Social Security and Risk Sharing

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Abstract

In this paper we identify conditions under which the introduction of a pay-as-you-go social security system is ex-ante Pareto-improving in a stochastic overlapping generations economy with capital accumulation and land. We argue that these conditions are consistent with many calibrations of the model used in the literature. In our model financial markets are complete and competitive equilibria are interim Pareto efficient. Therefore, a welfare improvement can only be obtained if agents' welfare is evaluated ex ante, and arises from the possibility of inducing, through social security, an improved level of intergenerational risk sharing. We will also examine the optimal size of a given social security system as well as its optimal reform.

The analysis will be carried out in a relatively simple set-up, where the various effects of social security, on the prices of long-lived assets and the stock of capital, and hence on output, wages and risky rates of returns, can be clearly identified.

Keywords: Intergenerational Risk Sharing, Social Security, Ex Ante Welfare Improvements, Interim Optimality, Price Effects.

1 Introduction

The pay-as-you-go social security system in the US was introduced as a tool to mitigate the effects of economic crises. In a special message to Congress accompanying the draft of the social security bill President Roosevelt said "No one can guarantee this country against the dangers of future depressions, but we can reduce those dangers. ... we can provide the means of mitigating their results. This plan for economic security is at once a measure of prevention and a measure of alleviation." (see Kennedy (1999) for a detailed historical discussion of social security). The idea that a pay-as-you-go social security system alleviates recessions by enhancing intergenerational risk-sharing dates back to at least Enders and Lapan (1982). Diamond (1977) already pointed out that market failures and missing asset markets might provide a normative justification for a pay-as-you-go public retirement plan because the absence of certain investment opportunities may lead to inefficient risk allocations.

To properly evaluate whether a social security system allows to improve welfare it is important to specify the welfare criterion that is used (and hence the market failures social security may address). If agents' utility is evaluated at an interim stage, conditionally on the state at their birth, an improvement can only be obtained if some financial markets are missing, or the economy is dynamically inefficient. While one might argue that in reality crucial markets are missing (in particular annuity markets and markets for securities that pay contingent on idiosyncratic shocks), this source of inefficiency is not specific to economies with overlapping generations and other insurance schemes could be introduced which are Pareto-improving (in particular new financial assets could be introduced). Hence the presence of some missing markets might provide a justification for some government intervention but does not directly point to social security as an ideal instrument. Using an interim welfare criterion, several authors have examined the potential benefits of pay-as-you-go social security systems in realistically calibrated, dynamically efficient economies with missing markets (see e.g. Imhoroglu et al. (1999) or Krueger and Kubler (2006)). They find that the negative effects of social security on the capital stock and wages clearly outweigh, quantitatively, any positive risk sharing effects of such a system.

However, if agents' welfare is evaluated at an ex ante stage competitive equilibria in stochastic overlapping generation models are generally suboptimal (even when markets are complete) because agents are unable to trade to insure against the realization of the uncertainty at their birth. There must then be some transfers between generations which improve intergenerational risk sharing and constitute a Pareto-improvement. It is then particularly of interest to investigate under what conditions a pay-as-you-go social security system (or, more generally, one-sided transfers from the young to the old) is Pareto-improving according to an ex ante welfare criterion in economies where equilibria are interim Pareto efficient. In such a framework, the only possible source of an improvement is the fact that intergenerational risk sharing is imperfect, given the limitations to trading (for unborn agents) imposed by the demographic structure.

In this paper we consider a class of overlapping generations economies where markets are complete, there is capital accumulation and land, an infinitely lived asset used in the production process together with labor and capital. The presence of land ensures that competitive equilibria are interim Pareto efficient. We show that, for a wide range of realistic specifications of the parameters of the economy, a pay-as-you go social security system is ex-ante Pareto improving and we demonstrate that general equilibrium changes of the price of land play a crucial role in enhancing the welfare benefits of social security.

We consider two period overlapping generations economies with a single[1] agent per generation and stochastic shocks to aggregate production, and analyze three different pay-as-you-go systems: a defined contribution system, where transfers from the young are proportional to their income level, a defined benefits system, where transfers from the young to the old are state independent, and an ideal system, where any state contingent transfer from the young to the old is allowed. We decompose the effects of a social security scheme into: i) a direct transfer from the young to the old (the one prescribed by the scheme), ii) an indirect transfer (which may have positive or negative sign) induced by the general equilibrium effects of social security on the stock of capital, and hence on equilibrium wages and return to capital, and on the price of long lived assets, iii) a change in the level of total resources available for consumption (due to the change in the stock of capital). In the rather simple set-up of the economy considered, we are then able to identify several crucial conditions (primarily on the covariance between the shocks affecting the agents when young and old, on their risk aversion, and on the stochastic properties of the production shocks) under which these different components of the effects of a social security scheme have a positive effect on welfare.

We proceed in steps, by examining first the effects of the direct transfer specified by the different social security schemes. To this end we consider the special case where there is no production, nor land, so that there is no trade in equilibrium and there are no general equilibrium effects. We find that in order for the introduction of a defined benefits social security system to be Pareto-improving, we need[2] consumption (and hence income) of the old agents to be positively correlated with the consumption of the young and, at the same time, to exhibit a higher variability. Similar conditions must hold for a defined contribution system if risk aversion is sufficiently large. A weaker condition suffices for an ideal system to be improving: the existence of at least one state where the old are poorer than the young (their discounted marginal utility for consumption exceeds the marginal utility when young).

Next, we consider the case where land is present, so that we do have a first general

[1]Since we assume markets are complete, perfect risk sharing can be attained within each generation. We can abstract then from the heterogeneity within each generation so as to better focus on intergenerational risk sharing.

[2]It should be kept in mind that such conditions are derived for competitive equilibria which are interim efficient.

equilibrium effect given by the changes in the price of long-lived assets generated by the introduction of the policy. We find that the price of land tends to decrease, so that we have an indirect transfer from young to old agents of negative sign, which in interim efficient economies has typically a positive effect on the welfare of future generations. On this basis, we show that the presence of long-lived assets improves the case for social security, making the conditions for such policy to be welfare improving less stringent.

With production the introduction of social security in dynamic efficient economies tend to crowd out the investment in capital and hence to lower output level. Furthermore, the stochastic structure of the production shocks determine the correlation between wages, affecting the income when young, and return to capital, affecting the income when old. We are then able to identify some conditions on the properties of the production shocks under which the various social security schemes are improving but show that in general it is harder to find a rationale of PAYGO-social security in models with capital accumulation but without land.

On the basis of such analysis, we can turn our attention to the more general class of economies under consideration, where there is both land and production. In this case we solve for equilibria numerically and show the existence of improving social security scheme for economies with somewhat realistic parameter specifications. It turns out that, while the general equilibrium effect of the change in the stock of capital tends to decrease welfare, this effect is overcompensated by the effect of the change of the price of land. The sign (and size) of the direct effect is then crucial for determining if a (defined benefits or defined contributions) social security scheme is Pareto-improving. As argued above, this direct effect is positive if the consumption when old is positively correlated, but sufficiently more volatile, with the consumption when young. In this set-up we analyze also the optimal size of the different social security systems and examine the benefits of reforming an existing social security system to improve its risk-sharing properties. We find that large welfare gains can be obtained when reforming a pay-as-you go defined contributions system and turning it into an ideal system. Such finding is in line with Shiller (1999)'s observation that, the US system's risk sharing potentials seem limited in that the young's transfers to the current old do not depend on the wealth of the young relative to that of the old.

Starting with Gordon and Varian (1988) several papers have examined the scope for a pay-as-you-go social security system under an ex ante welfare criterion. Shiller (1998), Ball and Mankiw (2001) and De Menil and Sheshinski approach this question by using a partial equilibrium analysis in that in their model there is no capital accumulation and no land, and agents have access to a risky storage technology. As argued in this paper, the general equilibrium effects induced by social security play a very important role in a proper assessment of the benefits of costs and benefits of social security; furthermore, to properly identify the role of social security in exploiting the possible gains in intergenerational risk sharing due to the fact that agents cannot trade before they are born, it is important to consider economies where competitive equilibria are interim efficient.

The second of the two comments above also applies to the work by Bohn (2003), probably the closest to ours (see also Olovsson (2004)), where the competitive equilibria with and without social security are compared, for a realistically calibrated version of an economy with capital accumulation. His findings seem to indicate that it is difficult to make in this set-up an argument in favor of social security. It should however be pointed out that Bohn abstracts from the presence of long-lived assets and restricts his attention to a specific form of production shocks.
The effects of other fiscal policy interventions on intergenerational risk sharing using an ex ante welfare criterion have then been studied by various authors. Gale (1990) characterizes the efficient design of public debt, while. Smetters (2004) discusses the possibility of improving intergenerational risk sharing from an ex ante perspective through capital taxation.

The paper is organized as follows. In Section 2 we describe the class of economies under consideration and give conditions for interim and ex-ante optimality. In Section 3 we introduce a social security system. Section 4 examines the direct effects of such a system. Section 5 focuses on the equilibrium effects on the price of land and Section 6 on the effects on capital accumulation. In Section 7 we analyze the effects of alternative social security schemes for the economies under consideration, with production and land, and derive a set of parameter specifications for which social security is Pareto-improving. In Section 8 we examine within the same framework the optimal size of a social security system as well as the welfare improving reforms of an existing system.

2 The OLG economy

We consider a simple stationary overlapping generations economy under uncertainty with 2 period - lived agents. Time runs from $t = 0$ to infinity. Each period a shock $s \in \mathcal{S} = \{1, ..., S\}$ realizes. Date-events, or nodes, are histories of shocks, and a specific date event at t is denoted by $s^t = (s_0...s_t)$. We collect all (finite) histories in an event tree Σ.

There are 4 commodities: capital, labor, a single consumption good, which is perishable, and a perfectly durable good, land.

