $\tilde{h} = -Q$ with equal probabilities. If h is positive (negative) the investor must sell the security in order to reduce her risk exposure and she is perfectly hedged if she sells (buys) Q shares. The investor privately learns the direction of her hedging need (h) at the beginning of each period. After learning the direction of the hedging need or receiving information, the investor can trade the risky security.

We assume that the investor starts each period with a constant endowment W_0 in the riskless asset. Furthermore we assume that the investor entirely consumes her wealth at the end of each period (no savings from one period to the next). The investor has a concave utility function, which is

$$U(W) = \gamma(W - W_0) \quad for \quad W > W_0,$$

$$U(W) = W - W_0 \quad for \quad W \leq W_0.$$

with $\gamma \in (0,1)$. Notice that the *lower* is γ, the larger is the investor's risk aversion.[8] For simplicity and without affecting the results, we normalize W_0 to zero.

To sum up, in each period, the investor may trade either to hedge or to benefit from her private information (as in Glosten (1989) for instance).[9] More precisely she has one of the five following *types* or *trading motives*

1. Positive hedging need, θ_1.
2. Negative hedging need, θ_2.
3. Informed that $\epsilon = -1$, θ_3.
4. Informed that $\epsilon = +1$, θ_4.
5. Informed that $\epsilon = 0$, θ_5.

The dealer is unable to observe the investor's trading motive, in any period and statements regarding this motive can not be verified.

The trading process

Once she knows her type, the investor decides to trade or not with her regular dealer. We model the trading process as a two stages game (the 'trading game').

In the first stage, the investor chooses the direction and the size of her trade. We denote by q the investor's order with the convention that $q > 0$ ($q < 0$) if the investor buys (sells) the security and $q = 0$ if the investor abstains from contacting her relationship dealer. The

[8] Dow (1998) uses a similar specification for investors' preferences.

[9] We exclude the case in which the investor would have both information and hedging needs. As argued in Seppi (1990), this case is not likely to occur in practice.

investor's trading strategy, $q(\theta, H)$, depends on her type and her trading history (H) with the dealer. If the dealer is contacted by the investor, he makes an offer, that is a price p at which he is willing to accommodate the investor's order. We refer to $p(q, H)$ as the dealer's bidding strategy. The dealer's offer depends on his belief regarding the investor's type. We denote by $\phi(q, H)$ the dealer's posterior probability distribution for the investor's type when she chooses to trade q and their trading history is H. The dealer's expected profit when the investor trades q shares is

$$\pi(p(q, H), q) = E_{\phi}(q(p(q, H) - \tilde{v}) \mid q, H).$$

The trading game has similarities with Glosten and Milgrom (1985)'s model. There are two significant differences, however. First the investor's trade size is endogenous (i.e. not restricted to a fixed number of shares). Second, the price charged to the investor can depend on her trading history. These two features play an important role in our analysis.

Outside Dealers.

There are other dealers (the outside dealers) with whom the investor has no established relationships. These dealers do not observe the investor's trading history. We assume that the investor can contact these dealers but that in this case, her relationship with her regular dealer is terminated. We discuss this assumption in details in Section 5. Figure 1 summarizes the sequence of events in each period.

3 Short-Term Trading Relationships

Market microstructure models have focused on dealers' pricing strategies in absence of enduring relationships. In this section, we analyze the equilibria which arise in such an environment. We start by considering (a) the case in which the investor can not commit on a trading strategy and (b) the case in which she can. The results provide useful benchmarks to assess the role of long-term trading relationships. They are also building blocks to solve for the equilibria in this case.

3.1 The equilibria with short-term trading relationships

All the dealers have the same information regarding the investor since she has no privileged relationship with a specific dealer. In this case, Bertrand competition among the dealers implies that they will charge a price such that they obtain a zero expected profit. Hence, when the investor announces that she wants to trade q shares, the dealers charge a price which satisfies

$$p(q) = E_{\phi}(\tilde{v} \mid q).$$

We focus on trading strategies and bidding strategies that form a Perfect Bayesian Equilibrium (PBE) of the trading game.[10] A PBE is a vector $(q^*(.), p^*(.), \phi^*(.))$ such that

1. The trading strategy $q^*(.)$ maximizes the investor's expected utility given the dealer's bidding strategy, $p^*(.)$.
2. The bidding strategy $p^*(.)$ is such that $p^*(q) = E_{\phi^*}(\tilde{v} \mid q)$.
3. The dealer's posterior belief $\phi^*(.)$ is derived from the investor's trading strategy using Bayes rule where possible.

Lemma 1 : *With short-term trading relationships, all the PBE of the trading game have the following properties:*

- *The investor buys the same amount, q^{buy}, of the security when she has type θ_2 or θ_4, i.e. $q^*(\theta) = q^{buy} \geq 0 \quad if \quad \theta \in \{\theta_2, \theta_4\}$.*
- *The investor sells the same amount, q^{sell}, of the security when she has type θ_3 or θ_4, i.e. $q^*(\theta) = -q^{sell} \leq 0 \quad if \quad \theta \in \{\theta_1, \theta_3\}$.*
- *The investor does not trade when she knows that there will be no change in the asset value, i.e. $q^*(\theta_5) = 0$.*
- *In the equilibria for which $q^{buy} > 0$, the dealers charge a price equal to $p(q^{buy}) = v_0 + s^{nc}$ when they receive the equilibrium buy order. In the equilibria for which $q^{sell} > 0$ the dealers charge a price equal to $p(-q^{sell}) = v_0 - s^{nc}$ when they receive the equilibrium sell order, with $s^{nc}(\alpha, \mu) \stackrel{def}{=} \frac{\alpha\mu}{\alpha\mu + (1-\alpha)}$.*

In line with intuition, the investor with type θ_1 (θ_2) sells (buys) the security in order to decrease her risk exposure. In order to avoid detection, an informed investor with bad (good) information mimics the behavior of an investor with type θ_1 by selling the same quantity. This creates adverse selection. In order to break-even, the dealers must price the security at a discount (a markup) relative to the security unconditional expected value

[10] This is the relevant equilibrium concept since the trading game is a signaling game. We just consider equilibria in pure strategies.

when the investor chooses to sell (buy) the security. This wedge, s^{nc}, can be interpreted as the 'spread' charged by the dealers. If the investor is informed that the innovation in the asset value is zero, she is left with no better choice than not trading. The investor's trade size may vary across equilibria (see the discussion below). However, the ask and bid prices posted by the dealers are always the same.

Proposition 1 *: When $s^{nc}(\alpha,\mu) \geq \bar{s}(\gamma,\mu)$, the unique equilibrium of the trading game is such that the investor never trades, i.e. $q^*(\theta) = 0, \forall\theta$, with $\bar{s}(\gamma,\mu) = \frac{(1-\gamma)\mu}{2-(1-\gamma)\mu} < 1$.*

Proposition 1 describes a market breakdown. This market breakdown occurs when the cost of hedging (s^{nc}) is large enough, so that the investor is better off not trading. The next corollary rewrites the condition under which a market breakdown occurs in terms of the exogenous parameters.[11]

Corollary 1 *: For a given risk aversion γ and a given probability α, a market breakdown occurs if $\mu \leq \mu^{nc}(\alpha,\gamma)$, where $\mu^{nc}(\alpha,\gamma) \overset{def}{=} \frac{\gamma}{1-\gamma} + \frac{3}{2} - \frac{1}{2\alpha}$.*

Figure 2 illustrates the corollary, for a fixed value of γ. As γ increases, the curve which separates the two areas (trading/no trading) shifts to the left. Hence a market breakdown occurs when (i) the probability of an informed trade is large or (ii) the investor's hedging need is weak because the asset is not very volatile or she is not very risk averse (μ small or γ large). In these cases, the utility gain to hedging is small compared to the cost of hedging and the investor is better off not trading.

The occurence of a market breakdown is of course not surprising given the adverse selection problem faced by the dealers. Furthermore other authors (e.g. Bhattacharya and Spiegel (1991) or Madhavan (1992)) have developed models where market breakdowns occur under similar conditions.[12] We show below that market breakdowns occur for *a smaller set* of parameters with enduring relationships.

Now consider the following trade size for the investor: $\bar{q} \overset{def}{=} \frac{Q}{1+s^{nc}} = \left[\frac{\mu\alpha+(1-\alpha)}{2\mu\alpha+(1-\alpha)}\right] Q$. The next proposition shows that the trading game has an equilibrium in which the investor trades an amount equal to $\bar{q}$. It also claims that this trading strategy yields the largest ex-ante expected utility to the investor.

[11] This is achieved by substituting s^{nc} and $\bar{s}$ with their expressions in the condition $s^{nc}(\alpha;\mu) \geq \bar{s}(\gamma;\mu)$.

[12] In these models a market breakdown designates a situation in which there are no equilibria. In our model, it refers to a situation in which the equilibrium exists but it features no trading.

Proposition 2 *: When $s^{nc} < \bar{s}(\gamma, \mu)$ (that is $\mu > \mu^{nc}$), the trading game has one PBE in which the investor trades an amount of the security equal to $\bar{q}$. In this equilibrium, the dealers' bidding strategy is*

$$p^{nc}(-\bar{q}) = v_0 - s^{nc},$$

$$p^{nc}(\bar{q}) = v_0 + s^{nc},$$

$$p^{nc}(-q) = v_0 - \infty \quad \forall q \neq \bar{q}, q > 0,$$

$$p^{nc}(q) = v_0 + \infty \quad \forall q \neq \bar{q}, q > 0.$$

The allocation of risk in this equilibrium Pareto-dominates ex-ante all the other possible equilibrium allocations in the equilibria with zero expected profits for the dealers.

When $s^{nc} < \bar{s}(\gamma, \mu)$, the trading game has multiple equilibria. As usual in signaling games, this reflects the absence of restrictions imposed by the PBE concept on the dealer's beliefs, ϕ^*, for trade sizes out-of-the equilibrium path. Among all the equilibria which yields a zero expected profit to the dealer, the investor's ex-ante expected utility is the largest in the equilibrium described in Proposition 2. The intuition for this result is that $\bar{q}$ is the trade size which maximizes the investor's expected utility when (a) she is uninformed and (b) the dealer offers to sell (or buy) the security at price $v_0 + s^{nc}$ (or $v_0 - s^{nc}$). Notice that in equilibrium, the investor does not perfectly hedge her risky endowment ($\bar{q} < Q$) if $\alpha > 0$. Actually, in this case, hedging is costly since the dealer sells (buys) the security at a markup (discount), s^{nc}.

The equilibrium described in Proposition 2 is the investor's favorite equilibrium. Hence, we use the investor's welfare in this equilibrium as a benchmark (see below and Section 4.4). Laffont and Maskin (1990) consider a model with a large informed investor and many small, uninformed, investors. They also focus on the large investor's favorite equilibrium, arguing that *"by virtue of his market power, the large trader ought to be able to influence other traders' beliefs and so ensure a favorable equilibrium for himself"* (Laffont and Maskin (1990), p.80). They provide several justifications for this claim.

