Income maintenance and labor force participation

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June 2002
Tentative and preliminary

*CREST-INSEE and CNRS URA 2200. I have benefited from numerous discussions with Philippe Choné, Christian Gouriéroux, Thierry Magnac and Bernard Salanié and from the remarks of seminar participants at CREST and Université de Cergy. A large fraction of the theoretical part of the paper comes from my joint work with Philippe Choné, where the interested reader will find the details of the proofs.

Abstract

Welfare reform has recently often focused on incentives to work. Optimal taxation, under a Rawlsian criterion, can bring useful insights in the analysis of these reforms. The theoretical part of the paper studies optimal taxation in an economy where the only decision of the agents is to participate, or not, to the labor force, drawing heavily on the work of Choné and Laroque (2001). The crucial feature of the economy that determines the shape of the optimal tax and benefit scheme is the joint distribution of the agents' productivities and aversions to work.

If one takes as given the maintenance income provided by the welfare state (the theorist has little to say on the relative weights that society puts on households depending on their composition), the Rawlsian optimum provides a benchmark: it maximizes government revenue and any utilitarian criterion would give larger financial incentives to work than the Rawlsian criterion.

Theory puts little restrictions on the shape of the optimal Rawlsian schedules. Essentially anything can happen provided that the financial incentives to work are nondecreasing with productivity and smaller than productivity. A qualitative analysis shows that a 100% marginal tax rate is likely to be optimal when the cumulative distribution function of work aversions has a kink. Positive work subsidies or *negative* marginal tax rates are optimal in a region where the c.d.f. has some discrete mass points.

Everything therefore hinges on the distribution of work aversions, which has to be recovered from the data. I posit a structural model which is estimated on a sample of French women aged 25-50. I compute the optimal Rawlsian financial incentives to work for single women, and for married women with two children or more. Quite surprisingly, the actual incentives to work appear to be very close to what a Rawlsian planner would recommend. It is as if the interactions between the multiple agencies that shape the income tax schedule in France manage to extract the maximum possible surplus from the population.

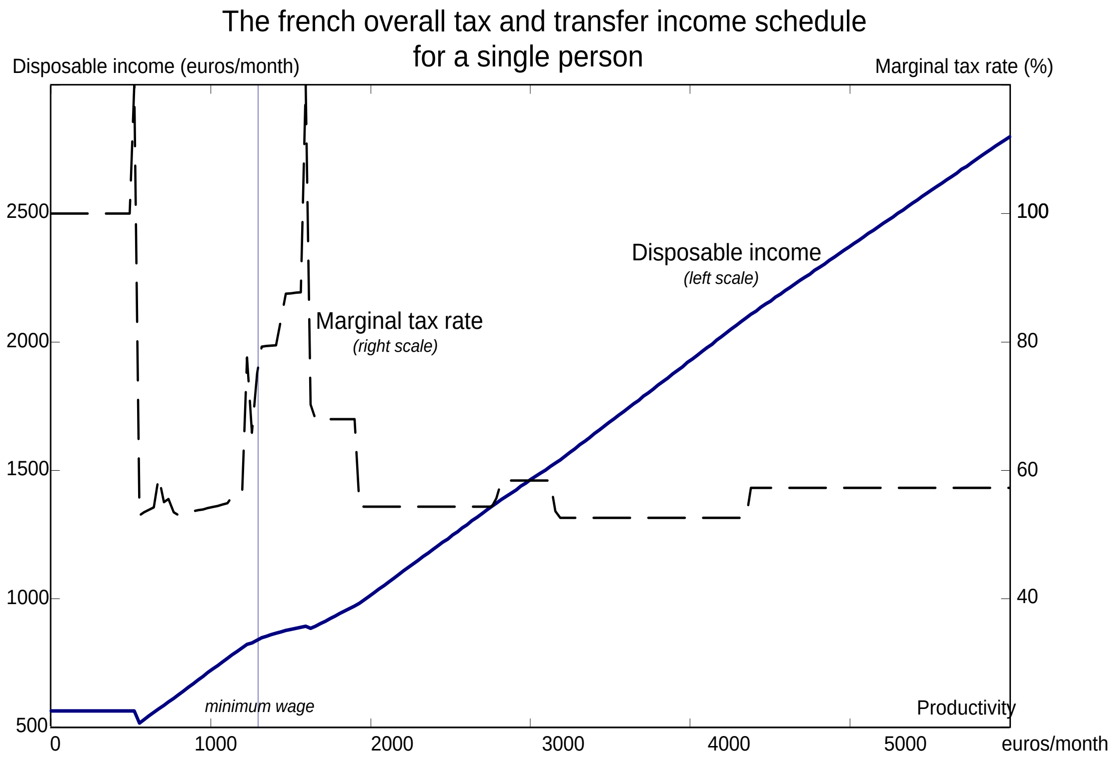


Figure 1: The French tax and transfer schedule in 1999

1 Introduction

The dark side of the welfare state is the inefficiencies that it creates. Figure 1, which represents the overall tax and transfer system in France in 1999, plots disposable income as a function of labor cost for a single person. When not working, the maintenance income (RMI), together with housing subsidies, amount to 560 euros per month. An unskilled worker, having the opportunity to take a full time job paid at the minimum wage (cost to the employer 1300 euros per month), would earn after transfers and taxes 840 euros per month, so that her financial incentive to work is 280 euros per month, or less than 2 euros per hour. The left part of the curve is horizontal, corresponding to a 100% marginal tax rate: working half time does not increase income at all. Unskilled workers may get stuck out of the labor force or induced to join the underground economy. Such poverty traps have been the subject of a lot of attention from policy makers in the past thirty years around the world. In the United States, the Earned Income Tax Credit, followed by the welfare reform of 1996, has been motivated in part by a willingness to *make work pay* and to reduce the undesirable side effects of the Welfare State. Canada has been in the forefront in the design of schemes to induce long term unemployed persons to participate full time in the labor force, see Robins and Michalopoulos (2001) or Card, Michalopoulos, and Robins (1999).

The equity-efficiency tradeoffs associated with the design of the tax-subsidy scheme have been studied by economists since the profession exists. In theory, optimal taxation, provided that society's preferences for redistribution are elucidated, should be a useful guide. In practice, the normative approach has not been very fruitful. Indeed, the relevant framework of optimal taxation, which goes back to the seminal paper of Mirrlees (1971), seems too far from the tax-benefit systems observed in practice to be a useful guide for policy[1]. When effort depends on financial incentives, at the *intensive* margin, the standard result has a zero marginal tax rate on the rich (which goes contrary to the common idea of equity, and is not observed, even in the US). The marginal tax rate is always non negative, which rules out pushing people to work through an earning subsidy, as intended by the EITC. The practical schemes have therefore mostly been influenced, one the one side by political pressures which in the recent years tend to ask for a retreat of the welfare state, and on the other hand, by empirical results which show the importance of targeting specific populations and the devil's role in the details of welfare implementation.

The purpose of the present paper is to make a step in bringing together the theoretical approach of social choice theory and optimal taxation with the empirical research on labor supply. On the theory side, the paper, drawing on joint results with Philippe Choné, focuses on an *extensive* model where the agents' decision is zero-one, to work or not to work, as in the studies of Diamond (1980), Saez (2000) and Beaudry and Blackorby (1997). Also, I consider a Rawlsian society, where the aim of the government is to maximize the well being of the poorer people in the society. In this circumstances, the optimal tax scheme is easily characterized. It depends on the joint distribution of productivities and *work aversions* (or monetary *disutilities for work*) in the economy. A qualitative analysis shows that a 100% marginal tax rate is likely to be optimal when the cumulative distribution function of work aversions has a kink. Positive work subsidies or *negative* marginal tax rates are optimal in a region where the c.d.f. is not log concave, for instance when it has some mass points. More generally, the shape of the optimal tax and transfer schedule directly derives from the distribution of work aversions: essentially any financial incentive scheme that is non decreasing and smaller than productivity can be rationalized with an appropriate distribution of work aversions (Choné and Laroque (2001)). Finally, it is unlikely that governments are Rawlsian in practice. But the Rawlsian optimum yields a useful benchmark: the subsistence income enacted by a utilitarian planner is smaller than that of a Rawlsian one; furthermore, when the work aversions are independent of the level of subsistence income, a utilitarian planner always gives larger financial incentives to work than the Rawlsian planner.

With this theoretical background, it is tempting to compare the Rawlsian tax

[1]Also, the optimal tax program is quite difficult to solve. These difficulties have been overcome by Saez (2001) who uses the available empirical evidence on the shape of the wage distribution and labor supply elasticities to compute optimal tax schedules.

scheme with the one observed in practice. The difficulty here is to get an estimate of the joint distribution of work aversions and productivity. The theoretical model, as often, is highly stylized, static. The empirical literature on labor supply has not given much attention to the distribution of work aversions. To provide an illustration of the theory, I use a model of labor force participation developed with Bernard Salanié on French data. The model accounts for the minimum wage and for frictional unemployment. It is estimated on a sample of women aged 25-49. Work aversion depends on the income of the spouse (if any) and on the number and ages of the children, as well as on an unobserved heterogeneity term. I discuss the identification of the distribution of this term, and implement an adaptive estimation procedure.

Once the joint distribution of productivities and work aversions is recovered from the data, it is easy to apply the theoretical computations to the particular case at hand to get the shape of the financial incentives associated with a Rawlsian criterion, holding fixed the (subsistence) income when not working. The results are presented for the single women, and for the married women with two children or more. Quite surprisingly, the actual incentives to work appear to be very close to the optimal Rawlsian recommendation. It looks as if the interactions between the multiple agencies that shape the income tax schedule in France lead to a Leviathan state that extracts the maximum possible surplus from the population.

These results call for independent confirmation. More generally, the present work raises more questions than it answers. A major step forward would be to deal with the intensive margin (part time work), incorporating some of the standard optimal taxation literature, along the lines of Saez (2001). It would be interesting to repeat the empirical exercise for other countries. The relative size of the government in the economy is much smaller in the US than in France. This type of analysis makes it possible to assess whether this contrast is all due to a difference in political attitudes as usually claimed, or whether part of it can be explained by a discrepancy in tastes for work on both sides of the Atlantic.

2 Theory

2.1 The model

We consider an economy made of a continuum of agents. A typical agent is described by a set of exogenous characteristics, denoted by $a = (w, x, y)$, which include her productivity w and other characteristics (x, y) which, together with productivity, influence her tastes for leisure. When working, the typical agent produces a quantity w of an undifferentiated commodity. The characteristics x of the agent are assumed to be observable by the government and verifiable, so that benefits and taxes can be conditioned on the values of x. For instance, x may include the number and ages of children in the household. On the other hand, the characteristics y are private, and the government only knows their distribution in the population conditional on the observables.

The utility function of a typical agent is a function of the non negative quantity c of commodity which she receives, and depends on the participation decision. The function $v(c, a)$ represents the utility of the non participating agent, while $u(c; a)$ is her utility when working.

To summarize formally, the characteristics $a = (w, x, y)$ of the agents belong to a set A in $\mathbb{R}^n$. Agent a has a (non negative) productivity w and utility functions $u(c; a)$ when she works and $v(c; a)$ when she does not work. The functions $v(.; a)$ and $u(.; a)$, for all a, are continuously differentiable on $\mathbb{R}_+$, with strictly positive derivatives, and they go to $+\infty$ with their argument. The functions $u(c; .)$ and $u'_c(c; .)$ are continuous on A. An economy is defined by a probability measure on A, with c.d.f. F. The proportion of agents whose characteristics have all their coordinates smaller than a is $F(a)$. The marginal distribution of w in the population, whose c.d.f. is noted $\tilde{F}$, is also of interest. We assume that the aggregate resources in the economy are finite, i.e. that w is integrable with respect to the measure F.

The only choice of the agent in our model is whether to participate, or not, in the work force. The participation status of agent a is described with a function $s(a)$, where $s(a)$ is equal to 0 or 1. When agent $a = (w, x, y)$ participates ($s(a) = 1$), she produces w units of commodity, while she does not produce anything when she does not participate ($s(a) = 0$).