As discussed in the introduction, our primary focus is on intergenerational risk sharing. We will abstract therefore from issues concerning intergenerational risk sharing by assuming there is one representative agent born at each date-event (hence we can think of a situation where agents, after they are born, can trade in a complete set of insurance markets). The representative agent in each generation has a unit endowment of labor when young and zero when old as well as an endowment of the consumption good which depends on age and the current shock; for an agent born in node s^t it is given by $e^y(s^t) = e^y_{s_t}$ when young and $e^o(s^{t+1}) = e^o_{s_{t+1}}$ when old. Agents' preferences are only defined over the consumption good and represented by the time-separable expected utility

$$U^{s^t}(c) = u(c^y(s^t)) + \beta E_{s^t} v(c^o(s^{t+1}))$$

where $u(.)$ and $v(.)$ are increasing, concave and smooth functions. They supply so their labor inelastically: $l_s^y = 1$ for all $s \in \mathcal{S}$. At a given node s^t, we denote the consumption of the young agent by $c^y(s^t)$ and consumption of the old agent who was born at s^{t-1} by $c^o(s^t)$.

At the root node, s^0 there is one initial old agent with utility $U(c) = v(c^o(s^0))$.

At each date event s^t, there is a representative firm which produces the consumption good using capital k, labor l and land b as inputs. The firm's production function, f, is subject to stochastic shocks. Given any shock $s \in \mathcal{S}$, $f(k, l, b; s)$ is assumed to exhibit constant returns to scale in capital, labor and land. Capital is obtained from the consumption good in the previous period via a storage technology: more precisely, one unit of the consumption good at $t-1$ yields one unit of capital in each state at t. On the other hand land is a perfectly durable commodity of which a fixed quantity exists, set equal to one.

After the shock is realized, the firm buys labor and capital and rents land from the households on the spot market so as to maximize spot-profits. We denote the price at s^t of capital, k by $r(s^t)$ (in terms of the consumption good whose price we normalize to 1), the price of labor, l, by $w(s^t)$, and the rental price of land, b, by $d(s^t)$, paid by producers to use land in the current production process. We denote the initial old's holdings of capital by $k(s^{-1})$ and refer to it as the 'initial condition'. The initial old also owns the entire amount of land; land is then traded by consumers in the market and $q(s^t)$ denotes the price of land at s^t.

Given an initial condition $k(s^{-1})$, a feasible allocation is $((c^y(s^t), c^o(s^t)), k(s^t))$ such that

$$c^y(s^t) + c^o(s^t) + k(s^t) \leq e_{s^t} + f(k(s^{t-1}), 1, 1; s_t) \text{ for all } s^t,$$

where $e_{s^t} = e^y_{s_t} + e^o_{s_t}$ denotes the agents' total endowment of the consumption good at s^t.

For simplicity we abstract from population growth or technological progress and assume the shocks to be i.i.d.; π_s denotes then the probability of shocks s occurring. Evidently, these are not innocuous assumptions when it comes to a quantitative analysis of social security. However, in this paper we focus largely on more qualitative issues.

In what follows it will be of interest to consider some special cases, (i) where capital is not productive, $\partial f(k, 1, 1; s)/\partial k = 0$ for all k, s, in which case the economy reduces to a pure exchange one, and/or (ii) where land is not productive (equivalently there is no land), $\partial f(k, 1, 1; s)/\partial b = 0$ for all k, s.

2.1 Optimality

As explained in the introduction, we distinguish between two welfare concepts. Given an initial condition $k(s^{-1})$, a feasible allocation $(c^y(s^t), c^o(s^t)), k(s^t))_{s^t \in \Sigma}$ is *conditionally Pareto optimal* (CPO) if there is no other feasible allocation $(\tilde{c}^y(s^t), \tilde{c}^o(s^t)), \tilde{k}(s^t))_{s^t \in \Sigma}$, with

$$U^{s^t}(\tilde{c}^y(s^t), \tilde{c}^o(s^{t+1})) \geq U^{s^t}(c^y(s^t), c^o(s^{t+1})) \text{ for all } s^t, t$$

with the inequality holding strict for at least one s^t. Thus, in this notion agents' welfare is evaluated at the interim stage, after the realization of the uncertainty at the time of birth.

On the other hand, a feasible allocation $((c^y(s^t), c^o(s^t)), k(s^t))_{s^t \in \Sigma}$ is *ex ante Pareto-optimal*[3] if there is no other feasible allocation $((\tilde{c}^y(s^t), \tilde{c}^o(s^t)), \tilde{k}(s^t))_{s^t \in \Sigma}$, with

$$E_0 U^{s^t}(\tilde{c}^y(s^t), \tilde{c}^o(s^{t+1})) \geq E_0 U^{s^t}(c^y(s^t), c^o(s^{t+1})) \text{ for all } t = 0, 1, ...,$$

with the inequality holding strict for at least one t.

The presence of an infinitely lived asset like land, yielding each period a dividend that is bounded away from zero ensures (see, e.g. Demange (2002)) that competitive equilibria are conditionally Pareto optimal, i.e. there is no possibility of welfare improvement conditionally on the state at birth of each generation. Hence the only possible source of inefficiency in the economy under consideration, when land is productive, is the fact that agents are unable to trade before they are born to ensure against the state at their birth. Because of this natural form of market incompleteness equilibria are not guaranteed to be ex ante Pareto optimal.

2.2 Competitive equilibria

A competitive equilibrium is a collection of choices for the households and firms such that households maximize utility, firms maximize spot profits and markets clear. It simplifies the characterization of equilibria to note that by market clearing, in equilibrium the firm will always buy the entire capital and labor and rent the entire amount of land from the households. Recalling that $l_s^y = 1$ for all s, an equilibrium is then characterized by the choices $\left\{c^y(s^t), c^o(s^t), k(s^t), b(s^t)\right\}_{s^t \in \Sigma}$, where $b(s^t)$ denotes the amount of land purchased by the young and $k(s^t)$ the amount of consumption good destined to capital at s^t, and prices $\left\{w(s^t), r(s^t), q(s^t), d(s^t)\right\}_{s^t \in \Sigma}$, such that:

i) at each s^t the young chooses $k(s^t), b(s^t)$ to maximize $u(c^y(s^t)) + \beta E_{s^t} v(c^o(s^{t+1}))$ subject to

$$c^y(s^t) = e^y(s_t) + w(s^t) - q(s^t)b(s^t) - k(s^t)$$

$$c^o(s^{t+1}) = e^o(s_{t+1}) + \left(q(s^{t+1}) + d(s^{t+1})\right) b(s^t) + k(s^t) r(s^{t+1});$$

ii) firms maximize profits, i.e. using the market clearing for labor and capital,

$$r(s^t) = \frac{\partial f(k(s^{t-1}), 1, 1; s_t)}{\partial k}, \quad w(s^t) = \frac{\partial f(k(s^{t-1}), 1, 1; s_t)}{\partial l}, \quad d(s^t) = \frac{\partial f(k(s^{t-1}), 1, 1; s_t)}{\partial b};$$

iii) and the land market clears:

$$b(s^t) = 1.$$

[3] In the following we will sometimes drop the qualification 'ex ante'.

2.3 Stationary equilibria

The analysis is obviously much simpler when the competitive equilibrium is stationary in the strong sense that individual consumption only depends on the current shock, s, i.e.

$$(c^y(s^t), c^o(s^t)) = (c^y_{s_t}, c^o_{s_t}), \quad \forall s^t.$$

In such situations we can easily derive the conditions for CPO and ex ante optimality of competitive equilibria and study the welfare effects of stationary taxes and transfers.

Given our assumption that shocks are i.i.d., stationary equilibria always exist in the special case where capital is not productive (pure exchange), whether or not land is productive. However, this is no longer true in the presence of production, in which case only the existence of ergodic Markov equilibria can be established under general conditions (see Wang (1993) for a proof of existence in an economy without land). We will consider some special cases where stationary equilibria still exist in the presence of production.

2.3.1 Conditional optimality

As shown by Aiyagari and Peled (1991), Chattopadhyay and Gottardi (1999) (see also Barbie et al. (2005)), a stationary equilibrium (in the above sense) is conditionally Pareto optimal if and only if the matrix of the representative agent's marginal rate of substitutions $\left\{\frac{\beta \pi_{s'} v'(c^o_{s'})}{u'(c^y_s)};\ s, s' \in \mathcal{S}\right\}$ has a maximal eigenvalue less or equal than 1. In our set-up, the separability of the agent's utility function and the fact that shocks are i.i.d. imply that this matrix has always rank 1, and its largest eigenvalue is given by the sum of its diagonal elements. It then follows that a stationary equilibrium is conditionally Pareto optimal if and only if:

$$\beta \sum_{s \in \mathcal{S}} \pi_s \frac{v'(c^o_s)}{u'(c^y_s)} \leq 1, \tag{1}$$

a condition that can be readily verified. We can then say the economy is 'at the golden rule' if $\beta \sum_{s \in \mathcal{S}} \pi_s \frac{v'(c^o_s)}{u'(c^y_s)} = 1$.

It is useful to rewrite condition (1) as follows:

$$\text{cov}(\beta v'(c^o), \frac{1}{u'(c^y)}) + \text{E}\left(\beta v'(c^o)\right) \text{E}\left(\frac{1}{u'(c^y)}\right) \leq 1; \tag{2}$$

which implies then, by Jensen's inequality:

$$\text{cov}(\beta v'(c^o), \frac{1}{u'(c^y)}) + \text{E}\left(\beta v'(c^o)\right) \left(\frac{1}{\text{E}u'(c^y)}\right) \leq 1. \tag{3}$$

The advantage of this formulation is that it clearly shows that CPO requires that at least one of the two following properties is satisfied: (i) on average the marginal utility of consumption when old is smaller than the marginal utility of consumption when young, (ii) the marginal utilities of consumption when old and the inverse of the marginal utility of consumption when young are negatively correlated (or equivalently, when c^y and c^o are co-monotonic,

c^y and c^o are positively correlated). While (i) is analogous to the condition for optimality found under certainty (agents should be on average richer when old than when young), (ii) identifies some specific properties of the allocation of risk within each generation.