To sum up, with short-term trading relationship, the investor's trading strategy in equilibrium is

$$q^{nc}(\theta) = 0, \quad \forall\theta \quad if \quad s^{nc} \geq \bar{s}$$

or, if $s^{nc} < \bar{s}$,

$$q^{nc}(\theta_1) = q^{nc}(\theta_3) = -\bar{q}$$

$$q^{nc}(\theta_2) = q^{nc}(\theta_4) = \bar{q}$$

$$q^{nc}(\theta_5) = 0.$$

3.2 The 'no informed trading' commitment

In order to better understand the benefit of establishing long-term relationships in our model, it is useful to compare the investor's welfare in her favorite equilibrium and her welfare if she could commit ex-ante (before learning her type) to refrain from trading when she is informed (for $\alpha \in (0,1)$). We refer to this commitment as the 'no informed trading commitment'.

Assume that the investor can enter into this commitment with her relationship dealer. Then the dealer can offer to sell (buy) at any price above (below) v_0 without losing money. Let s^c be the relationship dealer's spread. This means that the dealer sells the security at $p^c = v_0 + s^c$ and purchases it at $p^c = v_0 - s^c$. For this spread, the investor's optimal trade size when she is uninformed is $q = \frac{Q}{1+s^c}$ shares if $s^c < \bar{s}(\gamma, \mu)$ and zero otherwise (see the proof of Proposition 1). The trading strategy, $q^c(.)$ to which the investor commits is:

$$\begin{cases} q^c(\theta_1, H) = -\frac{Q}{1+s^c}, \\ q^c(\theta_2, H) = +\frac{Q}{1+s^c}, \\ No \quad Informed \quad Trading: \quad q^c(\theta, H) = 0 \quad \forall\theta \in \{\theta_3, \theta_4, \theta_5\}. \end{cases} \tag{1}$$

Let $\Delta U(s^c)$ be the difference between the ex-ante investor's expected utility with and without commitment, i.e.

$$\Delta U(s^c) = E_\theta\left(\bar{U}(q^c(.), \tilde{\theta})\right) - E_\theta\left(\bar{U}(q^{nc}(.), \tilde{\theta})\right),$$

where $\bar{U}(q(.), \theta)$ is the investor's expected utility when she uses the trading strategy $q(.)$

and her type is θ. The value of $\Delta U(s^c)$ measures the welfare gain generated by a 'no informed trading' commitment.

Proposition 3 *: There exists a spread $s^*(\alpha, \mu, \gamma) > 0$ such that a no-informed trading commitment improves the investor's welfare iff $0 \leq s^c < s^*(\alpha, \mu, \gamma)$ (that is $\Delta U(s^c) > 0$ iff $s^c \in [0, s^*)$). Furthermore:*

$$s^*(\alpha, \mu, \gamma) = \frac{\alpha\mu(1 - \gamma)}{\alpha\mu(1 + \gamma) + (1 - \alpha)} < s^{nc} \quad if \quad \mu > \mu^{nc},$$

$$s^*(\alpha, \mu, \gamma) = \bar{s}(\gamma, \mu) \quad if \quad \mu \leq \mu^{nc}.$$

Consider the case in which the parameters are such that $\mu > \mu^{nc}$ (no market breakdown). The ability to conduct informed trades is a double-edged sword for the investor. On the one hand, informed trading is a source of profit at the expense of the dealers in the states where the investor is informed. On the other hand, the risk of informed trading forces the dealers to charge a spread. The larger is this spread, the larger is the hedging cost for the investor. Hence the risk that she might be informed prevents the investor from hedging perfectly in the states where she receives a risky endowment. For this reason she is willing to give up the benefit from informed trading if in exchange she can substantially reduce her risk exposure when she has an endowment shock. For this the dealer's price must be smaller than $v_0 + s^*$.

The same intuition applies when there is a market breakdown in absence of a no-informed trading commitment. In this case, the dealer must offer a spread small enough so that the investor's demand for hedging is strictly positive (this requires $s^c < \bar{s}$).

Observe that the relationship dealer gets a per period rent equal to s^c when $s^c > 0$. Therefore there is a set of values for s^c such that the no-informed trading commitment generates a welfare gain both to the investor and the relationship dealer. As $\Delta U(s^c)$ decreases with s^c, the investor and the relationship dealer have conflicting views regarding the spread charged by the relationship dealer.

Unfortunately the no-informed trading commitment is not credible. If the relationship dealer naively trusts the investor, the latter is better off trading when she receives private information.[13] Furthermore it cannot be enforced with ex-post sanctions based on the investor's trading motivation (as in Benveniste et al. (1992) for instance) since this

[13] For instance, when she receives good information, the investor buys $\frac{Q}{1+s^c}$ shares at price $v_0 + s^c$ and she obtains a per share profit equal to $(1 - s^c) > (1 - s^*) > 0$.

motivation cannot be observed. In the next section we describe a mechanism by which the relationship dealer and the investor can overcome this problem and appropriate part of the mutual welfare gains that obtain with a no-informed trading commitment.[14]

4 Long Term Trading Relationships

At the end of each period, there is a fixed probability $(1 - \beta)$ that the trading game stops forever. Larger βs' are associated with relationships that last longer. After each encounter, the relationship dealer observes the direction of the last transaction and the innovation in the asset value, that is the pair $y = (i, \epsilon)$ where $i = +1$ (resp. $i = -1$) when the investor buys (sells) the security. Thus the public *trading history* after τ encounters is

$$H_\tau = \{y_0, ..., y_k, ..., y_{\tau-1}\}.$$

The investor's trading policy is the sequence of trading strategies used by the investor, that is $q = \{q_0, q_1, ..., q_\tau, ...\}$. The investor's *expected payoff* at date τ is

$$V(\theta_\tau, H_\tau) = \bar{U}(q_\tau(\theta_\tau, H_\tau)) + \sum_{n=\tau+1}^{\infty} \beta^t E(\bar{U}(q_n(\tilde{\theta}_n, \tilde{H}_n)) \mid \theta_\tau, H_\tau). \tag{2}$$

At each date, the investor chooses her trading strategy so as to maximize the value of the trading relationship, $V(.)$. The dealer's pricing policy p is the sequence of bidding strategies used by the relationship dealer ($p = \{p_0(.,.), p_1(.,.), ..., p_\tau(.,.), ...\}$). The dealer total expected profit at date τ when the investor wants to trade q_τ shares at this date is

$$\Pi(q_\tau, H_\tau) = \pi(p_\tau(q_\tau, H), q_\tau) + \sum_{n=\tau+1}^{\infty} \beta^t E(\pi(p_n(\tilde{q}_n, \tilde{H}_n), \tilde{q}_n) \mid q_\tau, H_\tau) \tag{3}$$

We focus our attention on perfect equilibria of the repeated game. A perfect equilibrium is a trading policy q^* and a pricing policy p^* such that from any date τ and given any trading history H_τ, **(a)** the trading policy from date τ onward (i.e. $\{q^*_\tau, q^*_{\tau+1}, ...\}$), maximizes the investor's expected payoff, **(b)** the pricing policy from date τ onward maximizes the relationship dealer's total expected profit and **(c)** the dealer's posterior belief at date τ, $\phi^*_\tau(q_\tau, H_\tau)$, is derived from the investor's trading strategy at this date using Bayes rule

[14] Admati and Pfleiderer (1991) consider a model in which uninformed traders can pre-announce the size of their trades (a practice called 'sunshine trading'.) They assume that informed investors can not make similar pre announcements or that they can commit not to make these announcements. In the present article, we explicitly analyze how to enforce this type of commitment.

where possible.

Recall that in each trading round, the investor has the option to start a new relationship with another dealer. For the time being, we take as exogenous the investor's total expected payoff if she switches dealers and starts a new relationship. We consider the worst possible outcome for the investor: if she starts a new relationship, she obtains the same total expected payoff as in the equilibrium with short-term trading relationships. We denote it $\bar{V}^{switch}$. We relax this assumption in Section 5.

The rest of this section is organized as follows. In a first step we define a specific type of pricing policy for the dealer, the 'scoring policy'. This pricing policy involves price improvements. In a second step we show that a scoring policy can induce the investor to optimally refrain from trading when she is informed (at least during some periods). In the last step, we establish that there are equilibria in which the regular dealer uses a scoring policy and the investor refrains from trading when she is informed. We call them cooperative equilibria.

4.1 Incentive Pricing

Consider the following pricing policy for the regular dealer. In some periods, he offers to buy (sell) $\frac{Q}{1+s^c}$ shares at price $p^c = v_0 - s^c$ ($p^c = v_0 + s^c$) with $s^c < s^*$. After each encounter, he assigns a 'score', $S \geq 0$, to the investor. This score depends on the profitability of the trades he conducts with her. When the dealer loses money, he increases the investor' score by 1. This happens when the investor sells the asset and subsequently the asset price decreases ($y = (-1, -1)$) or when the investor buys the asset and subsequently the asset price increases ($y = (+1, +1)$) since $s^c < s^* < 1$. For all the other trades, the dealer earns his posted spread (if the security value is unchanged) or even more (when $y = (-1, +1)$ or $y = (+1, -1)$). For these profitable trades or when there is no trade, he leaves the investor's score unchanged. We refer to the periods in which the dealer uses this bidding strategy as a *cooperative phase*. When the score reaches a specific threshold, S^*, the dealer behaves according to the short-term bidding strategy described in Section 3 for $T \geq 1$ periods. After this *non-cooperative phase*, the relationship dealer starts a new cooperative phase (with $S = 0$ at the beginning).

When the dealer chooses his prices as we just described, we say that he follows *a scoring policy*. This pricing policy is characterized by three parameters: (i) the 'trigger value' of the score (S^*), (ii) the length of the non cooperative phase (T) and (iii) the size of the spread during cooperative phases. We denote $p^{lt}(T, S^*, s^c)$ a specific scoring policy.

A scoring policy formalizes the idea that a dealer can offer prices contingent on his past profits (or losses) when he is engaged in enduring relationships. Here the dealer keeps offering good prices as long as cumulative losses are not too large since the score increases each time the dealer books a loss. The score can be seen as a reputational index. The investor's reputation deteriorates when the score increases. Observe that the spread is smaller during cooperative phases (since $s^{nc} > s^* > s^c$). Hence, in cooperative phases, the investor receives price improvements compared to non cooperative phases. As we will show below, this is crucial. Actually this is the fear of being denied price improvements which induces the investor to behave during cooperative phases.

The rationale for this pricing policy is as follows. The key intuition is that the probability distribution of the dealer's trading profits is in part under the control of the investor. To see this, consider the investor in a cooperative phase. She has two possible choices. On the one hand she can cooperate. This means that she behaves according to the trading strategy $q^c(.)$. In particular she does not exploit her private information against the relationship dealer. On the other hand the investor can act non-cooperatively in which case she always trades (informed or not)[15]. Let $Prob^c(Loss)$ (resp. $Prob^{nc}(Loss)$) be the probability of a trading loss for the dealer if the investor behaves cooperatively (non-cooperatively).[16] Let $r(Loss) = Prob^{nc}(y)/Prob^c(y)$ be the associated likelihood ratio. Computations yield

$$Prob^c(Loss) = \frac{\mu}{2},$$

$$Prob^{nc}(Loss) = \frac{(\alpha+1)\mu}{2(\alpha\mu + (1-\alpha))},$$

so that

$$r(Loss) = \frac{\alpha+1}{\alpha\mu + (1-\alpha)},$$

which is always larger than 1. Hence the distribution of trading profits depends on the investor's behavior. In line with intuition, the likelihood of a trading loss is larger when the investor exploits her private information. For this reason, in order to incentivize the investor, the dealer must 'punish' her after a sequence of trading losses and 'reward' her when her trades are profitable for the dealer.[17] In fact a scoring policy can be seen as an incentive scheme.