The analysis relies heavily on a measure of the disutility of work, which we call *work aversion*, and note $\Delta(c; a)$. The work aversion is the minimum (possibly negative) income supplement which makes agent a indifferent between working or living on resources c without working, i.e. the unique solution in Δ of the equation

$$u(c + \Delta; a) = v(c; a),$$

when such a solution exists. Otherwise we define it as $+\infty$, when the agent does not want to work, whatever the wage ($\lim_{z\to+\infty} u(z; a) < v(c)$), or to $-c$ when she always wants to work ($u(0; a) > v(c)$). Note that, by the implicit function theorem, the function Δ is continuously differentiable with respect to c when it is finite larger than $-c$. We postulate

Assumption 1 *Whenever defined, $\Delta(c; a)$ is a nondecreasing function of c. It is continuously differentiable whenever finite and larger than $-c$.*

The larger income when unemployed, the larger the required income *supplement* to make it worthwhile to take a job. Assumption (1) is the translation in our setup of the idea that leisure is a normal good: the supply of labor is a decreasing function of the level c of income when not working. Indeed, given a gross income at work $c + D$, the agent's labor supply is equal to zero when D is smaller than $\Delta(c; a)$, and equal to one otherwise. Then the fact that $\Delta(c; a)$ increases with c implies that labor supply decreases with c.

We note $G_{c,w,x}$ the c.d.f. of the distribution of work aversions $\Delta(c, a)$ conditional on the agent productivity w and on the observable x

$$G_{c,w,x}(D) = \Pr\ (\Delta(c, a) \leq D \mid w, x).$$

Assumption 1 implies that $G_{c,w,x}(D)$ is a nonincreasing function of c.

An allocation describes the employment status and the income of all the agents in the economy. Formally, it is defined as a pair of integrable functions $s(a)$ and $c(a)$ with values respectively in $\{0, 1\}$ and $\mathbb{R}_+$. An allocation $(s(.), c(.))$ is feasible when total consumption is equal to total production, i.e.:

$$\int c(a)\mathrm{d}F(a) = \int_{s(a)=1} w\mathrm{d}\tilde{F}(a). \tag{1}$$

At the *laissez-faire* allocation, an agent decides to work when her productivity makes it worthwhile, in comparison with a zero income when non participating, i.e. when

$$u(w; a) \geq v(0; a),$$

with indifference when there is equality.

Such an allocation can be very unequal, and it is of interest to look at redistribution schemes that tax the rich workers, with high w's, and give the proceeds to the unemployed. Such a redistribution scheme typically reduces the incentives to work. Indeed if $R(a)$, $R(a) \leq w$, is the income given to worker a, and r, $r \geq 0$, the subsistence level attributed to the unemployed, the decision to work under the redistribution scheme is associated with the inequality

$$u(R(a); a) \geq v(r; a),$$

which is always more stringent than at the market allocation. The purpose of the paper is to look at the tradeoff between equity (more equal utility levels) and efficiency (loss of output due to non participation generated by redistribution) depending on the government objective and to see whether the optimal taxation schemes exhibit some general properties.

There are a number of possible ways to represent society's preferences among equity-efficiency tradeoffs. The one most used in the optimal taxation literature, following the seminal work of Mirrlees (1971), is close to utilitarianism. There is an increasing concave function Ψ, whose concavity is an indicator of society's desire for equality, such that, when $c(a)$ is allocated to an agent of type a, welfare can be written as

$$W_U(c,s) = \int_{s(a)=1} \Psi[u(c(a);a)]\mathrm{d}F(a) + \int_{s(a)=0} \Psi[v(c(a);a)]\mathrm{d}F(a).$$

Diamond (1980) presents an example of an optimal utilitarian tax schedule with fixed hours of work. I focus in this paper on the Rawlsian criterion, which considers the utility of the worse off agents in the economy. Here it is equal to the lower bound of the support of the distribution of utilities in the population, i.e. its essential infimum:

$$W_R(c,s) = \operatorname{ess\,inf}\left\{u(c(a);a)\mathbb{1}_{s(a)=1} + v(c(a))\mathbb{1}_{s(a)=0}\right\}.$$

Take any (increasing in c) function $\bar{v}(c;a)$, and define $\bar{u}$ through $\bar{u}(c+\Delta(c;a);a) = \bar{v}(c;a)$. From Assumption 1, $c+\Delta(c;a)$ is increasing in c, so that $\bar{u}(.;a)$ is a well defined function of c, increasing on its domain, which can be extended over the whole of $\mathbb{R}_+$. From the point of view of the decentralized behavior of the agents, all couples of functions $(\bar{v},\bar{u})$ built in the above fashion are equivalent. By construction, whatever the function $\bar{v}$, the work aversion is unchanged. But the social optimum depends on the particular choice of utility functions. I shall restrict the analysis to the case where the utility when unemployed $v(c;a)$ only depends on the observable characteristics of the agent and therefore can be written as $v(c;x)$: the social planner objective only differentiates the unemployed persons according to their known characteristics, so that the policy instruments to maintain their incomes are in line with the objective.

Assumption 2 The social utility $v(c;a)$ of agent a when she does not work depends only on her observable characteristics x, so that in the sequel we shall note $v(c;a)$ as $v(c;x)$.

Following tradition, I study the social planner choice in stages, starting with the case of complete information of the planner (first best), following with the situation where the planner only observes part of the agents' characteristics (second best).

2.2 First best allocations

The first best allocations are obtained when the planner observes the agents' types a. With my previous notations, this amounts to (temporarily) discarding the unobserved component y from the model. The planner decides whether an agent of type a works or not and attributes her an income $c(a)$ under the feasibility condition.

With a Rawlsian criterion, all efforts are made so that everybody gets the same level of utility. It then is worthwhile putting someone to work if and only if her productivity is larger than the extra income necessary to compensate her for the penibility of work.

The first best Rawlsian allocation $(c(a), s(a))$ can be derived in two steps. First, I take the value of the welfare W_R as a parameter and derive properties of an optimal allocation $(c(a), s(a))$. In a second step, the endogenous value of W_R is obtained through the government budget constraint.

The qualitative properties of an optimal allocation follow from the simple argument below:

- For each agent, let $R(a)$ (resp. $r(x)$) be the *minimum* nonnegative income that ensures that agent a's utility is at least as large as W_R when she works (resp. does not work). Since all transfers between agents are possible, at the optimum, agent a collects $c(a) = R(a)$ when she works and $c(a) = r(x)$ when she does not;
- It must be the case that the government cannot obtain a higher revenue without deteriorating the welfare (otherwise one could use this gain to improve the welfare). The working rule $s(a)$ therefore maximizes government revenue. The problem is simply to compare the government revenue when agent a works $(w - R(a))$ and when she does not $(-r(x))$.

Formally, this is summarized in the following (the elementary proof is left to the reader):

Theorem 1 *Suppose that the optimal Rawlsian allocation* $(c(a), s(a))$ *leads to a social utility level* W_R*,* $W_R \geq W_{\min} = \min_x v(0, x)$*.*

Let $r(x)$ *be equal to zero if* $v(0; x) \geq W_R$ *and to the unique solution of the equation* $v(r; x) = W_R$ *otherwise. Similarly, let* $R(a)$ *be equal to zero if* $u(0; a) \geq W_R$ *(or* $\Delta(r(x); a) = -r(x)$*) and to the unique solution of the equation* $u(R(a); a) = W_R$ *otherwise. Then the incomes* $c(a)$ *of the agents are equal to* $R(a)$ *when they work and* $r(x)$ *when they do not work. Furthermore their employment status* $s(a)$ *is given by*

$$\begin{aligned} w > \Delta(r(x); a) &\Longrightarrow s(a) = 1 \\ w < \Delta(r(x); a) &\Longrightarrow s(a) = 0. \end{aligned} \tag{2}$$

The work status and allocation are indeterminate on the border line, when $w = \Delta(r(x); a)$*. Then society is indifferent between having agent* a *working with income* $R(a)$ *or not working with income* $r(x)$*.*

The workers are exactly compensated for the penibility of their labor and receive an income $R(a) = r(x) + \Delta(r(x); a)$. All agents such that $w > \Delta(r(x); a)$

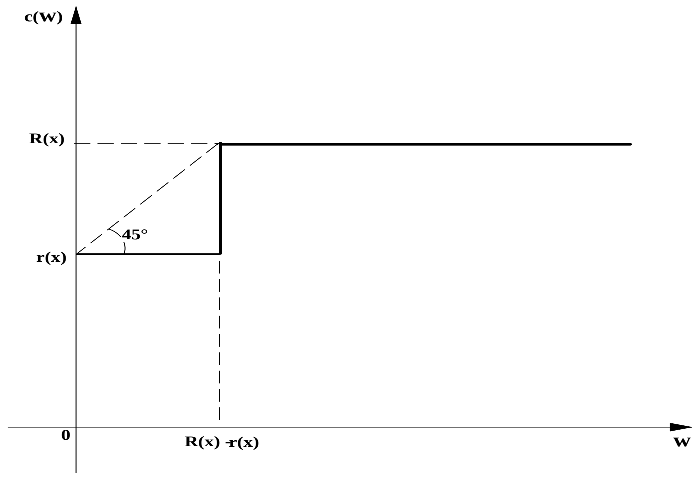


Figure 2: First best allocation: $R - r = w$ for the pivotal agent

are working at the first best Rawlsian optimum, while all agents such that $w < \Delta(r(x); a)$ are not working and receiving $r(x)$. On the other hand, income is disconnected from productivity away from the pivot, being equal either to the subsistence level, or to the reservation wage. As soon as $r(x)$ is positive, under Assumption 1, the first best employment rate is smaller than the market rate.

It is of interest to look more explicitly at how the first best employment status of the agents depends on their types. Fix x, and let productivity w vary. The inequalities (2) determine who is employed and who is not. Since in general the work aversion varies with w, these inequalities are implicit in w: the set of productivities of the working agents is not necessarily a half line, of the form $w \geq \omega$, but can be a union of intervals. At the optimum, around any pivotal agent ω, income is discontinuous (but, of course, utility is constant, equal to W_R). If the agents do not work for w smaller than ω, they receive $c(a) = r(x)$, while the agents who work with w larger than ω get $c(a) = r(x) + \Delta(r(x), a) = R(\omega_+, x)$. The inequality (2) expresses the fact that the discontinuity $R(\omega_+, x) - r(x) = \Delta(r(x); a)$ is equal to the productivity ω of the pivotal agent.

Consider the particular case where the utility functions u and v do not depend on the productivity w, but only on x. Then the work aversion Δ only depends on x. Also the incomes R and r, as defined in Theorem 1, depend on x, but not on w. In that case, inequality (2) gives explicitly the set of workers: the workers are all the agents with productivity w higher than the threshold $\Delta(r(x); x)$. Figure 2 represents the income collected by agent (w, x) as a function of w, for a fixed x. Agents with a productivity lower than $R(x) - r(x)$ do not work, while those with a larger productivity are in the labor force.

It is easy to find the Rawlsian optimum. It is the allocation which satisfies Theorem 1 and balances the government budget. Let W be the unknown optimal social utility level. W is at least as large as $W_{\min} = \min_x v(0; x)$: it is feasible to have everyone not working, with zero income. For $W \geq W_{\min}$, let $\rho(W;x)$ be the unique positive real number which satisfies $v(\rho(W;x)) = W$ when $v(0;x) \geq W$, and 0 otherwise. Then the excess of commodity, or government budget surplus, if one were to implement a Rawlsian allocation satisfying Theorem 1 for this level of W, is

$$T(W) = \int \left\{ [w - \Delta(\rho(W;x);a)] \mathbb{1}_{w \geq \Delta(\rho(W);a)} - \rho(W;x) \right\} \mathrm{d}F(a). \tag{3}$$

Theorem 2 *The government net revenue $T(W)$ is a continuous and decreasing function of W. There exists a unique value $W^\star$ such that $T(W^\star) = 0$. This value determines the first best Rawlsian welfare. The corresponding allocation is given by Theorem 1 above, with $W_R = W^\star$.*

Proof First note that the functions under the sign $\int$ are continuous with respect to W. Furthermore, since ρ is increasing in W by construction and Δ is increasing in its first argument by Assumption 1, $T(W)$ is decreasing in W. For $W = W_{\min}$, $T(W) = \int \max[w - \Delta(0;a), 0] \mathrm{d}F(a)$ is positive. For $W = \operatorname{ess\,sup} v(w;x)$, $\rho(W;x)$ is larger than w, so that $T(W)$ is negative. The Rawlsian optimum corresponds to the unique solution of the equation $T(W) = 0$. ∎

2.3 Second best allocations

When all the characteristics of the agents, productivities and aversions to work, are known to the government, the optimal Rawlsian allocation is very abrupt: every one with a work aversion larger than her productivity is kept out of the work force with an income that brings her to the social objective; all those whose work aversions are smaller than their productivities work, and get the minimal income to reach the social objective. The agents on the border line receive an income supplement when they work, compared with the situation where they would not work, which is just enough to compensate them for their disutility of work.

