The Market for Lawyers: the Value of Information on the Quality of Legal Services.*

Elisabetta Iossa† and Bruno Jullien‡

19 January 2007

Abstract

We study the value of information on the quality of legal services by analyzing the incentives of litigants to hire high-quality lawyers and the effect of legal representation on the decision-making behaviour of adjudicators.

In a setting where adjudicators have reputational concerns and where the quality of lawyers is reflected in their knowledge of legal principles, we show that better information over the quality of legal representation generates a trade-off. It allows for a better match between the value of a legal dispute and the quality of the legal representation. But it induces a bias in the decisions of adjudicators in favour of the litigant with the highest-quality lawyer. For a given distribution of the quality of lawyers, the social value of public information on the quality of lawyers may then be negative.

We discuss the implications of these effects on the desirability of quality certification system (such as the Queen’s Counselor system) in the market for the legal professions. Certification also has the effect of increasing the incentives of lawyers to invest in quality-enhancing training. We show that free certification leads to excessive supply of certified lawyers. We also show that the main insights are robust to the accounting for proof-taking activities by lawyers.

*For helpful comments we wish to thank Julian Greenhill, Paul Grout, Ian Jewitt, William Kovacic, Gilat Levy, Neil Rickman, Kathryn Spier and seminars participants at Brunel University, Birkbeck College, University of Essex, University of Rome Tor Vergata and at the CMPO Workshop on the Economics of Legal Services and Justice (Bristol).

†Economics and Finance Section, Brunel University; DEI, University of Rome Tor Vergata, CEDI and CMPO.

‡Toulouse School of Economics (IDEI & GREMAQ) and University of Bristol (CMPO).
1 Introduction

Efficient legal systems are key for the legal enforcement of contractual relationships and thus for the development of market transactions. This paper analyzes the functioning of the market for lawyers, focusing on how information over the quality of legal services impacts on the efficiency of the legal system.

Legal services are credence goods. Due to the sheer complexity of the law, litigants in a dispute are generally unable to gauge the quality of the legal service they receive even if they can observe the legal strategy undertaken by their legal counsels. Gauging the quality of legal services requires knowledge of principles of law and educated reasoning, for which specific training is necessary.

Information over the quality of legal services can however be affected by legal market regulation. For example, restrictions on entry conditions in the legal profession, in the form of minimum qualifications and experience, can contribute to ensure that a minimum quality standard prevails in the market. Quality considerations are in fact at the heart of the justification provided by legal associations and regulatory bodies for restricting access to the profession (see OFT, 2001).¹

Also, distinctive layers of quality may exist within the legal profession. In England and Wales, for example, the lawyers who appear in court to argue a case before an adjudicator or tribunal are divided into juniors and Queen’s Counsels (QCs). Appointment as QC is a mark of quality and brings a number of formal privileges.² In court, QCs wear a distinctive uniform: a short wig, wing collar and bands and silk gown over a special court coat. Furthermore, QCs tend to focus on more complex cases and usually benefit from an increased fee rate per case. Schemes equivalent to the QC system exists e.g. also in Scotland, Northern Ireland, Canada, New Zealand and South Australia.

¹Shaked and Sutton (1981) explicitly analyze the rationale for self-regulation and whether the profession should, or should not, be allowed to retain monopolistic powers for quality purposes. They show that self-regulating profession that maximizes either the relative or the absolute incomes of its members will choose a size which is socially sub-optimal.

²Currently, following consultation with the judiciary and the profession, the Lord Chancellor recommends for appointment those practitioners whom have marked themselves out as leaders of the profession. QCs are then appointed annually by the Queen.
In other countries the legal profession has no mark of distinction or quality comparable to the QC mark, yet quality layers can still be identifiable. As described for example by Rosen (1992) or by Garicano and Hubbard (2003) for the US, one can distinguish between two categories of lawyers. The first category comprises lawyers who graduated from elite institutions, who are well-connected and influential in the profession. These lawyers tend to serve business clients, charge high fees and earn high incomes. The second category serves more individual clients and comprises lawyers who graduated from lower-tier schools, charge lower fees and provide largely routine, non-contested legal services. Depending on the category, lawyers are then employed in different law companies, with most reputable companies employing the most talented and well trained lawyers.

Studying the Chicago bar, Spurr (1987) provides evidence that information, such as academic performance and quality of law school, matters as the market tends to assign larger claims to the higher quality lawyers. Moreover the information available has improved considerably as specialized journals such as Legal Times, The American Lawyer or National Law Journal now provide detailed information on the performance of law firms and individual lawyers.

In this paper we study the value of public information on the quality of lawyers and thus the desirability of regulation that provides for quality certification. We consider the interaction between three groups of agents: the litigants, their lawyers and the adjudicator in charge of resolving the dispute. We focus on the incentives of litigants to hire high-quality lawyers, the incentives of lawyers to appeal the adjudicator’s decision and how the decision-making behaviour of the adjudicator is affected by the quality of lawyers in courts.

A lawyer’s quality is a multidimensional concept that encompasses knowledge of the principles of law and their interpretation, capacity to identify aspects of the law useful to make the case in court and advocacy skills. In any systems of law committed to precedent, such as the common law system, the art of legal reasoning is also the art of
drawing out distinctions and similarities between cases; in adversarial systems it is also
the art of fact finding and evidence gathering. We focus on the quality dimension that
is related to the ability to identify the underlying ‘state of the world’ (i.e., the ‘correct
decision’, which can also be viewed as the decision that the appeals court would take).
In most of the paper we abstract from other quality dimensions in order to highlight
the role played by knowledge of the underlying state. Thus a high-quality lawyer in our
model is neither a better advocate nor is he able to substantiate his knowledge with
production of verifiable evidence. If quality matters, this is only because of the way
knowledge of the underlying state affects the appeals strategy of a lawyer. We extend
the analysis to other dimensions in the discussion section.

An important element in our analysis is that adjudicators have reputational concerns and wish to appear competent, i.e. able to identify the correct decision. Careerist
adjudicators take into account how their decision affects the likelihood of an appeal
and how the appeal outcome in turn affects the inference on the adjudicator’s competency. Contrary to standard settings in the career concerns literature, in judicial
systems whether an adjudicator’s decision is correct may never be found out. However, in expectations higher courts’ adjudicators are more competent than lower courts’
one. By providing information as to the correctness of lower-court decisions, appeals
outcome then provide information as to the competency of lower-court adjudicators.\(^3\)

The idea that adjudicators may have reputational concerns is now well received in
the law and economics literature (see related literature below). Adjudicators may care
about how others perceive their quality either because of a general concern for prestige
or influence or because their reputation can directly influence their career and future
income. Empirical evidence indeed shows that the perceived quality of judges plays
an important role in their promotion to higher courts (see Levy 2005 and references
therein). Also, reputable judges often take prestigious and well paid positions upon retirement from the judicia. It is not unusual for example for retired judges to become

\(^3\)See for example Shavell, 2004, and Iossa and Palumbo, 2007, for an in depth discussion of the
role of appeals as monitoring mechanism of adjudicators.
arbitrators in commercial disputes or international transactions.

To study the value of public information on lawyers’ quality, we build a stylized model where lower-court adjudicators have private information about their competency and wish to appear competent. Parties to a dispute choose whether to hire a certified or an uncertified lawyer. Certified lawyers hold more precise signals about the state of the world than uncertified lawyers. Signals are soft information and cannot be credibly transmitted. A dispute is resolved through an initial stage where a lower-court adjudicator makes an initial decision, and, if the losing litigant appeals, an appeals stage where the appeals court makes a final decision. Appeals are costly. We focus on the reputational concerns and the behaviour of the lower court; the appeals court is assumed to comprise only competent adjudicators who take the correct decision.

In this setting, the gain for a litigant from hiring a high-quality lawyer stems from more efficient appeals and from a decision bias effect. A certified lawyer discovers and appeals incorrect decisions more often than an uncertified lawyer. When a certified lawyer observes an incorrect and unfavorable decision, he correctly advises his client to appeal even if appeals costs are high, as he is confident about the possibility to win in appeal. Uncertified lawyers unsure about the rightness of the decision do not appeal for high appeals cost.

Critically, hiring a certified lawyer also generates a decision bias effect: less competent adjudicators bias their decisions in favour of litigants with certified lawyers, due to their reputational concerns. This occurs because high-quality lawyers hold more precise signals over the state of the world than low-quality lawyers and this translates into a more informed appeals strategy. As appeals decisions and appeals outcome reveal information about the competency of the adjudicator, incompetent adjudicators fear appeals from certified lawyers more than appeals from uncertified lawyers. They bias their decisions in favour of certified lawyers in order to minimize inference about their competency.

The decision-bias effect that we find rationalizes the perception described by re-
spondents to a consultation paper issued by the Department of Constitutional Affair (DCA) in the UK investigating the desirability of QC systems or in general of marks of quality. As reported by the DCA (DCA, 2003, p.23)

"There was a perception that QCs were now instructed in circumstances where their particular skills were not really needed: for example because it might be thought that judges would pay more attention to a QC’s argument, or because a simple equality of arms was needed -just because the other side had already instructed a QC"

The decision-bias effect thus rationalizes the incentives to "pay more attention" to a QC’s argument as one of reputational concerns and suggests that it will be less competent adjudicators who are more likely to favour litigants with a QC.

An implication of our results is that whilst higher-quality of lawyers increases welfare, knowledge of lawyers’ quality is not necessarily welfare improving. Due to more efficient appeals, higher quality of lawyers improves the likelihood that the correct decision will be taken whilst optimizing on appeals cost. Quality certification however generates two contrasting effects. On the one hand, it allows for a better match between the value of a legal dispute and the quality of legal representation. In equilibrium low-quality lawyers are hired by litigants with low-value cases and receive a basic wage whilst high-quality lawyers are hired by litigants with high-value cases and receive a high wage. This matching effect is welfare increasing since better legal representation improves the likelihood that the correct decision is taken and better legal outcomes are more valuable for high-value cases.

On the other hand, public information over the quality of legal services generates the decision-bias effect and this has efficiency consequences. At the market equilibrium there is misallocation of lawyers: cases inefficiently arise where litigants’ behaviour is asymmetric, with one litigant hiring a certified lawyer and the other one hiring an uncertified lawyer. Thus the matching effect that knowledge of lawyers’ quality generates does not lead to a first best allocation, due to the decision-bias effect. When
the *matching effect* and the *decision-bias effect* are compared we find that the social value of information over the quality of legal representation may be negative.

Furthermore due to the potential advantage that the appointment of a high-quality lawyer generates through the bias effect, a market equilibrium may be reached where there is over-demand of quality; the decision bias effect is then also the source of excessive fees for lawyers. Litigants can end up spending excessive resources on legal representation exacerbating the prisoner’s dilemma problem of any legal process.4

In the last section of the paper we extend our model in a number of directions. First we introduce the possibility that lawyers are also in charge of the task of information gathering, as in adversarial systems of law, and that higher-quality lawyers are more likely to find evidence about the correct decision. A new an interesting effect is now generated by knowledge of lawyers’ quality, but our main insights carry on. Being partisans, lawyers have incentives to disclose only evidence that is favourable to their case. No reported evidence should then be interpreted as bad news for the lawyer, raising the suspicion that he conceals some information. However, the adjudicator may not choose the efficient decision because of the reputational effect.