[15] Of course when she is informed the investor claims to be uninformed and trades $\frac{Q}{1+s^c}$ shares.
[16] These probabilities are conditional on a trade taking place.
[17] The reward is that the investor's score is unchanged.

Notice that the likelihood ratio $r(Loss)$ decreases with μ and increases with α. This means that past revenues are poor indicators of the investor's behavior when the volatility of the asset is large or when the investor has a small probability of being informed. Consider α for instance. When α is small, the investor is uninformed most of the time. This means that the investor is rarely in the position of being able to 'control' the dealer's trading profits. Hence the frequency of trading losses is not significantly larger when the investor misbehaves. The same problem arises when the probability of large price movements (μ) is high. In this case, the probability of a trading loss for the dealer is large, be the investor informed or not.[18] Thus when μ is large and/or α is small, we expect a scoring policy to be less effective in disciplining the investor.

Remark. The trading history up to date τ does not provide information on the investor's trading motives in the forthcoming periods. Actually, there is no persistence in these motives from one period to the next in our model. Hence Bayesian learning of the trading motive is impossible. The dealer's prices are contingent on the trading history because this disciplines the investor, not because the history contains information on the investor's future behavior. This makes our approach distinct from reputation models à la Kreps and Wilson (for instance Benabou and Laroque (1992)). In these models investors are given by Nature a given type of behavior (cooperative or non cooperative). In our model, the dealer' s pricing strategy *induces* investors to be cooperative.

4.2 Cooperative Trading

Now we study the optimal trading policy for the investor when her regular dealer uses a scoring policy. During non-cooperative phases, the investor's reputation does not affect the prices posted by the relationship dealer. Hence, in these phases, it is optimal for the investor to behave as if her relationship with the dealer were short-term (that is she follows the trading strategy $q^{nc}(.)$ defined in Section 3.1).[19] During cooperative phases, the investor can enjoy the benefits of good prices for a long time if she follows the trading strategy $q^{c}(.)$ (the trading strategy associated with a no-informed trading commitment). We derive below the conditions under which this is optimal.

More formaly consider the following trading policy for the investor:

$$q_{\tau}^{lt}(\theta, H) = q^{c}(\theta) \quad in \quad cooperative \quad phases,$$

[18] The probability of a trading loss conditional on the investor being non-informed (resp. informed) is $\mu/2$ (resp. 1).

[19] Observe that if $s^{nc} \geq \bar{s}$, the investor does not trade during the non-cooperative phase.

$$q^{lt}_{\tau}(\theta, H) = q^{nc}(\theta) \quad in \quad non-cooperative \quad phases.$$

This defines the cooperative trading policy, that we denote $q^{lt}(T, S^*, s^c)$.

Let $V(in, S)$ be the investor's expected payoff, in a cooperative phase, when (a) the investor is informed (b) her score is equal to $S \leq S^* - 1$ and (c) she follows the cooperative trading policy.[20] In the same way, let $V(he, S)$ be the investor's expected payoff, in a cooperative phase, when she has a hedging need and she follows the cooperative trading policy. Hence, the ex-ante investor's expected payoff in a cooperative phase when she follows the cooperative trading policy is

$$V(S) = \alpha V(in, S) + (1 - \alpha)V(he, S) \quad for \quad S \leq S^* - 1. \tag{4}$$

In a cooperative phase, when the investor is informed, she refrains from trading with her relationship dealer. She misses a profit opportunity but she maintains her score unchanged. Therefore,

$$V(in, S) = \beta V(S) \quad for \quad S \leq S^* - 1. \tag{5}$$

Now consider the case in which the investor needs to hedge. In the cooperative phase, she trades $\frac{Q}{1+s^c}$ shares and she obtains an expected utility that we denote $\bar{U}^c$ for brevity.[21] Following this transaction, the investor's score may deteriorate if the dealer loses money. The probability of this event is $\frac{\mu}{2}$. Otherwise her score is unchanged. Therefore,

$$V(he, S) = \bar{U}^c + \beta \left(\frac{\mu}{2} V(S+1) + (1 - \frac{\mu}{2})V(S) \right) \quad for \quad S \leq S^* - 1. \tag{6}$$

Let $V(S^*)$ be the investor's expected payoff when the cooperative phase comes to an halt ($S = S^*$). In this case the per period expected utility of the investor is the same as in the equilibrium with short-term trading relationship and it takes T periods before reverting to a cooperative phase. The next equation follows.

$$V(S^*) = \sum_{t=0}^{t=T-1} \beta^t E(\bar{U}(q^{nc}(\tilde{\theta})) + \beta^T V(0) = \frac{(1 - \beta^T)}{(1 - \beta)} E(\bar{U}(q^{nc}(\tilde{\theta}))) + \beta^T V(0). \tag{7}$$

Finally let $V(\tau)$ be the investor's expected payoff during a non-cooperative phase when τ

[20] All the value functions $V(.)$ depend on S^*, T and s^c. We dropped these arguments to simplify the notations. Furthermore the value functions do not depend on the direction of the signal received by the investor or the direction of her hedging need because of the model symmetry.

[21] Using a notation introduced in Section 3, $\bar{U}^c = \bar{U}(q^c(\theta_1); \theta_1) = \bar{U}(q^c(\theta_2); \theta_2)$, where the second equality is due to the model symmetry. Computations yield $\bar{U}^c = \frac{-Q s^c}{1+s^c}$.

periods remain before the end of this . Observe that

$$V(\tau) \geq V(S^*) \geq \bar{V}^{switch},$$

since $\bar{V}^{switch}$ is the investor's total expected payoff in the equilibrium with short-term trading relationships. The next lemma establishes that the investor cares about her reputation.

Lemma 2 *: The value function $V(.)$ decreases with S. Furthermore*

$$V(S+1) - V(S+2) > V(S) - V(S+1) \quad for \quad S \leq S^* - 2.$$

This means that an increase in the investor's score has a negative impact on the value of her relationship with the dealer. This implies that

$$V(0) > V(1) > ... > V(S^*)$$

Hence, since $V(S^*) \geq \bar{V}^{switch}$,it is never optimal for the investor to terminate her relationship with the dealer if she expects him to follow a scoring policy.

In the cooperative phase, when the investor is informed, she faces the following trade-off. If she refrains from contacting her relationship dealer, she misses a profit opportunity but she maintains in good stand her relationship with the dealer (her score is unchanged). If she contacts the dealer and masquerades as being uninformed, she derives a utility equal to $\frac{\gamma(1-s^c)Q}{(1+s^c)}$.[22] However her reputation deteriorates (her score is increased by one unit) and she expects a lower total payoff from continuation of this relationship. The loss in the value of her relationship reflects the fact that (1) a non cooperative phase becomes more likely when the score is high and (2) risk sharing is less efficient during non cooperative phases. The investor is better off **not** ripping her relationship dealer off iff

$$\underbrace{V(in,S)}_{Informed\ refrains\ from\ trading} \geq \underbrace{\frac{\gamma(1-s^c)Q}{(1+s^c)} + \beta V(S+1)}_{Informed\ trades} \quad \forall S \leq S^* - 1,$$

which is equivalent (using Equation (5)) to

$$\beta(V(S) - V(S+1)) \geq \frac{\gamma(1-s^c)Q}{(1+s^c)} \quad \forall S \leq S^* - 1. \tag{8}$$

This Incentive Compatibility Constraint means that the investor refrains from using her

[22] The investor trades $\frac{Q}{1+s^c}$ shares and the dealer's spread is s^c

private information if the loss in the value of her relationship with the dealer is larger than the immediate gain from informed trading.

From Lemma 2, we know that $V(.)$ decreases at an increasing rate. This means that if the previous inequality is satisfied for $S = 0$, then it is satisfied for any $S \leq S^* - 1$. Hence, we can replace the S^* previous inequalities by the condition

$$\beta(V(0) - V(1)) \geq \frac{\gamma(1 - s^c)Q}{(1 + s^c)}. \tag{9}$$

The next lemma summarizes the discussion.

Lemma 3 : *The cooperative trading policy $q^{lt}(T, S^*, s^c)$ is optimal when the dealer follows the scoring policy $p^{lt}(T, S^*, s^c)$ if and only if*

$$\beta(V(0) - V(1)) \geq \frac{\gamma(1 - s^c)Q}{(1 + s^c)}. \tag{10}$$

It remains to identify the set of parameters for which this Incentive Compatibility Constraint (IC) holds true. To this end, we compute the value function $V(.)$. This is achieved by solving the system of Equations (5), (6) and (7). In order limit, the number of parameters, we only report the results for the case in which $\beta \approx 1$. More generally our results hold for β large enough.[23] We obtain the following result.

Lemma 4 : *When β goes to 1, the IC constraint (10) is equivalent to*

$$\Delta U(s^c) \geq \left(\frac{(1 - s^c)\gamma Q}{1 + s^c}\right)\left(Prob(\Delta S = +1) + \frac{S^*}{T}\right), \tag{11}$$

with $Prob(\Delta S = +1) = Prob(S_{t+1} - S_t = +1) = \frac{(1-\alpha)\mu}{2}$.

Condition (11) has an intuitive interpretation. First observe that it requires $\Delta U(s^c) \geq 0$. This means that the investor behaves only if she benefits from the no-informed trading commitment. This necessary condition is satisfied when $s^c < s^*$. The cut-off s^* is strictly smaller than the competitive spread charged by the dealers in presence of adverse selection (that is $s^* < s^{nc}$). Hence the dealer *must* grant a price improvement ($s^{nc} - s^* > 0$) to the investor during cooperative phases. Otherwise the IC contsraint does not hold.

The R.H.S of Condition (11) increases with S^* and decreases with T. This means that, other things equal, it is more difficult to discipline the investor when (a) cooperative

[23] As it is well known, repeated interactions can sustain cooperation only if players sufficiently value future payoffs. This requires a small probability that the game comes to an end.

phases are long or (b) non-cooperative phases are short. Actually, in these two cases the threat of being denied price improvement is less effective since the cost of entering into a non-cooperative phase is smaller. The expected length of the cooperative phase is

$$EL^c(S^*) = \frac{S^*}{Prob(\Delta S = +1)} \tag{12}$$

Hence the larger is the probability of an increase in the score ($Prob(\Delta S = +1)$), the shorter is the average duration of a cooperative phase. This implies that the benefit of maintaining good relationship with the dealer is smaller. This effect impairs the investor's incentive to refrain from using her private information during cooperative phases and explains that the R.H.S of Condition (11) increases with $Prob(\Delta S = +1)$.

Finally observe that the incentive effect of s^c is ambiguous. On the one hand, a small spread during cooperative phases results in more efficient hedging for the investor (larger $\Delta U(s^c)$). On the other hand, a small spread makes informed trading more profitable ($\frac{(1-s^c)Q}{1+s^c}$ decreases with s^c). However, the first effect always dominates the second effect in our model. This is stated formally in the next lemma.