2.3.2 Ex Ante Improving Transfers

At a stationary equilibrium the welfare, evaluated at the ex ante stage, is the same for all generations and given by:

$$E_0 U^{s^t}(c^y(s^t), c^o(s^{t+1})) = \sum_{s \in \mathcal{S}} \pi_s (u(c^y_s) + \beta v(c^o_s)).$$

We derive a simple conditions under which which a stationary allocation cannot be improved by direct stationary transfers. If there exist non-stationary improving transfers, there must also exist stationary ones. However, considering direct transfers is only a necessary condition for the ex ante optimality of an allocation, since a welfare improvement could also be found by changing the level of production and investment. But since our focus is on the possibility that the a stationary social security system is welfare improving, it is of particular interest to consider first the conditions where an improvement can be found only with stationary transfers.

The non existence of welfare improving stationary transfers requires that there does not exist a vector $(T_s)_{s \in \mathcal{S}}$ such that an infinitesimal net transfer from the young to the old agents in the direction of $(T_s)_{s \in \mathcal{S}}$ has a (weakly) positive effect on the (ex ante) welfare of the representative generation:[4]

$$\sum_{s \in \mathcal{S}} \pi_s (-u'(c^y_s) + \beta v'(c^o_s)) T_s \geq 0 \tag{4}$$

as well as on the agents who are old when the transfers are introduced:

$$\sum_{s \in \mathcal{S}} \pi_s v'(c^o_s) T_s \geq 0. \tag{5}$$

with one at least of the two inequalities being strict.

Obviously, a vector $(T_s)_{s \in \mathcal{S}}$ satisfying conditions (4) and (5) exists if for some s, $(-u'(c^y_s) + \beta v'(c^o_s)) > 0$. Moreover, one can easily see that since the transfers are not restricted to be positive, an improvement is also possible if the vectors of the marginal utilities when young and old are are not collinear. Therefore focusing on direct transfers, one can characterize ex-ante efficiency as follows.

Proposition 1 *At any stationary allocation $\left(c^y_s, c^o_{s'}\right)_{s,s' \in \mathcal{S}}$, a necessary and sufficient condition for the non-existence of welfare improving stationary transfers is that the vectors $(u'(c^y_s))_{s \in \mathcal{S}}$ and $(v'(c^o_s)_{s \in \mathcal{S}})$ are collinear and that for all $s \in \mathcal{S}$, $\beta v'(c^o_s) \leq u'(c^y_s)$.*

[4] In the case of stationary Markov allocations a similar condition holds, though the expression is now evaluated with the invariant distribution.

Condition (4) can be rewritten as follows:

$$Cov\left\{\beta v'(c^o) - u'(c^y), T\right\} \geq \mathbb{E}\left(T\right)\left[\mathbb{E}\left(u'(c^y)\right) - \beta\mathbb{E}\left(v'(c^o)\right)\right] \tag{6}$$

Thus an improving transfer T should be characterized by a sufficiently high covariance with $v'(c^o)$ and a low covariance with $u'(c^y)$.

When ex ante welfare is considered the timing of the introduction of the transfers also plays a role. In particular, (5) applies to the case where, at the time in which the transfer scheme is introduced, the transfer starts operating at a given date *in all possible states*. We can understand this as describing a situation where the transfer scheme is announced one period in advance, say at the end of some date t, after some history s^t, and will be implemented starting from date $t+1$ at every successor node of s^t (hence there will be a transfer from the young to the old at date $t+1$ for each possible realization s of the uncertainty at date $t+1$).

If on the other hand the transfer scheme were not announced in advance, but began to operate at date $t+1$, when say the current shock is $\bar{s}$, the scheme would be welfare improving if the following conditions hold, in addition to (4):

$$\pi_{\bar{s}} v'(c^o_{\bar{s}}) T_{\bar{s}} \geq 0, \tag{7}$$

saying that the agents who are old at $t+1$ are not worse off, and:

$$-u'(c^y_{\bar{s}}) T_{\bar{s}} + \sum_{s \in \mathcal{S}} \beta \pi_s v'(c^o_s)) T_s \geq 0. \tag{8}$$

stating that the agents who are young at $t+1$ are also not worse off. Note that (7) is equivalent to $T_{\bar{s}} \geq 0$, and given this it follows from (8) that (5) holds. The reverse however is not necessarily true. We conclude that the set of transfer schemes which are welfare improving when announced one period before their introduction includes the set of transfer schemes which are improving when they are introduced at the time of their announcement.

FACT 1 *Announcing a stationary transfer scheme one period before its introduction is always optimal.*

3 Social Security

We model social security as a system of non-negative transfers from the young to the old. In general the pattern of transfers is described by $(\tau(s^t))_{s^t \in \Sigma}$, where $\tau(s^t) \geq 0$. We let $\nu \geq 0$ denotes the size of the system, so that at any node s^t the current young transfers $\nu\tau(s^t)$ units of the consumption good to the current old.

Actual social security systems in most developed countries are characterized by the fact that neither the specification of taxes nor benefits seem to vary across states of the world. In various countries, like the US, a social security trust-fund stabilizes imbalances between benefits and contributions over the business cycle. We will abstract from this feature and not allow for the presence of a trust-fund, so as to focus on the pure intergenerational transfer component of social security systems. Evidently, this fact will lead us to underestimate the welfare benefits of social security systems.

In what follows, we will therefore restrict our attention to stationary social security systems that maintain budget balance in every period; i.e., current benefits coincide with current taxes and the specification of the transfer at each node s^t depends at most on the current state s_t, not on past history. We will consider three different kinds of stationary social security systems:

1. In the first case, the social security transfer is a suitably designed function of the current shock. The transfer pattern is thus given by $(\tau_s)_{s\in\mathcal{S}}$, where τ_s can be any non-negative number: in each state $s \in \mathcal{S}$ the current young makes then a transfer proportional to τ_s units of the consumption good to the current old. We will refer to this as an *'ideal' (stationary) social security system.*

2. In the second case the contributions paid by the young are proportional to their income. Since the latter may vary with the node, so will the level of the tax paid, but the tax rate is state invariant. That is, for all s^t, we have:

$$\tau(s^t) = e^y_{s_t} + w(s^t).$$

   We will refer to this as a '*defined contributions*' social security system since the social security tax-rate remains constant across states.

3. In the third case considered in this paper, it is benefits who are state invariant. We call a social security system a '*defined benefits*' one if, for all $s \in \mathcal{S}$, $\tau_s = 1$.

Feldstein and Liebman (2001) characterize the US pay-as-you-go system as a defined benefits system and argue that some countries such as Sweden and Italy have defined contribution programs. The fact however that we require transfers to balance in each period constitutes, as we already argued, a departure from the features of the social security systems present in most industrial countries.

The ideal system is an idealization, which provides though an important reference point by allowing us to identify the welfare maximizing stochastic structure of the transfers from young to old agents and can be contrasted with the welfare improving transfers $(T_s)_{s\in\mathcal{S}}$ discussed in the previous section, where no sign restriction was imposed on T_s. It also allows us to explain to how far existing defined benefit or contribution systems are from an welfare-maximizing system.

In the presence of a social security system $(\nu\tau(s^t))_{s^t\in\Sigma}$ a competitive equilibrium is again given by a collection of choices $\left\{c^y(s^t), c^o(s^t), k(s^t), b(s^t)\right\}_{s^t\in\Sigma}$ and prices $\left\{w(s^t), r(s^t), q(s^t), d(s^t)\right\}_{s^t\in\Sigma}$ such that households maximize utility, firms maximize spot profits and markets clear. The only difference with respect to conditions i)-iii) of Section 2.2 is the expression of the consumers' budget constraint, now given by:

$$c^y(s^t) = e^y(s_t) + w(s^t) - q(s^t)b(s^t) - k(s^t) - \nu\tau(s^t)$$
$$c^o(s^{t+1}) = e^o(s_{t+1}) + \left(q(s^{t+1}) + d(s^{t+1})\right) b(s^t) + k(s^t)r(s^{t+1}) + \nu\tau(s^{t+1}).$$

We intend to analyze the welfare effects of an infinitesimal increase of the scale of the social security system, $d\nu > 0$. To this end, it is convenient to decompose such effects into those of the direct transfer prescribed by the social security scheme, $\tau(s^t)$, and the effects generated by the changes in the equilibrium price of land as well as in the stock of capital (and hence in wages, returns on capital and rental price of land), i.e. by the general equilibrium effects of the change in the scheme. The latter as we will see can be reduced to those of an additional, indirect net transfer from the young to the old (which may have positive or negative sign).

In the next three Sections, we will consider the case where a stationary equilibrium exists. In such situation, the transfers prescribed by all the three types of stationary social security systems described above are also stationary and, as we will show, the economy reaches a new stationary equilibrium in at most one period after the policy change is introduced. As a consequence, we can focus our attention on the total transfers generated by the policy change, evaluated at the new stationary equilibrium. These can be described by a vector $(T_s)_{s\in\mathcal{S}}$ where, in each state $s \in \mathcal{S}$, T_s is given by the sum of the direct transfer τ_s prescribed by the policy and the indirect transfer induced by the general equilibrium effects of the policy change. Once the pattern $(T_s)_{s\in\mathcal{S}}$ of the total transfers induced by an infinitesimal increase of the scale of a social security scheme $(\tau_s)_{s\in\mathcal{S}}$ is identified, we can use the analysis in the previous Section to ascertain whether or not such policy change is improving welfare (in the ex ante sense). It suffices in fact to verify whether conditions (4) and (5) are both satisfied for the values of $(T_s)_{s\in\mathcal{S}}$ we derived, one at least as a strict inequality.

We will derive the expressions of $(T_s)_{s\in\mathcal{S}}$ for the case where the effects of $d\nu$ are evaluated at $\nu = 0$, i.e. an infinitesimal amount of social security is introduced, starting from a situation without social security. It should be clear that the analysis can be immediately extended to the case where the policy change is evaluated at $\nu > 0$.