A second extension that we analyze is related to effect of quality certification on the incentives of lawyers to train and raise their ability. We show that indeed certification helps to raise quality in the market. As lawyers anticipate that quality certification brings higher income, they will have incentives to make investments that enhance their legal abilities. Since investment (or equivalently certification) is costly, only the most capable lawyers will obtain certification and enjoy indeed higher equilibrium salaries. When the individual benefit of winning a case is larger than the social value of reaching the correct decision, we show that free certification would lead to excessive investment by lawyers and excessive supply for certified lawyers.

Finally we also briefly discuss the effect of relaxing some of our assumptions on the cost of appeals, and show that if a high quality lawyer raises the likelihood of

---

4 The presence of a prisoner’s dilemma in litigation has been first pointed out by Ashenfelter and Bloom (1993).
socially inefficient appeals, the main conclusions are robust, although the bias may not be observed in equilibrium..

The paper is organized as follows. In Section 2 we review the related literature. Section 3 presents the basic model; section 4 and 5 discuss the decision-making behaviour of the adjudicator and the outcome of the decision process. Section 6 considers the incentives of the parties in the dispute to hire a certified lawyer and the equilibrium in the market for lawyers. Section 7 studies the private and social value of quality of legal services whilst the private and social value of certification is analyzed in section 8. Section 9 discusses some extensions whilst section 10 concludes. All proofs missing from the text are in the Appendix.

2 Related literature

The legal literature has long debated the impact of lawyers’ capabilities in adjudication (see for example Galenter, 1974). Consensus and evidence has been gathered on how legal representation makes for a significant difference both in likelihood of recovery and in amount recovered (e.g. Ross, 1970). A number of empirical papers have then analyzed the dynamics of the market for lawyers and quantified the rewards from training and specialization (see for example Sauer 1998, Rosen, 1992, and Pashigian, 1977, Garicano and Hubbard, 2007).

The theoretical literature on the value and quality of legal representation however remains slim (see Cooter and Rubinfeld, 1989, Spier, 2005 and Shavell, 2006 for a general discussion on the economics of litigation). Lawyers can affect the probability of winning through their (trial) effort (see Hirshleifer and Osborne, 2001) or through their information gathering (see Dewatripont and Tirole, 1999). They may then efficiently inform the court (see e.g. Bundy and Elhauge, 1993 and more recently Che and Severinov, 2006), or mislead it (see Kaplow and Shavell, 1989 ). We are not aware of any study that looks at how knowledge over the quality of legal representation affects the functioning of the market for lawyers. To the extent of our knowledge, our paper is
also the first paper to derive the value of quality of legal representation endogenously, from the interaction between career concerns of adjudicators and a lawyer knowledge of the underlying state.\footnote{There is of course an extensive literature on the role of information and certification for consumption goods and services. References include, among others, Shapiro (1986), Biglaiser (1993) and Lizzieri (1999). These papers focus on incentives and asymmetric information issues. We abstract from these issues and focus on the interaction between the parties on a case and the behaviour of the adjudicator.}

Our paper is also related to literature on careerist decision makers, such as regulators, managers or experts who try to prove their ability to make the correct decision. For example, in Levy (2005) careerist adjudicators contradict precedents too often in order to signal their ability, whilst in Iossa (2007) arbitrators with career concerns bias their decision in favour of long-term players. Career concerns may also induce less able bureaucrats to use soft policies so as to keep interests groups quiet and mistakes out of public eyes, (Leaver, 2004). See also Bourjade and Jullien (2005), and Ottaviani and Soerensen (2006) for recent studies on bias in experts’ advice.\footnote{The paper is in general related to the carrier concerns literature, although in most of this literature, and contrary to our approach, the quality of the decision is verifiable ex post.}

\section{The model}

\textit{The general setting}

We consider a setting where high-quality lawyers hold a qualification to certify that their quality is above a certain threshold. When there is a dispute, litigants choose whether to hire a certified lawyer or not. A dispute is resolved through an initial stage where a lower-court adjudicator makes an initial decision, and, if the losing litigant appeals, an appeals stage where the appeals court makes a final decision.

For simplicity, we treat the occurrence of a dispute as an exogenous event and we model a dispute as disagreement between two parties, $P_1$ and $P_2$ over the realization of a state of the world $\theta$. There are two states, 1 and 2, and it is common knowledge that $\Pr(\theta = 1) = 1/2$. We denote by $d$ the decision of the lower-court adjudicator and by $D$ the one of the appeals court. We assume that there are only two possible
decisions, $d, D \in \{1, 2\}$. A decision $d = 1$ (or equivalently $D = 1$) favours $P_1$ and it is the most appropriate in state $\theta = 1$, whilst a decision $d = 2$ favours $P_2$ and it is the most appropriate in state $\theta = 2$. We refer to $d = \theta$ as the correct decision and we interpret the correct decision as the decision that the appeals court would choose. Thus we assume that following an appeal, the appeals court always chooses $D = \theta$. We assume that $\theta$ is independent across disputes.

The lawyers

There is a mass of lawyers who may work for litigants on a dispute or on some alternative activities leading to an expected utility normalized to zero. The mass is large enough for all litigants to obtain legal representation. Lawyers may be of two types "certified" or not. Certified lawyers have quality $r$, whilst uncertified lawyers have quality equal to zero.\(^7\) We denote by $S$ the mass of certified lawyers. In a dispute a certified lawyer observes a perfectly informative and unverifiable signal of the state of the world $\theta$ with probability $r$. With probability $(1 - r)$ he observes nothing.

We denote by $w_0$ the salary of an uncertified lawyer and by $w$ the salary of a certified lawyer. $w_0$ is exogenously given by the productivity on some outside option. For simplicity we set $w_0 = 0$.

Remark 1 Notice that given our interpretation of $\theta$ as the would-be decision of the appeals court, the concept of competent lawyer refers to the ability to predict accurately the behavior of the appeals court.

The adjudicator and the appeals court

There are two types of lower-court adjudicators: the competent ($C$) and the incompetent ($I$). Type $C$ privately observes the state of the world $\theta$ with probability 1, type $I$ observes nothing. Types are private information and we let $\gamma$ denote the proportion of competent adjudicators. The model can be generalized to continuous types.

\(^7\)The analysis would be similar for a positive expected quality of uncertified lawyers, provided that this is smaller than the one of certified lawyers.
The adjudicator has reputational concerns in the sense that he wishes to appear competent to an evaluator $E$, his payoff being equal to the posterior belief held by $E$ about the competency of the adjudicator. The assumption that the the appeals court chooses $D = \theta$ allows us to focus our attention on the lower-court adjudicator (hereafter simply referred to as the ‘adjudicator’) and the interplay between adjudicators’ competency and lawyers.

The litigants

There is a mass 1 of disputes. The litigants in a dispute (also referred to as ‘parties’), $P_1$ and $P_2$, value a favourable decision at $V(1 + \delta)$ if it is correct, $V$ if it is incorrect, and an unfavourable decision at zero; the value $V$ varies across disputes and has a cumulative distribution function $F(V)$. The litigants sequentially choose whether to hire a certified lawyer at salary $w$ or an uncertified lawyer at salary $w_0$. Sequentiality is innocuous and avoids mirror equilibria in our model. Further, in practice the game is indeed sequential with the plaintiff initiating the case. In order to focus on the value of information on the quality of the legal representation, we assume away agency problems between lawyers and their clients and assume throughout that the hired lawyer acts in the best interest of his client and this is public information.

Remark 2 An alternative interpretation of the model is a situation of self-litigation versus professional litigation. In this interpretation certification is the minimal quality requirement to be a member of the legal profession. Then agents choose to be represented by a lawyer or not (self-litigation).

Upon observing the adjudicator’s decision $d$, the party who loses the dispute can choose to appeal the decision. This party then incurs costs $A.V$ for the appeal, where $A$ is a random variable that is realized after a decision $d$ is made and before an appeal is

---

8The parameter $\delta$ can capture for example the sense of justice of the parties or (in a reduced form) the value for contractual parties of better enforcement of contractual terms (see Anderlini, Felli and Postelwaite, 2007, for a discussion of the role of court decisions on ex ante incentives). Ex ante (i.e. before a dispute arises) better enforcement can help to ensure better incentives for relationship-specific investments and to increase the surplus from the contractual relationship.
demanded. For simplicity we assume that $A$ can take two values, $A = 0$ or $A = \bar{A}$ with respective probabilities $q$ and $1 - q$.\footnote{The extension to a general distribution does not add much to the main argument.} Moreover we assume that $1 + \delta > \bar{A} > \frac{1 - \gamma}{1 + \gamma} (1 + \delta)$, which is sufficient to ensure that faced with a high appeals cost $\bar{A}$, an uninformed party never appeals, and an informed party appeals only if the decision is not correct.

We also assume that the private value of the correct decision is not too large, namely $\delta < \bar{A}$. We relax this assumption in section 8. To simplify the exposition, we also rule out the possibility that the certified lawyer appeals an unfavorable decision when appeals cost is low, $A = 0$, and he is informed that the decision is correct.

The evaluator

An evaluator ($E$) observes the decisions $d$ and $D$, whether the lawyers of the parties in the dispute are certified or not and whether they appeal. Using this information, the evaluator $E$ updates his beliefs about the competency of the adjudicator rationally.

Timing

1. Disputes arise and states of the world $\theta$ are realized. The litigants sequentially choose whether to hire a certified lawyer or an uncertified lawyer. $P1$ chooses first, then $P2$ chooses.

2. The lower-court adjudicator observes $\theta$ if he is type $C$ and nothing otherwise. Then the adjudicator makes decision $d$.

3. A lawyer with quality $r$ observes a perfectly informative signal with probability $r$ and nothing otherwise. A lawyer with quality zero observes nothing. Appeals cost $A$ are privately realized. Based on his information, the losing party decides whether to appeal.

In the event of an appeal, the appeals court observes $\theta$ and chooses $D = \theta$.

Equilibrium Concept
We use the concept of Sequential Equilibrium to solve the model. Beliefs are derived from players’ equilibrium strategies and the strategies are rational given those beliefs.

4 The judicial game

In this section we study the behaviour of the adjudicator, given the choice of the parties as to whether to hire a certified lawyer. To this purpose we consider a dispute where party $P_1$ hires a lawyer with quality $r_1$, and the other party, $P_2$, hires a lawyer with quality $r_2$. In the game of course $r_i$ ($i = 1, 2$) will take value $r$ or $0$, depending on whether the lawyer is certified or not.

In this setting, consider the following strategies for the adjudicator: $C$ chooses $d = \theta$, whilst $I$ randomizes between $d = 1$ and $d = 2$, choosing $d = i$ with probability $z_i$. We now show when these strategies constitute an equilibrium. To simplify notation, let $\pi(d, D)$ denote the posterior belief of $E$ following a decision $d$ of the adjudicator, an appeal and a decision $D$ of the appeals court. When there is no appeal, instead, let $\pi(d, 0)$ denote the posterior belief of $E$ following a decision $d$.