Lemma 5 *: If Condition (11) holds for $s^c = s_0$ then it holds true for $s^c < s_0$, other things equal.*

This implies that larger price improvements (smaller values of s^c) stiffens the investor's incentive to cooperate. Incentives are therefore maximal when $s^c = 0$ (the smallest possible value for the spread), $S^* = 1$ and $T = \mathbf{1}$. This defines the harshest scoring policy. For this scoring policy, Condition (11) is:

$$\Delta U(0) \text{ ¸ } \gamma Q Prob(\Delta S = +1). \tag{13}$$

If (α, γ, μ) are such that Inequality (13) is satisfied then there is at least one scoring policy (namely the harshest scoring policy) which induces the investor to follow a cooperative trading strategy. Furthermore, if the inequality is strict, by continuity, there are strictly positive values for s^c and $\frac{S^*}{T}$ (i.e finite values of T) such that Condition (11) holds true. This means that there are scoring strategies with $s^c > 0$ and $T < \mathbf{1}$ which sustains cooperation. If Inequality (13) is not satisfied then there is no scoring policy which can induce the investor to behave. These remarks yield the next proposition.

Proposition 4 *: Suppose that the relationship dealer uses a scoring policy $p^{lt}(T, S^*, s^c)$ with $s^c > 0$ and $T < \mathbf{1}$. In this case the cooperative trading policy $q^{lt}(T, S^*, s^c)$ is optimal* *for the investor if and only if* $\gamma < \frac{1}{2}$ *and* $\mu < Max\,\{\mu^c(\alpha,\gamma), \mu^{nc}(\alpha,\gamma)\}$ *with* $\mu^c(\alpha,\gamma) = \frac{(1-\gamma)}{\gamma} + \frac{1}{2} - \frac{1}{2\alpha}$.

Figure 3 describes the set of parameters for which it is possible to design a profitable scoring policy which induces the investor to cooperate (we denote this set F^c). This set is non-empty but there are also values of the parameters (α small or μ large) for which it is not possible to find a scoring policy which induces the investor to refrain from using her private information. The intuition is as follows. The expected length of a cooperative phase is inversely related to

$$Prob(\Delta S = +1) = Prob(y \in \{(-1, -1), (+1, +1)\}) = \frac{(1-\alpha)\mu}{2},$$

which decreases with α and increases with μ. This means that when α is small or μ is large, cooperative phases are short. This effect decreases the long run payoff the investor expects from adhering to a 'no informed trading' commitment. Accordingly it becomes more difficult to sustain a cooperative equilibrium.

It is worth stressing that the effect of parameters α and μ are counter-intuitive. One would expect that it is easy to discipline the investor when she is rarely in possession of private information (α small) or her risk exposure is high (μ large). The opposite result is obtained. Actually, recall that in these two cases, dealer's trading profits or losses are poor indicators of the investor's unobserved behavior. Hence the scoring policy performs badly in disciplining the investor.

More generally, the result shows that imperfect observation of the investor's trading motivation can invalidate the view that long-term relationships mitigate informational asymmetries. There are two problems. First, penalty phases have to occur when the investor's trading motivation cannot be observed. This reduces the benefit from cooperation. Second the variables that are used to detect misbehavior (here the dealer's profits) may be poor indicators of the investor's true behavior. Notice that the two problems are related. When α is small, cooperative phases are not substantially shorter if the investor misbehaves because the probability distribution of trading profits is not very sensitive to the investor's behavior.

Finally consider the effect of risk aversion (γ). The frontier which is depicted in Figure 3 shifts to the left when γ decreases. This means that the set of parameters for which cooperation arises is larger when the investor becomes more risk averse. Risk aversion does not affect the likelihood of trading profits (losses) for the dealer. But risk sharing gains from cooperation are larger as risk aversion increases. Hence the investor assigns a larger value to her relationship with the dealer. Accordingly, cooperation is easier to sustain when the investor becomes more risk averse.

In general, for given values of the parameters (α, μ, γ) 2 F^c, there exists a continuum of values for s^c such that the cooperative trading strategy is optimal, provided that the ratio $(\frac{S^*}{T})$ is not too large.

Corollary 2 *Suppose that* (α, μ, γ) 2 F^c. *Let* H^* *be the value of the ratio* $H = (\frac{S^*}{T})$ *such that the IC contsraint is binding for* $s^c = 0$. *For* H · H^*, *there exists a cut-off* $\bar{s}(H)$ *such that the cooperative trading strategy is optimal for all* s^c *such that* $0 < s^c$ · $\bar{s}$. *Furthermore the maximal spread* $\bar{s}$ *decreases with* H.

The threshold $\bar{s}(H)$ is the spread s^c which makes Inequality (11) binding for a given ratio $H = \frac{S^*}{T}$. This is the largest possible spread that can be charged by the dealer for fixed values of the other parameters of the scoring policy. It defines the smallest price improvement that must be offered by the dealer to induce the investor to behave. Interestingly, the largest possible spread becomes smaller when cooperative phases last longer or non-cooperative phases are shorter. Actually, in these cases, the investor is more tempted to deviate for a fixed price improvement. In order to maintain incentive, the smallest price improvement must be increased.

4.3 Cooperative Equilibria.

When she follows a cooperative trading strategy, the investor is locked-in since she loses the value of her relationship with the dealer if she terminates this relationship. This endows the dealer with some bargaining power. However the investor can terminate her relationship with the dealer if her price is too large. In this case, the dealer loses the profits he derives from his relationship with the investor. This threat forces the dealer to balance the one shot profit from charging a supra-normal spread against the future profits he derives with a spread equal to s^c.

In order to formalize this idea, consider the following policies for the investor and the dealer. The dealer follows a scoring policy and the investor follows a cooperative policy as long as the dealer's spread is s^c during cooperative periods and s^{nc} during non cooperative periods. If this is not the case in one period, at the next period, the dealer follows the short-term pricing policy forever and the investor terminates her relationship with the dealer.[24]

[24] This will not happen in equilibrium. The speci¯cation of the dealer's pricing policy after a deviation implies that implementing her threat is optimal for the investor.

Now consider the dealer in a cooperative phase when the investor's score is S. If he charges a spread equal to s^c, his total expected profit is

$$\Pi(S) = s^c + \beta(\frac{\mu}{2}\Pi(S+1) + (1 - \frac{\mu}{2})\Pi(S)),$$

where $\Pi(S)$ is the dealer's ex-ante total expected profit when the investor's score is S. If instead the dealer charges a spread equal to $s^d > s^c$, he obtains a total expected profit equal to s^d since from the next period onward, the dealer will obtain zero profits. Hence the dealer is better off charging s^c iff

$$s^d \leq s^c + \beta\left[\frac{\mu}{2}\Pi(S+1) + (1 - \frac{\mu}{2})\Pi(S)\right].$$

If $s^c > 0$ and non-cooperative phases have finite length, the value of the future profits earned with the investor (the term inside the brackets) grows with β. In the limit case in which β goes to one, this value become infinite.[25] Since s^d is bounded (otherwise the investor would simply refuse to trade), the condition for no opportunistic behavior on the dealer's side is satisfied for β large enough. The same reasoning applies during non cooperative phases. This yields the following result.

Lemma 6 *: If $T < \infty$, $s^c > 0$, and $\beta \approx 1$, $p^{lt}(T, S^*, s^c)$ is the optimal pricing policy for the relationship dealer when the investor uses the cooperative trading policy $q^{lt}(T, S^*, s^c)$.*

The next proposition immediately follows from this result and Proposition 4.

Proposition 5 *: There exists a cooperative equilibrium if and only if $\gamma < \frac{1}{2}$ and $\mu < Max\{\mu^c, \mu^{nc}\}$.*

The previous proposition provides the conditions under which there exist values of T, S^* and s^c such that the course of actions prescribed by scoring policies and cooperative trading policies are self-enforcing, for both the dealer and the investor. Observe that in general there is a multiplicity of cooperative equilibria. For instance for a given value of the ratio $\frac{S^*}{T}$, all the values of $s^c \in (0, \bar{s}(\frac{T}{S^*}))$ can constitute the spread charged by the dealer in a cooperative equilibrium.

[25] This is clearly not true when $s^c = 0$ since in this case the dealer's per profit is always zero. This is not true as well if a non cooperative phase lasts forever. In this case, $\Pi(S)$ is finite for all values of β since the probability of entering into a non cooperative phase is 1 and the dealer's expected profit during a non cooperative phase is zero. These statements can be checked using the closed-form solution for $\Pi(S)$ given in Section 4.4.

4.4 Welfare

Consider $(1-\beta)V(0)$, the average 'per period' ex-ante expected utility[26] obtained by the investor when her score is equal to zero in a cooperative equilibrium. In the same way, $(1-\beta)$ is the average per period expected profit for the relationship dealer.

Proposition 6 : *Consider a cooperative equilibrium in which the dealer uses a scoring policy with parameters T, S^* and s^c.*

$$lim_{\beta\to1}(1-\beta)V(0) = E_\theta\left(\bar{U}(q^{nc}(\tilde{\theta}),\tilde{\theta})\right) + \left(\frac{1}{1+\frac{T}{EL(S^*)}}\right)\Delta U(s^c). \tag{14}$$

and

$$\lim_{\beta\to1}(1-\beta)\Pi(S) = s^c - \frac{s^c\frac{T}{EL(S^*)}}{1+\frac{T}{EL(S^*)}}. \tag{15}$$

Hence a cooperative equilibrium always Pareto-dominates an equilibrium obtained with short-term trading relationships but it never achieves the welfare level associated with a 'no informed trading' commitment.

A scoring policy induces the investor to behave as when she can enter into a no-informed trading commitment during cooperative phases but not during non-cooperative phases. It follows that the traders can capture only part of the welfare gains created by a no-informed trading commitment. As shown by Equations (14) and (15) , the welfare gains associated with scoring strategies crucially depends on the ratio $\frac{T}{EL(S^*)}$. The larger is this ratio, the smaller is the welfare gain brought up by scoring strategies because cooperative phases are shorter on average and non cooperative phases last longer. Hence, for a given s^c, the favorite cooperative equilibrium for both the investor and the dealer is obtained by choosing the largest value of $\frac{S^*}{T}$ such that the investor's IC is satisfied.

Recall that there are parameters' values such that no trading is possible between the dealer and the investor when their relationship is short term. It is readily checked that, for these parameters, there always exists a cooperative equilibrium when $\gamma < \frac{1}{2}$.

Corollary 3 : *When $\gamma < \frac{1}{2}$, the set of parameters for which a cooperative equilibrium exists always includes the set of parameters for which a market breakdown occurs in absence of long-term trading relationships.*

[26] Actually this is the total expected payoff ($V(0)$) divided by the expected length of the relationship.

Hence there are cases in which a market breakdown occurs with short-term trading relationships but not with long-term trading relationships. In this sense, a trading mechanism which enables dealers and investors to establish enduring relationships is more robust than a trading mechanism which does not. This observation vindicates the view that non-anonymous trading venues, like upstairs markets, sustain trades which could not otherwise occur (see Madhavan and Cheng (1996)). Notice however that market breakdowns do not completely disappear when traders have long-term relationships. They are still possible when the investor's risk aversion is small ($\gamma > 1/2$).

4.5 Empirical Implications

The previous analysis has some implications that can be useful for empirical investigations. Our results concur with a basic prediction of the extant literature on non anonymous markets. In these markets, some traders must be able to negotiate price improvements.[27] The novelty here is that price improvements are part of an incentive mechanism which induces investors to refrain from ripping off their relationship dealer when they are informed. This makes our explanation of this phenomenon distinct from the explanation provided by Bernhardt et al. (1999) or Rhodes-Kropf (1998).