4 Direct Effects of Social Security

We will consider first the case where there is neither land nor production: there is a unique equilibrium, which is stationary, and given by autarky. While this case is almost trivial to analyze, it helps in identifying some of the main conditions needed for social security to be Pareto-improving. In this case in fact the total net transfer T_s induced by the policy in equilibrium in any state s coincides with the direct transfer τ_s prescribed by the policy: there are no additional transfers generated by price effects or other changes in the equilibrium variables. The welfare consequences of social security can then just be determined on the basis of the relationship between the direct transfers $(\tau_s)_{s\in S}$ and the stochastic pattern of the agents' marginal utility for consumption (in this case coinciding with their endowments). Hence to determine whether the scheme is Pareto improving, we only have to verify that conditions (4) and (5) are satisfied when T_s is replaced by τ_s, for all s:

$$\begin{aligned}\sum_{s\in\mathcal{S}} \pi_s(-u'(e^y_s) + \beta v'(e^o_s))\tau_s &> 0,\\ \sum_{s\in\mathcal{S}} \pi_s v'(c^o_s)\tau_s &\geq 0,\end{aligned}$$

(one of the two inequalities being strict).

Since the direct transfers τ_s from young to old agents prescribed by a social security scheme are required to be non-negative, the initial old are always at least weakly better off as a result of the introduction of the scheme $(\tau(s))_{s\in\mathcal{S}}$, i.e. the second of the two above inequalities is always satisfied.

4.1 Ideal social security system

All what is required for the existence of a Pareto improving ideal social security system is that the first of the two above condition holds.

PROPOSITION 2 *At an autarkic equilibrium, a Pareto improving ideal social security system exists if and only if there is at least one state $\bar{s}$ for which*

$$\beta v'(e^o_{\bar{s}}) > u'(e^y_{\bar{s}}).$$

Intuitively this is a very weak condition: it only requires the existence of one state, where the time (*but not* probability) discounted marginal utility of the old is larger than the marginal utility of the young, i.e. where we can say the old are 'poorer' than the young. The Pareto improving social security system prescribes then a transfer from the young to the old only when state $\bar{s}$ is realized: such transfer scheme can be viewed as providing some insurance to the old from the young.

It is therefore useful to contrast this with the necessary and sufficient condition (1) for the conditional optimality of the competitive equilibrium which, in the case of an autarkic equilibrium, reduces to:

$$\beta \sum_{s\in\mathcal{S}} \pi_s \frac{v'(e^o_s)}{u'(e^y_s)} \leq 1$$

Obviously, for a large set of economies for which the (autarkic) equilibrium is CPO we can find a Pareto-improving social security system. Conditional optimality, as we saw, requires that on average old are 'richer' than young (or their consumption is positively correlated). As long as there is one shock $\bar{s}$ for which the old are 'poorer' than young, social security can be Pareto-improving. Since the improvement can be attained with nonzero transfers only in one state $\bar{s}$, for all $s \neq \bar{s}$ the young agents must also be better off conditionally on the state at birth. If the initial allocation is CPO the utility of the agents born in state $\bar{s}$ must then decrease.

While this shows that an optimally designed social security system can be Pareto-improving for a large range of parameter values, actual social security systems are somewhere between defined benefits and defined contribution.

4.2 Defined benefits

When transfers are constant across all shocks, the simple analysis of the autarky case reveals one surprising necessary condition for a defined benefits social security system to be Pareto improving.

The necessary and sufficient condition for a defined benefits system to be Pareto improving, at an autarkic equilibrium, is again readily obtained from (4), setting $T_s = 1$ for all s:

$$\sum_{s\in\mathcal{S}} \pi_s \left(-u'(e^y_s) + \beta v'(e^o_s)\right) > 0. \tag{9}$$

Thus the average marginal utility of consumption has to be larger when old than when young. Recall that the first of the two alternative necessary conditions for CPO found in (3) prescribes that the opposite inequality holds. Hence, the other necessary condition, $\text{cov}(\beta v'(c^o), \frac{1}{u'(c^y)}) < 0$, must hold for a defined benefits system to be welfare improving at a CPO equilibrium.

An additional condition for such scheme to be improving can be found by rewriting (9) as:

$$\sum_{s\in\mathcal{S}} \pi_s \left(-1 + \beta\frac{v'(e^o_s)}{u'(e^y_s)}\right) = \mathbb{E}\left(u'(e^y)\right)\mathbb{E}\left(-1+\beta\frac{v'(e^o)}{u'(e^y)}\right) + Cov\left(u'(e^y), \beta\frac{v'(e^o)}{u'(e^y)}\right) > 0. \tag{10}$$

Since the necessary and sufficient condition for CPO, (1), requires that $\mathbb{E}\left(-1+\beta\frac{v'(e^o)}{u'(e^y)}\right) \leq 0$, the second term in (10) has to be strictly positive for that condition to hold.

We have thus shown:

PROPOSITION 3 *At a conditionally Pareto optimal autarkic equilibrium, a defined benefits social security system can be Pareto improving only if:*[5]

$$cov(\beta v'(e^o), \frac{1}{u'(e^y)}) < 0 < cov(u'(e^y), \beta\frac{v'(e^o)}{u'(e^y)}). \tag{11}$$

The first inequality in (11) says that a welfare improving defined benefits system can only be found when marginal utilities of the old and the inverse of the marginal utilities of the young are *negatively* correlated. Hence, when the variables describing the endowment

[5] Note that, at the golden rule, the inequality on the right hand side of (11) is also a sufficient condition for the existence of an improving policy.

when young and when old are co-monotone, marginal utilities when young and when old must be positively correlated, i.e., in all states where the old are rich, the young must also be rich and vice versa! This may come a bit as a surprise as we might have expected that the margins for welfare improving risk sharing trades between young and old would be greater when their income is negatively correlated. We should bear in mind though that we are limiting our attention here to deterministic transfers, so that mutual insurance cannot be properly achieved; moreover,.the conditional optimality of the equilibrium imposes some restriction on the pattern of the variability of consumption when young and when old[6].

Furthermore, when utility is linear-concave or concave-linear (in that either $u(c) = c$ or $v(c) = c$), or when e^y or e^o are deterministic, this condition can never be satisfied, so that an improvement will never be possible with a defined benefit system.

The second inequality in (11) requires that $u'(e^y)$ and $\frac{v'(e^o)}{u'(e^y)}$ are positively correlated. Thus, when endowments when young and when old vary co-monotonically, we must have that not only $u'(e^y)$ and $v'(e^o)$ are positively correlated, as shown above, but that whenever $u'(e^y)$ increases, $v'(e^o)$ also increases, and more than $u'(e^y)$. Endowment when old must then vary in the same direction of the endowment when young, as we saw, but also have to exhibit a greater variability and/or the old must be more risk averse than the young. Given this feature, the fact a deterministic transfer from the young to the old is welfare improving should not be surprising, since the old are bearing more risk than the young and it is beneficial for the young to provide them some insurance, even in the form of a deterministic transfer of income. Since adding a riskless stream of consumption to risky consumption tends to decrease the risk, there is a sense in which a defined benefits scheme shifts risk from the old to the young.

If the necessary condition (11) above is satisfied, it is indeed easy to construct examples, where the introduction of a defined benefit system is Pareto-improving. Consider for example the case where $u(c) = v(c) = \log(c)$, $\beta = 1$, $S = 2$ and $\pi_1 = \pi_2 = 1/2$. If endowments are

$$(e^y(1), e^y(2)) = (1, 2), \quad (e^o(1), e^o(2)) = (0.75, 4)$$

the economy is clearly conditionally optimal and (9) is satisfied. The introduction of a defined benefits social security system is then Pareto improving. The wealth (and hence the consumption) of young and old agents are clearly positively correlated, and exhibits a clearly higher variability for the old than for the young agents. In this case, a state independent transfer from the young to the old is welfare improving.

4.3 Defined contributions

Under a defined contributions system the young pay, in each state, a constant fraction of their income, i.e. $\tau_s = e^y_s$ for all shocks $s \in \mathcal{S}$. Hence the necessary and sufficient condition

[6] As we already noticed, the first inequality in (11) is in fact one of the two alternative necessary conditions for CPO we obtained from (3).

for a defined contributions system to be welfare improving, at an autarkic equilibrium, is obtained from (4), setting $T_s = e^y_s$ for all s :

$$\sum_{s\in\mathcal{S}} \pi_s(-u'(e^y_s) + \beta v'(e^o_s))e^y_s > 0. \tag{12}$$

The existence of an improving policy requires in this case a condition on the joint pattern of the agent's endowment and its marginal utility, so that the elasticity of the agents' utility function will now play a role.

Proceeding along similar lines to the previous subsection, we note first that the CPO condition (1), applied to the autarkic equilibrium allocation, can then also be rewritten as

$$1 \geq \mathrm{cov}(\beta v'(e^o)e^y, \frac{1}{u'(e^y)e^y}) + \mathrm{E}\left(\beta v'(e^o)e^y\right) \mathrm{E}\left(\frac{1}{u'(e^y)e^y}\right), \tag{13}$$

and implies, by Jensen's inequality, an expression analogous to (3) above:

$$1 \geq \mathrm{cov}(\beta v'(e^o)e^y, \frac{1}{u'(e^y)e^y}) + \mathrm{E}\left(\beta v'(e^o)e^y\right) \left(\frac{1}{\mathrm{E}\left(u'(e^y)e^y\right)}\right), \tag{14}$$

where, if (12) holds, the second term is greater than one and hence the first one has to be negative.

Moreover, using the CPO condition (1) we obtain this other implication:

$$\mathrm{cov}(u'(e^y)e^y, \beta\frac{v'(e^o)}{u'(e^y)}) \geq \mathrm{E}\left(e^y\beta v'(e^o)\right) - \mathrm{E}\left(u'(e^y)e^y\right), \tag{15}$$

whose term on the right hand side is positive, whenever the necessary and sufficient condition (12) for the policy to be improving is satisfied.