Notice first that following an appeal, if reversal occurs, $E$’s posterior belief is $\pi(1, 2) = \pi(2, 1) = 0$, since under our candidate equilibrium only type $I$ in equilibrium takes incorrect decisions.

- **Behavior of $I$**

Consider now the behavior of an adjudicator of type $I$. Since $I$ is uninformed and he wishes to appear competent, he will attempt to minimize the chance of his decision being reversed in appeal, taking into account the incentives of the litigants to appeal. The appeals strategies of the two lawyers thus play an important role in determining the incentives of $I$.

Consider the expected payoff of $I$ when he chooses $d = i$. With probability $r_j$ the lawyer of $P_j$ is informed and if he observes $\theta = j$, he successfully appeals the
unfavourable decision \( d = i \) for all realizations of \( A \). If instead the lawyer observes an unfavourable state \( \theta = i \), he does not appeal. With complementary probability, \( 1 - r_j \), the lawyer of \( P_j \) is uninformed. In this case, he appeals the unfavourable decision only if the appeals cost is low, which occurs with probability \( q \).

In light of these appeals strategies, consider \( E' \)'s posterior beliefs for each possible outcome. Following a confirmation of the decision \( d = i \) in appeal, \( E' \)'s belief is given by

\[
\pi (i, i) = \Pr (C \mid d = i, \theta = i) = \frac{\gamma}{\gamma + (1 - \gamma) z_i}
\]

since conditional on \( \theta = i \), a correct decision \( d = i \) is taken by \( C \) with probability 1 and by \( I \) with probability \( z_i \).

Suppose that \( z_i < 1 \). Following no appeal when \( d = i \), \( E \) can infer that either the lawyer of the losing party is informed that the decision is correct or that the appeals cost \( A \) is high and the lawyer is uninformed. In particular, conditional on \( A \) being low, \( E' \)'s belief is the same as when there is a confirmation in appeal, \( \pi (i, i) \), as only an informed lawyer who observes \( d = \theta \) does not appeal. Conditional on \( A \) being high, \( E \) does not know whether the lack of an appeal is due to appeals cost being high and the lawyer being uninformed or instead it is due to the lawyer being informed and having observed \( d = \theta \). The posterior belief on the adjudicator’s type conditional on this latter case is given by

\[
\Pr (C \mid d = i, D = 0, A = \bar{A}) = \frac{\gamma}{\gamma + (1 - \gamma) z_i (2 - r_j)}
\]

and we note that the higher the probability \( r_j \) that the lawyer of \( P_j \) is informed the more no appeal signals competency.

The appeals cost is not publicly observed, so that overall, when there is no appeal, the posterior is

\[
\pi (i, 0) = \frac{g \frac{1}{2} r_j \pi (i, i) + (1 - q) \left( 1 - r_j + \frac{1}{2} r_j \right) \Pr (C \mid d = i, D = 0, A = \bar{A})}{\frac{1}{2} r_j + (1 - r_j) (1 - q)}
\]
and the expected payoff of \( I \) when he chooses \( d = i \) is given by

\[
v(d = i) = \frac{1}{2} q (1 - r_j) \pi(i, i) + \left( \frac{1}{2} r_j + (1 - r_j) (1 - q) \right) \pi(i, 0)
\]

\[
= \frac{1}{2} q \pi(i, i) + (1 - q) \frac{1}{2} \gamma (2 - r_j) \gamma (2 - r_j)
\]

Let us now consider the case where \( z_i = 1 \). In this case a decision \( d = j \) reveals that the adjudicator is competent so that \( \pi(j, 0) = 1 \). In this case \( P_i \) is indifferent between appealing or not if the appeals cost is \( A = 0 \), and can appeal \( d = j \) with some probability if not informed that the decision is correct.

- **Behaviour of \( C \).**

Consider the expected payoff of \( C \) when he observes \( \theta = i \) and takes the correct decision \( d = i \). With probability \( (1 - r_j) \) the lawyer of losing party \( P_j \) is uninformed and appeals when the appeals cost is low. The adjudicator’s decision is then confirmed in appeal. In all other cases there is no appeal. It follows that \( C \) obtains

\[
v(d = i| \theta = i) = q (1 - r_j) \pi(i, i) + (1 - q (1 - r_j)) \pi(i, 0)
\]

If instead \( C \) takes the incorrect decision \( d = j \), then with probability \( r_i \) the lawyer of \( P_i \) is informed and always appeals the decision taken by \( C \). With probability \( 1 - r_i \) the lawyer of \( P_i \) is uninformed and appeals the decision only when appeals cost is low. Thus, by taking the incorrect decision, \( C \) obtains

\[
v(d = j| \theta = i) = (1 - q) (1 - r_i) \pi(j, 0)
\]

5 The equilibrium strategies

5.1 Asymmetric case

In this section we consider the case where one party, say \( P_1 \), hires a certified lawyers whilst the other party, \( P_2 \), hires an uncertified lawyer. We then have: \( r_1 = r \) and \( r_2 = 0 \), and in light of the analysis in the previous section we obtain the following proposition.
Proposition 1 When party \( P_1 \) hires a certified lawyer and \( P_2 \) does not, (i) in equilibrium the competent adjudicator chooses \( d = \theta \), whilst the incompetent adjudicator chooses \( d = 1 \) with probability \( z_{hl}^1 \in (\frac{1}{2}, 1] \) and \( d = 2 \) with probability \( z_{hl}^2 \).

Proof. See the Appendix

The proposition above highlights the presence of a ‘decision-bias effect’ that arises from the interaction between career concerns of adjudicators and a lawyer’s knowledge of the underlying state. Whilst the competent adjudicator always takes the correct decision, the incompetent adjudicator biases his decision in favour of the party with the certified lawyer. Intuition follows from the fact that the appeal strategy of a certified and informed lawyer depends on the underlying state of the world \( \theta \) and it is therefore more informative about the state of the world than the appeal strategy of the uninformed lawyer. Since the decision of \( C \) is correlated with the underlying state, the appeals strategy of the certified and informed lawyer reveals information about the type of the adjudicator that the appeals strategy of the uninformed lawyer does not. \( I \) has then more to fear from appeals by a certified lawyer than by an uncertified lawyer, which yields the decision bias of Proposition 1.

There are two types of equilibrium that arise and lead to the decision bias, depending on parameters value. The first one is a mixed strategy equilibrium where the incompetent adjudicator chooses \( d = 1 \) with probability \( z_{hl}^1 \in (\frac{1}{2}, 1] \). The second one is a pure strategy equilibrium where \( z_{hl}^1 \in (\frac{1}{2}, 1] \). Intuitively, when \( z_{hl}^1 = 1 \) a decision \( d = 2 \) perfectly signals the competency of the adjudicator. However, when an incompetent adjudicator deviates and announces \( d = 2 \), there is the possibility that the certified lawyer learns that \( \theta = 1 \). In this case there is a conflict in beliefs and Bayes rule doesn’t apply. As we focus on a sequential equilibrium, the litigant decides to appeal when informed that \( \theta = 1 \), in which case the decision is reversed. For some parameters, this effect limits the incentives to deviate to \( d = 2 \), and yields a pure strategy equilibrium rather than a mixed strategy one.

The corollary below highlights the implication of an increase in the probability \( r \) that a certified lawyer is informed, that is of an increase in the quality of certified
lawyers.

**Corollary 1** When party $P_1$ hires a certified lawyer and $P_2$ does not, the decision bias in favour of the party with the certified lawyer is non-decreasing in the probability $r$ that a certified lawyer is informed.

Since the difference between a certified and an uncertified lawyer rests with $r$, the probability that the certified lawyer is informed, then as $r$ increases so does the difference in appeals probability between the two types of lawyers. Other things being equal, this increases the risk for the incompetent adjudicator of facing a reputational loss through reversal in appeal when he takes a decision unfavourable to the party with the certified lawyer. To reestablish indifference in the mixed strategy equilibrium, the decision bias in favour of that party must increase.

To further highlight the role of appeals, consider now what happens to the decision bias when the cost of appeals decreases.

**Corollary 2** When party $P_1$ hires a certified lawyer and $P_2$ does not, the decision bias in favour of the party with the certified lawyer is non-increasing in the probability $q$ of low appeals cost.

As $q$ increases, appeals when $d = 1$ and when $d = 2$ become more likely (for a given $z_{hl}^1$). However, other things being equal, the probability of an appeal increases more when the decision is unfavourable to the party with the uncertified lawyer than to the one with the certified lawyer. This is because $q$ does not affect the appeals strategy of a certified and informed lawyer who always and only appeals incorrect decisions; $q$ only affects the appeals from an uninformed (certified or not) lawyer. Thus as $q$ increases, the incentives of the incompetent adjudicator to make a decision in favour of the party with the certified lawyer decrease. To reestablish indifference in the mixed strategy equilibrium, the decision bias in favour of that party must decrease.
Before concluding this section we note that the decision bias here discussed is given by a combination of reputational concerns for the adjudicator and his knowledge of lawyers’ quality. Lawyers’ quality as such does not lead to bias.

**Corollary 3** When the adjudicator has symmetrical belief about the quality of the lawyers, at the equilibrium there is no decision bias.

Corollary 3 then identifies the source of bias in the public nature of the quality mark. Were the adjudicator uninformed over the quality of the lawyers representing the litigants in the dispute, no decision bias would arise in equilibrium. It is the knowledge about the lawyers’s quality that matters. It also follows immediately from corollary 1 that the less precise is the information of the adjudicator as to the lawyers’ quality the lower the decision bias. Intermediate cases can be viewed as representative of situations where the system of certification is imprecise.

### 5.2 Symmetric cases

Suppose now that both parties hire a certified lawyer and thus $r_1 = r_2 = r$. In this case, with probability $r$ the certified lawyer is informed and an incorrect decision is always reversed in appeal. With probability $1 - r$ the certified lawyer is uninformed and an incorrect decision is reversed only if the appeals cost is low. Since the typology of lawyers for each side in the dispute is the same so is their appeals strategy and there is no incentive for $I$ to bias his decision in favour of one party. Moreover, a decision bias in favour of $P_i$ would generate countervailing incentives to $I$ because a decision favourable to $P_i$ would then signal incompetency.

The same conclusion holds also when both parties hire an uncertified lawyer and thus $r_1 = r_2 = 0$.

**Proposition 2** When both parties hire a certified lawyer or when both parties hire an uncertified lawyer, at the equilibrium there is no decision bias: $z_{1h}^{hh} = z_{1l}^{ll} = \frac{1}{r}$. 
6 The market for lawyers

In this section we derive the demand for certified lawyers. To this purpose we calculate the payoffs of the parties when they choose a certified lawyer and study the incentives of litigants to hire certified lawyers. We denote by \( l \) a low quality, uncertified lawyer, and by \( h \) a high quality, certified lawyer.