Our main predictions regard the relationship between price improvements and each individual investors' trading history. First the model predicts that a given trader should not constantly receive price improvements from a given dealer. Hence for a given trader, the size of price improvements should vary over time, even after controlling for trade size. Second, according to the model, phases in which the investor do not receive price improvements should be observed after a sequence of suspicious trades. Suspicious trades are either a purchase followed by a price increase or a sale followed by a price decrease. More generally this result suggests to consider the frequency of suspicious trades by investors. Those who conduct suspicious trades more frequently should receive price improvements less frequently.

5 Competition between Relationship Dealers

Until this point we have assumed that the value of the investor's outside option (switching to outside dealers) is equal to her total expected payoff in the equilibrium with short-term relationships. This is tantamount to assume that the investor cannot establish relationship

[27] In practice, the price improvement can be measured relative to the quotes posted by the dealer just before the transaction or relative to the best quotes posted in a competing, anonymous, trading system.

with these dealers. We now relax this assumption. Our main purpose is to analyze the effect of outside competition on the provision of incentives by the relationship dealer. We assume throughout this section that $\gamma < \frac{1}{2}$ and $\mu < Max\{\mu^c, \mu^{nc}\}$.

To simplify the analysis, assume that there are two relationship dealers A and B. Each dealer has established relationships with two different investors, respectively 1 and 2. The two investors have the same characteristics (same Q, α, γ) for simplicity as well. We search the conditions for a *symmetric* and *stable* cooperative equilibrium. A symmetric cooperative equilibrium is a cooperative equilibrium in which the two dealers use the same scoring policy. A stable cooperative equilibrium is a cooperative equilibrium where no investor is better off switching.

We denote V_{ij} the value of switching from dealer i to dealer j. This value does not depend on the investor's trading history with i (for instance her score) since dealer j does not observe it. Observe that V_{ij} depends on the way the investor and dealer j would build up their relationship if the investor switched. To simplify, we assume that they start their relationship in a non -cooperative phase which lasts $T^{obs} \geq 0$ periods (the observation phase). We allow the length of the observation phase, T^{obs}, to be different from the length of a regular non-cooperative phase. This captures the idea that the beginning of a relationship is special. The dealer may offer good prices very quickly in order to attract the investor ($T^{obs} < T$). In contrast, it may take long before a trust relationship establishes between dealer j and the investor ($T^{obs} \geq T$). A scoring policy is now defined by 4 parameters (T^{obs}, T, S^*,s^c).

For a symmetric and stable cooperative equilibrium to exist, there are two types of constraints that must be satisfied. First the IC contsraint that we have derived in the previous section must still be satisfied. Second the no-switching constraint $V(S^*) \geq V_{ij}$ must be satisfied. Using this second remark we obtain the following lemma.

Lemma 7 *: A necessary condition for the existence of a symmetric and stable equilibrium is that* $T^{obs} \geq T$.

The intuition is straightforward. If observation phases are shorter than non-cooperative phases then an investor is better off switching dealers when she is about to enter into a non-cooperative phase (i.e. when $S = S^*$). But then the no-switching constraint is not satisfied and the equilibrium can not be stable. The implication is that an observation phase is necessary for cooperative equilibria to exist. Actually, $T \geq 1$ is required for a cooperative equilibrium. Hence $T^{obs} = 0$ is incompatible with cooperation. Intuitively, if the investor immediately obtains good prices when she terminates her relationship then the threat of refusing price improvements cannot be effective and cooperation is impossible.

This shows that, in a competitive dealership market, incentive pricing requires some coordination among dealers. They should not compete on the length of observation periods. Actually this type of competition destroys the possibility of sustaining cooperative equilibria in the first place. In practice, dealers may voluntarily adhere to the convention that they should wait several rounds before offering price improvements to a new client. For this reason, from now on we take T^{obs} as given and we consider the equilibrium which is likely to arise if dealers choose non-cooperatively the other aspects of their scoring policies.

Suppose that dealer A chooses a scoring policy (T, S^*, s^c). In equilibrium, dealer B should not be able to propose another scoring policy such that (a) it induces the investor to behave (the IC constraint is satisfied), (b) the investor is better off switching ($V(S^*) < V_{AB}$) and (c) dealer B makes profit. For a given s^c, the higher is $\frac{S^*}{T}$, the higher is the investor's welfare. Hence for a given s^c, the ratio $H = \frac{S^*}{T}$ must be fixed at the largest possible value so that the IC constraint holds. But this is not sufficient. Actually, in this situation, dealer B can attract the investor by offering a smaller spread during cooperative phases. Intuitively this implies that competition should drive dealers to offer a scoring policy so that $s^c = 0$ and $H = H^*$. However, recall that $s^c = 0$ is not a feasible solution (it does not sustain cooperation). This problem is easily fixed by assuming that the set of prices is discrete and that for this reason the spread cannot be smaller than ε,where ε is arbitrarily small but strictly positive. Hence, dealer B cannot improve upon dealer's A offer iff $s^c = \varepsilon$ and $\frac{S^*}{T}$ is the largest value which satisfies the IC constraint when $s^c = \varepsilon$. There are several values of S^* and T which satisfy this second constraint. Among these values we pick the pair with the smallest T and the smallest S^*. We denote these values $T^{\min}$and $S^{\min *}$ and we call the scoring policy, $p^{lt}(\varepsilon, T^{\min}, S^{\min *})$, the competitive scoring policy.

Proposition 7 *: The cooperative equilibrium in which the relationship dealers use the competitive scoring policy and the investors use the cooperative trading policy* $q^{lt}(\varepsilon, T^{\min}, S^{\min *})$ *is a stable and symmetric cooperative equilibrium when* $T^{\min} \cdot T^{obs}$.

6 Conclusions

In this article we have studied a specific pricing policy by which a dealer induces his regular clients to refrain from trading when they have private information. This pricing policy consists in offering price improvements to traders who do not repeatedly inflict losses to the dealer and to stop offering these improvements otherwise. Hence our model offers an explanation for why price improvements are a pervasive phenomenon in dealership markets (e.g. the London Stock Exchange or Nasdaq). In our model, the prospect of losing price improvements deters investors from ripping off their regular dealer when they are informed. In turn, this behavior enables the dealer to offer price improvements without losing money. Our explanation for price improvements does not rely on excess profits for the dealers or on dealers' ability to distinguish informed and non informed traders.[28] Rather price improvements are part of an incentive mechanism which induces investors to honor a 'no-informed trading' commitment.

Furthermore we assess the benefits of long-term trading relationships. First we show that they can effectively remedy no trading outcomes due to adverse selection. Second we show that the ability to forge long-term trading relationships can generate substantial welfare gains, for traders with large risk aversions or/and a large probability of being informed. In our model, the investor conducts her liquidity trades with the relationship dealer. She should trade with other counter parties when she has private information. This suggests that when an anonymous and a non anonymous trading system co-exist, long term trading relationships in the non anonymous system will exacerbate adverse selection in the parallel trading system. In this case, the welfare gains for traders engaged in long term relationships may obtain at the expense of traders who only have access to the anonymous trading system (or for whom cooperative equilibria can not be sustained). This issue is an interesting topic for future research.

[28] We assume that the dealer never observes the investor's trading motive.

7 Bibliography

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8 Appendix

Proof of Lemma 1. Clearly, the price posted by the dealer must belong to $[v_0 - 1, v_0 + 1]$. Hence an investor with type θ_3 (θ_4) never submits an order to buy (sell) the asset. It follows that, in any equilibria, the dealer must accommodate sell orders (resp. buy orders) at a discount (resp. at a markup) relative to v_0. This fact has several implications. First, a trader with type θ_5 is better off not trading so that $q^*(\theta_5) = 0$. Second, in any equilibria, a trader with type θ_2 (resp. θ_1) never submits a sell order (resp. a buy order). Actually this would reinforce the investor's risk exposure without compensation for risk taking. Third, in equilibrium, when she has type θ_3 the investor must submit the same sell order as when she has type θ_1. Actually this is the only way for the investor to conceal her private information in this case. It follows that

$$q^*(\theta_1) = q^*(\theta_3) = -q^{sell} \leq 0,$$

and

$$q^*(\theta_2) = q^*(\theta_4) = q^{buy} \geq 0.$$

In equilibrium, the dealer's bidding rule is competitive. Hence

$$p(-q^{sell}) = E(\tilde{v} \mid q^*(\theta) = -q^{sell}) = E(\tilde{v} \mid \theta \in \{\theta_3, \theta_1\}) = v_0 - s^{nc},$$

where s^{nc} is as defined in Lemma 1. In the same way, when $q^{buy} > 0$,

$$p(q^{buy}) = E(\tilde{v} \mid q^*(\theta) = q^{buy}) = E(\tilde{v} \mid \theta \in \{\theta_4, \theta_2\}) = v_0 + s^{nc}. \blacksquare$$

Proof of Proposition 1. We define

$$\bar{s}(\gamma, \mu) \stackrel{def}{=} \frac{2(1-\gamma)}{2-(1-\gamma)\mu}.$$

The next lemma gives the investor's demand (supply) function when she has type θ_2 (θ_1) at a price larger (lower) than v_0.

Lemma 8 *: For $s \in [0, 1)$,*

1. *The demand of an investor with type θ_2 at a price $p = v_0 + s$ is $d^b(v_0 + s; \theta_2) = \frac{Q}{1+s}$ if $s < \bar{s}(\gamma, \mu)$ and zero if $s \geq \bar{s}(\gamma, \mu)$.*

2. *The supply of an investor with type θ_1 at a price $p = v_0 - s$ is $d^s(v_0 - s; \theta_1) = -\frac{Q}{1+s}$ if $s < \bar{s}(\gamma, \mu)$ and zero if $s \geq \bar{s}(\gamma, \mu)$.*

Proof: The final wealth of the investor when she has type θ_2 and purchases d shares of the security at price $p = v_0 + s$ is

1. $W_U(d) = -Q + d(v_0 + 1 - p(\theta_2)) = -Q + d(1 - s) \quad if \quad \epsilon = +1,$
2. $W_0(d) = d(v_0 - p(\theta_2)) = -ds \quad if \quad \epsilon = 0,$
3. $W_D(d) = Q + d(v_0 - 1 - p(\theta_2)) = Q - d(1 + s) \quad if \quad \epsilon = -1,$

with $d \geq 0$. Notice that $W_0(d) \leq 0$. Furthermore $W_U(d) \leq 0$ if and only if $d \leq \frac{Q}{1-s}$ and $W_D(d) \leq 0$ if and only if $d \geq \frac{Q}{1+s}$. Notice that $s \in [0, 1)$ so that $1/(1+s) \leq 1/(1-s)$. Now using the definition of the utility function, it is straightforward to show that, if $s < \bar{s}(\gamma, \mu)$, then

$$\frac{\partial E(U(\tilde{W}(d))}{\partial d} > 0 \quad if \quad 0 \leq d < \frac{Q}{1+s},$$

and

$$\frac{\partial E(U(\tilde{W}(d))}{\partial d} < 0 \quad if \quad d > \frac{Q}{1+s},$$

and

$$\frac{\partial E(U(\tilde{W}(d))}{\partial d} = 0 \quad if \quad d = \frac{Q}{1+s}.$$