We have proved so the following:

Proposition 4 *A necessary condition for a defined contributions system to be Pareto improving, at a conditionally Pareto optimal autarkic equilibrium, is:*[7]

$$\mathit{cov}(\beta v'(e^o)e^y, \frac{1}{u'(e^y)e^y}) < 0 < \mathit{cov}(u'(e^y)e^y, \beta\frac{v'(e^o)}{u'(e^y)}). \tag{16}$$

Note that such condition does not rule out the possibility that in this case an improvement may be found even when the agents' utility function is linear - concave (in which case, e^y and e^o, if co-monotonic, have to be negatively correlated), or concave - linear, i.e. whatever the pattern of the risk aversion over the agents' lifetime. The same is true when the endowment when old is deterministic. On the other hand, a defined contributions policy can never be welfare improving if the endowments when young are riskless.

[7] In this case too, if we are at the golden rule, the inequality on the right hand side of (16) is also a sufficient condition for the existence of an improving, now defined contributions policy.

Condition (16) is somewhat harder to interpret than the analogous condition we obtained in the case of defined benefits. It is useful to consider the special case when agents have the same constant relative risk aversion utility function when young and old, given by

$$u(c) = v(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{17}$$

when the coefficient of risk aversion is $\sigma \neq 1$ and by $u(c) = v(c) = \log(c)$ when $\sigma = 1$. In this case, condition (16) simplifies to:

$$\mathrm{cov}(\beta \frac{e^y}{(e^o)^\sigma}, \frac{1}{(e^y)^{1-\sigma}}) < 0 < \mathrm{cov}((e^y)^{1-\sigma}, \beta \left(\frac{e^y}{e^o}\right)^\sigma). \tag{18}$$

Note that when $\sigma = 1$, i.e. when consumers have logarithmic preferences, this inequality can never hold, thus a defined contributions social security system can never be Pareto-improving.

When $\sigma < 1$, an improvement is only possible, when the endowment of the young and the old vary co-monotonically, if in all states where the endowment when young is high (resp. low), the endowment when old is low (resp. high), i.e. the two are negatively correlated, or the endowment when old is also high but exhibits less variability than the endowment when young. Observe that, somewhat surprisingly, in this case an improvement is always possible if endowments when old are riskless, endowments when young are risky and the economy is sufficiently close to the golden rule. With social security, consumption when old becomes risky, however, the 'representative agent' is compensated for this by less risk when young.

On the other hand, when $\sigma > 1$ the situation is somewhat analogous to the one we found in the case of defined benefits: an improvement is only possible (again under the comononicity assumption) if whenever the endowment when young is high the endowment when old is even higher, thus when endowments when young and old are *positively* correlated and the endowments when old fluctuate more than the endowments when young. Note that in this case an improvement is impossible if the endowments when old are riskless. Here the intuition from the defined benefits case carries over.

5 Effects on the Price of Long-lived Assets

As explained in the introduction, the pure exchange case without land neglects many important general equilibrium effects of the introduction of a social security system. The first such effect we want to examine is the consequence of the introduction of social security on the price of long-lived assets such as land. Hence we maintain here the restriction that capital and labor are not productive but suppose now that land is productive, $\partial f(k, l, b; s)/\partial b = d_s$ for all $b, l \leq 1, k$, and constitutes then an infinitely-lived asset in unit net supply paying each period a dividend d_s whenever shock s realizes.

In the presence of land a stationary equilibrium still exists both without and with a (stationary) social security system. We will again examine the effects on the stationary equilibrium of introducing an infinitesimal amount of social security, starting from a situation with zero social security transfers, $\nu = 0$. We consider the case where the introduction of social security is announced at some date t, after some history s^t, and after all trades have taken place at that date, and will start being implemented from date $t+1$, at every successor node of s^t. The argument used to establish Fact 1 still applies here but because of the change of the price of land, we can go even further and establish:

FACT 2 *In the presence of land, if the policy is announced more than one period in advance, it can never be welfare improving.*

The argument is as follows. Suppose the policy were announced at date t and starts being implemented only at, say, $t+Z$, for $Z > 1$. Then at date $t+Z-1$ the price of land will settle at its new stationary equilibrium level which, as we show below, will be lower. The prices at all intermediate dates (between $t+1$ and $t+Z-2$) may then also vary; whatever the direction in which they vary, since the price at $t+Z-1$ will be lower, we can say that for at least one generation the price, compared to the initial equilibrium, will be greater or equal when young and lower when old. The welfare of this generation will thus necessarily decrease, so that a welfare improvement cannot be attained in this case.

The net transfer T_s induced by the introduction of social security is now no longer equal to the direct transfer τ_s prescribed by it. To this we have to add the indirect transfer induced by the change in the equilibrium price of land (the price effect) which, since the total outstanding amount of land is 1, is given by:

$$T_s = \tau_s + \frac{dq_s}{d\nu}.$$

To simplify the analysis, we will consider first the case in which agents' preferences are linear concave, i.e. $u(x) = x$, $v(x)$ concave. In such case we can solve explicitly for the equilibrium price of land:

$$q = \beta \sum_{s \in \mathcal{S}} \pi_s (q + d_s) v'(e_s^o + q + d_s + \nu\tau_s). \tag{19}$$

The price of land is constant across states and the price change is given by

$$\frac{dq}{d\nu} = -\frac{\beta \sum_{s\in\mathcal{S}} \pi_s \tau_s (q + d_s) v''(e_s^o + d_s + q)}{-1 + \beta \sum_{s\in\mathcal{S}} \pi_s (v'(e_s^o + q + d_s) + (q + d_s) v''(e_s^o + q + d_s))}.$$

The indirect transfer induced by the introduction of social security is thus deterministic. Moreover, since the economy is conditionally optimal, from (1) we get in this case $-1 + \beta \sum_{s\in\mathcal{S}} \pi_s v'(c_s^0) \leq 0$, so that we can say that the price effect is always negative. Thus in the current set-up the introduction of a social security transfer scheme might even make the initial old (i.e., the agents who are old at date t) worse off, since the transfer they receive from the policy may be more than offset by the reduction in the value of their land holdings.

On the basis of the above, we can explicitly determine the total transfer generated by the policy. For any $\bar{s} \in \mathcal{S}$:

$$T_{\bar{s}} = \tau_{\bar{s}} + \frac{dq}{d\nu} = \frac{\tau_{\bar{s}}(1 - \beta \sum_{s \in \mathcal{S}} \pi_s v'(c_s^o)) + \sum_{s \in \mathcal{S}} \pi_s (\tau_s - \tau_{\bar{s}}) \beta v''(c_s^o)(q + d_s)}{1 - \beta \sum_{s \in \mathcal{S}} \pi_s \left[v'(c_s^o) + (q + d_s) v''(c_s^o)\right]}, \tag{20}$$

where $c_s^o = e_s^o + q + d_s$ is the equilibrium consumption of the agents when old in the initial equilibrium. Substituting this expression into (5), and noting that its denominator is always positive (given CPO), and independent of $\bar{s}$, we find that the introduction of social security improves the initial old if and only if

$$\sum_{\bar{s} \in \mathcal{S}} \pi_{\bar{s}} v'(c_{\bar{s}}^o) \left(\tau_{\bar{s}}(1 - \beta \sum_{s \in \mathcal{S}} \pi_s v'(c_s^o)) + \sum_{s \in \mathcal{S}} \pi_s \beta (\tau_s - \tau_{\bar{s}}) v''(c_s^o)(q + d_s) \right) > 0 \tag{21}$$

Evidently, the old can be made better off by introducing social security uniformly across states. However, this will not help future generations.

If we then substitute (20) into (4), we obtain the following necessary and sufficient condition for the introduction of social security to improve of the future generations:

$$\sum_{\bar{s} \in \mathcal{S}} \pi_{\bar{s}} (-1 + \beta v'(c_{\bar{s}}^o)) \left(\tau_{\bar{s}}(1 - \beta \sum_{s \in \mathcal{S}} \pi_s v'(c_s^o)) + \sum_{s \in \mathcal{S}} \pi_s \beta (\tau_s - \tau_{\bar{s}}) v''(c_s^o)(q + d_s) \right) > 0. \tag{22}$$

As we saw in the previous section, the necessary and sufficient condition for an optimally designed ideal social security system to be Pareto improving, at an autarkic equilibrium, with linear concave preferences, is that there is some state $\hat{s}$, for which: $\beta v'(c_{\hat{s}}^o) > 1$. We will show that, in the presence of land, this same condition also suffices for the existence of a welfare improving ideal system:

Proposition 5 *In the presence of land, with linear-concave preferences, a welfare improving ideal social security system exists if in the equilibrium without social security there exists some shock $\hat{s}$ for which $\beta v'(c_{\hat{s}}^o) > 1$.*

The proofs of the propositions in this section can be found in the appendix.

In the presence of land, the condition stated in the above Proposition is only sufficient, no longer necessary for the existence of an improving ideal social security welfare scheme. This is because the indirect transfer generated by the policy is a negative transfer from the young to the old. Hence in this case it is possible to design social security schemes which have better insurance policies by implying a total transfer from the young to the old in the states where the young are rich, and a transfer from the old to the young when the latter are poor. As a consequence, a welfare improving scheme may exist even when, for all s, we have $\beta v'(c_s^o) \leq 1$. To see this consider the following simple example.

Suppose $v(c) = \log(c)$ and $e_s^o = 0$ for all $s \in \mathcal{S}$. From (19) we find that, in the absence of social security, $q = \beta$. It is clear that in this case, since $d_s \geq 0$ for all s, $\beta / v'(c_s^o) = \frac{\beta}{\beta + d_s} \leq 1$

for all s. Consider then an ideal social security system with $\tau_{\bar{s}} = 1$ and $\tau_s = 0$ for all $s \neq \bar{s}$; with such system we have:

$$\frac{\partial q}{\partial \nu} = -\frac{\pi_{\bar{s}}\beta}{\beta + d_{\bar{s}}},$$

and the condition for an improvement of a representative generation, equation (4), becomes

$$\sum_{s=1}^{S} \pi_s (1 - \frac{\beta}{\beta + d_s}) \frac{\pi_{\bar{s}}\beta}{\beta + d_{\bar{s}}} + \pi_{\bar{s}}(-1 + \frac{\beta}{\beta + d_{\bar{s}}}) > 0.$$

This is obviously satisfied if $d_{\bar{s}}$ is sufficiently small compared to other dividends (in particular, if $d_{\bar{s}} = 0$ and $d_s > 0$ for all $s \neq \bar{s}$). The condition then for the welfare of the initial old to increase, (5), becomes in this case:

$$\frac{\pi_{\bar{s}}}{\beta + d_{\bar{s}}} > \frac{\beta \pi_{\bar{s}}}{\beta + d_{\bar{s}}} \sum_{s=1}^{S} \frac{\pi_s}{\beta + d_s},$$

which is always true.