- **Case ll**

  If both \( P_1 \) and \( P_2 \) choose \( l \), we know from Proposition 2 that \( z_{1l} = \frac{1}{2} \) and the behaviour of the parties is symmetrical. Since \( \Pr(D = j \mid d = i) = \frac{1}{2}(1 - \gamma) < \frac{\pi}{(1 + \delta)} \), the losing party appeals only if \( A \) is low; \((1 - \gamma)(1 - q)\frac{1}{2} \) is therefore the probability that the final decision is not correct and it follows that the expected payoff of a party in case \( ll \) is given by (normalized by \( V \))

  \[
  u_{ll} = \frac{1}{2} + \left( 1 - (1 - \gamma)(1 - q) \right) \frac{1}{2} \frac{\delta}{2}.
  \]

- **Case hh**

  If both \( P_1 \) and \( P_2 \) choose \( h \), the behaviour of the parties is also symmetrical and \( z_{1h} = \frac{1}{2} \) (Proposition 2). Compared to the case \( ll \) now the lawyer may know that the decision is incorrect and appeal it also when \( A \) is high. The expected payoff of a party in case \( hh \) is therefore given by

  \[
  u_{hh} = u_{ll} - (1 - \gamma)(1 - q) \frac{r}{4}(\bar{A} - \delta)
  \]

- **Case hl and lh**

  If \( P_1 \) chooses \( h \) and \( P_2 \) chooses \( l \), we know from Proposition 1 that \( z_{1hl} > 1/2 \). Compared to case \( ll \), hiring a certified lawyer when the opponent’s lawyer is uncertified affects the legal outcome when the adjudicator is incompetent and the appeal cost is high, which occurs with probability \((1 - \gamma)(1 - q)\). Then the chance of obtaining a favourable decision in the lower court raises from \( 1/2 \) to \( z_{1hl} \). Furthermore, when
the lower court’s decision is unfavourable, the certified lawyer may discover that the decision is incorrect and appeal. This raises the winning probability further by $\frac{r_z h}{2}$.

Taking also into account the benefit from a correct decision, the expected payoff of $P1$ in case $hl$ is

$$u^{hl} = \frac{1}{2} + (1 - \gamma)(1 - q) \left(z^{hl}_1 - \frac{1}{2} + \frac{r_z h}{2} z^{hl}_2\right) + (1 - (1 - \gamma) z^{hl}_2 (1 - q)(1 - r)) \frac{\delta}{2} - (1 - \gamma) z^{hl}_2 \frac{r}{2} (1 - q) \bar{A}$$

whilst the expected payoff of $P2$ is

$$u^{lh} = \frac{1}{2} - (1 - \gamma)(1 - q) \left(z^{hl}_1 - \frac{1}{2} + \frac{r_z h}{2} z^{hl}_2\right) + (1 - (1 - \gamma) z^{hl}_1 (1 - q)) \frac{\delta}{2}$$

Notice that the quality of lawyers is only relevant when the cost of appeal is high, which occurs with probability $(1 - q)$, and the decision maker is incompetent, which occurs with probability $(1 - \gamma)$. We now define

$$w^h \equiv u^{hh} - u^{lh} \text{ and } w^l \equiv u^{hl} - u^{ll},$$

thus $w^h$ and $w^l$ capture respectively the gain for a party from hiring a certified lawyer when the other party has a certified lawyer and when she does not. It is easy to show that

$$w^l - w^h = (1 - \gamma)(1 - q) \left(z^{hl}_1 - \frac{1}{2}\right) \frac{r_z h}{2} (\bar{A} - \delta)$$

and we are now in a position to characterize the demand for certified lawyers.

**Lemma 1** $w^h$ and $w^l$ are positive, and the decision-bias effect implies that $w^l > w^h$.

For a given salary $w$, the demand for certified lawyers is characterized as follows.

(i) If $V \geq \frac{w}{w^h}$, both sides hire a certified lawyer;

(ii) If $\frac{w}{w^h} > V \geq \frac{w}{w^l}$, only one side hires a certified lawyer;

(iii) If $\frac{w}{w^l} > V$, no side hires a certified lawyer.
Public information over the quality of legal service generates a ‘matching effect’. Litigants to a dispute form different pools based on the amounts at stake. Those with high-value cases hire high-quality lawyers and pay a high salary, whilst those with low-value cases hire low-quality lawyers and pay a low salary.

The gain for a litigant from hiring a certified lawyer stems from the decision-bias effect and from more informed appeals. By hiring a certified lawyer when the opponent does not, a party gains a decision bias in her favour; when the other party also has a certified lawyer, the party gains that no decision bias against her will arise. Also a certified lawyer discovers and appeals incorrect decisions more often than an uncertified lawyer.

The gain from a certified lawyer is however greater when the other party does not have a certified lawyer than when she does (i.e., \( w^l > w^h \)). This is due to the decision-bias effect. While the increase in the probability of winning is independent of the quality of the other party’s lawyer, the cost of appeal is borne less often when the rival’s lawyer is uncertified, i.e. when favoured by the bias than when on equal foot with the competitor. With \( w^l > w^h \), if the cost \( w \) of a certified lawyer is sufficiently low relatively to the value of the dispute, both parties hire a certified lawyer. If instead \( w \) is sufficiently high, both parties hire an uncertified lawyer. For intermediate values of \( w \), \( P1 \) who moves first hires a certified lawyer whilst \( P2 \) who moves second does not. If it was not for the decision-bias effect, \( w^b \) would be equal to \( w^l \) i.e. the value of a certified lawyer to a party would be independent of whether either side has hired a certified lawyer or not. Only cases with two certified lawyers or with none would then arise, depending on the cost \( w \).

In the light of Lemma 1, the total demand for certified lawyers is given by

\[
D (w) = 2 - F \left( \frac{w}{w^h} \right) - F \left( \frac{w}{w^l} \right).
\] (4)

**Corollary 4** For \( r \) below some threshold \( \bar{r} \), the demand for certified lawyers is increasing with \( r \).
For $r$ not too high, the gain for a party from hiring a certified lawyer is increasing in the quality of the lawyer both when the other party’s lawyer is certified and when he is not (i.e., $w^l, w^h$ are both increasing in $r$). Over this range an increase in $r$ raises the efficiency of appeals and the decision-bias effect. The condition on $r$ is then explained as follows. When there is only one certified lawyer and $r$ is sufficiently high that $z^{hl}_i = 1$, the certified lawyer never intervenes since he always receives a favourable decision by the adjudicator. Thus the parties’ utility levels, $u^{hl}$ and $u^{lh}$, are independent of $r$, implying that the value $w^l$ of hiring a certified lawyer when the opponent’s lawyer is not certified is also independent of $r$. Moreover, when there are two certified lawyers, from equation (2), the marginal effect of $r$ on $w^{hh}$ is $(1 - \gamma) (1 - q) \frac{1}{4} (\delta - A) < 0$. The utility of both parties decreases with the quality of certified lawyers because the winning probability is unchanged but the probability of appeal increases. Therefore, in this case the value $w^h$ of hiring a certified lawyer when the other party has hired a certified lawyer decreases with $r$. As a result demand decreases (weakly) with $r$.

The market equilibrium then equates demand $D(w)$ to supply $S$. In the light of Corollary 4, we obtain.

**Proposition 3** The equilibrium salary $w(r)$ exists and is positive. $w(r)$ is unique and increases with $r$ for $r < \bar{r}$.

The market wage increases with the quality of certified lawyers reflecting the higher demand from parties in a dispute. However, as we shall see, this does not imply a higher value for the parties as a whole or for society.

7 The value of quality

In the previous section we have discussed the gain for a litigant from hiring a certified lawyer, given the choice of the other litigant. In the corollary below we find however that, when it is calculated at the equilibrium, the value of quality for the parties is negative.
Corollary 5 (i) For $V \geq \frac{w}{wh}$ both sides obtain $u^{hh}V - w < u^{ll}V$. (ii) For $V \in [\frac{w}{wh}, \frac{w}{wh}]$ one side obtains $u^{hl}V - w > u^{ll}V$ and the other side obtains $u^{hh}V < u^{ll}V$, and $u^{hl}V - w + u^{lh}V < 2u^{ll}V$. (iii) For $\frac{w}{wh} > V$, both sides obtain $u^{ll}V$.

In our model there is a prisoners’ dilemma problem: the litigants would collectively be better off if they each hired an uncertified lawyer. Intuitively, consider case $hh$. Each party has an incentives to pay a higher salary $w$ to hire a certified lawyer in order to obtain a decision bias in her favour. In equilibrium both parties hire a certified lawyer, pay the additional cost $w$ but obtain no decision bias. The effects of hiring a certified lawyer are then reduced to an increase in the probability that the correct decision is reached (due to more informed appeals) and to an increase in the appeals cost (due to certified lawyers appealing also when $A$ is high). With $\delta$ sufficiently low, as under to the assumption $\delta < \bar{A}$, the cost of additional appeals dominates the benefit from a better decision and the public knowledge of quality negatively affects the private parties’ payoffs.

Let $\lambda$ be the social value of a correct decision. Typically society will care about the correct decision being taken but will not attach the same value to it than the private parties. As pointed out by Shavell (1997) there is a divergence between the private and the social motive to use the legal system. When a party makes a litigation decision, she does not take into account the legal costs that she induces the opponent to incur (a negative externality), nor does she recognize associated effects on deterrence and other social benefits (a positive externality). Consequently, the privately determined level of litigation can either be socially excessive or inadequate. For this reason we assume no relationship between $\lambda$ and $\delta$.

We assume however that $\lambda > \bar{A}$, which implies that appeals costs are always worth incurring if the appeal leads to the correct decision. This seems reasonable as otherwise it would be optimal not to have an appeals system; we shall however discuss the opposite case briefly below. Then the normalized social value from the judicial procedure is $\omega V$, with $\omega = \Pr(D = \theta)\lambda - E(A)$, where with a slight abuse of notation, $\Pr(D = \theta)$ is
the probability that the final decision is correct while $E(A)$ is the expected cost borne by the parties for appeals. In this function we ignore the utility of the adjudicator and more generally the social value of the information generated on the adjudicator.\footnote{Notice that in our model the adjudicator’s preferences are linear in posterior beliefs on his ability so that his ex-ante expected utility is independent of the equilibrium and equal to $\gamma$. A value of the information on the adjudicator could be accounted for by incorporating a componant $\kappa(\pi)$ convex in $\pi$.} The social welfare is then

$$\Omega = \omega^I \int_0^\infty V dF V + \omega^H \int_0^\infty \omega^H V dF (V) + \omega^{hh} \int_0^\infty \omega^{hh} V dF V,$$

(5)

where $\omega^I$ denotes the value of $\omega$ for cases where both litigants hire uncertified lawyers, and so on.

Before analyzing the value of information, we notice that quality has a positive social value.

Proposition 4 The social welfare is higher if there are certified lawyers with positive quality, i.e if $r > 0$, than if all lawyers are of low quality, i.e. if $r = 0$.

Recall that the quality of lawyers is only relevant when the cost of appeal is high, i.e. when $A = \bar{A}$ and the decision maker is incompetent. From a social perspective, a high-quality lawyer generates one benefit and one cost. The benefit is given by the increase in the probability of reversing incorrect decision that more informed appeals generate. The cost is the increase in the cost of appeals, as certified lawyers appeal also when $A$ is high. This explains Proposition 4. If it were the case that $\lambda < \bar{A}$, then high-quality lawyers would have negative social value and the social optimum would be to have $S = 0$.