It follows that

$$d^b(v_0 + s; \theta_2) = \frac{Q}{1+s},$$

is the optimal trade size for the investor. If $s \geq \bar{s}(\gamma, \mu)$ then

$$\frac{\partial E(U(\tilde{W}(d))}{\partial d} \leq 0 \quad \forall d \geq 0.$$

It follows that the investor's demand at price $p = v_0 + s$ is $d^b(v_0 + s; \theta_2) = 0$. The same argument can be used to derive the supply function of the investor when she has type θ_1.∎

Consider an investor with type θ_1. We know from Lemma 1 that, in equilibrium, (a) an investor with type θ_1 sells the security or does not trade and that (b) in any equilibria in which the investor sells the amount q^{sell}, the price charged by the dealer for this order is $p(-q^{sell}) = v_0 - s^{nc}$. But when the dealer charges this price, the investor's optimal supply when she has type θ_1 is zero if $s^{nc} \geq \bar{s}(\gamma, \mu)$ (see Lemma 8). It follows that the investor is better off not contacting her relationship dealer. This implies that when $s^{nc} \geq \bar{s}(\gamma, \mu)$, the only possible equilibrium outcome is

$$q^*(\theta_1) = q^*(\theta_3) = 0.$$

In the same way we establish that

$$q^*(\theta_2) = q^*(\theta_4) = 0,$$

is the unique possibility when $s^{nc} \geq \bar{s}(\gamma, \mu)$. ■

Proof of Corollary 1. There is a market breakdown iff

$$s^{nc}(\alpha, \mu) \geq \bar{s}(\gamma, \mu).$$

Using the expressions for $\bar{s}(\gamma, \mu)$ and s^{nc}, we rewrite this inequality as

$$\mu \leq \frac{\gamma}{(1-\gamma)} \frac{(1-\alpha)}{2\alpha} - \frac{1}{2\alpha(1-\gamma)} + \frac{3}{2(1-\gamma)},$$

which is equivalent to

$$\mu \leq \frac{\gamma}{1-\gamma} + \frac{3}{2} - \frac{1}{2\alpha}. \blacksquare$$

Proof of Proposition 2.

Step 1. We first show that the case in which the investor's trade size is $\bar{q}$ is an equilibrium outcome. Consider an investor with type θ_2. It is clearly not optimal for this investor to sell the security at a price below v_0 (this would reinforce her exposure to risk without any compensation). Thus she should buy shares or abstain from contacting her relationship dealer. The investor is better off buying $\bar{q}$ shares at price $v_0 + s^{nc}$ rather than not trading. Actually

$$\bar{q} = \frac{Q}{1 + s^{nc}},$$

is the investor's optimal demand at price $v_0 + s^{nc}$ since $s^{nc} < \bar{s}(\gamma, \mu)$ (see Lemma 8). At the prices charged by the dealers for the other quantities, the investor's demand is zero. Hence the investor chooses to buy $\bar{q}$. In the same way, we show that an investor with type θ_1 chooses to sell $\bar{q}$ shares.

Trades sizes different from $\bar{q}$ have a probability zero to be observed in equilibrium. Consequently, for these trade sizes, the dealers' beliefs on the investor type can not be computed by Bayes rule. In this case we are free to let the dealer have any beliefs. Suppose that the dealer believes the investor to have type θ_3 (θ_4) when the latter wants to sell (buy) a quantity $q \neq \bar{q}$. This yields the dealer's offer for $q \neq \bar{q}$.The rest of the proposition follows from Lemma 1.

Step 2. Now we show that the investor's ex-ante expected utility is maximum when her trade size is equal to $\bar{q}$. Let $\bar{U}(q^*(.), \theta)$ be the investor's expected utility when she uses the trading strategy $q^*(.)$ and her type is θ. We define $f(q_1, q_2) = E_\theta(\bar{U}(q^*(.), \tilde{\theta})$, the investor's ex-ante expected utility in an equilibrium in which $q^*(\theta_1) = q^*(\theta_3) = -q_1 < 0$ and $q^*(\theta_2) = q^*(\theta_4) = q_2 > 0$. We show below that $f(.,.)$ reaches its maximum when $q_1 = \bar{q}$ and $q_2 = \bar{q}$. For this we compute $E_\theta(\bar{U}(q^*(.), \tilde{\theta}))$ as a function of q_1 and q_2 when the price at which the investor sells q_1 is $v_0 - s^{nc}$ and the price at which she buys q_2 is $v_0 + s^{nc}$ (since these are the equilibrium prices in any equilibria). The proof relies on the fact that $\bar{q}$ is the trade size which maximizes the investor's expected utility when the investor is non-informed and the dealer charges a spread equal to s^{nc} (see Lemma 8 in the proof of Proposition 2).

First suppose that $q_2 < \bar{q}$. Notice that

$$E_\theta(\bar{U}(q^*(.), \tilde{\theta})) \,|\theta \in \{\theta_2, \theta_4\}) \times Prob(\theta \in \{\theta_2, \theta_4\}) = \frac{\alpha\mu}{2}\bar{U}(q_2, \theta_4) + \frac{(1-\alpha)}{2}\bar{U}(q_2, \theta_2).$$

Clearly this expression does not depend on q_1. It is straightforward that $\bar{U}(q_2, \theta_4)$ increases with q_2 (when she is informed, the investor is better off trading the largest possible quantity since $s^{nc} < 1$). As $s^{nc} < \bar{s}(\gamma, \mu)$ and $q_2 < \bar{q}$, we also obtain that $\bar{U}(q_2, \theta_2)$ increases with q_2 (Lemma 8). It follows that

$$E_\theta(\bar{U}(q^*(.), \tilde{\theta}) \,|\theta \in \{\theta_2, \theta_4\}) \times Prob(\theta \in \{\theta_2, \theta_4\}),$$

increases with q_2. In the same way we show that

$$E_\theta(\bar{U}(q^*(.), \tilde{\theta})) \,|\theta \in \{\theta_1, \theta_3\}) \times Prob(\theta \in \{\theta_1, \theta_3\}),$$

increases with q_1 for $q_1 < \bar{q}$. As

$$f(q_1, q_2) = E_\theta(\bar{U}(.) \,|\theta \in \{\theta_2, \theta_4\}) Prob(\theta \in \{\theta_2, \theta_4\}) + E_\theta(\bar{U}(.) \,|\theta \in \{\theta_1, \theta_3\}) Prob(\theta \in \{\theta_1, \theta_3\}),$$

we have shown that

$$\frac{\partial f(., q_2)}{\partial q_1} > 0 \quad for \quad q_1 < \bar{q} \tag{16}$$

and

$$\frac{\partial f(q_1, .)}{\partial q_2} > 0 \quad for \quad q_2 < \bar{q}. \tag{17}$$

Now suppose that $q_2 > \bar{q}$. We obtain (after some algebra) that

$$E_\theta(\bar{U} \mid \theta \in \{\theta_2, \theta_4\}) Prob(\theta \in \{\theta_2, \theta_4\}) = \begin{cases} (\alpha\mu(1 - s^{nc})(\gamma - 1))\, q_2 & if \quad q_2 \in [\bar{q}, \frac{Q}{1-s^{nc}}], \\ \left((\alpha\mu + \frac{\mu}{2})(1 - s^{nc})(\gamma - 1)\right) q_2 + \frac{(1-\gamma)\mu Q}{2} & if \quad q_2 > \frac{Q}{1-s^{nc}} \end{cases}$$

Recall that $\gamma < 1$ and $s^{nc} < 1$. Thus

$$E_\theta(\bar{U}(q^*(.), \tilde{\theta})) \mid \theta \in \{\theta_2, \theta_4\}) Prob(\theta \in \{\theta_2, \theta_4\}),$$

decreases with q_2 for $q_2 > \bar{q}$. In the same way we show that

$$E_\theta(\bar{U}(q^*)(.), \tilde{\theta}) \mid \theta \in \{\theta_1, \theta_3\}) Prob(\theta \in \{\theta_1, \theta_3\}),$$

decreases with q_1 for $q_1 > \bar{q}$. Hence

$$\frac{\partial f(., q_2)}{\partial q_1} < 0 \quad for \quad q_1 > \bar{q} \tag{18}$$

and

$$\frac{\partial f(q_1, .)}{\partial q_2} < 0 \quad for \quad q_2 > \bar{q}. \tag{19}$$

It follows from Equations (16), (17), (18), (19) that $f(.,.)$ reaches its maximum for $q_1 = \bar{q}$ and $q_2 = \bar{q}$.$\blacksquare$

Proof of Proposition 3.

Case 1: $\mu > \mu^{nc}$.

In the equilibrium described in Proposition 2, the investor's ex-ante expected utility is

$$E_\theta\left(\bar{U}(q^{nc}(.), \tilde{\theta})\right) = \left(\frac{\alpha(1 - \alpha)\mu(1 - \gamma)}{2\mu\alpha + (1 - \alpha)}\right) Q.$$

Now consider the case in which the investor follows the trading strategy $q^c(.)$. When the investor is informed, she does not trade. Hence she does not improve upon her reservation utility which is zero. If the investor is uninformed, she buys or sells (depending on her type) $\frac{Q}{1+s^{nc}}$ shares. She obtains an expected utility equal to $-\frac{Qs^c}{1+s^c}$. Hence

$$E_\theta\left(\bar{U}(q^c(.), \tilde{\theta})\right) = -(1 - \alpha)\frac{Qs^c}{1 + s^c}.$$

It follows that

$$\Delta U(s^c) = \left(\frac{\alpha(1-\alpha)\mu(1-\gamma)}{2\mu\alpha + (1-\alpha)}\right) Q - (1-\alpha)\frac{Qs^c}{1+s^c} \quad if \quad \mu > \mu^{nc}. \tag{20}$$

Observe that ΔU decreases with s^c. The spread $s^*(\alpha, \mu, \gamma)$ is the spread which solves

$$\Delta U(s^*) = 0.$$

This yields

$$s^*(\alpha, \mu, \gamma) = \frac{\alpha\mu(1-\gamma)}{\alpha\mu(1+\gamma) + (1-\alpha)}.$$

Case 2: $\mu \leq \mu^{nc}$.