Since at CPO allocations, as we saw, the old tend to be richer than the young, we can conclude that the indirect transfer is in the 'right direction' and its negative sign, combined with the positive sign of the direct transfer, allows to generate a richer pattern of transfers between young and old and hence to make an improvement in intergenerational risk sharing easier.

5.1 Defined benefits and defined contributions

As before, we now turn our attention to the two cases which are a more realistic description of actually observed social security systems, defined benefits and defined contribution.

The overall transfer, as we saw, amounts to a combination of the transfer prescribed by the policy and an indirect transfer like in a negative defined benefit system. We can then use our findings for the autarky case to show:

PROPOSITION 6 *In the presence of land, with linear-concave preferences:*

- the introduction of a defined benefits social security system is never Pareto-improving;

- the introduction of a defined contributions scheme, on the other hand, will be Pareto-improving under weaker conditions (on the pattern of covariances of consumption) than under autarky.

5.2 General preferences

When also the utility over consumption when young is described by a general function, the price effect of the introduction of social security will typically no longer be deterministic. Its stochastic structure, and in particular its correlation with consumption when young and old, will then also play an important role in that case in determining whether or not the introduction of social security is welfare improving.

For the case of utility functions strictly concave both in consumption when young and old, one cannot obtain, in general, closed-form solutions for the price of land across states. However it is easy to see from the expression of the first order conditions,

$$q_s u'(e_s^y - q_s - \nu\tau_s) = \beta \sum_{s\in\mathcal{S}} \pi_s(q + d_s)\beta v'(e_s^o + q + d_s + \nu\tau_s). \qquad s \in \mathcal{S},$$

that with iid shocks the price of land is higher when consumption of the young is higher.

To study the exact properties of equilibria one has then to revert to numerical solutions. In all the examples we considered the price of land decreases after the introduction of social security, and the magnitude of . the absolute change in the price is positively correlated with consumption when young. The overall effect of the introduction of social security is then a combination of the transfer (from the young to the old) prescribed by the policy and another transfer in the opposite direction, which is bigger when young agents' consumption is higher, i.e. somewhat analogous to the direct transfer of a negative defined contributions system. We can then use again the arguments developed in Section 3 to determine when such transfers are improving.

To demonstrate that our findings obtained for the simpler economies studied above carry over to more general environments, we consider one such example, where agents have constant relative risk aversion utility with coefficient of risk aversion $\sigma = 2$, $u(c) = v(c) = -c^{-1}$, and $\beta = 1$. There are 2 states with $\pi_1 = \pi_2 = 0.5$ and land's dividends are deterministic: $d_1 = d_2 = 0.05$. Let $e_1^y = 1$, $e_2^y = 2$, $e_1^o = 0.1$, $e_2^o = 1$.

The first two columns of Table 1 below show the consumption allocation as well as the prices of land. Note that consumption when old and when young are positively correlated and consumption when old is more volatile.

We can show that in this economy both defined contributions and defined benefits can be Pareto-improving.

Consider the introduction of a defined contributions system at the scale $\nu = 0.01$ (i.e. a social security tax of 1 percent of young agents' income e_s^y in each state s and the use of its revenue to pay the current old)[8]. In the third column of Table 1 we see the effect of this scheme on the equilibrium price of land: the price of land always decreases and the magnitude of its change is larger in state 2, when young agents' consumption is higher. The large drop in the land price in state 2 leads to a reversal of the sign of the transfer – the total transfer to the old which is induced by this policy is positive in state 1 and negative in state 2. Thus we have a transfer from the young to the old in state 1 (where the young are richer) and from the old to the young in state 2 (where the old are richer), with a clear improvement in intergenerational risk sharing. Even though the direct transfer to the old agents is positively correlated with the young's, as well as the old's, consumption, the total transfer is negatively correlated with it (the indirect transfer induced by the price effect proves then stronger than the direct transfer prescribed by the policy) - thus

[8]Unlike in the previous analysis, the change in policy is here discrete, though small.

helping the old in hedging their risk. The total transfer is then also negatively correlated to consumption when young, so the young will face altogether more risk, but as we noticed their consumption was less volatile than that of the young to begin with. As a consequence, it can be verified that both the initial old and all future generations gain from the introduction of such scheme.

Consider next the introduction of a defined benefits system, also at the scale $\nu = 0.01$, characterized then by a tax $\tau_s = 0.01$ in each state s. We can see in the last column of Table 1 that, as in the case of defined contributions, the strong negative price effect in state 2 leads to a reversal of the sign of he total transfer to the old. Since the direct transfer is here constant and the size of the (negative) indirect transfer is again, in absolute value, positively correlated with the young's, as well as the old's, consumption, the stochastic properties of the total transfer are here unambiguously those of the price effect. The total transfer to the old is then smaller (in fact negative) when the old are richer. We have so an improvement in intergenerational risk sharing which increases the utility of all future generations (and we can verify that the welfare of the initial old also improves).

States	c^y	c^o	q	T(0.01) defined contrib.	T(0.01) defined benefit
1	0.635	0.515	0.365	0.001 ($\Delta q_1 = -0.009$)	0.001 ($\Delta q_1 = -0.009$)
2	1.037	2.013	0.967	-0.001 ($\Delta q_2 = -0.021$)	-0.004 ($\Delta q_2 = -0.014$)

Table 1: Social security with land.

6 Effects on Capital and Output

The preceding analysis abstracted from one important negative effect of social security. If the equilibrium is conditionally Pareto optimal, and if the introduction of a pay-as-you-go social security system leads to a reduction in savings, the stock of capital and hence aggregate equilibrium output and consumption will be reduced for future generations. In this section we will explore, within a simple set-up, the conditions under which this effect does not offset the positive risk-sharing effects of social security. In addition, when the output is subject to productivity shocks, their properties contribute in an important way to determine the nature of the correlation between consumption when old and when young and, also, the stochastic structure of the indirect transfers generated by social security.

To better focus on the effects of social security on capital and output, we consider here the case where there is no land and agents have no endowments of the consumption good, only of labor. Since agents supply inelastically their (unit) endowment of labor, the production function can be written here more simply as $f(k, s)$; we denote then its first and second derivatives with respect to k by $f_k(k, s)$ and $f_{kk}(k, s)$, its derivative with respect to l by $f_l(k, s)$ and the cross derivative by $f_{kl}(k, s)$.

Again, we consider first the case where agents have a quasi-linear (linear concave) utility function. Under this condition a stationary equilibrium still exists[9], both without and with a (stationary) social security system. We can therefore use many of the results obtained in Sections 3 and 4 above. A slight difference is that the transition to the new steady state is now not immediate but takes one period. If social security is introduced (unanticipated) at some time t the current old only receive the direct transfers, there is no additional transfer induced by general equilibrium effect at this time. The current young agents have to pay the transfer but their wage does not change; when old, next period, they will receive the transfer prescribed by the policy and will also be affected by the change in the interest rate induced by the change in the stock of capital) . The new steady state is only reached at $t+1$.

Let k denote the level of savings of an agent when young (or, equivalently, the amount invested in the firms' technology, which will yield the same amount of capital next period). Since shocks are i.i.d., the first order condition of a member of the representative generation is

$$-1+\beta\sum_{s\in\mathcal{S}}\pi_s f_k(k,s)v'(f_k(k,s)k+\nu\tau_s)=0. \tag{23}$$

The market clearing condition is then:

$$\theta = k,$$

i.e. the supply of capital by agents have to equal its demand by firms. We see from (23) that the agents' supply of capital is state invariant; as a consequence the stationary equilibrium is characterized by a constant amount of capital.

The effect on the equilibrium stock of capital of the introduction of an infinitesimal amount of social security is then:

$$k_\nu = \frac{\partial k}{\partial \nu} = -\frac{\sum_{s\in\mathcal{S}}\pi_s f_k(k,s)v''(c_s^o)\tau_s}{\sum_{s\in\mathcal{S}}\pi_s[f_{kk}(k,s)v'(c_s^o)+f_k(k,s)(f_{kk}(k,s)k+f_k(k,s))v''(c_s^o)]}.$$

where c_s^o is again the equilibrium level of consumption of the representative agent when old before the introduction of social security, now given by $c_s^o = f_k(k,s)k$. Observe that this effect is negative whenever $\frac{f_{kk}k}{f_k}\geq -1$, a condition that in what follows we will assume is always satisfied[10].

What is the effect of the change in k on the level of the young and old agents' consumption? For the young, since the equilibrium wage is given by f_l and $c_s^y = f_l(k,s)-k$, it is $(f_{lk}(k,s)-1)\,k_\nu$, while for the old (whose consumption, as we saw, is $c_s^o = f_k(k,s)k$) it is $(f_k(k,s)+kf_{kk}(k,s))\,k_\nu$. The constant returns to scale property of the production function implies that $kf_{kk}=-f_{lk}$. Hence the total change in the amount of resources available for

[9] With production this is no longer true with general preferences.

[10] The condition is equivalent to capital income, $f_k k$, being increasing in k and is always satisfied, for instance, by Cobb-Douglas and in fact by all CES production functions as long as the elasticity of substitution is not too small.

consumption of the agents when young is:

$$-\tau_s - k_\nu \left(k f_{kk}(k,s) + 1\right), \tag{24}$$

and for the old it is:

$$\tau_s + k_\nu \left(k f_{kk}(k,s) + f_k(k,s)\right). \tag{25}$$

Note that in this case the changes do not add to zero, but to $k_\nu(f_k(k,s)-1)$, thus we do not only have a transfer but a change in available resources. Since, whenever competitive equilibria are CPO we have[11] $\mathbb{E}(f_k) > 1$, we can say that the introduction of social security, by reducing the stock of capital, also lowers the expected value of output and hence the average amount of resources available to agents for consumption.