8 The social value of certification

In the previous section we have shown that quality of certified lawyers has a positive impact on welfare. In this section we show however that information over quality is not necessarily welfare improving.
The first question is whether certification induces a first-best allocation of lawyers. This would be true if there were no bias introduced by certification in the judicial process. But the bias induces a distortion in the allocation of lawyers among cases.

**Proposition 5** Due to the decision bias, the equilibrium allocation involves too many cases with only one certified lawyer and too few cases with two certified lawyers.

This result is the consequence of the decision bias that makes $w^l > w^h$ and raises the mass of disputes with only one certified lawyer (Lemma 1). Consider two cases with value $V' > V$ each with one certified lawyer. The welfare is $\omega^{hl} V + \omega^{hl} V'$. Suppose we allocate the two certified lawyers to the case $V'$ so as to obtain $\omega^{hl} V + \omega^{hh} V'$. We prove in appendix that $V' (\omega^{hh} - \omega^{hl}) > V (\omega^{hl} - \omega^h) > 0$ which implies that the welfare is higher when both lawyers are on the same case. Welfare would be maximized by allocating lawyers on the highest value case, which would occur if there was no decision bias and $w^l$ were equal to $w^h$ (from expression 3).

From a social point of view the marginal value of having a certified lawyer is higher when the other party also has a certified lawyer than when she does not. This is in contrast with what we have seen for the private value of quality to a litigant, where the gain from a certified lawyer is greater when the other party does not have a certified lawyer than when she does (from Lemma 1). It stems in part from the value of a correct decision being smaller for private parties than for society as a whole ($\delta < \bar{A}$ but $\lambda > \bar{A}$). It stems also from the decision bias reducing the social value of certified lawyers by lowering the likelihood that the lawyer’s information will be used in the appeal process.\(^{11}\)

As raising $r$ amplifies the decision bias, the distortion of the allocation of lawyers is more severe for large quality of lawyers.

\(^{11}\)Notice that when a case involves only one certified lawyer, social welfare would be maximized by inducing the incompetent adjudicator to be biased against him so as to induce efficient use of information.
Lemma 2 \textit{Increasing the quality of certified lawyers raises the mass of cases with one certified lawyers and reduces the mass of cases with two certified lawyers.}

Proposition 5 and Lemma 2 suggest that a system of quality certification is not necessarily welfare enhancing. The matching effect that knowledge of lawyers’ quality generates, described in Lemma 1, does not lead to a first best allocation, due to the decision bias effect. To study the social value of certification, consider then the allocation that is obtained in the absence of a certification system. The parties are now unable to distinguish the high-quality lawyers from the low-quality lawyers; absent the matching effect, lawyers are allocated randomly across cases. To highlight the matching effect, we assume here that, in the absence of certification, high-quality lawyers will all be hired on some disputes. Of course one other cost of not having certification is that they may choose other activities. We endogenize the mass of certified lawyers in Section 9.2.

Thus the $S$ certified lawyers are allocated to some case. As a result a fraction $\frac{(2-S)^2}{4}$ of cases have no certified lawyers, a fraction $\frac{(2-S)S}{2}$ of cases have only one certified lawyer and the remaining fraction of cases have two certified lawyers. Furthermore, in all asymmetric cases no \textit{decision-bias effect} arises. Indeed, in the absence of a system of certification, the adjudicator is unable to distinguish high-quality lawyers from low-quality lawyers, and we know from Corollary 3 that in this case $I$ chooses $d = 1$ with probability $1/2$.

The welfare in the absence of a system of certification is then given by

$$\Omega = \left( \frac{(2-S)^2}{4} \omega_{hl} + \frac{S^2}{4} \omega_{hh} + \frac{(2-S)S}{2} \tilde{\omega}_{hl} \right) \int_0^\infty V dF(V) \tag{6}$$

where $\tilde{\omega}_{hl}$ denotes the social value for cases with just one certified lawyer but no decision bias, i.e where $z_1^{hl} = z_2^{hl} = \frac{1}{2}$. By decomposing the welfare under certification given by $\Omega$ in expression (5), the \textit{matching effect} and the \textit{decision-bias effect} can be measured.
respectively by $\Omega^m$ and $\Omega^{db}$ where

$$
\Omega^m = \omega^l \int_0^{\bar{V}} VdF(V) + \tilde{\omega}^h \int_{\bar{V}}^{\infty} VdF(V) + \omega^{hh} \int_{\bar{V}}^{\infty} VdF(V),
$$

$$
\Omega^{db} = (\omega^{hl} - \tilde{\omega}^{hl}) \int_{\bar{V}}^{\infty} VdF(V),
$$

and $\Omega = \Omega^m + \Omega^{db}$.

**Proposition 6** The matching effect that arises under a system of quality certification increases social welfare whilst the decision-bias effect reduces it: $\Omega^m > 0$, $\Omega^{db} < 0$.

Information over the quality of legal services generates a trade-off. On the one hand, it affects the allocation of lawyers by inducing high-quality lawyers to serve clients with the high-value cases. Ceteris paribus, this is positive since high-value cases are those where a correct decision is most valuable. On the other hand, a system of certification creates decision bias and this is costly in welfare terms as it generates a misallocation of lawyers and it reduces the likelihood that the certified lawyer’s information will be used. When this second effect prevails a system of certification reduces social welfare.

For the case of a uniform distribution of $V$, we show as follows.

**Corollary 6** Suppose that $V$ is uniformly distributed on $[\bar{V} - 1, \bar{V}]$, that $\bar{V}$ is close to $\frac{\bar{W}}{w - \bar{w}}$ and that $S$ is smaller but close to 1, then the social value of certification is negative.

Under the conditions in the corollary, certification results in only cases with one certified lawyers which exacerbates the negative impact of the decision-bias effect on the efficiency of allocation. Moreover, this also minimizes the benefit of the matching effect: as all the lawyers are assigned to $P1$, certified lawyers are reallocated from $P2$ to low-value cases.
9 Extensions

9.1 Proof taking

We have so far focused on the interaction between the role of lawyers in the appeal process and the adjudicator’s reputation concerns. In legal systems however lawyers are also in charge of the task of proof taking and it is natural to ask how our results would be affected by this. In particular a lawyer’s quality should be also reflected in his ability to produce evidence in court. To address this issue, let us focus on the case where $r = 1$, i.e. certified lawyers observe the true state of the word. Assume that in addition a certified lawyer observes a verifiable signal of the state of the world with probability $\mu$. For simplicity, we assume that the verifiable signal is perfectly informative. It can be costlessly and publicly transmitted to the court but it can be concealed. In particular the report is observed by outside observers.

It is straightforward that the certified lawyer will transmit the signal when it is favourable to his client and not otherwise. The analysis of the judicial game is unchanged when the two lawyers are both certified or both uncertified. The reason is that there is perfect inference by the judge when a verifiable signal is transmitted and no new information on the state of the world if no signal is transmitted.

But consider the case where the party $P_1$ has hired a certified lawyer whilst $P_2$ has not. We denote by $h$ the public report on the verifiable signal made by $P_1$. In this setting, with probability $\frac{1}{2}\mu$, $P_1$ (finds and) reports evidence in his favour (denoted by $h = 1$) and the decision maker chooses $d = 1$. $P_1$ never reports $h = 2$ as he would lose the case. Thus, with probability $(1 - \frac{1}{2}\mu)$ $P_1$ reports that he found nothing (denoted by $h = \phi$). The decision maker anticipates that a report $h = \phi$ comes either from the certified lawyer not finding any verifiable signal or from him concealing evidence against his case. Thus, following $h = \phi$, the posterior is

$$\Pr(\theta = 1 \mid \phi) = \frac{1 - \mu}{2 - \mu} < \frac{1}{2}.$$ 

Since $\Pr(\theta = 1 \mid \phi) > \Pr(\theta = 2 \mid \phi)$ for any $\mu > 0$, the efficient decision for the incompetent adjudicator when there is no report is $d = 2$. The reason is that no report
should be interpreted as bad news for $P_1$, raising the suspicion that $P_1$ conceals some information. However, the adjudicator may not choose the efficient decision because of the reputation effect.

**Proposition 7** When only $P_1$ hires a certified lawyer and $\mu < 2 - \gamma - q$, the incompetent adjudicator facing no report chooses $d = 1$ with a positive probability $z_1$; thus a decision bias in favour of $P_1$ arises. The probability $z_1$ is decreasing in $\mu$.

The decision-bias effect is still present, but it has to be adjusted to account for the fact that $d = 2$ is more likely to be the correct decision. Indeed if $\mu$ is close to 1 and the appeal cost is likely to be low ($q$ high), the adjudicator is almost sure that a decision $d = 1$ will be reversed in appeal. Thus the decision bias remains only for $\mu$ below some threshold.

### 9.2 Incentives to train and the supply of lawyers

Consider now the supply of certified lawyers by looking at the incentives of lawyers to train and obtain certification.

Suppose there is a training stage where lawyers choose whether to train in order to raise their individual quality. There is a test certifying that quality is above a threshold $r$. Untrained lawyers have quality 0. Raising quality from zero to some positive level $r \in (0, 1]$ is costly. The cost of training is heterogenous among lawyers and we denote by $S(c)$ the mass of lawyers with cost of training to level $r$ smaller than $c$. We assume that $S(c)$ is continuous and increasing from 0 to $\infty$ on the real line.

Lawyers maximize their expected salary net of training cost. In our setup this implies that a lawyer will either train to raise his quality to $r$ and be certified or he will not train at all in which case his quality will be zero.

In this setting a system of certification generates an *investment effect*: it induces lawyers to train to raise their salary from 0 to $w$. In particular, a lawyer chooses to train if $w - \tau - c \geq 0$ where $\tau$ is the price of a certification test. Then the market
equilibrium wage solves

\[ D(w) = S(w - \tau) \]

and the total number of certified lawyers in the market is \( S(w - \tau) \).

In this setting we can analyze the effect of changing the supply of certified lawyers by adjusting \( \tau \). The effect is shown in appendix to be

\[
\frac{\partial \Omega}{\partial S} = x_1 (\omega_{hh} - \omega_{ll}) z^{hl}_2 V_1 + x_2 (\omega_{hh} - \omega_{ll}) z^{hl}_1 V_2 - (w - \tau) \tag{7}
\]

where \( (w - \tau) \) is the marginal training cost.

As \( S \) increases, the additional lawyers will be randomly allocated between the two types of cases, those with no certified lawyer and those with only one certified lawyer. Then the increase in the mass of cases with one lawyer bring a value \((\omega_{hh} - \omega_{ll}) V_1 = (\omega_{hh} - \omega_{ll}) z^{hl}_2 V_1\), whilst the increase in the mass of cases with two lawyers bring a value \((\omega_{hh} - \omega_{ll}) V_2 = (\omega_{hh} - \omega_{ll}) z^{hl}_1 V_2\). The increase of supply \( S \) is then transferred to the two margins with proportion \( x_1 \) and \( x_2 \). The first term in expression (7) is thus the expected social benefit from an increase in \( S \). The second term is the expected social cost from the increase in the cost of training.