In this case, in absence of the no-informed trading commitment, there is a market breakdown and $q^{nc}(\theta) = 0, \forall\theta$. It follows that

$$E_\theta\left(\bar{U}(q^{nc}(.), \tilde{\theta})\right) = \left(\frac{(1-\alpha)\mu(1-\gamma)}{2}\right) Q \quad if \quad \mu \leq \mu^{nc}.$$

We deduce that

$$\Delta U^c(s^c) = \left(\frac{(1-\alpha)\mu(1-\gamma)}{2}\right) Q - (1-\alpha)\frac{Qs^c}{1+s^c} > 0 \quad if \quad \mu \leq \mu^{nc}. \tag{21}$$

The rest of the proof is identical to Case 1. In this case the cut-off value for the spread is

$$s^*(\alpha, \mu, \gamma) = \bar{s}(\gamma, \mu). \blacksquare$$

Proof of Lemma 2. Using Equation (4) and Equation (5), we obtain that

$$V(he, S-1) = \left(\frac{1-\alpha\beta}{1-\alpha}\right) V(S-1) \quad for \quad 1 \leq S \leq S^*.$$

Substituting $V(he, S-1)$ by this expression in Equation (6) and rearranging, we obtain

$$V(S) = kV(S-1) - \frac{2\bar{U}^c}{\mu\beta} \quad for \quad 1 \leq S \leq S^*, \tag{22}$$

with $k = \left(1 + \frac{2(1-\beta)}{\beta(1-\alpha)\mu}\right)$. We deduce that

$$V(S) - V(S-1) = k(V(S-1) - V(S-2)) \quad for \quad 2 \leq S \leq S^*, \tag{23}$$

which yield

$$V(S) - V(S-1) = k^{S-1}(V(1) - V(0)) \tag{24}$$

Furthermore

$$V(1) - V(0) = (k-1)V(0) - \frac{2\bar{U}^c}{\mu\beta} = \left(\frac{2(1-\beta)}{\beta\mu(1-\alpha)}\right)\left(V(0) - \frac{(1-\alpha)\bar{U}^c}{(1-\beta)}\right).$$

Observe that $\frac{(1-\alpha)\bar{U}^c}{(1-\beta)}$ is the investor's expected payoff with the trading strategy q^{lt} when a cooperative phase lasts forever. However cooperative phases do not last forever since a non cooperative phase occurs with probability 1. Furthermore in non cooperative phases the investor's per period expected payoff is less than $(1-\alpha)\bar{U}^c$ since $s^c < s^*$. We deduce that

$$V(0) - \frac{(1-\alpha)\bar{U}^c}{(1-\beta)} < 0.$$

This implies that $V(1) - V(0) < 0$. In turn, this yields (using Equation (24)) that

$$V(S) - V(S-1) < 0 \quad for \quad 2 \leq S \leq S^*.$$

Hence $V(.)$ decreases with S. Furthermore since $k > 1$, Equation (23) implies that

$$V(S-1) - V(S) > V(S-2) - V(S-1) \quad for \quad 2 \leq S \leq S^*. \blacksquare$$

Proof of Lemma 3.

Part 1.

According to the Optimality Principle of Dynamic Programming, the investor has no incentive to deviate from the trading strategy q^{lt} iff there is no circumstances in which a one shot deviation is profitable. Hence, we just need to identify conditions under which a one shot deviation from the trading strategy $q^{lt}(.)$ is non optimal for the investor, in a cooperative phase or in a non cooperative phase.

Step 1. In the cooperative phase.

For each possible value of $S \leq S^* - 1$, three types of deviations are possible.

1. The investor is informed and she has good (bad) information. She buys (sells) the security (instead of not trading).

2. The investor is informed and she has good (bad) information. She sells (buys) the security (instead of not trading).

3. The investor is uninformed and she does not contact the dealer.

We consider each of these deviations in turn. We have already explained in the text that the first deviation is not optimal iff

$$\beta(V(0) - V(1)) \geq \frac{\gamma(1 - s^c)Q}{(1 + s^c)}, \tag{25}$$

If the investor sells (buys) the security when she has good information, she incurs a loss and she leaves her score unchanged. She is clearly better off not trading. Hence the second deviation cannot be optimal.

Now consider the case in which the investor is uninformed. When she has score $S \leq S^* - 1$, the investor is better off trading iff

$$\bar{U}_h^r + \beta V(S) \leq V(he, S), \tag{26}$$

where $\bar{U}_h^r = \frac{(\gamma-1)\mu Q}{2} < 0$ is the investor's expected utility when she has a hedging need and she does not trade. Using Equation (6), we rewrite Condition (26) as

$$\bar{U}_h^r \leq \bar{U}^c + \frac{\beta\mu}{2}\left(V(S+1) - V(S)\right).$$

This inequality is satisfied for all $S < S^* - 1$ iff

$$\bar{U}_h^r \leq \bar{U}^c + \frac{\beta\mu}{2}\left(V(S^*) - V(S^* - 1)\right), \tag{27}$$

since $V(S+1) - V(S) < V(S) - V(S-1) \quad \forall S \leq S^* - 1$. In the proof of Lemma 2, we have shown that $V(S) = kV(S - 1) - \frac{2\bar{U}^c}{\mu\beta}$ with $k = 1 + \frac{2(1-\beta)}{\beta(1-\alpha)\mu})$. Using this result, after straightforward manipulations, we can rewrite Equation (27) as

$$\frac{(1 - \alpha)\bar{U}_h^r}{(1 - \beta)} \leq V(S^* - 1). \tag{28}$$

Observe that the L.H.S of this inequality is the total expected payoff to the investor if she never trades (uninformed or not). When she follows the trading strategy q^{lt}, the investor trades sometimes and when she trades, she obtains a per period expected utility which is strictly larger than $\bar{U}_h^r$. Hence, for a given score, the value of the relationship for the investor, must be larger than her total expected payoff if she never trades. In particular Equation (28) holds true. Hence Condition (25) is necessary and sufficient for q^{lt} to be a best response for the investor in the cooperative phase.

Step 2. Non Cooperative Phase.

In the non cooperative phase, the dealer's pricing policy is not influenced by past outcomes and is identical to the pricing strategy with short-term trading relationships. Therefore, $q^{nc}(.)$ is the optimal trading strategy for the investor during a non cooperative phase.$\blacksquare$

Proof of Lemma 4.

In this proof we denote $E_\theta(\bar{U}(q^{nc}(.), \tilde{\theta}))$ as $E\bar{U}^{nc}$ for brevity.

Step 1. We first compute $V(0)$. Using Equation (22), we write $V(S)$ as a function of $V(0)$. We obtain

$$V(S) = k^S V(0) - \frac{2(\sum_{j=0}^{j=S-1} k^j)\bar{U}^c}{\mu\beta} \quad for \quad 1 \leq S \leq S^*, \tag{29}$$

where $k = (1 + \frac{2(1-\beta)}{\beta(1-\alpha)\mu})$. For brevity we define $f(S) = \sum_{j=0}^{j=S-1} k^j$. Recall that

$$V(S^*) = \frac{(1-\beta^T)}{(1-\beta)} E\bar{U}^{nc} + \beta^T V(0).$$

It follows from Equation (29) that

$$k^{S^*} V(0) - \frac{2f(S^*))\bar{U}^c}{\mu\beta} = \frac{(1-\beta^T)}{(1-\beta)} E\bar{U}^{nc} + \beta^T V(0).$$

Solving this equation for $V(0)$, we obtain that

$$V(0) = \frac{\frac{(1-\beta^T)}{1-\beta} E\bar{U}^{nc} + \frac{2f(S^*))\bar{U}^c}{\mu\beta}}{k^{S^*} - \beta^T}. \tag{30}$$

Step 2. Using equation (29) for $S = 1$, the condition

$$\beta(V(0) - V(1)) \geq \frac{\gamma(1-s^c)Q}{(1+s^c)},$$

can be written as

$$\beta\left((1-k)V(0) + \frac{2\bar{U}^c}{\mu\beta}\right) \geq \frac{\gamma(1-s^c)Q}{(1+s^c)}. \tag{31}$$

Observe that k is a function of β and that

$$lim_{\beta\to 1} \frac{(1-k)}{k^{S^*} - \beta^T} = \frac{-2}{T\mu(1-\alpha) + 2S^*}.$$

Furthermore

$$lim_{\beta \to 1} f(S^*) = S^*.$$

The last two equations and Equation (30) imply that

$$lim_{\beta\to 1}(1-k)V(0) = \left(\frac{-2}{T\mu(1-\alpha)+2S^*}\right)(TE\bar{U}^{nc} + \frac{2S^*\bar{U}^c}{\mu})$$

which yields

$$lim_{\beta\to 1}(1-k)V(0) = \left(\frac{2T}{T\mu(1-\alpha)+2S^*}\right)((1-\alpha)\bar{U}^c - E\bar{U}^{nc}) - \frac{2\bar{U}^c}{\mu}$$

Hence when β goes to 1, Condition (31) is

$$\left(\frac{2}{T\mu(1-\alpha)+2S^*}\right)\left[(1-\alpha)\bar{U}^c - E\bar{U}^{nc}\right] \geq \frac{\gamma(1-s^c)Q}{(1+s^c)},$$

which is equivalent to

$$\Delta U(s^c) = (1-\alpha)\bar{U}^c - E\bar{U}^{nc} \geq \left(\frac{\gamma(1-s^c)Q}{(1+s^c)}\right)\left(\frac{(1-\alpha)\mu}{2} + \frac{S^*}{T}\right). \blacksquare$$

Proof of Lemma 5.

Condition (11) can be written

$$\frac{1+s^c}{1-s^c}(\Delta U(s^c)) \geq \gamma Q\left(Prob(\Delta S = +1) + \frac{S^*}{T}\right), \tag{32}$$

Using the expression for $\Delta U(s^c)$ given by Equations (20) or (21) (depending on μ), we obtain that the L.H.S of this inequality decreases with s^c. $\blacksquare$

Proof of Proposition 4. The discussion which precedes the proposition implies that

$$\Delta U(0) - Prob(\Delta S = +1)\gamma Q > 0, \tag{33}$$

is a necessary and sufficient condition for the existence of a scoring strategy with $s^c > 0$ and $T < \infty$ which induces the investor to follow the cooperative strategy $q^{lt}(T, S^*, s^c)$. From Equations (20) and (21) (proof of Proposition 3), we deduce that

$$\Delta U(0) = \frac{\alpha(1-\alpha)\mu(1-\gamma)}{2\mu\alpha + (1-\alpha)}Q \quad if \quad \mu > \mu^{nc}(\alpha,\gamma),$$

and that

$$\Delta U(0) = \frac{(1-\alpha)\mu(1-\gamma)}{2}Q \quad if \quad \mu \leq \mu^{nc}(\alpha,\gamma).$$

It follows that if $\mu > \mu^{nc}(\alpha,\gamma)$, Equation(33) can be written as

$$\frac{\alpha(1-\alpha)\mu}{2\mu\alpha+(1-\alpha)}\left(\frac{(1-\gamma)}{\gamma}-\frac{2\mu\alpha+(1-\alpha)}{2\alpha}\right) > 0,$$

which yields

$$\frac{\alpha(1-\alpha)\mu}{2\mu\alpha+(1-\alpha)}[\mu^{c}(\alpha,\gamma)-\mu] > 0. \tag{34}$$

If $\mu \leq \mu^{nc}$, Equation(33) can be written

$$\frac{(1-\alpha)\mu}{2}[\frac{1-\gamma}{\gamma}-1] > 0. \tag{35}$$

Case 1. $\gamma \leq \frac{3-\sqrt{5}}{2}$. Computations show that under this condition $\mu^{nc}(\alpha,\gamma) \leq \mu^{c}(\alpha,\gamma)$.

Case 1.1: Suppose that $\mu > \mu^{nc}(\alpha,\gamma)$. In this case the condition for cooperation is given by Equation (34). Since $\mu > 0$, this condition can be satisfied iff $0 < \mu \leq \mu^{c}$.

Case 1.2: Now suppose that $0 \leq \mu \leq \mu^{nc}(\alpha,\gamma)$. In this case the existence condition is given by Equation (35). The L.H.S of this equation is positive since $\gamma \leq \frac{3-\sqrt{5}}{2} < \frac{1}{2}$. Hence, we have shown that

$$0 < \mu \leq \mu^{c}(\alpha,\gamma),$$

is a necessary and sufficient condition for an equilibrium with cooperation to exist when $\gamma \leq \frac{3-\sqrt{5}}{2}$.