When everyone has the same (constant) disutility Δ of work and the only information unknown to the government is the individual productivities, the first best result translates directly into an income schedule with a shape similar to that of Figure 2. After tax income is equal to r for all before tax income smaller than Δ, to $r + \Delta$ when before tax income is larger than Δ. This amounts to a marginal tax rate equal to $-\infty$ at the switching point, a large downward tax discontinuity. Such negative income taxes are not seen in practice. Quite the contrary, in France, apart from a temporary subsidy, there is a 100% marginal tax rate on earnings when one takes a job; in the US, a similar feature was associated with the Aid to Families with Dependent Children program before the 1996 reform. It was partially mitigated by the Earned Income Tax Credit. The

EITC can amount to 40% of earnings, i.e. each dollar earned yields 1.4 dollar for the wage earner. After the welfare reform and the replacement of AFDC with TANF, there may exist some income schedules in some states with a zone of negative marginal tax rates. Still, this is far from the kind of discontinuities described above. But of course in practice work aversions are heterogeneous in the population and unobserved by the fiscal authorities: this fact is likely to smooth the shape of the optimal subsidy scheme, as will be seen below.

2.3.1 Characterization of second best Rawlsian allocations

I assume that agent a's productivity w is observed by the government *only when agent a works*, but that the unobservable individual characteristics y of the agent cannot be used to base the tax-subsidy scheme. The government, however, observes the characteristics x and knows the (typically non degenerate) distribution of unobservables y in the economy, conditional on (w, x).

Transfer schedules

The government, without loss of generality[2], posts a menu describing the disposable income that an agent of characteristics x gets either if she does not work, or if t she works with productivity w. A menu is a couple $(r(x), D(w;x))$, with $r(x) \geq 0$, the subsistence revenue of the non worker and $R(w;x) = r(x) + D(w;x)$ the income of the worker. The *financial incentives to work* provided by the government policy are $D(w;x)$. Since $R(w;x) \geq 0$, the incentive $D(w;x)$ is larger than $-r(x)$. Facing such a menu, an agent a chooses either to work and receive $R(w;x)$ or not to work and receive $r(x)$. She decides to work when $\Delta(r(x), a) \leq D(w;x)$, with indifference in case of equality.

Recall that $G_{r,w,x}$ is the c.d.f. of the distribution of work aversions $\Delta(r, a)$ conditional on the agent productivity w

$$G_{r,w,x}(D) = \Pr\ (\Delta(r, a) \leq D \mid w, x).$$

Suppose now that the government posts a schedule $(r(x), D(w;x))$. Then the probability that an agent of type x with productivity w works when she faces this schedule is $G_{r,w,x}(D(w;x))$. The government revenue under the scheme $(r(x), D(w;x))$ can be written as

$$\begin{aligned} T(r(.), D(.)) &= \int [w - D(w;x)] \mathbb{1}_{\Delta(r(x);a) \leq D(w;x)} \mathrm{d}F(a) - \int r(x) \mathrm{d}F(a) \\ &= \int [w - D(w;x)] G_{r,w,x}(D(w;x)) \mathrm{d}\tilde{F}(w, x) - \int r(x) \mathrm{d}F(a), \end{aligned} \quad (4)$$

[2]Any menu yields an allocation $(c(a), s(a))$. It is easy to check that this allocation satisfies the standard incentive compatibility definition. Conversely, Choné and Laroque (2001) show that any incentive compatible allocation such that the utility of any type x agent is at least as large as $v(0;x)$ is dominated by an allocation associated with a well chosen menu.

where $\tilde{F}$ is the joint distribution of productivities and observable characteristics in the population. The pair $(r(x), D(w; x))$ is feasible when it satisfies the budget constraint

$$T(r(.), D(.)) = 0. \tag{5}$$

To characterize the second best allocations, I first take the value of the welfare W_R as given and determine the associated functions $r(x)$ and $D(w; x)$. Then the value of W_R is chosen so that the budget constraint is binding: government revenue must be zero at the optimum.

The maintenance incomes $r(x)$

A first useful remark is that, without loss of generality, if the government wants to obtain a Rawlsian utility level W_R, it can do so while announcing a subsistence income such that the utility of the (potentially) unemployed is equal to W_R, and this for every type x.

Theorem 3 *Let $(\bar{r}(x), \bar{D}(w; x))$ be a schedule, associated with a value W of the Rawlsian criterion.*

Let $\rho(W; x)$ be equal to zero when $v(0; x) \geq W$ and to the unique solution in r of the equation $v(r; x) = W$ otherwise. Then there exists a schedule $(r(x), D(w; x))$, with $r(x) = \rho(W; x)$, such that

$$T(r(.), D(.)) \geq T(\bar{r}(.), \bar{D}(.)).$$

Proof: 1) Consider the types x such that $v(\rho(W; x); x) < W$. All agents of type x work. Let $D(w; x)$ be such that $\rho(W; x) + D(w; x) = \bar{r}(x) + \bar{D}(w; x)$. Faced with the schedule $(\rho(W; x), D(w, x))$, they get the same utility level, with the same employment status, while government income is unchanged.
2) From above, I need only consider schedules such that $\bar{r}(x) \geq \rho(W; x)$ for all x. Now first replace $\bar{D}(w; x)$ with w whenever w is larger than $\bar{D}(w; x)$: this does not decrease welfare, because the subsistence option is open to everyone, and it increases government income if there were agents employed at a cost larger than their productivity. Finally, replace $\bar{r}(x)$ with $\rho(W; x)$, whenever the inequality is strict. Government spending on subsistence income decreases, and government revenue increases under assumption 1, $\Delta(\rho(W; x); a) \leq \Delta(\bar{r}(x); a)$, so that the set of employed agents, which bring non negative receipts (recall that $w \geq \bar{D}(w; x)$), is larger. ∎

It follows that when looking for a Rawlsian optimum, one can restrict the attention to schedules where $r(x)$ is of the form $\rho(W; x)$, for some W, as in Theorem 3. Furthermore, by construction, increasing $r(.)$ increases the social welfare.

Characterization of incentives $D(w;x)$

Given the subsistence income, the following theorem provides a characterization of the least costly financial incentives that guarantee a welfare level equal to W_R. We let $\bar{d}(r|w,x)$ (possibly equal to $+\infty$) denote the maximum of the support of $G_{r,w,x}$.

Theorem 4 *If $(r(x), D(w;x))$ is an optimal schedule, then $D(w;x)$ is an element of*

$$\text{argmax}_{\,D}(w-D)G_{r,w,x}(D). \tag{6}$$

As a consequence, we have, for all w, $D(w;x) \le w$ and $D(w;x) \le \bar{d}(r|w,x)$.

Proof: Suppose that changing the incentive scheme from $D(w;x)$ to $D_1(w;x)$ increases government revenue by some $\alpha > 0$. In that case, it would be possible to redistribute this gain to all the agents (replacing $r(x)$ with $r(x)+\alpha$) and to increase the value of the Rawlsian welfare. We conclude that the schedule $D(w;x)$ must maximize the government revenue. The result follows directly from equation (4). ■

An interesting property of the optimal allocation is that, for each $w \ge 0$ and each x, there exists an agent with productivity w whose utility is equal to $v(r(x);x)$. This is a consequence of Theorem 4:

- if there exists an unemployed agent with productivity w, we know that this agent's utility is $v(r(x);x)$;
- otherwise[3], all agents of type x and productivity w are employed. Then $G_{r,w,x}(D(w;x)) = 1$. Thus $D(w;x) \ge \bar{d}(r|w,x)$. Theorem 4 gives the inequality in the other direction. Therefore the agent of type x with productivity w and work aversion $\bar{d}(r|w,x)$ is just indifferent between working and not working.

All the workers with productivity w and work aversion strictly lower than $D(w;x)$ get a rent, i.e. their utilities are strictly larger than the social norm. In the second best environment, the government cannot extract all the rent from the agents. Hereafter, we denote $K_{r,x}(w)$ the value of the maximum

$$K_{r,x}(w) = \max_{D\le w}(w-D)G_{r,w,x}(D). \tag{7}$$

[3]Note that it may happen that all agents work in the second best optimal allocation. A necessary condition for no unemployment at the optimum is $w \ge \bar{d}(r|w,x)$ for all w (this requires in particular that $\bar{d}(r|w,x) < +\infty$ for all w,x). Welfare W is then the unique solution to the government budget constraint, which writes, in that case

$$\int w \mathrm{d}\tilde{F}(w) = \int [\rho(W;x) + \bar{d}(r|w,x)]\mathrm{d}\tilde{F}(w,x).$$

The quantity $K_{r,x}(w)$ is the share of the total surplus w collected by the government on agents of type x with productivity w (the government leaves $D(w;x)$ to the workers as an incentive to work). I can now complete the characterization by determining the value of the social welfare.

Determination of social welfare

Theorems 3 and 4 give a procedure to construct a Rawlsian optimum. For any W in $[W_{\min}, \operatorname{ess\,sup} v(w;x)]$, note $T(W)$ (with a slight abuse of notation) the government net revenue

$$T(W) = \int_w K_{\rho(W;x)}(w)\mathrm{d}\tilde{F}(w) - \int \rho(W;x)\mathrm{d}F(a). \tag{8}$$

Choné and Laroque (2001) show that, under Assumptions 1 and 2, the surplus $K_r(w)$ is a continuous and nonincreasing function of r. It follows that the government revenue $T(W)$ is a continuous and decreasing function of W. It is non negative at $W_{\min}$, negative at the upper end of its domain, so that it has a unique zero, equal to social welfare at the optimum. The subsistence revenue and the incentives to work at the Rawlsian optimum are then given by Theorems 3 and 4.

Some simple comparative statics results follow. Using the definition (7) of K_r, we see that the function K_r decreases when the distribution of work aversions first order stochastically increases ($G_{r,w}$ *decreases*). Therefore the government revenue also decreases for all r (see equation (8)).

> *Under Assumptions 1 and 2, the second best Rawlsian optimum utility level W_R decreases when the distribution of work aversions $G_{r,w,x}$ first order stochastically increases for all (w,x).*

When the distribution of work aversions does not depend on w, using definition (7), we see that K_r is a nondecreasing function of w. It follows that the government revenue T (and consequently the optimum W_R) increases when the distribution of w stochastically increases ($G_{\rho(W;x),x}$ being fixed). Consequently, *Under Assumptions 1 and 2, when the distribution of work aversions does not depend on productivity, the second best Rawlsian optimum utility level W_R increases when the distribution of productivities $\tilde{F}(w)$ first order stochastically increases.*

2.3.2 Qualitative analysis

The Rawlsian problem exhibits a particular structure, which makes many qualitative properties of optimal schedules easy to derive for a large class of distributions of work aversions. This structure is most transparent when the distribution of work aversions is independent of the productivity of the agents:

Assumption 3 *The conditional distribution of work aversions $G_{r,w}(.)$ is independent of w.*

I first describe the fundamental structure of the problem and give a geometric representation. I then look at some particular cases, in particular at the circumstances under which it is optimal to use negative marginal tax rates.

The basic structure of the problem

The optimization problem has two important features: the objective is linear with respect to productivity w and it depends in a simple way on the distribution of work aversions.