**Proposition 8** When \( 1 + \delta > \lambda \), if \( \tau = 0 \) in equilibrium there is an excessive supply of certified lawyers.

Thus, for a given certification standard, whenever the private benefit of reversing an incorrect and unfavourable decision is larger than the social benefit of a correct decision, the number of lawyers who choose to invest in training is excessive compared to the social optimum. The parties have here excessive incentives to win, hence an excessive incentives to hire certified lawyers.

### 9.3 Case where \( \delta > \bar{A} \)

We have assumed throughout the paper that the private value of a correct decision is lower than the appeals cost. When this assumption is relaxed, there is no bias
in decisions in equilibrium, but still the decision-bias effect is present and generates excessive incentives to hire certified lawyers.

**Proposition 9** When $\delta > \overline{\alpha}$: (i) $u^{hh} - u^{ll} < u^l = u^{hl} - u^{lh} < w^h = u^{hh} - u^{lh}$; (ii) for $V > \frac{w}{w+},$ the equilibrium is $hh$, whilst for $V < \frac{w}{w+}$, the equilibrium is $ll$; (iii) both sides obtain $u^{hh}V - w < u^lV$ for $V \in \left(\frac{w}{w+}, \frac{w}{w-w+}\right)$.

At the equilibrium there are now either cases with no certified lawyers or cases with two certified lawyers. The asymmetric equilibrium, where only one party hires a certified lawyer, ceases to exist. The reason is that a certified lawyer is now more valuable if the other party has a certified lawyer than if the other party’s lawyer is not certified. Furthermore, the prisoner dilemma problem always arises but only for some intermediate values of $V$. Provided that the value $V$ is large compared to the salary $w$, the parties are now better off from both hiring a certified lawyer than not, as the gain from better quality decisions is now greater than the appeals cost. Indeed the probability of winning is $1/2$ in both cases, but with certified lawyers, parties benefit from winning the "right" case more often. However, for intermediate values of $V$, they end up hiring certified lawyers at salaries higher than their values, due to competition to create or correct the decision bias.

In this case, certification yields the first-best allocation when the supply of lawyers is fixed since there are asymmetric disputes. However we show in Appendix that there is still n excessive supply of lawyers if the supply is endogenous.

10 Conclusions

We have studied the value of information on the quality of legal services by analyzing how quality certification affects the incentives of litigants to hire high-quality lawyers, the incentives of lawyers to invest in training and the decision-making behaviour of adjudicators. In this context, the presence of a decision bias has adverse effect on the value of information, both for private parties and for society. We have shown that
quality certification is more likely to be beneficial when the social value of a correct
decision is high or when training costs are low or when the appeals cost is low.

To the extent that the social value of a correct decision is higher in systems based
on precedent, such as the common law system, our results suggest that a QC system is
more likely to be beneficial in a common law system than in a civil law system. This
is in line with casual observation that quality certification in the form of a QC system
is prevalent in countries with common law tradition rather than in counties with civil
law tradition of codified law.

We have assumed throughout that the lower court comprises only one adjudicator
who may be more or less competent. It would be interesting to extend the analysis to
account for the possibility that the lower court comprises a panel of judges (or that there
is a jury) and then study whether cases with certified lawyers are better adjudicated
by a single judge as opposed to a panel (or a jury). Two contrasting effects would play
a role here. On the one hand, assigning panels to cases with one certified lawyers is
beneficial because it reduces the incidence of the bias effect. On the other hand, panels
are more valuable when incorrect decisions are a priori more likely, that is when there
are no certified lawyers on the case.

Finally, we have considered a setting where, absent certification, information is
symmetric. It would be interesting to extend the results to settings where absence
certification, asymmetry of information exists between lawyers.

11 Appendix

Proof of Proposition 1. Consider the case of a mixed strategy equilibrium where
0 < z₁ < 1. This requires that

\[ v(d = 1) = v(d = 2) \]

or

\[ \frac{1}{2} \pi(1, 1) + (1 - q) \pi(1, 0) = \frac{1}{2} q (1 - r) \pi(2, 2) + \left(\frac{1}{2} r + (1 - r) (1 - q)\right) \pi(2, 0) \]
where the RHS decreases with $z_1$ and the LHS increases with $z_1$.

For $z_1 = 1/2$ we have

$$v(d = 1) = q \frac{\gamma}{1 + \gamma} + (1 - q) > v(d = 2) = q \frac{\gamma}{1 + \gamma} + (1 - q) \frac{\gamma (2 - r)}{2\gamma + (1 - \gamma)(2 - r)}$$

since $\frac{\gamma (2 - r)}{2\gamma + (1 - \gamma)(2 - r)} < 1$.

Thus an equilibrium with mixed strategy exists if at $z_1 = 1$, we have

$$q \frac{\gamma}{2 - \gamma} + (1 - q) \left( 1 - \frac{r}{2} \right) = 1 - r \left( 1 - r \right) \frac{q}{2}$$

where $1 - \frac{r}{2} - (1 - r) \frac{q}{2} < 1 - \frac{r}{2}$.

Consider now an equilibrium with $z_1 = 0$, then $\pi (1, 0) = \pi (1, 1) = 1$ and there can only be appeal with some probability $x$ after $d = 1$ if the cost is zero:

$$v(d = 1) \geq q \frac{\gamma}{2} x + q (1 - x) + 1 - q \geq 1 - \frac{q}{2}$$

but

$$v(d = 2) = \frac{1}{2} q \gamma + (1 - q) \left( 1 - \frac{1}{2} r \right) \frac{\gamma}{\gamma + (1 - \gamma)(2 - r)} < 1 - \frac{q}{2}$$

Thus this cannot be an equilibrium.

Suppose that in equilibrium the competent adjudicator chooses $d = \theta$ and consider the payoff of the incompetent one if he chooses $z_1 = 1$ in equilibrium; we have

$$v(d = 2, z_1 = 1) = 1 - \frac{r}{2} + \frac{r}{2} \pi (2, 1)$$

where $\pi (2, 1)$ is arbitrary.

There is an equilibrium with $z_1 = 1$ if $v(d = 1, z_1 = 1) = q \frac{1}{2} \gamma + (1 - q) \frac{\gamma}{2 - \gamma} \geq v(d = 2, z_1 = 1)$, for some $\pi (2, 1)$, thus if

$$q \frac{1}{2} \gamma + (1 - q) \frac{\gamma}{2 - \gamma} \geq 1 - \frac{r}{2}$$

which is obtained for $\pi (2, 1) = 0$.

Now suppose that $1 - \frac{r}{2} > q \frac{1}{2} \gamma + (1 - q) \frac{\gamma}{2 - \gamma} > 1 - \frac{r}{2} - (1 - r) \frac{q}{2}$. Then it must be the case that the equilibrium has $z_1 = 1$ and the uncertified lawyer appeals when $d = 2$.
with some probability $x > 0$ when the cost is $A = 0$. We then have

\[
v(d = 2, z_1 = 1, x) = 1 - \frac{r}{2} - (1 - r) \frac{qx}{2}
v(d = 1, z_1 = 1) = q\frac{1}{2}\gamma + (1 - q) \frac{\gamma}{2 - \gamma}
\]

and the equilibrium involves

\[
q\frac{1}{2}\gamma + (1 - q) \frac{\gamma}{2 - \gamma} = 1 - \frac{r}{2} - (1 - r) \frac{qx}{2},
\]

which concludes the proof. ■

**Proof of Corollary 1.** First notice that $z_1^{hl} = 1$ for $q\frac{1}{2}\gamma + (1 - q) \frac{\gamma}{2 - \gamma} > 1 - \frac{r}{2} - q\frac{\gamma}{2} + r\frac{\gamma}{2}$; which writes:

\[
r > \frac{2 - q - q\gamma - (1 - q) \frac{2\gamma}{2 - \gamma}}{1 - q} = 1 + \frac{1 - (2 - q) \frac{\gamma}{2 - \gamma}}{1 - q}.
\]

The RHS is bigger than 1 if $1 > (2 - \gamma q) \frac{\gamma}{2 - \gamma}$. Thus for $1 > (2 - \gamma q) \frac{\gamma}{2 - \gamma}$ which holds if $\gamma < \frac{2}{3}$ or $q$ is large, there is always an equilibrium with $z_1^{hl} < 1$.

On the range $z_1^{hl} < 1$ we have

\[
\frac{1}{2} q\pi(1, 1) + (1 - q)\pi(1, 0) = \frac{1}{2} q\pi(2, 2) + (1 - q) \frac{1}{2}\gamma + (1 - \gamma) \frac{\gamma}{2} (2 - r)
\]

where the $\pi$ are independent of $r$. Then the term $\frac{\gamma^2 (2 - r)}{\gamma + (1 - \gamma) z_2^{hl}(2 - r)}$ is decreasing in $r$ which implies that $z_1^{hl}$ is increasing with $r$. ■

**Proof of Corollary 2.** On the range $z_1^{hl} < 1$ we have

\[
q \left(\frac{1}{2}\pi(1, 1) - \pi(1, 0) - \frac{1}{2} \pi(2, 2) + \frac{\gamma}{2\gamma + (1 - \gamma) z_2^{hl}(2 - r)}\right) = \frac{1}{2} \frac{\gamma}{2\gamma + (1 - \gamma) z_2^{hl}(2 - r)} - \pi(1, 0)
\]

where the $\pi$ are independent of $q$.

Thus $z_1^{hl}$ decreases with $q$ if the LHS is negative

\[
\frac{1}{2} \frac{\gamma}{2\gamma + (1 - \gamma) z_2^{hl}(2 - r)} - \frac{1}{2} \frac{\gamma}{\gamma + (1 - \gamma) z_2^{hl}(2 - r)} < 0
\]

and

\[
\frac{\gamma}{2} \left(\frac{2 - (2 - r)(1 - \gamma)(2z_1^{hl} - 1) - r\gamma}{(\gamma + (1 - \gamma) z_2^{hl}(2 - r))(\gamma + (1 - \gamma) 2z_1^{hl})}\right) < 0
\]
So \( z_1^{hl} \) decreases with \( q \) if

\[
z_1^{hl} < \frac{1}{2} + \frac{\gamma r}{4 (2 - r)(1 - \gamma)}.
\]

which holds for \( z_1^{hl} = \frac{1}{2} \). For \( q \) close to 1 we have

\[
\frac{1}{2} \pi (1, 1) \simeq \frac{1}{2} \pi (2, 2)
\]

which implies that \( z_1^{hl} \) is close to 1/2. This along with the fact that \( z_1^{hl} \) increases if it is above some threshold implies that \( z_1^{hl} \) decreases with \( q \). ■

**Proof of Corollary 3 and Proposition 2.** In a symmetric case the incompetent judge chooses the action that yields the highest probability \( \pi \) of being competent in case of no appeal. If one decision is more likely to be chosen by the incompetent, it will yield the lowest \( \pi \) which would yield a contradiction. Thus both decisions are chosen with the same probability. ■

**Proof of Lemma 1.**

Straightforward computations yield

\[
w^h = (1 - \gamma)(1 - q) \frac{1}{2} \left( (2z_1^{hl} - 1 + rz_2^{hl}) + \left( z_1^{hl} - \frac{1}{2} (1 - r) \right) \bar{A} \right)
\]

\[
> (1 - \gamma)(1 - q) \frac{r}{4} (1 + \delta - \bar{A}) > 0
\]

and

\[
w^l = (1 - \gamma)(1 - q) \frac{1}{2} \left( (2z_1^{hl} - 1 + rz_2^{hl}) + \left( z_1^{hl} - \frac{1}{2} (1 - r) \right) \delta - z_2^{hl} r \bar{A} \right)
\]

\[
> (1 - \gamma)(1 - q) \frac{r}{4} (1 + \delta - \bar{A}) > 0.
\]

This leads to \( w^l - w^h = (1 - \gamma)(1 - q) \frac{1}{2} (z_1^{hl} - \frac{1}{2}) r (\bar{A} - \delta) \). Suppose \( w < V u^{hh} - V u^{lh} \equiv V w^h \). Then the second mover always chooses \( h \). The first mover chooses \( h \) since it prefers to pay \( w \) to induce \((h, h)\).