Case 2. $\frac{3-\sqrt{5}}{2} < \gamma$. Under this condition:

$$\mu^{c}(\alpha,\gamma) < \mu^{nc}(\alpha,\gamma).$$

If $\mu > \mu^{nc}(\alpha,\gamma)$, the existence condition is given by Equation (34)and it cannot be satisfied since $\mu > \mu^{nc} > \mu^{c}$. If $\mu \leq \mu^{nc}$, the existence condition is given by Equation (35). It is satisfied if and only if $\gamma < \frac{1}{2}$. ∎

Proof of Corollary 2.

We define

$$G(s^c, H) = \Delta U(s^c) - \left(\frac{(1 - s^c)\gamma Q}{1 + s^c}\right)\left(Prob(\Delta S = +1) + \frac{S^*}{T}\right).$$

The IC constraint is satisfied iff $G(s^c, H) \geq 0$. For a given $H = S^*/T$, we define $\bar{s}(H)$ the spread such that $G(\bar{s}(H), H)) = 0$. It is immediate that G decreases with s^c and H. This implies that $\bar{s}$ decreases with H. This also implies that for $s^c > \bar{s}$, the IC constraint cannot be satisfied since $G(s^c, H) < 0$ for $s^c > \bar{s}(H)$. Furthermore for $H = H^*$, $\bar{s} = 0$ by definition of H^*.$\blacksquare$

Proof of Proposition 6. To be written.

Proof of Corollary 3. Recall that a market breakdown occurs when $\mu < \mu^{nc}(\alpha, \gamma)$. For $\gamma < \frac{3-\sqrt{5}}{2}$, a cooperative equilibrium exists if an only if $0 < \mu < \mu^c(\alpha, \gamma)$. In this case, $\mu^{nc}(\alpha, \gamma) < \mu^c(\alpha, \gamma)$. Hence the set of parameters for which a market breakdown occurs is included in the set of parameters for which a cooperative equilibrium exists. For $\frac{3-\sqrt{5}}{2} < \gamma \leq \frac{1}{2}$, a cooperative equilibrium exists iff $\mu < \mu^{nc}$. Hence the set of parameters for which a market breakdown occurs and the set of parameters for which a cooperative equilibrium exists are identical.$\blacksquare$

Figure 1: Sequence of Decisions in a Period

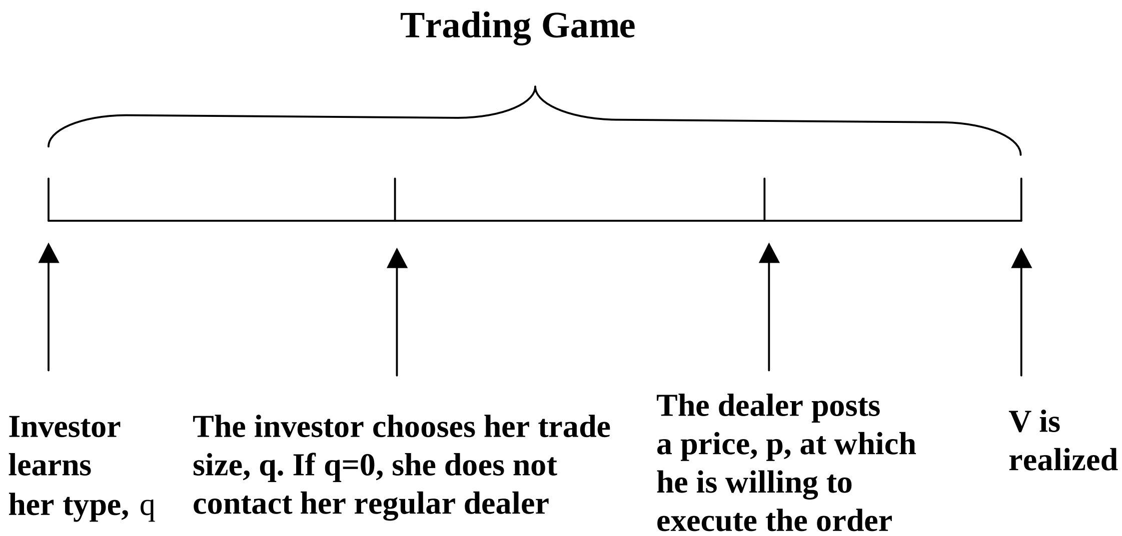

Figure 2: Market Breakdown with Short-Term Trading Relationships

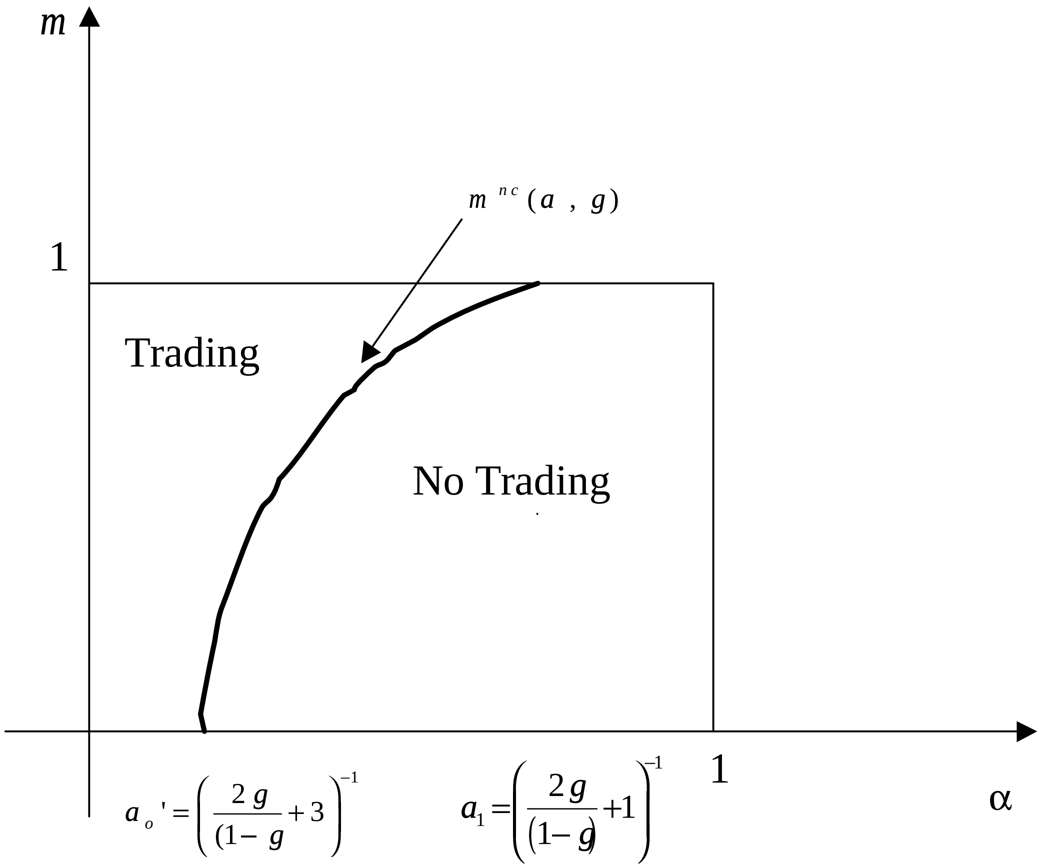

Figure 3: Existence of Cooperative equilibria

Figure 3-a : $g \leq \frac{3-\sqrt{5}}{2}$

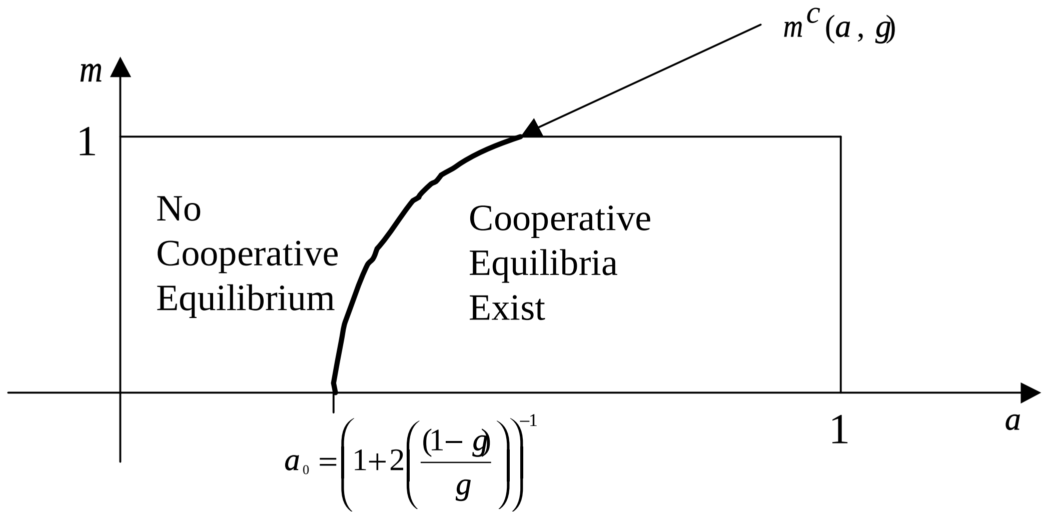

Figure 3-b : $\frac{3-\sqrt{5}}{2} \leq g \leq 1/2$

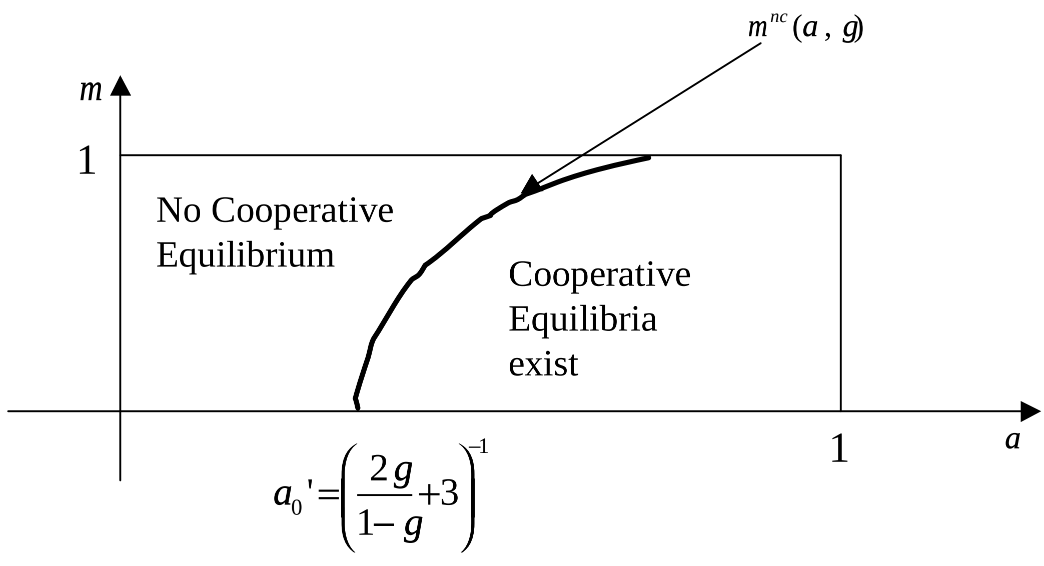

Figure 3 (ctd)

Figure 3-c : $\gamma > 0{,}5$