Theorem 5 *Consider a second best Rawlsian allocation. Under Assumptions 1 to 3, we have*

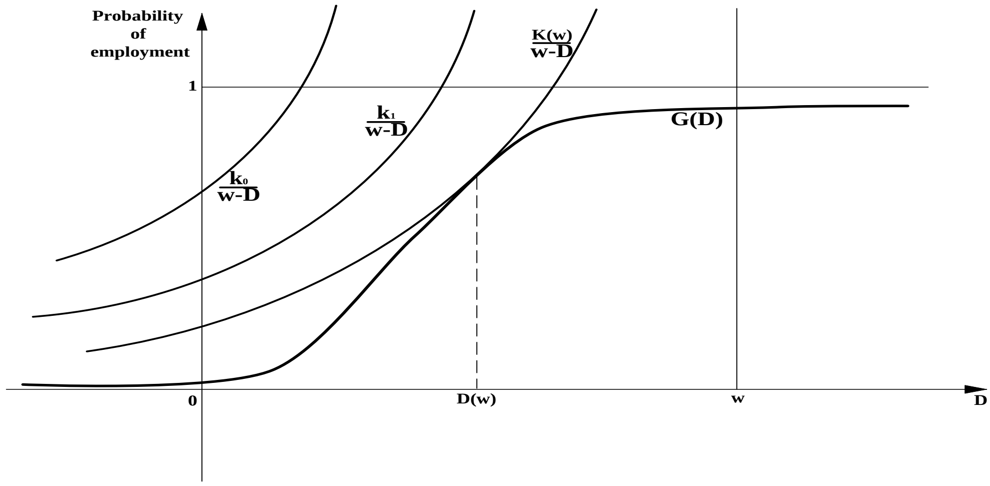
1. *The surplus $K_r(w) = (w - D(w))G_r(D(w))$ raised by the government at the optimum is a non decreasing convex positive function of w, of slope at most equal to 1.*

2. *$D(w)$ is a nondecreasing function of w, with $D(w) \leq w$. The proportion of agents of productivity w at work, $G_r(D(w))$, is also nondecreasing in w.*

Proof From Theorem 4, $K(w)$ is the supremum of the set of linear mappings $(w - d)G_r(d)$, where d is any real number. It is positive ($d = w$ is possible), convex as the supremum of convex functions. $G_r(D(w))$ is a subgradient of $K(w)$, whose slope cannot thus exceed 1. Convexity implies that the subgradient is nondecreasing, which implies that $G_r(D(w))$ is nondecreasing in w, and $D(w)$ as well. ■

The theorem shows that, under Assumption 3, the marginal tax rates $1 - D'(w)$ are less than or equal to 1. The fact that $D(w)$ is nondecreasing implies that it would not be in the interest of an agent to announce a productivity lower than the truth, if this were allowed. The tax schedule is incentive proof to the mimicking of agents with lower productivities.

A graphical representation helps to understand the structure of the problem. On the top panel of Figure 3, the c.d.f. $G_r(D)$ is plotted: if D is selected by the government, $G_r(D)$ is the proportion of agents that are willing to work. For a given value of w, the problem (see the definition of K_r (7)) is to find the maximum value of k such that $k/(w - D)$ intersects the graph of the c.d.f.. Therefore, for a given w, I draw a bunch of isoquants of the form $k/(w - D)$, all arcs of hyperbolas whose asymptotes are the negative D axis and the vertical line of abscissa w. The solution is at the highest isoquant which is tangent to the c.d.f.. When



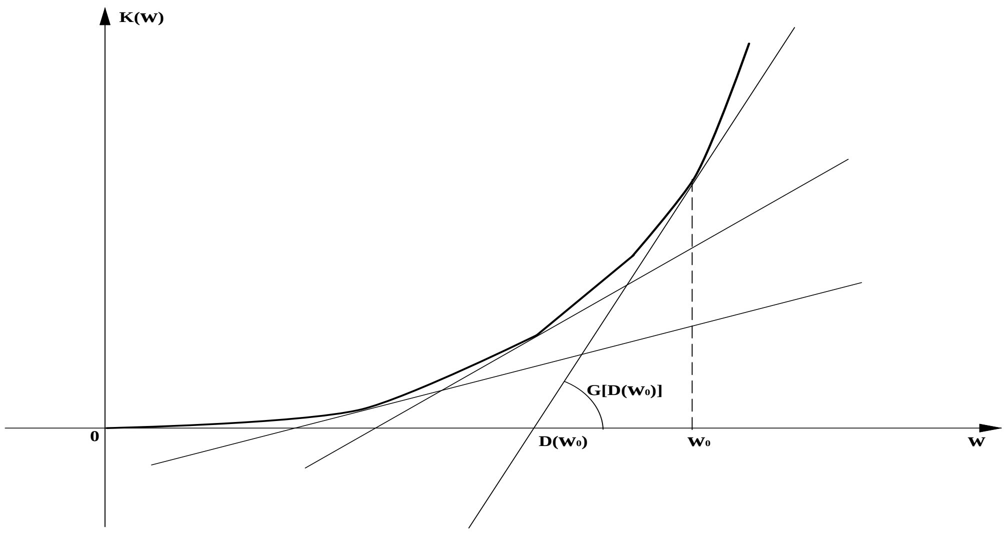


Figure 3: The optimization program

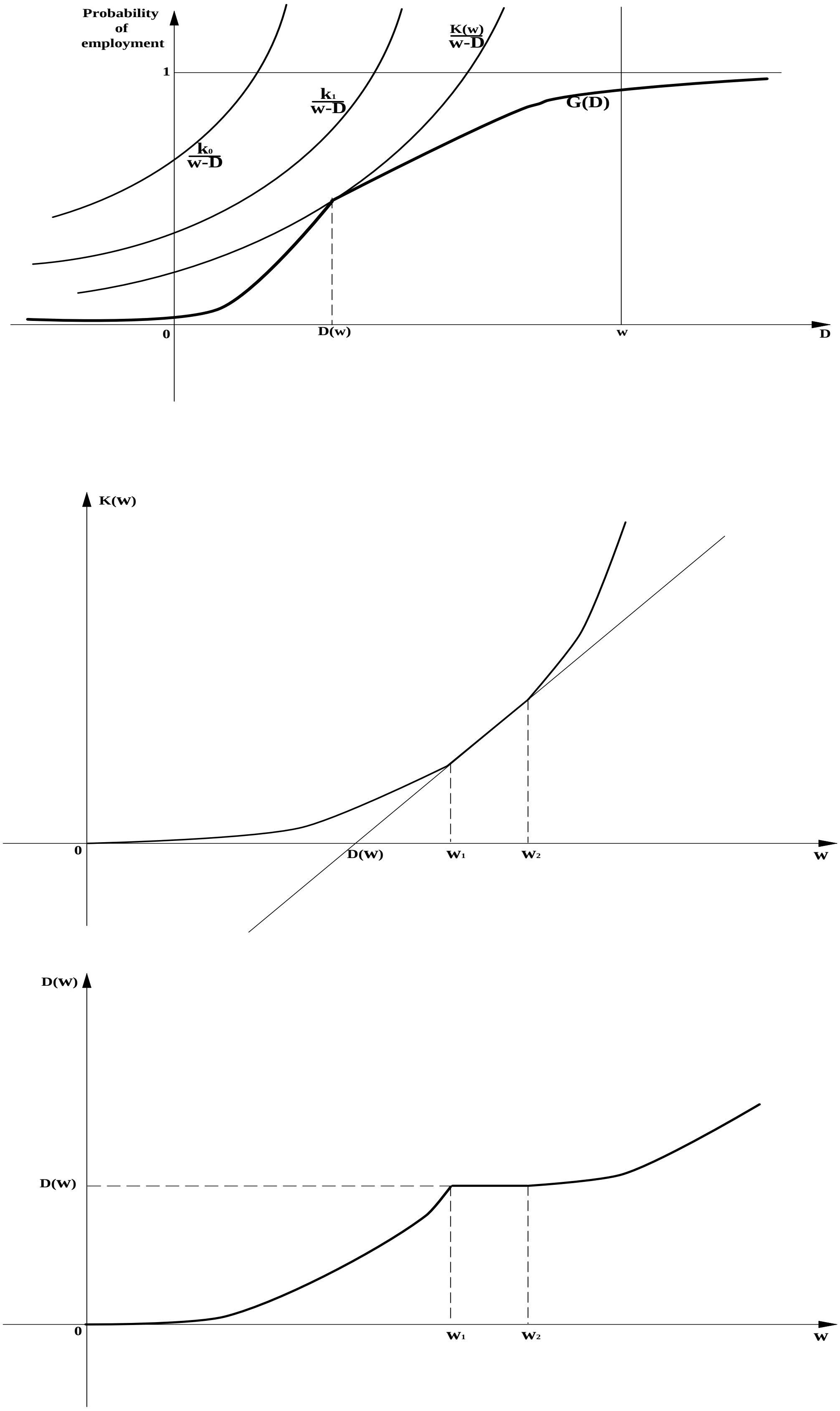


Figure 4: 100% marginal tax rate

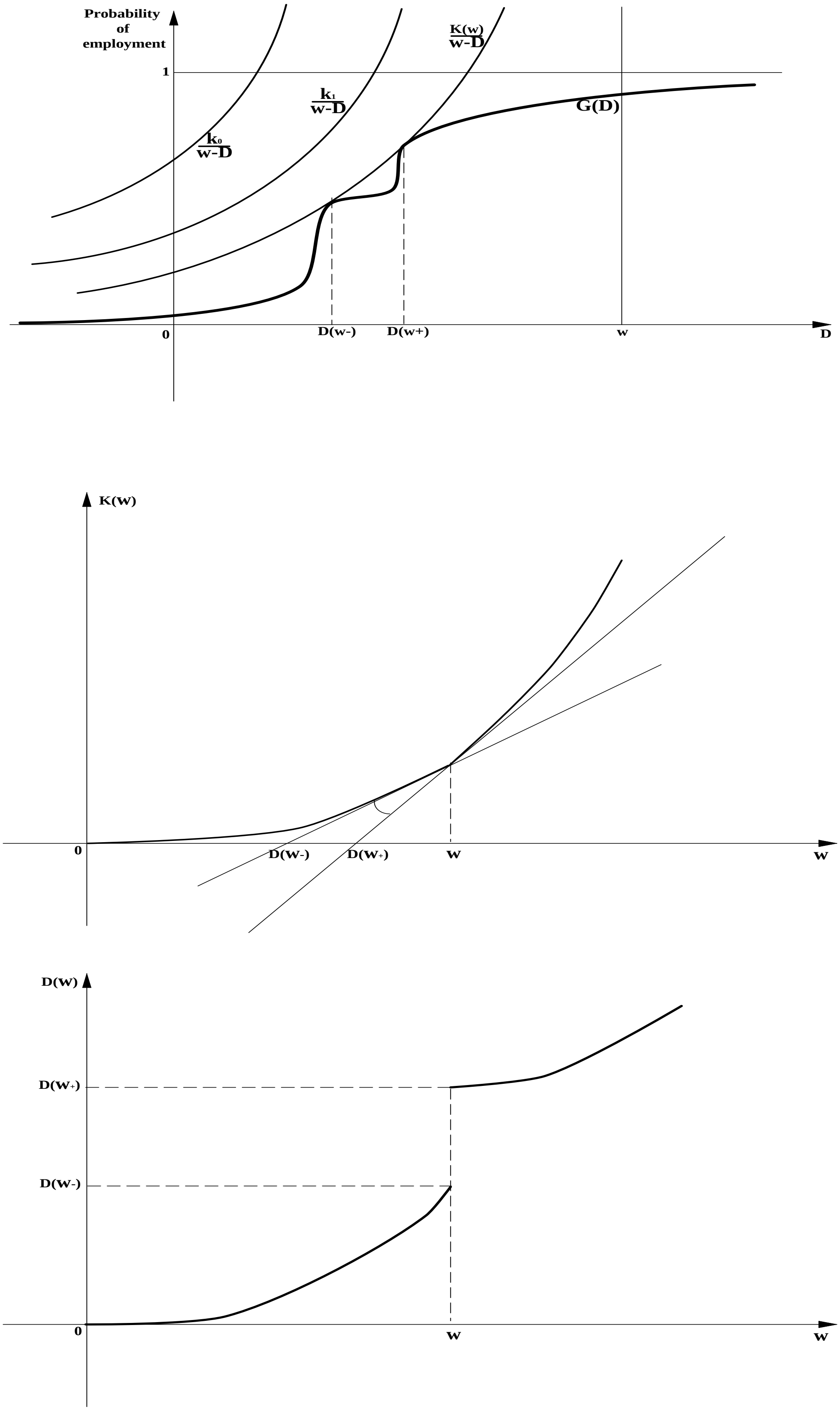


Figure 5: Discontinuity of the tax scheme

w increases, the hyperbolas translate to the right, so that both $D(w)$ and $K_r(w)$ increase. It is also of interest to locate the market allocation on the graph. For r equal to zero, all the agents with a work aversion larger than w, the top of the distribution, do not work and get 0. The rest of the population works and enjoys a surplus from work equal to the horizontal distance between the graph of G and the vertical line of abscissa w. Unless the distribution is concentrated on a single point, a Rawlsian optimum always increases unemployment, provides the unemployed with a positive income r rather than 0 (note that under Assumption 1 this pushes the graph to the right, also in the direction of increased unemployment), and reduces the surpluses of the employed to the distance of the (new) graph of G to the financial incentive to work D, $D \leq w$.

As we will see later, the point-wise optimization program, for a specific value of w, needs not be well behaved. However, the overall optimization is simple, as shown on the bottom panel of Figure 3 drawn in the plan $(w, K(w))$. The maximization involves taking the upper envelope $K(w)$ of a set of straight lines of equation $(w - D)G_r(D)$, when D varies. The typical line intersects the w axis at D, and has slope $G_r(D)$, a number between 0 and 1. The function $K_r(w)$ is increasing convex (and therefore continuous), and has a slope everywhere smaller than 1.