Suppose \( w > V u^{hl} - V u^{ll} \equiv V w^l \), then the second mover always chooses \( l \). The first mover then prefers \( l \) since \( V u^{ll} > V u^{hl} - w \).
Suppose now that $Vu^{hl} - Vu^{ll} > w > Vu^{hh} - Vu^{lh}$. The second mover chooses $h$ if the first mover chooses $l$ and $h$ otherwise. The first mover chooses between $Vu^{hl} - w$ and $Vu^{lh}$. But $Vu^{hl} - Vu^{lh} > Vu^{hl} - Vu^{ll} > w$ (because $u^{lh} < u^{ll}$) so she prefers $(h, l)$.

Proof of Corollary 4.

Since $D$ is non-increasing it suffices to show that $u^h$ and $u^l$ are non-decreasing with $r$.

$$\frac{\partial u^l}{\partial r} = (1 - \gamma) (1 - q) \frac{1}{2} \left[ (2 - r) \frac{\partial z_1^{hl}}{\partial r} + (1 - r) \delta z'(r) + r \bar{A} \frac{\partial z_1^{hl}}{\partial r} + z_2^{hl} (1 + \delta - \bar{A}) \right]$$

is positive since $\frac{\partial z_1^{hl}}{\partial r} \geq 0$.

$$\frac{\partial u^h}{\partial r} = (1 - \gamma) (1 - q) \frac{1}{2} \left[ (2 - r) \frac{\partial z_1^{hl}}{\partial r} + \frac{\partial z_1^{hl}}{\partial r} \delta + 1 - z_1^{hl} + \frac{1}{2} (\delta - \bar{A}) \right]$$

is positive if $z_1^{hl} < 1 + \frac{1}{2} (\delta - \bar{A})$. Since $z_1^{hl}(r)$ is non-decreasing and $z_1(0) = 1/2$ and $1 - \frac{1}{2} + \frac{\delta}{2} - \frac{1}{2} \bar{A} > 0$, there exists $\bar{r}$ such that $\frac{\partial u^h}{\partial r} \geq 0$ if $r \leq \bar{r}$.

Proof of Proposition 3. An equilibrium verifies $S(w, r) = D \left( \frac{w}{w^*} \right) + D \left( \frac{w}{w^*} \right)$ and exists at a positive salary since $S(0, r) = 0 < 2D(0)$ and $D \left( \frac{w}{w^*} \right) + D \left( \frac{w}{w^*} \right)$ converges to 0 when $w$ goes to infinity. On the range where $r \leq \bar{r}$, $S(w, r)$ decreases with $r$ while $D \left( \frac{w}{w^*} \right) + D \left( \frac{w}{w^*} \right)$ is non-decreasing with $r$. Thus the equilibrium salary increases with $r$.

Proof of Corollary 5.

$$u^{hh} = u^{ll} + (1 - \gamma) \frac{1}{4} (1 - q) r (\delta - \bar{A}) < u^{ll}$$

On the range $V \in \left[ \frac{w}{w^*}, \frac{w}{w^*} \right]$ we have $u^{hl}V - w > u^{ll}$. Moreover

$$u^{lh} = u^{ll} - (1 - \gamma) (1 - q) \left( z_1^{hl} - \frac{1}{2} + \frac{r}{2} z_2 \right) - (1 - \gamma) (1 - q) \left( z_1^{hl} - \frac{1}{2} \right) \frac{\delta}{2} < u^{ll}$$

and

$$u^{hl} + u^{lh} = 2u^{ll} + (1 - \gamma) z_2^{hl} (1 - q) \frac{r}{2} (\delta - \bar{A}) < 2u^{ll},$$
which give the results.

**Proof of Proposition 4.**

First note that in the absence of certification, welfare is $\omega^l$ since quality is unobservable and lawyers have no incentives to train. Then note

$$\omega^l = \left(1 - (1 - \gamma) \frac{1}{2} (1 - q)\right) \lambda$$

$$\omega^{hh} = \left(1 - (1 - \gamma) \frac{1}{2} (1 - q) (1 - r)\right) \lambda - r \frac{1}{2} (1 - \gamma) (1 - q) \bar{A}$$

$$\omega^{hl} = (1 - (1 - \gamma) z_2^{hl} (1 - q) (1 - r)) \frac{\lambda}{2} + (1 - (1 - \gamma) z_1^{hl} (1 - q)) \frac{\lambda}{2} - (1 - \gamma) z_2^{hl} \frac{r}{2} (1 - q) \bar{A}$$

$$\omega^l = \omega^{hl} + (1 - \gamma) (1 - q) z_2^{hl} \frac{r}{2} (\lambda - \bar{A})$$

with

$$2 \omega^{hl} - \omega^{hh} - \omega^l = (1 - 2 z_1^{hl}) \frac{r}{2} (1 - \gamma) (1 - q) (\lambda - \bar{A})$$

And if $\lambda > \bar{A}$: $\omega^{hh} > \omega^{hl} > \omega^l$ and $2 \omega^{hl} < \omega^{hh} + \omega^l$. For $\lambda > \bar{A}$ we then have

$$\int_0^{\bar{w}} \omega^l V f (V) dV + \int_{\bar{w}}^{\bar{w}_l} \omega^{hl} V dF (V) + \int_{\bar{w}_l}^{\bar{w}_h} \omega^{hh} V f (V) dV > \int_0^{\bar{w}} \omega^l V f (V) dV.$$

**Proof of Proposition 5.** Define $W (S; r)$ as the level of $w - \tau$ solving $S(w - \tau; r) = S$. Given $S$, let $V_1 = \frac{w}{w_l}$ and $V_2 = \frac{w}{w_h}$ denote the cutoff values. They are solutions of

$$\int_{V_1}^{\bar{V}} f (V) dV = S$$

$$V_1 w^l = V_2 w^h \quad (= W (S; r) + \tau)$$

37
and we can define the level of welfare achieved by the equilibrium allocation as
\[
\Omega = \int_0^{V_1} \omega^H f(V) \, dV + \int_{V_1}^{V_2} \omega^{hl} f(V) \, dV + \int_{V_2}^{V_f} \omega^{hh} f(V) \, dV - \int_0^{W(S,r)} c dS(c,r).
\] (10)

Note that for \( \lambda > \bar{A} : \omega^{hh} > \omega^{hl} > \omega^H \) and \( 2 \omega^{hl} < \omega^{hh} + \omega^H \). Consider an allocation in the thresholds \( V_1 \) and \( V_2 \), where cases with value \( V \in [V_1, V_2] \) have one certified lawyer and cases with value \( V > V_2 \) have two certified lawyers. The allocation must satisfy \( S = D(V_1) + D(V_2) \). Suppose we change \( V_1 \) and adjust \( V_2 \) to maintain the supply, then
\[
\frac{\partial V_2}{\partial V_1} = \frac{-f(V_1)}{f(V_2)}
\]
and
\[
\frac{\partial \Omega}{\partial V_1} \bigg|_{S=\text{cst}} = \left( (\omega^H - \omega^{hl}) V_1 f(V_1) + (\omega^{hl} - \omega^{hh}) V_2 f(V_2) \left( -\frac{f(V_1)}{f(V_2)} \right) \right)
\]
Evaluated at the equilibrium we obtain
\[
\frac{\partial \Omega}{\partial V_1} \bigg|_{S=\text{cst}} = \left( - (\omega^{hl} - \omega^H) V_1 + (\omega^{hh} - \omega^{hl}) V_2 \right) f(V_1) f(w) w > 0,
\]
thus increasing \( V_1 \) would raise welfare.

**Proof of Lemma 2.**

Since \( V_1 w^l = V_2 w^h \)
\[
\frac{1}{V_1} \frac{\partial V_1}{\partial r} - \frac{1}{V_2} \frac{\partial V_2}{\partial r} = \frac{1}{w^h} \frac{\partial w^h}{\partial r} - \frac{1}{w^l} \frac{\partial w^l}{\partial r}
\]
Since \( \frac{\partial V_1}{\partial r} \) and \( \frac{\partial V_2}{\partial r} \) have opposite signs, \( \frac{\partial V_1}{\partial r} < 0 \) if the RHS is negative which is equivalent to \( \frac{\partial}{\partial r} \left( \frac{w^l}{w^h} \right) > 0 \). Now \( \frac{w^l}{w^h} = 1 + \frac{r(z_1^h - \frac{1}{2})(A - \delta)}{(z_1^h - r + 1)(1 + \delta - A) + \frac{1}{2} z_1^h r (1 + \delta - A)} \), so that
\[
\frac{\partial}{\partial r} \left( \frac{w^l}{w^h} \right) = \frac{\partial}{\partial r} \left( \frac{\bar{A} - \delta}{(2 + \delta - r) + \frac{1}{2} (1 + \delta - A)} \right) > 0
\]
Hence the result.