However, it is important to notice that, as long as the policy is introduced at an infinitesimal level, by the envelope theorem the welfare consequences of the induced change in output are zero. The first order conditions for an agent's optimum (23) imply in fact that the effect on the agent's expected utility of increasing consumption when young by $d\nu$ and lowering it when old by $f_k(k,s)d\nu$ is zero. Thus, in evaluating the welfare consequences of the changes in the consumption when young and old given in (24) and (25), we can ignore the last term, so that the change in resources available to the young is exactly equal to the opposite of the change in resources available to the old. We can then say that the overall effect of the policy is a pure transfer effect from the young to the old, given by:

$$T_s = \tau_s + k_\nu k f_{kk}(k,s), \tag{26}$$

strictly positive for all s.

We should stress that the above property follows from the fact that we are considering the introduction of an infinitesimal amount of social security, starting from a situation where its level is zero. When on the other hand a discrete change in policy is considered, its effect is not simply that of a transfer among generations, but the welfare consequences of the change in the output level, and hence of the resources available for consumption, have also to be taken into account. This will become clear in the next Section.

On the basis of the above argument, equation (4) can still be used in this case, now with T as in (26), to evaluate whether or not the introduction of a social security scheme is welfare improving. We see from (26) that the indirect transfer induced by the policy has always a positive sign and varies with the state, according to the stochastic properties of f_{kk}. This is quite different, if not the opposite of what we found in the case of land, where the indirect transfer was negative and, with linear concave preferences, deterministic. The fact that the indirect transfer is always non-negative makes the possibility of an improvement harder

[11] Recalling the necessary and sufficient condition for the equilibrium to be CPO, with linear - concave preferences given by $1 \geq \beta \sum_{s \in \mathcal{S}} \pi_s v'(c^o_s)$, and using the first order conditions for the agents' optimization problem, (23), we obtain $-Cov(\beta v'(c^o), f_k) \leq \mathbb{E}(f_k) - 1$. Since $c^o = f_k k$, the variables $v'(c^o)$ and f_k are clearly comonotonic and negatively correlated, so we get $\mathbb{E}(f_k) > 1$.

(since the overall transfer will then also be non-negative in every state, which limits the possibilities of improving intergenerational risk sharing; moreover, at conditionally Pareto efficient allocations, improving transfers are typically from the old to the young). We can then say that, while the presence of long-lived assets as land altogether helps the case for the introduction of social security to be welfare improving, the presence of production generally hurts.

The stochastic properties of this indirect transfer, which depend on how the technology shocks affect f_{kk}, also matter. To see this more precisely, notice first that the necessary and sufficient condition for the introduction of social security to be improving the utility of the representative generation in the new steady-state generation can be obtained by substituting (26) for T in equation (4):

$$\sum_{s\in\mathcal{S}} \pi_s(-1+\beta v'(c_s^o))\,(\tau_s + k_\nu k f_{kk}(k,s)) > 0. \tag{27}$$

As discussed above, the initial old cannot lose since they only obtain the direct transfer. For the initial young (i.e. born in the period of the introduction, before the new steady state is reached) the analogous condition is obtained:

$$-\sum_{s\in\mathcal{S}} \pi_s\tau_s + \sum_{s\in\mathcal{S}} \pi_s\beta v'(c_s^o)\,(\tau_s + k_\nu k f_{kk}(k,s)) > 0, \tag{28}$$

and is always satisfied whenever (27) holds, since k_ν is negative. Hence in the presence of production, to find an improvement it suffices to consider equation (27).

The fact that the total transfer T_s is always nonnegative implies:

Proposition 7 *A necessary condition for a stationary (ideal) social security scheme to be welfare improving (when preferences are linear - concave) is that for at least one state $\bar{s}$*

$$\beta v'(c_{\bar{s}}^o) > 1.$$

In the case of autarky the condition of the lemma is also sufficient for the existence of an ideal welfare improving scheme[12]. However, this is no longer true in the presence of a (strictly concave) production function, because the (positive) additional transfer induced by the policy implies it is now harder to fully control the risk sharing characteristics of the total transfer generated by the policy, unlike in the case of autarky.

Having determined in (26) the value of the total transfer associated to any social security scheme, by a very similar argument to the one of the proof of Proposition 4 we can show that a necessary condition for scheme $(\tau_s)_{s\in\mathcal{S}}$ to be welfare improving is:

$$\mathrm{cov}(\beta v'(f_k k)\,(\tau + k_\nu k f_{kk})\,, \frac{1}{\tau + k_\nu k f_{kk}}) < 0 < \mathrm{cov}(\tau + k_\nu k f_{kk}, \beta v'(f_k k)). \tag{29}$$

From (29) we see that:

[12]On the other hand with land the condition is no longer necessary since the total transfer may be negative in some state.

PROPOSITION 8 *At a CPO equilibrium with production, when preferences are linear - concave:*

- an improving defined benefits social security system only exists if the production shocks are such that $(-f_{kk})$ *and* $v'(c^o)$ *are positively correlated;*

- an improving defined contributions system only exists if $(-f_{kk})$*, or* $w = f - f_k k$*, is positively correlated with* $v'(c^o)$*.*

When f_k and f_{kk} are co-monotonic, the above necessary condition for defined benefits to be improving is equivalent to the condition that $(-f_{kk})$ and $f_k k$ are negatively correlated. A similar property holds for defined contribution.

On this basis we can look at various alternative specification of the production function, and in particular of the form of the technology shocks.

1. We examine first the case of TFP shocks, with a Cobb-Douglas production function with capital share $\alpha \in (0,1)$:

$$f(k,s) = \xi_s k^\alpha \text{ for } \xi_s + 1 - \delta > 0, \; s \in \mathcal{S}.$$

   Note that in this case $f_k = \alpha k^{\alpha-1}\xi$, $-f_{kk} = \alpha(1-\alpha)k^{\alpha-2}\xi$ and $w = (1-\alpha)k^\alpha \xi$ are all perfectly and positively correlated, so the above necessary conditions are all violated, which implies that neither defined benefits nor defined contributions can ever be improving in this set-up. The fundamental problem lies in the fact that with TFP shocks the marginal utility when old and the total transfer-payment induced by a defined benefits or defined contributions scheme are negatively correlated: when the old are rich, the transfer-payment is high and vice-versa.

2. Alternatively, consider the case where technological shocks are given by a combination of shocks to the depreciation rate of capital and of TFP shocks (as in 1.):

$$f(k,l,m;s) = \xi_s k^\alpha l^{1-\alpha} + (1-\delta_s), \quad s \in \mathcal{S}$$

   If ξ and $1-\eta$ are sufficiently negatively correlated, also $-f_{kk} = \alpha(1-\alpha)k^{\alpha-2}\xi$ and $f_k = \xi\alpha k^{\alpha-1} + 1 - \eta$ may be negatively correlated. So we can show that in this case we can have an improvement both with defined benefits and defined contributions. Consider for example $v(c) = \log(c)$, $\beta = 1$, $\pi_1 = 0.093$, $\alpha = 0.3$, $\xi_1 = 1.1$, $\xi_2 = 0.9$, $\delta_1 = 1$, $\delta_2 = 0$. The equilibrium values are reported in the following table:

States	c^o	w	$-f_{kk}$	T
1	0.33	0.77	0.231	1.230
2	1.27	0.63	0.189	1.188

TABLE 2: Defined Benefits SS with shocks to TFP and capital depreciation.

   The consumption of the old agents c^o is now negatively correlated with the indirect transfer induced by the policy (which, as we argued, is proportional to $-f_{kk}$). So will be then the total transfer $(T_s)_{s\in\mathcal{S}}$, reported in the last column of the table, in the case of a defined benefits scheme, which is thus improving in this example.Since wages are negatively correlated with c^o a defined contributions system will also be improving.

Another possible specification sometimes found in the literature but not examined further in this paper is

$$f(k,s)=k^{\alpha_s},\ \text{for } \alpha_s>0,\ s\in\mathcal{S},$$

where shocks are to the factors' shares. Assume $\alpha_s \leq 0.5$ for all s. Now $f_k = \alpha k^{\alpha-1}$ is positively correlated with $-f_{kk} = \alpha(1-\alpha)k^{\alpha-2}$ but possibly negatively correlated with $w=(1-\alpha)k^{\alpha}$. Hence we can have an improvement with defined contributions but there cannot be one with defined benefits.

We assumed so far that utility is linear-concave. If on the other hand the utility is strictly concave both with respect to the consumption when young and when old, the stochastic structure of the production shocks also affects the correlation between the marginal utility of consumption when young and when old which, as we saw in Section 3, plays a crucial role for the welfare effects of social security.

It is useful to relate this results to the findings obtained by Bohn's (2003) for a more general specification of the economy, where there is production and the technology is characterized by multiplicative shocks to the output level and a deterministic depreciation rate of capital[13]. Suppose $\widetilde{e}^y = \alpha\widetilde{z}$, $\widetilde{e}^o = \gamma\widetilde{z}+\eta$, for $\alpha,\gamma,\eta>0$; it can be easily shown that these are essentially the consumption patterns one obtains in a model with production where the technology has the above form, if one also assumes log-utility, i.e. $u(c)=v(c)=\log(c)$. In this case $\mathrm{cov}(u'(e^y),\beta\frac{v'(e^o)}{u'(e^y)})$ reduces to $-\mathrm{cov}(\frac{1}{\alpha/\gamma e^o-\eta\alpha/\gamma},\frac{\alpha\eta}{\gamma e^o})$. This last term is always negative since the covariance is between two co-monotonic variables and therefore is positive. As a consequence, the second of the two necessary conditions for the existence of an improving defined benefit scheme can never hold here. This also shows that it is difficult to make a case for a defined benefits social security system in the framework considered by Bohn. The intuition behind this is that a defined benefits social security system shifts risks from the old to the young. When income when old is less risky to begin with and risk aversion when old and young are the same, this can never be improving.