The case of a 100% marginal tax rate

Figure 4 shows a case where a 100% marginal tax rate is optimal. This occurs when the optimum is at a kink of the graph of the c.d.f.(top panel): $D(w)$ and $G_r(D(w))$ stay constant on a range of productivities. Let w_2 be such that the corresponding isoquant is tangent to the left piece of the graph of the c.d.f.: $K(w_2)/(w_2 - D(w))^2 = G_r'(D(w)-)$. Similarly define w_1 for a tangency from the right: $K(w_1)/(w_1 - D(w))^2 = G_r'(D(w)+)$. All the agents a with the same work aversion $\Delta(r; a) = D(w)$ and productivity $w_1 \leq w(a) \leq w_2$ receive the same income $r + D(w_1)$: the marginal tax rate is equal to 100%. This translates into a linear portion of K (middle panel of Figure 4), and yields a flat bit in the income schedule with a 100% marginal tax rate (bottom panel).

When are negative marginal tax rates optimal?

The top panel of Figure 5 shows a situation where there are two tangency points. For this particular value of w, both $D_1(w)$ and $D_2(w)$ yield the optimum $K_r(w)$. All the agents with work aversions between these two values are always treated in the same way, either being non employed (for productivities smaller than w) or employed (for productivities larger than w)[4]. This results in an upward

[4] The choice of agents with productivity *equal* to w and work aversion between $D_1(w)$ and $D_2(w)$ depends on whether the government posts $D(w) = D_1(w)$ or $D_2(w)$ (the government is

discontinuity of $D(w)$ or, equivalently, an infinite negative marginal tax rate at w.

Two remarks are necessary at this stage. First, such discontinuities have nothing pathological: they will occur as soon as the c.d.f. has pieces that are flatter than the arc of hyperbola going through them, for instance for discrete distributions. Second, I have represented the extreme case of an infinite negative tax rate. This should not induce the reader to believe that *finite* negative marginal tax rates are impossible. Actually, almost every nondecreasing schedule is indeed optimal for some distribution of work aversion (see 2.3.3 below).

To understand intuitively why negative marginal tax rates can be optimal, suppose there is an accumulation of agents with work aversion close to d (d being known to the planner). Recall that work aversion is unobserved in the second best environment: the only available screening variable is w. For small w, it is too costly to put these agents to work; and it is optimal to do so for large w. If the distribution of work aversion is very concentrated around d, the second best solution is such that the incentives strongly increase ($D'(w) > 1$) precisely at the point w such that $D(w) = d$.

There exists a simple regularity assumption on the distribution of work aversion guaranteeing that negative marginal tax rates are never optimal. In particular, this assumption rules out mass points in the distribution G.

Proposition 6 *When G is log concave, the optimal marginal tax rate is everywhere nonnegative.*

Proof The problem (6) can be rewritten $\max_{D \leq w} \ln(w-D)+H(D)$, with $H(D) = \ln G$. Since H is concave, the function $D \to \ln(w-D)+H(D)$ is strictly concave and has a unique maximum, characterized by the first order conditions[5]

$$H'(D) = \frac{1}{w-D}.$$

Since D is nondecreasing and H' is nonincreasing, it follows that $w - D(w)$ increases in w, which gives the result. ∎

As mentioned in the introduction, one feature of the intensive model à la Mirrlees is that marginal tax rates are nonnegative at the optimum. This property

indifferent between these two possibilities). Note that, for some w, the hyperbola and the c.d.f. graph of G could be tangent along a continuous portion of the hyperbola. In that case, the government would be indifferent between all the corresponding values of D for the productivity w.

[5]When G has a kink, the first order condition is that 0 is in the subgradient of $\ln(w-D)+H(D)$.

holds in the current framework *provided that the distribution of work aversions is log-concave.* In this circumstance, the use of negative tax rates (like with EITC in the US or WFTC in the UK) is not justified by incentive purposes under a Rawlsian optimality criterion.

To sum up, we have found two polar cases: the log-concave case, where the problem $\max(w - D)G(D)$ is concave for all w and the marginal tax rates are nonnegative on the one hand, the case represented in Figure 5 where the problem $\max(w - D)G(D)$ has distinct solutions and infinite negative marginal tax rates can happen, on the other hand. Between these polar cases, virtually every intermediate pattern is possible, including finite marginal negative tax rates as discussed below.

2.3.3 The inverse problem

For policy recommendations, it is of interest to know whether theory restricts the type of income schedules that may arise at a Rawlsian optimum. Consider a schedule $(r(x), D(w; x))$. Under which conditions is it possible to find an economy (i.e. utility functions u and v and distributions of work aversion $G_{r,w,x}$) such that $(r(x), D(w; x))$ is a Rawlsian second best optimal scheme for some level T of government income? To be more in line with real life circumstances, I want to give little advanced knowledge to the planner: I restrict the attention to economies where the distribution of work aversions is independent[6] of w, as in Assumption 3. Choné and Laroque (2001) show that theory by itself imposes very little restrictions on the tax schedule: if the productivity distribution has an upper bound, essentially any increasing function $D(w; x)$, with $D(w; x) < w$, is the incentive schedule of a well chosen economy. Therefore one cannot have a marginal tax rate larger than 100%, but 100% tax rates are allowed, as well as negative taxes, including marginal tax rates equal to $-\infty$ corresponding to an upward discontinuity in the incentives.

2.3.4 The utilitarian case revisited

The utilitarian case has been explored by Diamond (1980). Although it has a similar structure as the Rawlsian problem, there are additional terms in the objective function and the above qualitative analysis does not carry over. These additional terms come from the fact that the planner now values the utility level of all the agents: the surplus of the workers explicitly enters the Lagrangian.

[6] The problem has an easy, economically uninteresting, solution, if one allows for distributions of work aversions that depend on w. Indeed, consider the economy where all the agents of type x with productivity w have the same work aversion, equal to $D(w; x)$: formally $G_{w;x}$ is the Dirac mass at the aversion level $D(w; x)$. Then its optimal tax scheme is $D(w; x)$. It puts everyone to work, which maximizes the government revenue to be distributed through $\rho(W; x)$ among the population. This is rather unrealistic.

This contrasts with the Rawlsian case, where the planner needs only consider the labor supply *behavior* of the agents.

Utilitarianism takes into account society's desire for equality through an increasing concave function Ψ, which is applied to the agents' utilities. To illustrate the argument, consider the situation where the government does not differentiate the tax scheme according to the characteristics x. The utilitarian welfare criterion then can be written as

$$W_U = \int \{\Psi[u(r + D(w(a)); a)]\mathbb{1}_{\Delta(r,a)\leq D(w(a))} + \Psi[v(r)]\mathbb{1}_{\Delta(r,a)>D(w(a))}\}\mathrm{d}F(a).$$

The Lagrangian associated with the optimization problem is

$$\begin{aligned}\mathcal{L}_{\mathrm{U}} &= \int \{\Psi[u(r + D(w(a)); a)] - \lambda(r + D(w(a)) - w(a))\}\mathbb{1}_{\Delta(r,a)\leq D(w(a))}\mathrm{d}F(a) \\ &+ \int \{\Psi[v(r)] - \lambda r\}\mathbb{1}_{\Delta(r,a)>D(w(a))}\mathrm{d}F(a).\end{aligned}$$

Let $H(a|w)$ be the c.d.f. of the distribution of a in the population, conditional on the productivity level $w(a) = w$. One can substitute $\mathrm{d}F(a)$ with $\mathrm{d}H(a|w)\mathrm{d}\tilde{F}(w)$ and rewrite the objective function conditionally on w. It follows that for each w, the optimal financial incentive to work $D(w)$ is solution to

$$K_r(w) = \max_D \{(w - D + \tau_{r,w}(D))G_{r,w}(D)\} \tag{9}$$

where $\tau_{r,w}(D)$ denotes the average rent of the employed agents with productivity w

$$\tau_{r,w}(D) = \frac{1}{\lambda}\frac{1}{G_{r,w}(D)}\int \{\Psi[u(r + D, a)] - \Psi[v(r)]\}\mathbb{1}_{\Delta(r,a)\leq D}\mathrm{d}H(a|w).$$

The function τ, however, cannot be expressed in terms of the c.d.f. G_r only, so that the simple geometric interpretation given in the Rawlsian context does not carry over to the utilitarian case.

A utilitarian version of Theorem 5 may, however, be stated. Under a stronger independence condition, so that the distribution of $u(c; a)$ and $\Delta(r; a)$ are both independent of w, the function $K_r(.)$ defined by (9) is convex with respect to w, and its derivative is equal to the employment rate in the productivity class w; that rate, therefore, is increasing with productivity.

It is possible to compare the optimal second best Rawlsian and utilitarian allocations. First, as we have seen, the Rawlsian criterion amounts to maximize the subsistence income level r. It follows that

> *The subsistence income r_U^{SB} at a second best utilitarian optimum is smaller than the corresponding Rawlsian subsistence income r_R^{SB}.*

The comparison of the financial incentives in the two cases is not as straightforward. Given r, the programs which determine the financial incentives, respectively (9) and (7), yield always a level $D_{U,r}(w)$ in the utilitarian situation that is larger than the Rawlsian outcome $D_{R,r}(w)$. This is natural, since the utilitarian planner puts weight on the rents enjoyed by the workers[7]. However, as just seen, the subsistence level is smaller at the utilitarian optimum, which reduces the need for financial incentives under Assumption 1. The two effects play in opposite directions, which makes it impossible to conclude in general. However, in the case where r does not modify the work aversion

> *When the distribution of work aversions is independent of the subsistence level r, the financial incentives $D_R(w)$ at the Rawlsian optimum are smaller than at the utilitarian optimum $D_U(w)$.*

3 An empirical illustration

Theory in itself is of little guidance as to the shape of the optimal income support schedules, which are determined by the distributions of work aversions in the population. To use the above theory in practice, one has to postulate and estimate a labor supply model to derive the distributions we are looking for.

In the remainder of the paper, I shall rely on a model developed on French data in joint work with Bernard Salanié. The model abstracts from a number of important features of real life, and the results below should be considered illustrative. The model is static and is applied to women who either do not work or have a full time job[8]. It specifically accounts for the high level of the minimum wage in France. The structure of the model is as follows. The typical woman's productivity satisfies :

$$\ln w = X\alpha + \sigma_\varepsilon \varepsilon, \tag{10}$$

where X includes age at end of studies and its square, work experience and its square and diploma in six categories. A woman has a job if the three following inequalities are satisfied:

1. Her productivity is higher than the cost to an employer of the minimum wage

$$w \geq w_{\min},$$

[7]This property follows from the fact that $\tau_{r,w}(D)G_{r,w}(D)$ is an increasing function of D. Therefore, for all D smaller than D_R, $\tau_{r,w}(D_R)G_{r,w}(D_R)$ is larger than $\tau_{r,w}(D)G_{r,w}(D)$. This implies that the maximum of (9) is reached for some D larger than D_R.

[8]The same model has been estimated for various subsets of the French population of working age (see e.g. Laroque and Salanié (2000)). For lack of information on their incomes, households with a self employed person are excluded from the analysis. Also the civil servants who have tenure are excluded from the sample under study.

2. She is not subject to frictional or keynesian unemployment

$$\nu \leq P_k(Y\beta), \tag{11}$$

where ν is uniformly distributed in the interval $[0, 1]$, P_k is a given function from $\mathbb{R}$ into $[0, 1]$, and the variables Y include diploma and age;

3. Finally, she is willing to work, i.e.

$$R(w) \geq R(0) + Z\gamma + \rho\varepsilon + \sigma_\eta \eta. \tag{12}$$

Here $R(.)$ is a known highly non linear function: $R(w)$ is the net after tax and subsidies income when the cost of labor to the employer is equal to w. The variables Z include the out of work income $R(0)$ itself, as well as the family composition (presence of a spouse, number of children by age range).

The unobserved heterogeneity is described by the triple (ε, ν, η). The model is estimated by maximum likelihood under the assumption that the three random terms are independently distributed. It is assumed that ε is distributed as a standard normal, ν as a uniform on the interval $[0, 1]$, and η as a logistic. Under this parametric assumption, it is easy to recover the distribution of the work aversion Δ, equal to $Z\gamma + \rho\varepsilon + \sigma_\eta \eta$, which we are interested in (see below).