**Proof of Proposition 6.** Recall
\[
\omega^{hl} = z_1^h \omega^H + z_2^h \omega^{hh} = \frac{1}{2} \omega^H + \frac{1}{2} \omega^{hh} - \left( z_1^h - \frac{1}{2} \right) \left( \omega^{hh} - \omega^H \right)
\]
In the case of no certification we have:
\[ \tilde{\omega}^{hl} = \frac{1}{2} \omega^l + \frac{1}{2} \omega^{hh} \]

We then have
\[
\tilde{\Omega} = \int_0^{\bar{V}} V dF(V) \left[ \left(1 - \frac{S}{2}\right) \omega^l + \frac{S}{2} \omega^{hh} \right] \]
\[ = \int_0^{\bar{V}} \omega^l V dF(V) + \left(\frac{\omega^{hh} - \omega^l}{2}\right) S \int_0^{\bar{V}} V dF(V) \]

\[
\Omega = \int_0^{\bar{V}} \omega^l V dF(V) + \left(\frac{\omega^{hh} - \omega^l}{2}\right) \left(\int_0^{\bar{V}} V dF(V) + \int_0^{\bar{V}} V dF(V) \right) \]
\[ - \int_0^{\bar{V}} \left(\frac{z_{hl} - 1}{2}\right) (\omega^{hh} - \omega^l) V dF(V) \]

\[
\frac{\Omega - \tilde{\Omega}}{\omega_{hh} - \omega^l} = \int_0^{\bar{V}} V dF(V) + \int_0^{\bar{V}} V dF(V) - S \int_0^{\bar{V}} V dF(V) - (2z_{hl} - 1) \int_0^{\bar{V}} V dF(V) \]

Because \(2 - F\left(\frac{w}{w^h}\right) - F\left(\frac{w}{w^l}\right) = S\), we have
\[
S \int_0^{\bar{V}} V dF(V) = \left(1 - F\left(\frac{w}{w^h}\right)\right) \int_0^{\bar{V}} V dF(V) + \left(1 - F\left(\frac{w}{w^l}\right)\right) \int_0^{\bar{V}} V dF(V) \]

But
\[
\frac{1}{1 - F\left(\frac{w}{w^h}\right)} \int_0^{\bar{V}} V dF(V) > \int_0^{\bar{V}} V dF(V) \]
\[
\frac{1}{1 - F\left(\frac{w}{w^l}\right)} \int_0^{\bar{V}} V dF(V) > \int_0^{\bar{V}} V dF(V) \]

We thus have the allocation effect, proportional to \(\Omega \).
\[
\int_0^{\bar{V}} V dF(V) + \int_0^{\bar{V}} V dF(V) - S \int_0^{\bar{V}} V dF(V) > 0 \]

which vanishes when the distribution converges to a Dirac. On the other side the bias introduces an efficiency cost, proportional to \(\Omega_{db}\):
\[
- (2z_{hl} - 1) \int_0^{\bar{V}} V dF(V) < 0. \]

39
For the case of a uniform distribution between $\bar{V} - 1$ and $\bar{V}$, we have for an interior solution,\footnote{An interior solution is such that $\bar{V} \geq \frac{w}{w^h}$ and $\frac{w}{w^l} \geq \bar{V} - 1$.} using $2\bar{V} - \frac{w}{w^h} - \frac{w}{w^l} = S$:

$$w = \frac{w^h w^l}{w^h + w^l} (2\bar{V} - S)$$

The solution is interior if

$$\bar{V} \geq \frac{w}{w^h} = \frac{w^l}{w^h + w^l} (2\bar{V} - S)$$

$$\frac{w}{w^l} = \frac{w^h}{w^h + w^l} (2\bar{V} - S) \geq \bar{V} - 1$$

which occurs for

$$\frac{w^l S}{w^l - w^h} \geq \bar{V}$$

$$\frac{w^h (1 - S) + w^l}{w^l - w^h} \geq \bar{V}$$

At $\frac{w^l S}{w^l - w^h} = \bar{V}$ and $S \leq 1$, we obtain $w = \frac{w^l w^h S}{w^l - w^h}$, $\bar{V} = \frac{w}{w^h}$ and $\bar{V} - S = \frac{w}{w^l}$ so that all cases have either 1 or 0 certified lawyers. Notice that this requires that $S > 1 - \frac{w^h}{w^l}$ for the support to be positive.

We evaluate $\frac{\Omega - \bar{\Omega}}{2}$ at $\bar{V} = \frac{w^l S}{w^l - w^h}$ and obtain

$$\frac{\Omega - \bar{\Omega}}{2^{\frac{w^h - w^l}{2}}} = \int_{\bar{V} - S}^{\bar{V}} VdF (V) - S \int_{\bar{V} - 1}^{\bar{V}} VdF (V) - (2z_1^{hl} - 1) \int_{\bar{V} - S}^{\bar{V}} VdF (V)$$

$$= \frac{S}{2} \left( 1 + \frac{w^l + w^h}{w^l - w^h} (1 - z_1^{hl}) - \frac{2w^l S}{w^l - w^h} \right).$$

This is negative for

$$S > 1 - \left( \frac{w^l + w^h}{w^l} \right) \left( z_1^{hl} - \frac{1}{2} \right).$$

\begin{itemize}
  \item **Derivation of expression 7.** To analyze the effect of a change in $S$ whilst keeping $r$ constant suppose there is a tax $\tau$ on certified lawyers that is paid upon successful
certification. Then, defining \( V_1 = \frac{w}{w^l} \) and \( V_2 = \frac{w}{w^h} \), the equilibrium conditions are

\[
\int_{V_1}^V f (V) dV + \int_{V_2}^V f (V) dV = S (w - \tau; r);
\]
and choosing \( \tau \) at given \( r \) is equivalent to choosing the supply \( S \) and consequently the wage. Indeed we have

\[
\tau = w - W (S; r)
\]
where \( W (S; r) \) defines the level of \( w \) solving \( S (w; r) = S \).

Then differentiating (10) with respect to \( S \), using (??) and (??) we obtain

\[
\frac{\partial \Omega}{\partial S} = - (\omega^h - \omega^l) \left( z_2^h V_1 f (V_1) \frac{\partial V_1}{\partial S} + z_1^l V_2 f (V_2) \frac{\partial V_2}{\partial S} \right) - W (S; r)
\]
which equates the gain in judicial decisions with the cost of additional certified lawyers.

Using (8) and (9), the optimal level of \( S \) solves

\[
f (V_1) \frac{\partial V_1}{\partial S} + f (V_2) \frac{\partial V_2}{\partial S} = -1;
\]
and

\[
\frac{\partial V_1}{\partial S} w^l = \frac{\partial V_2}{\partial S} w^h.
\]

which yields

\[
\frac{\partial V_1}{\partial S} = \frac{-w^h}{f (V_1) w^h + f (V_2) w^l} \quad \frac{\partial V_2}{\partial S} = \frac{-w^l}{f (V_1) w^h + f (V_2) w^l}
\]
and substituting back into \( \frac{\partial \Omega}{\partial S} \) we have expression 7.

Following the same reasoning as in Section 4, we have

\[
\pi (i, i) = \frac{\gamma}{\gamma + (1 - \gamma) z_i}, \quad i = 1, 2
\]
\[
\pi (2, 0) = \pi (2, 2)
\]
\[
\pi (1, 0) = \frac{(1 - \mu) \gamma}{(1 - \mu) \gamma + (2 - \mu) (1 - \gamma) z_1}
\]
and

\[ v(d = 1) = \Pr(\theta = 1 | \phi)q\pi(1,1) + (1-q)\pi(1,0) = \frac{1 - \mu}{2 - \mu \gamma + (1 - \gamma) z_1} + \frac{(1-q)(1-\mu)}{(1-\mu)\gamma + (2 - \mu) (1 - \gamma) z_1} \]

and

\[ v(d = 2) = \Pr(\theta = 2 | \phi)\pi(2,0) = \frac{1}{2 - \mu \gamma + (1 - \gamma) z_2} \]

However, I will choose \( z_1 = 0 \) only if at \( z_1 = 0 \) we have \( v(d = 1) < v(d = 2) \), i.e.

\[ \frac{(1-\mu)q}{\gamma} + \frac{(2-\mu)(1-q)}{\gamma} < 1 \]

which holds iff \( (2 - q - \mu) < \gamma \).

When \( 0 < z_1 < 1 \), we have \( v(d = 2) = v(d = 10) \) which writes as

\[ \frac{1}{\gamma + (1 - \gamma) z_2} = \frac{(1-\mu)q}{\gamma + (1 - \gamma) z_1} + \frac{(1-q)(2-\mu)}{\gamma + (2-\mu)(1-\gamma)z_1} \]

where the RHS is decreasing with \( \mu \). Thus \( z_1 \) is decreasing with \( \mu \).  ■

**Proof of Proposition 8.** From (7), at the optimal supply we have

\[ (\omega^{hh} - \omega^l) \frac{f(V_1) w^h z_2^{hl} V_1 + f(V_2) w^l z_1^{hl} V_2}{f(V_1) w^h + f(V_2) w^l} = W(S; r) \]

Using \( W(S; r) = w - \tau \) and \( w = V_1 w^l = V_2 w^h \), we have

\[ (\omega^{hh} - \omega^l) \frac{f(V_1) \frac{w^h}{w^l} z_2^{hl} + f(V_2) \frac{w^l}{w^h} z_1^{hl} V_2}{f(V_1) w^h + f(V_2) w^l} w = w - \tau \]

or

\[ \tau = \left( 1 - (\omega^{hh} - \omega^l) \frac{f(V_1) \frac{w^h}{w^l} z_2^{hl} + f(V_2) \frac{w^l}{w^h} z_1^{hl} V_2}{f(V_1) w^h + f(V_2) w^l} \right) w \]

The tax is positive if

\[ \frac{\omega^{hl} - \omega^l}{w^l} f(V_1) w^h z_2^{hl} + \frac{\omega^{hh} - \omega^l}{w^h} f(V_2) w^l z_1^{hl} V_2}{f(V_1) w^h + f(V_2) w^l} < 1 \]
Then
\[ \frac{\omega_{hh} - \omega_{hl}}{w^h} = \frac{2z_{hl}}{(2z_{hl} - 1) \left( \frac{2 + \delta - A}{r} \right) + (1 + \delta - A)} (\lambda - \tilde{A}) \]
\[ \frac{\omega_{hl} - \omega_{ll}}{w^l} = \frac{2z_{hl}^2}{(2z_{hl} - 1) \left( \frac{2 + \delta - A}{r} \right) + 1 + \delta - A + (2z_{hl} - 1) (\tilde{A} - \delta)} (\lambda - \tilde{A}) \]

Noticing that \(1 + \delta - \tilde{A} < \frac{2 + \delta - A}{r}\), it follows that the first ratio decreases with \(z_{hl}^1\). Thus it is smaller than 1 if the value at \(z_{hl}^1 = 1/2\) is less than 1. This is the case if \(\frac{\lambda - \tilde{A}}{1 + \delta - \tilde{A}} \leq 1\) or if \(\lambda \leq 1 + \delta\). Moreover \(\frac{\omega_{hl} - \omega_{ll}}{w^l} \leq \frac{\omega_{hh} - \omega_{hl}}{w^h}\) because \(z_{hl}^2 < \frac{1}{2} < z_{hl}^1\) and \(\tilde{A} > \delta\). Thus both ratios are smaller than 1 if \(\lambda < 1 + \delta\). It follows that
\[ \frac{\omega_{hl} - \omega_{ll}}{w^l} f(V_1) w^h z_{hl}^2 + \frac{\omega_{hh} - \omega_{hl}}{w^h} f(V_2) w^l z_{hl}^1}{f(V_1) w^h + f(V_2) w^l} < z_{hl}^2 + z_{hl}^1 = 1. \]
as required for positive tax to be optimal.

Case of \(\delta > \bar{A}\)

Proof of Proposition 9. (i) and (ii) follow from the proofs of Lemma 1 and of Corollary 5. The difference is that in the range \(\frac{w}{w^h} < V < \frac{w}{w^h}\), the second mover follows the choice of the first one. The first mover then chooses \(l\) since \(u^l V > u^h V - w > u^{hl} V - w\).

The invariance of the other results is proven by following the same procedures as for the case of \(\delta < \bar{A}\).

References