3.1 Semiparametric identification and estimation

The above model depends heavily on the assumed distributions of heterogeneity. It is of interest to see whether the data allow to identify these distributions, while keeping with the chosen exogenous variables and the functional forms used to describe their influence. The model is complicated and I shall follow a piecemeal approach, using intuitions from results in the literature obtained in simpler setups for models with a single or two equations, without formal proofs.

3.1.1 Identification

1. *Minimum wage.* The cost of the minimum wage relative to the distribution of observed wages appears to be high in France, by comparison with other developed countries. This potentially creates difficulties to identify the distribution of heterogeneity in the wage equation. Indeed, if one does not have some observed exogenous variables determining productivity (here mainly the diploma), there is no hope to know the distribution of potential wages *below* the minimum wage (Meyer and Wise (1983a) or Meyer and Wise (1983b)). Semiparametric identification here relies on the assumption that the distribution of heterogeneity does not depend on the diploma, and that the more skilled agents have a wage distribution with support above the minimum wage.

2. *Frictional or keynesian unemployment.* The specification of this component of unemployment is ad hoc in the model. Exclusion restrictions make the distribution identifiable. Indeed, we assume that the heterogeneity in this type of unemployment does not depend on some of the exogenous variables which determine productivity (age at end of school, its square and experience squared). Then provided long studies and a lot of experience characterize a set of persons who want to work and are not barred by the level of the minimum wage, their only reasons for being unemployed are frictional or keynesian, which allows to identify the distribution I am looking for.

3. *Participation equation.* The coefficients α and γ of the labor demand and labor supply equations are identified from exclusion restrictions: the diplomas appear in labor demand, not in labor supply, while family composition, spouse income and the tax scheme are determinants of labor supply, not of labor demand. The issue of interest is whether the distribution of η, initially assumed to be logistic, can be recovered from the data. By analogy with the familiar analysis of single index models (see e.g. Horowitz (1998)), everything else being given, the distribution of η in equation (12) seems to be identified under location and scale normalizations, since there is a continuous variable among the Z's, the income of the spouse which is an implicit argument of $R(0)$.

The above arguments discuss in turn the semiparametric identification of each of the distributions of ε, η and ν, the other two being given. I do not know whether the joint distribution of (ε, η, ν) is identified. In any case, I limit the attention here to the study of the distribution of η, maintaining the assumption that ν and ε are independently distributed as uniform on $[0, 1]$ and standard normal. It may be of some comfort to know that, as far as ε is concerned, lognormality is not rejected by the data holding the distributions of ν and η at the maintained hypothesis (see Laroque and Salanié (2002) for a test against a mixture of lognormals).

3.1.2 Estimation of the distribution of η

According to the above argument, one should be able to estimate the distribution of η. At the very least, it seems desirable to check whether the parametric assumption of a logistic under which the first estimation has been carried out is acceptable. This is potentially of practical importance, since in the simulations below, the standard error of the estimated work aversion is of the order of 1150 euros, while the standard error of the unobserved heterogeneity term amounts to 850 euros. I have explored some of the various possible directions[9] that seem

[9] A natural route which I did not follow yet is to describe the distribution of η as a mixture of distributions in an *appropriate* basis space. Since it is difficult to pin down what is *appropriate* in this setup, I started within an agnostic semiparametric framework.

open at this stage. It turns out that the unknown distribution of η only enters the likelihood function through its cumulative distribution function. This makes it particularly easy to use an adaptive estimation technique[10]: start with a maximum likelihood estimation of the parametric model; then estimate the c.d.f. of η, given the parameter values from the previous step (ignoring the assumed distribution of η) through a suitable semiparametric procedure; iterate the maximum likelihood estimation with the computed c.d.f., numerically interpolated, instead of the initial logistic, keeping fixed the location and scale parameters in equation (12) at their first step values. The crucial element of the procedure of course is the estimation of the unknown c.d.f.. I have looked at several possibilities.

1. The first idea that comes to mind is to use equation (12) to directly estimate the distribution of $R[W(\varepsilon)] - \rho\varepsilon - \sigma_\eta\eta$, where according to (10) $W(\varepsilon) = \exp(X\alpha + \sigma_\varepsilon\varepsilon)$. Indeed, if e is the indicator variable for employment, equal to 1 when the woman is employed and to zero otherwise, the following equality holds:

   $$E\left\{\frac{e}{P_k(Y\beta)}|Y\right\} = \Pr\{[R[W(\varepsilon)] - \rho\varepsilon - \sigma_\eta\eta \geq R(0) + Z\gamma]|W(\varepsilon) \geq w_{\min}, Y\}.$$

   A non parametric regression of $e/P_k(Y\beta)$ on $R(0) + Z\gamma$ therefore yields the cumulative distribution of $\rho\varepsilon + \sigma_\eta\eta - R[W(\varepsilon)]$, conditional on $W(\varepsilon)$ being larger than the cost of the minimum wage, and on the variables Y (diploma, age). To obtain the distribution of η, one faces a deconvolution problem, since the distribution of $R[W(\varepsilon)] - \rho\varepsilon$, conditional on $W(\varepsilon)$ larger than $w_{\min}$ and on Y, is known. Note that the function $R[W(\varepsilon)]$ is highly nonlinear and depends on a number of exogenous variables, such as the household composition and the spouse income. Unfortunately, this makes the deconvolution problem untractable in practice.

2. To handle the difficulties associated with the function $R[.]$, a more promising road is to simulate the residuals of the structural model, conditional on the observations. For each observation, it is relatively easy to draw the residuals (ε, ν, η) in their joint distribution conditional on the observed employment status, and on wage when employed[11]. In the tradition of the generalized residuals literature (Gouriéroux, Monfort, Renault, and Trognon (1987)),

[10] A similar technique might be used more generally, in the estimation of structural models, when the distributions of the random terms are semiparametrically identified. However the implementation would typically involve more complicated functions of the unknown distributions than the mere c.d.f..

[11] This was done through the Gibbs sampling algorithm. For an unemployed person, given η and a value of ν smaller than $P_k(Y\beta)$, the difficulty is to draw a value of ε such that either $W(\varepsilon) < w_{\min}$ or

$$R[W(\varepsilon)] < R(0) + Z\gamma + \rho\varepsilon + \sigma_\eta\eta. \qquad (*)$$

Let $\varepsilon_{\min}$ be the value of ε such that $W(\varepsilon)$ is equal to w_{min}. The half line $\varepsilon \geq \varepsilon_{\min}$ is divided into

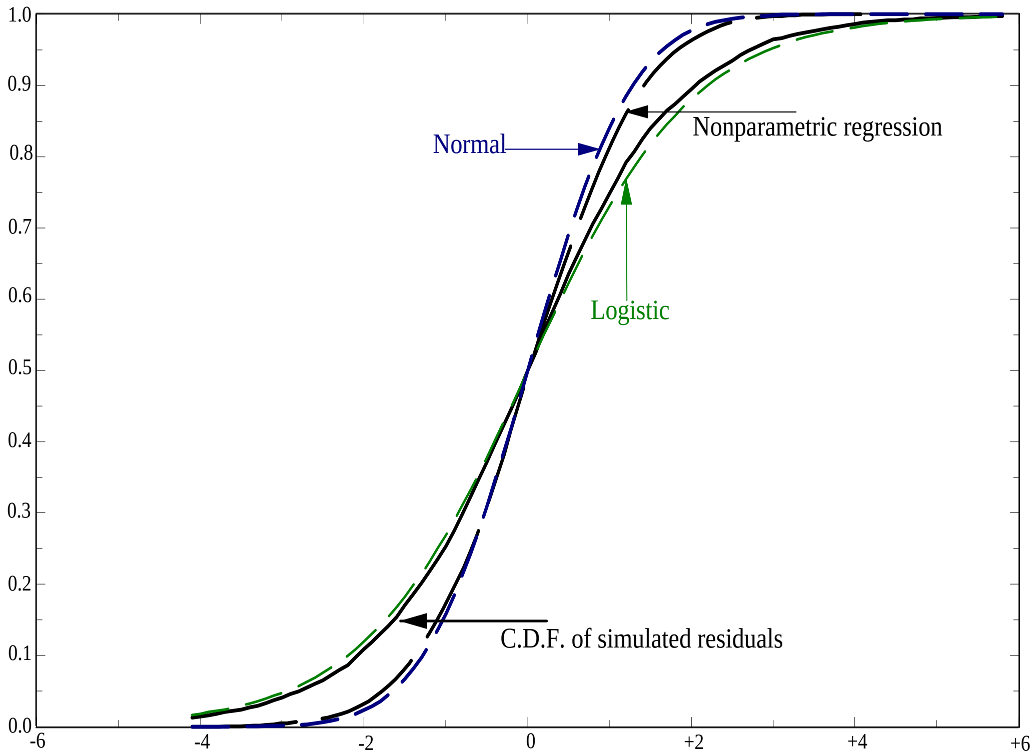


Figure 6: Simulated residuals vs. nonparametric regression

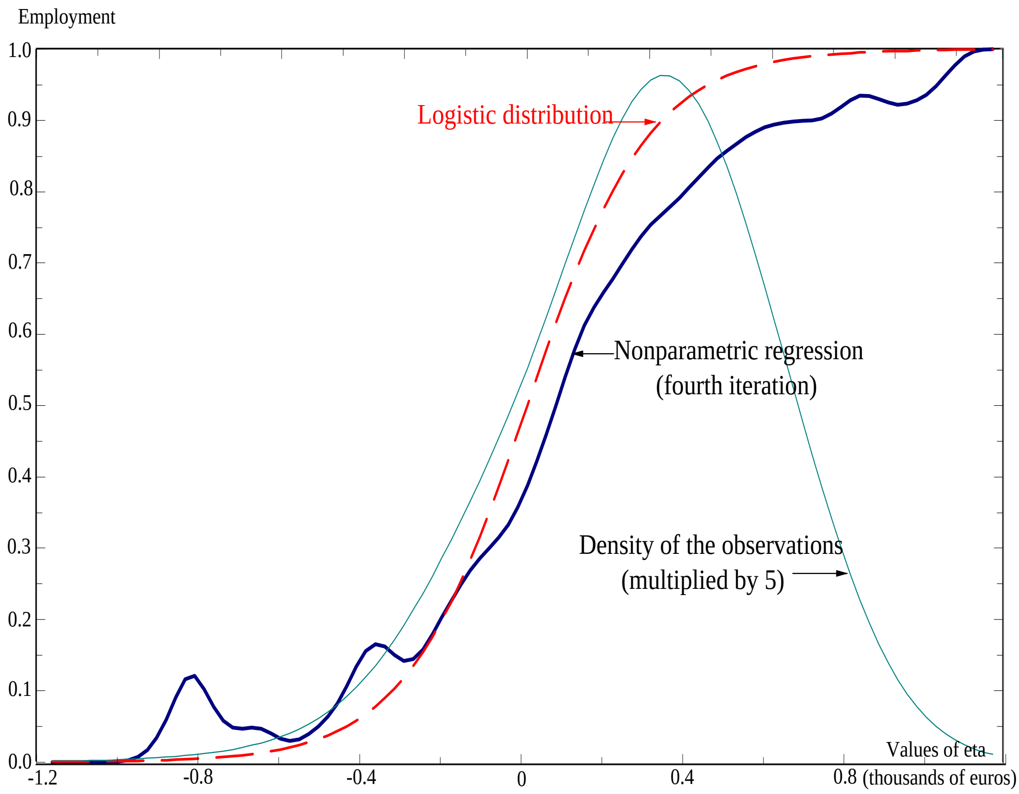
one then can build the c.d.f. of the simulated η, and, if different from the logistic, take it as a new distribution to estimate new parameters and iterate the procedure. However this estimation technique seems to be under too much influence from the starting point. To check its efficiency, I simulated a probit model with data generating process :

$$e = \mathbb{1}_{X+\eta>0},$$

where X has a centered normal distribution with standard error equal to 3, and η is an independent standard centered normal. I then computed the simulated residuals of this model, knowing X (there is no parameter to estimate), under the assumption that the distribution of η is logistic (standard error $\pi/\sqrt{3}$). The distribution of the simulated residuals, shown on Figure 6, is close to the logistic, and thus quite far from the true (normal) distribution.

3. In the preceding example, a nonparametric regression of e on X, $E(e|X) = 1 - \Pr(\eta < -X)$, works wonders, as shown on Figure 6. This suggests

50 equal probability intervals and $R[W(\varepsilon)]$ is tabulated at the median points of these intervals. Then ε is drawn from a conditional normal restricted to the union of $\varepsilon \leq \varepsilon_{\min}$ with the intervals such that $(*)$ is satisfied at their median points.



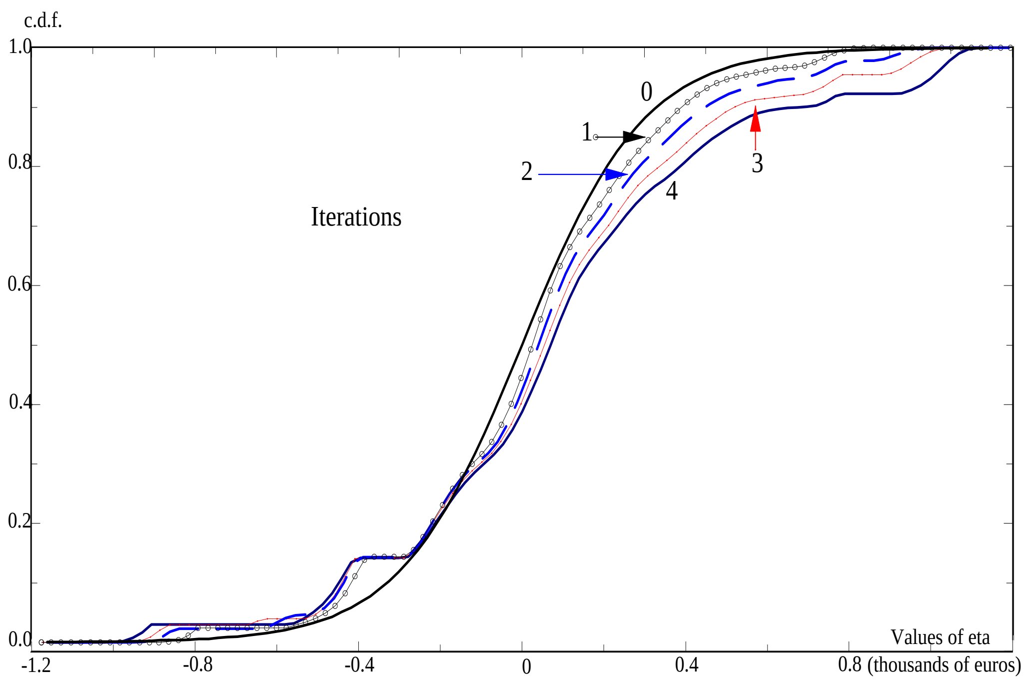


Figure 7: Estimation with simulated residuals and non parametric regression

to use simulated residuals to implement a nonparametric regression of the observed employment status on the simulated value of

$$\frac{R(w) - R(0) - Z\gamma - \rho\varepsilon}{\sigma_\eta},$$

on the subset of observations such that, when a woman is unemployed, her simulated productivity is larger than $w_{\min}$ and she is not subject to frictional unemployment ($\nu \leq P_k(Y\beta)$). Two difficulties pop out when implementing this estimation strategy. First, the nonparametric regression is not constrained to be nondecreasing, while the c.d.f. has to be. Since the decreasing parts of the curve are located at the edges and are minor, I just fix the problem by taking the largest nondecreasing function that is everywhere smaller than the nonparametric regression[12]. Second, similarly, the nonparametric regression lacks precision in the range of values where there are few observations, here at the tails of the distribution. In the present situation, this eventually makes it impossible to pursue the algorithm. The upper tail of the distribution gets more and more weight as shown on the lower panel of Figure 7, but this comes from very few points since the density of the observations is of course low in this region (upper panel of Figure 7). The values of the (semiparametric) loglikelihood function at the maximum along the iterations are: -7853.44 (starting point), -7849.81 (first iteration), -7855.46 (second), -7864.04 (third). I have proceeded using the results of the last iteration, while checking how much difference this makes from the starting point.

3.2 Rawlsian incentive schemes

The simulation of the model, conditional on the observations, yields a measure of the work aversion of each individual in the sample, which depends both on the observed characteristics of the household (presence or not of a spouse, income of the spouse, number of children, etc.) and on the simulated unobserved heterogeneity. Figure 8 shows a kernel based estimation of the cumulative distribution function of work aversions, for the sample as a whole and for some subcategories of the population. The c.d.f.s appear to be smooth. In conformity with my priors, the distribution of work aversions seems to be first order increasing with the number of children in the household.

According to both the theoretical and empirical specifications, these c.d.f.'s depend both on the disposable income when not working and on the productivity of the agents. In all the computations below, I shall take income when out

[12]This explains the difference between the nonparametric regression at the fourth iteration in the upper panel of Figure 7, which has some decreasing pieces, and the corresponding nondecreasing c.d.f. in the lower panel.

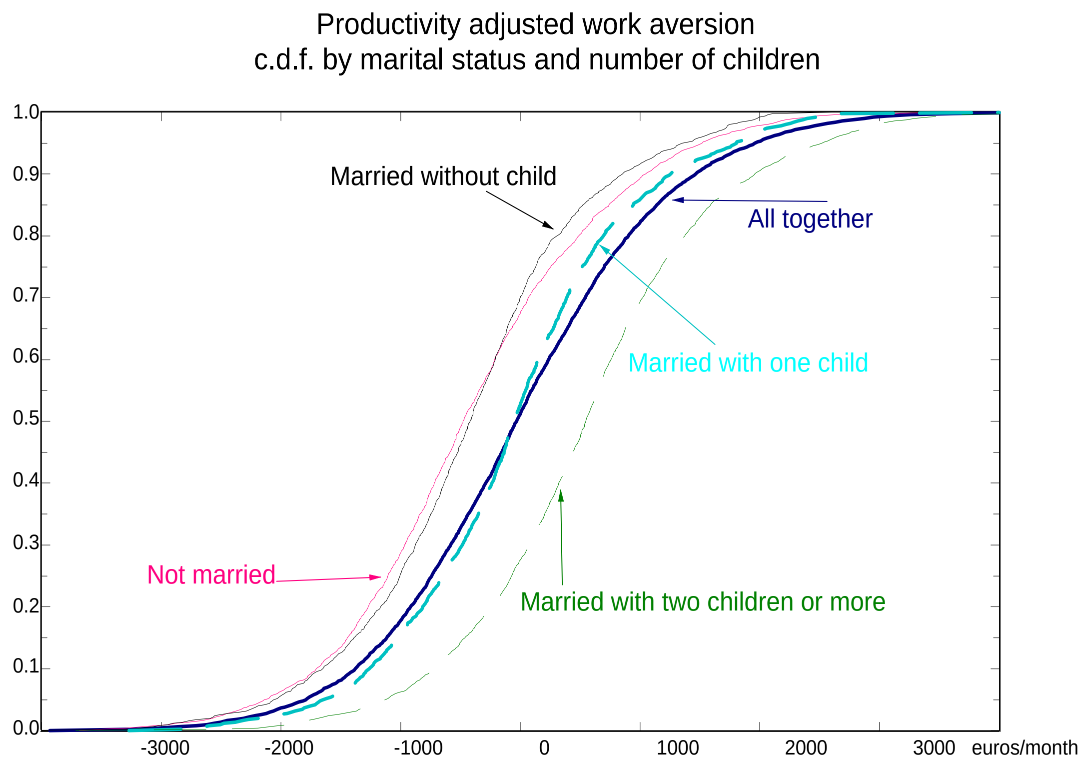


Figure 8: Distribution of work aversions by marital status and number of children

of work, the subsistence income, as given. The government provides different maintenance incomes to different households, depending for instance on family composition and the ages of the children. Also, income when out of work depends on the spouse's income. A full Rawlsian optimization would involve recovering the implicit equivalence scales used by the government, which is out of the scope of this paper. I only undertake a *partial* optimization, looking for optimal incentives to work taking as given the actual household subsistence income when the woman does not work.

The distribution of work aversions depends on productivity. Indeed, from equations (10) and (12),

$$\Delta(a) = Z\gamma + \rho\frac{\ln w - X\alpha}{\sigma_\varepsilon} + \sigma_\eta\eta. \tag{13}$$

If the government could design a different tax schedule for each value of (X, Z), one could just use the above formula, together with the estimated distribution of η, to compute the optimal schedule. In practice, the government is unable to discriminate in such detail, and I look at broader categories. Here, I shall concentrate on two cases: single women, and women with two children or more.

The final difficulty comes from the fact that productivity w is correlated with the (unobservable to the government) exogenous variables X and Z. To tackle the issue in a way that puts less weight on specific functional forms than equa-

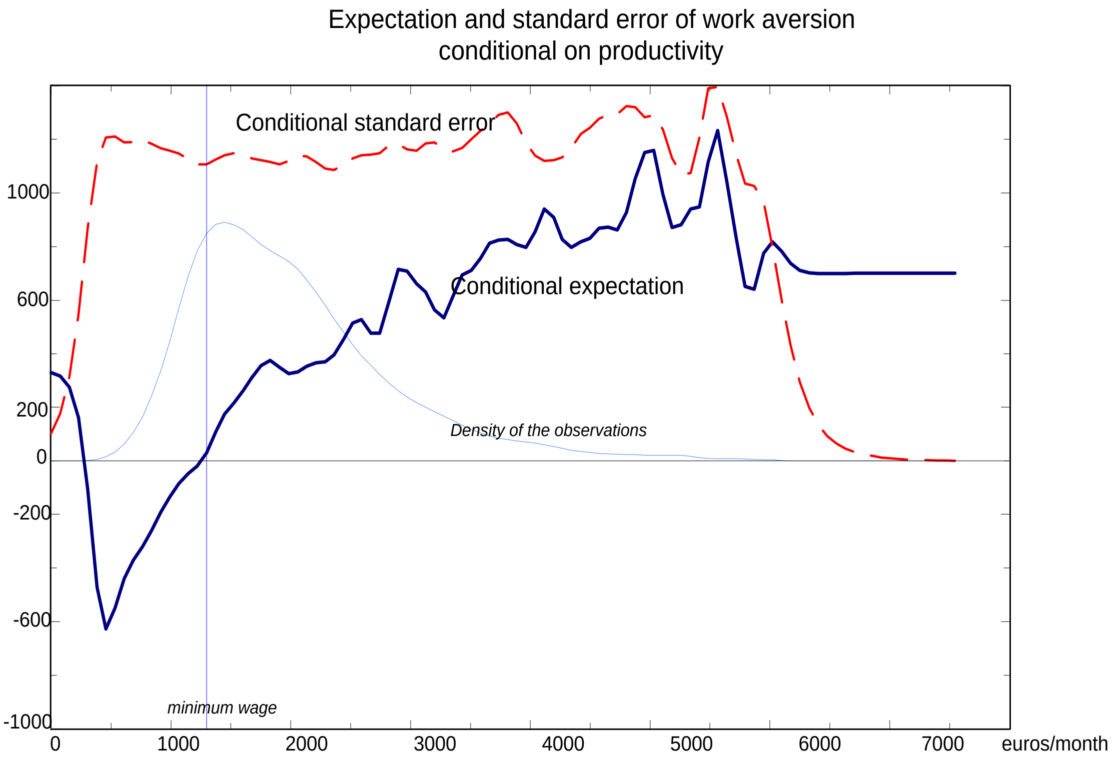
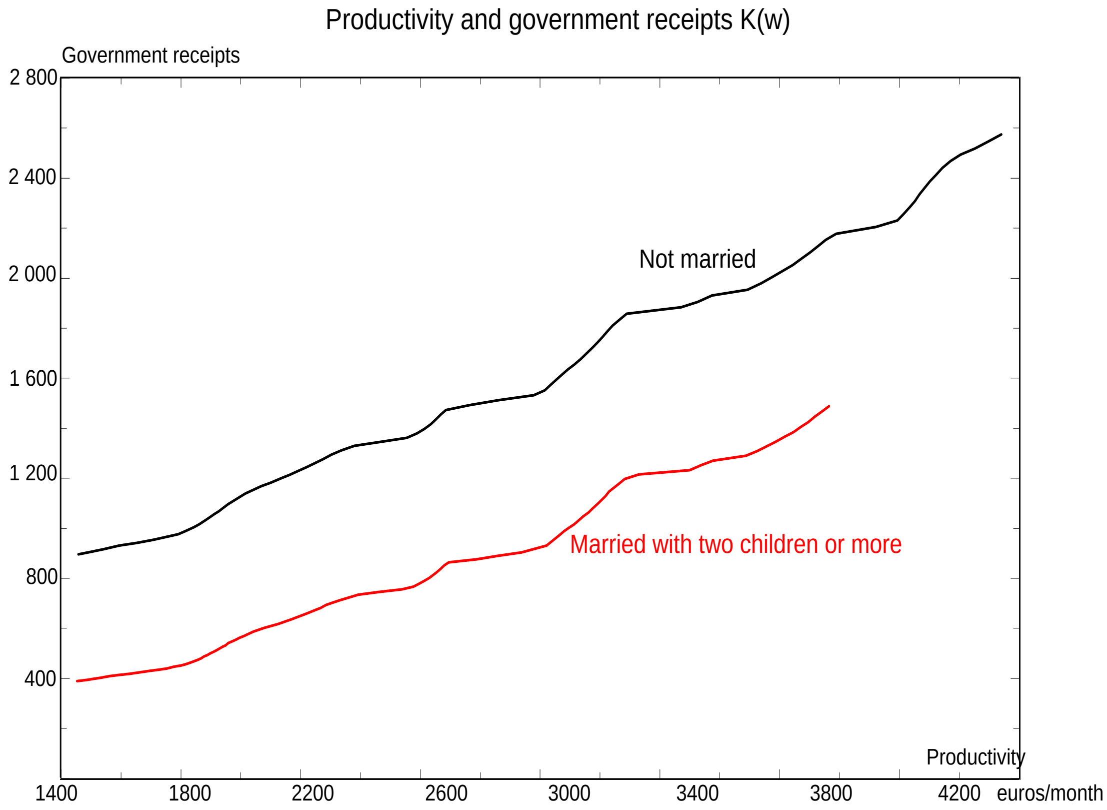


Figure 9: Non parametric regression of work aversion on productivity



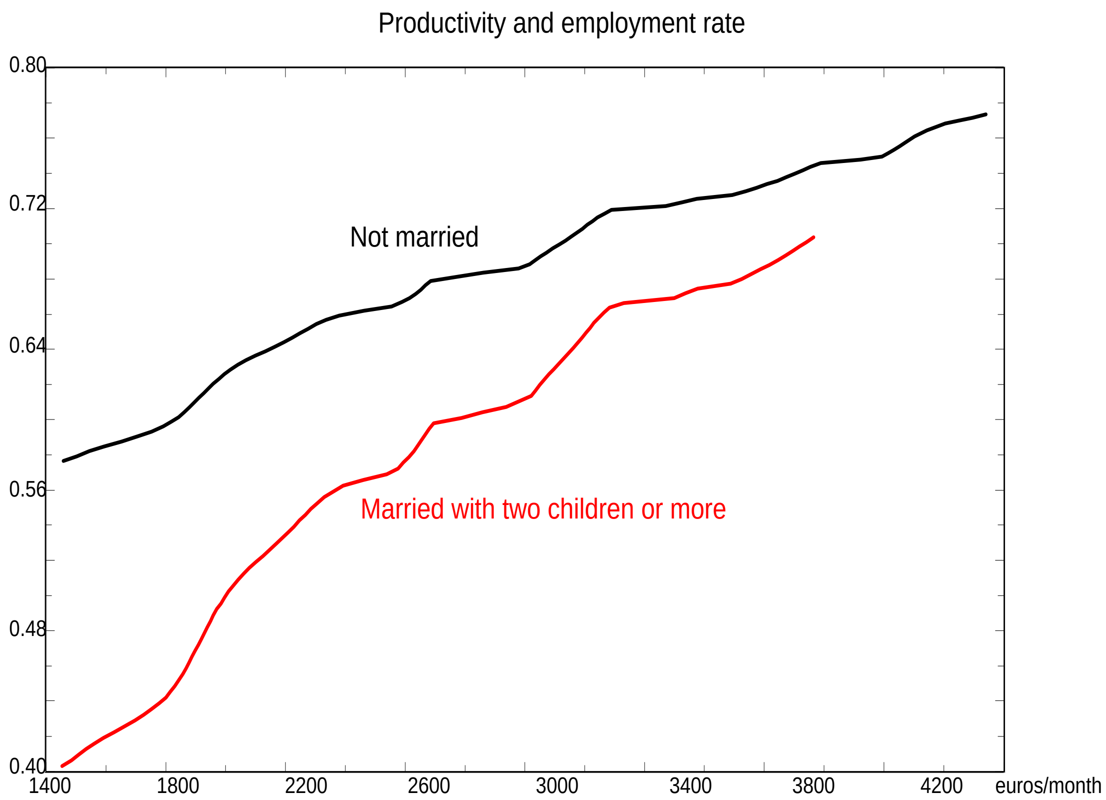


Figure 10: Optimal Rawlsian schemes for single women and for married women with two children or more

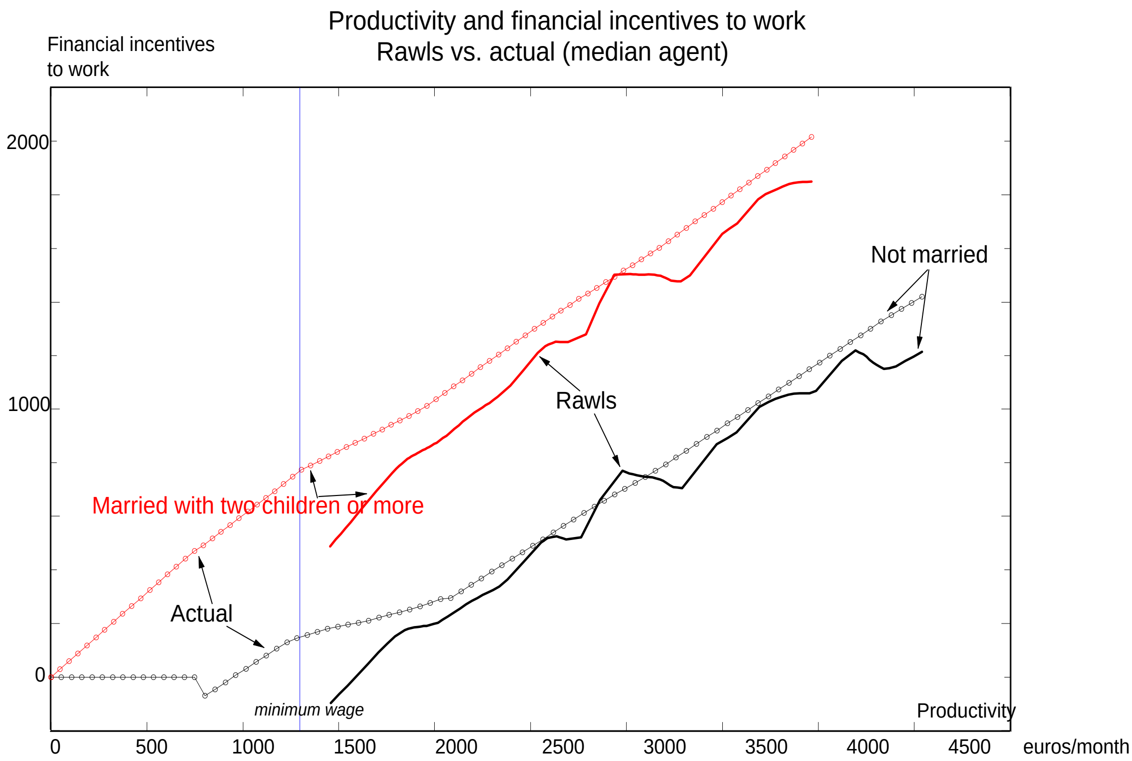


Figure 11: Optimal Rawlsian incentives vs. actual incentives for the median agent

tion (13), I directly take the values of the disutilities of work and productivities out of the conditional simulations used in the estimation process, and undertake a nonparametric regression of work aversion and its square on productivity, postulating that

$$\Delta(a) = m(w) + s(w)\delta,$$

where δ has a distribution that does not depend on w. Figure 9 shows that, while $m(w)$ does vary with w, there is no noticeable heteroscedasticity: $s(w)$ can be taken as a constant. Let G_w be the c.d.f. of Δ and G the c.d.f. of δ. Then $G_w(D) = G[D - m(w)]$. The Rawlsian problem can be rewritten, letting $d = D - m(w)$,

$$K(w) = \max_D (w - D)G_w(D) = \max_d (w - m(w) - d)G(d) = \kappa[\omega(w)],$$

where

$$\kappa[\omega] = \max_d (\omega - d)G(d) \quad \text{and} \quad \omega(w) = w - m(w).$$

The function $\kappa[\omega]$ has all of the theoretical properties shown for $K(w)$ when the distribution does not depend of w in the qualitative analysis of 2.3.2: it is convex, increasing, with slope smaller than one, and the optimal d is nondecreasing in ω and everywhere smaller than ω. It can be easily numerically computed, which yields the optimal Rawlsian incentives to work[13] . By construction, the optimal $D(w) = d[w - m(w)] + m(w)$ is also smaller than w, but this is the only property that is preserved in general, everything else depending on the shape of the function $m(w)$.

Figure 10 presents the values of the government receipts $K(w)$ and of the associated employment rates both for single women and for women with two children or more. As expected, government receipts and employment rates are higher for the less work averse category, here for single women without children.

Figure 11 shows the Rawlsian financial incentives to work for the same two categories of women (broken lines), together with the actual current incentives provided by the French taxes and social transfers to the median[14] women in the category (solid lines). Several comments are in order:

1. It is comforting to see that the French system seems mostly to be on the right side of the Laffer curve. However, the distance is small. It looks as if, through competition between the various government agencies, the transfer schedule is close to maximizing government income on these two categories of women.

2. A utilitarian criterion would yield larger incentives than the Rawlsian one, holding constant income when not working. It follows that the implicit social welfare criterion of French politicians, again concerning the women in the sample, is not far from Rawls.

3. This appraisal needs to be qualified: it is highly dependent on the information that the government is entitled to use in the design of the transfer scheme.

[13] There are (at least) two ways to do this computation. Brute force involves evaluating the function G^{-1} at, say, a thousand quantiles, and computing the maximum on a grid of points w of interest. Another possibility, given the smoothness of the distribution, is to use the first and second order conditions associated with the maximization. The first order condition is

$$\omega = d + \frac{G(d)}{G'(d)},$$

and the second order condition, taken at the point ω which satisfies the first order condition, is

$$-2[G'(d)]^2 + G(d)G''(d) < 0.$$

It is easy to get the derivatives of G by differentiating the kernel. The second order condition then yields a range of admissible values of d and the corresponding ω's follow from the first order condition. This makes for smoother figures than the brute force technique and the figures shown below are drawn according to the latter method.

[14] The values taken for each component of X and Z are the median values of this component in the subsample under consideration.

References

Beaudry, P., and C. Blackorby (1997): "Taxes and Employment Subsidies in Optimal Redistribution Programs," Discussion Paper 22, University of British Columbia.

Card, D., C. Michalopoulos, and P. K. Robins (1999): "When Financial Incentives Pay for Themselves: Early Findings from the Self-Sufficiency Project's Applicant Study," Discussion paper, SRDC, Social Research and Demonstration Corporation.

Choné, P., and G. Laroque (2001): "Optimal Incentives for Labor Force Participation," Discussion Paper 25, CREST-INSEE.

Diamond, P. (1980): "Income Taxation with Fixed Hours of Work," *Journal of Public Economics*, 13, 101–110.

Gouriéroux, C., A. Monfort, E. Renault, and A. Trognon (1987): "Generalized Residuals," *Journal of Econometrics*, 34, 5–32.

Horowitz, J. (1998): *Semiparametric Methods in Econometrics*, no. 131 in Lecture Notes in Statistics. Springer.

Laroque, G., and B. Salanié (2000): "Une décomposition du non-emploi en France," *Economie et statistique*, 331, 47–66.

——— (2002): "Labour Market Institutions and Employment in France," *Journal of Applied Econometrics*, 17(1), 25–48.

Meyer, R., and D. Wise (1983a): "Discontinuous Distributions and Missing Persons: The Minimum Wage and Unemployed Youth," *Econometrica*, 51, 1677–1698.

——— (1983b): "The Effect of the Minimum Wage on the Employment and Earnings of Youth," *Journal of Labor Economics*, 1, 66–100.

Mirrlees, J. (1971): "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175–208.

Robins, P. K., and C. Michalopoulos (2001): "Using Financial Incentives to Encourage Welfare Recipients to Become Economically Self-Sufficient," *Federal Reserve Bank of New York Economic Policy Review*, 7(2), 105–123.

Saez, E. (2000): "Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses," Discussion Paper 7708, NBER.

——— (2001): "Using Elasticities to derive Optimal Income Tax Rates," *Review of Economic Studies*, 68, 205–239.