7 Pareto-improving introduction of social security

We now investigate the possibility of Pareto improving social security schemes in more realistic set-ups where there is production which uses labor, capital and land as inputs. The model is explained in detail in Section 2.

Since we consider a model with two period lived agents (i.e. a period corresponds to 30 years) and without population or technology growth it is not sensible to calibrate the model to match historic prices and quantities. However, we do want to consider a specification of

[13] See also Section 6.

preferences and technology which is roughly consistent with the calibrations of stochastic olg models in the existing literature (e.g. Bohn (2003), Smetters (2004) or to some extend Constantinides et al (2003)).

There are 4 i.i.d. shocks, $s = 1, ..., 4$, preferences are age-invariant: $u(.) = v(.)$ and exhibit constant relative risk aversion, of the form (17), with $\beta = 1$ and $\sigma = 2$. In order to be able to control the correlation of returns to capital and wages, we consider a specification of the production function as in case 3. of the previous Section, where there is stochastic depreciation, in addition to TFP shocks. We denote the land input to the production function by b and have

$$f(k, l, b; s) = \xi_s k^\alpha l^\gamma b^{1-\alpha-\gamma} + (1 - \delta_s)k, \quad s = 1, \ldots, 4.$$

Consistent with the existing literature (e.g. Imrohogolu et al (2002)) we consider $\alpha = 0.28$ and $\gamma = 0.69$, i.e. the land share is 3 percent, the capital share 28 percent. We fix the TFP shocks to be $\xi_1 = \xi_2 = 1.15$, $\xi_3 = \xi_4 = 0.85$ and the depreciation shocks to be $\delta_1 = \delta_3 = \bar{\delta} + \zeta$ and $\delta_2 = \delta_4 = \bar{\delta} - \zeta$. We set average depreciation $\bar{\delta}$ to be equal to 0.9, given an average annual depreciation of 5 percent this is a bit too low for a 30-year time-interval. However, as we will discuss below a literal interpretation of δ as depreciation is difficult in the two period model. The size of the TFP shock is roughly consistent with what is usually assumed in the literature, the resulting coefficient of variation of wages as well. In Section 7.4 below we discuss how sensitive the findings are with respect to tfp shocks and preference parameters.

Given our previous analysis, it is clear that the welfare implications of different social security schemes will crucially depend on the vector (π, ζ), since this will govern the volatilities and covariance of consumption when old and consumption when young. In the following we will show how different values for this vector will result in different welfare implications. There is no clear-cut empirical guidance in the choice of these parameters. First, it not possible to obtain good estimates of prices or quantities for 30-year periods. Secondly, it is well known that it is impossible to match both the Sharpe ratio and volatility of consumption in this model. Smetters (2003) who matches average returns (and considers a model very similar to ours) takes ζ to be around 5 which in turn leads to unrealistically high consumption volatility. Furthermore, allowing for δ to take values larger than 1 makes it difficult to interpret it as actual depreciation. Bohn (2003) considers non-stochastic depreciation close to 1 (i.e. $\zeta = 0$). Therefore, we consider a variety of different parameter specifications; three cases for the size of the depreciation shock, $\zeta \in \{0, 1, 2\}$ and three cases for the specification of probabilities, $\pi \in \{(1/4, 1/4, 1/4, 1/4), (0, 1/2, 1/2, 0), (0, 1/2, 0, 1/2)\}$. Since there is no stationary equilibrium in this model we have to compute equilibria numerically (we describe the algorithm in Appendix A). For each value of the 'free parameters' we compute equilibrium and evaluate the welfare effects of a small (but finite) introduction of social security. In order to get a first idea how the different specifications imply different equilibrium prices and allocations, we first report the resulting summary statistics for average returns to capital, coefficient of variation of returns, coefficient of variation of aggregate consumption and wages and correlation of returns and wages and compare them to estimates from the literature.

7.1 Equilibrium prices and allocations

Smetters (2003) estimates the 'true' average return to capital to be 1056 percent (for a 30-year horizon, this corresponds to 8.5 percent p.a.), and the coefficient of variation to be 0.87. He estimates the correlation between returns and wage-income to be 0.75. (However, these estimates are largely uninformative; for example Krueger and Kubler (2006) estimate the correlation to be -0.4 for a 7-year period). We will not match any of these numbers in our specifications below, but it should be instructive to see how the different choices of parameters could be judged more or less realistic, depending on the resulting prices.

In the following table π^1 refers to $(1/4, 1/4, 1/4, 1/4)$, π^2 refers to $(0, 0.5, 0.5, 0)$ and π^3 to $(0.5, 0, 0, 0.5)$. Since for $\zeta = 0$ these 3 cases are identical, we only report $(0, \pi^1)$.

(ζ, π)	Avg return	coeffvar return	coeffvar wages	corr returns wages	coeffvar agg. cons
$(0, \pi^1)$	1.32	0.05	0.20	0.27	0.25
$(1, \pi^1)$	1.84	0.54	0.19	0.08	0.29
$(2, \pi^1)$	2.71	0.74	0.18	0.10	0.30
$(1, \pi^2)$	2.02	0.57	0.19	0.996	0.38
$(2, \pi^2)$	2.96	0.76	0.17	0.999	0.40
$(1, \pi^3)$	1.67	0.56	0.19	-0.9998	0.04
$(2, \pi^3)$	2.31	0.79	0.18	-0.9998	0.054

The table shows that in all specifications we are far from matching average return to capital or its variation. However, it is also clear that higher ζ lead to more realistic values for these statistics. In this sense, higher ζ seem more realistic. However, they also lead to unrealistically high coefficient of variation in aggregate consumption. With the second specification of probabilities, the correlation is counterfactual, but it is still interesting in the light that part of the literature takes the correlation to be significantly positive. The third specification leads to very low variation in consumption and a correlation of -1 is quite unrealistic. However, it will be interesting to see what role the correlation plays of the welfare effects of social security.

Given our previous analysis, we are also interested in how the correlation of returns affects the correlation of consumption when young and consumption when old. The correlation of consumption when young and consumption when old in the case of no social security and for the intermediate depreciation shock is as follows.

$(1, \pi^1)$	$(1, \pi^2)$	$(1, \pi^3)$
0.4897	0.9856	-0.8374

7.2 Measuring direct and indirect effects

Since we approximate equilibria numerically, we need to consider a small but finite change in the social security system. As described in Section 4 above, we assume that the social security policy starts operating after the end of a given period, at all possible direct successor (i.e. at all shocks). Its introduction is not anticipated at the previous date. We trace the effects of the introduction of the policy along the event tree. We report welfare gains and losses (in wealth equivalents – the exact computations of welfare changes is reported in Appendix A) for the current generation and for the next 6 generations. After 5-6 periods welfare changes seem to stabilize.

In order to understand welfare changes for generations far in the future, we consider a first order approximations of these changes and decompose this approximation into changes induced by intergenerational transfers (in turn generated by direct and indirect transfers) and changes induced by a crowding-out of the capital investment induced by social security. The fact that we consider a change from zero taxes to positive taxes implies that the welfare effects of these changes in the output level can no longer be ignored. Suppose social security is introduced immediately following some node $s^{t'}$. For all $t > t'$, denote the consumption in the equilibrium without social security by $c(s^t)$ and the consumption in the equilibrium with social security by $\tilde{c}(s^T)$. Denote the total transfer (direct plus indirect effects) to the old as $T^o(s^t) = \tilde{c}^o(s^t) - c^o(s^t)$ and the one from the young by $T^y(s^t) = c^y(s^t) - \tilde{c}^y(s^t)$. In the presence of capital, we also need to define the total change of aggregate consumption, $L(s^t) = T^o(s^t) - T^y(s^t)$.

A first order approximation of the effects of an introduction of social security on the welfare of generation t is given by

$$\mathbb{E}_{s^0}(-u'(c^y(s^t))T^y\left(s^t\right) + \beta v'(c^o(s^t, s_{t+1}))T^o\left(s^t, s_{t+1}\right)).$$

This expression can be rewritten as the sum of three effects:

$$\mathbb{E}_{s^0}\left\{\left(-u'(c^y(s^t, s_{t+1})) + \beta v'(c^o(s^t, s_{t+1}))\right)T^y(s^t, s_{t+1})\right\} + \tag{30}$$

$$+\mathbb{E}_{s^0}\left\{\beta u'(c^o(s^t, s_{t+1}))[T^o\left(s^t, s_{t+1}\right) - T^y\left(s^t, s_{t+1}\right)]\right\} + \tag{31}$$

$$\mathbb{E}_{s^0}\left\{-u'(c^y(s^t))T^y\left(s^t\right) + u'(c^y(s^t, s_{t+1}))T^y\left(s^t, s_{t+1}\right)\right\} \tag{32}$$

The last term, (32), captures the welfare effect due to non-stationarity, i.e. the difference between the effect on the young agents of generation t and the effect on the welfare of the agents who will be young at date $t+1$. Under the assumption that there exists a ergodic Markov equilibrium, as $t \to \infty$, this last term tends to zero because for any ergodic Markov process (x_t) we have that $\mathbb{E}_0 x_t - \mathbb{E}_0 x_{t+1} \to 0$

The second term, (31), captures the welfare effect of the change in the aggregate level of resources available for consumption as a result of the policy (crowding out effect):

$$C(t+1) = \sum_{s^t \succ s^{t'}} \pi(s^{t+1}|s^{t'})\beta u'(c^o(s^{t+1}))L(s^{t+1}).$$

On the basis of the previous analysis we can easily verify that change in resources $L(s^{t+1})$ is approximately equal to $\Delta k(s^t) f_k(s^{t+1}) - \Delta k(s^{t+1})$.

Finally, the first term, (30), measures the welfare effect of the total transfer from the young to the old at date $t+1$; it is then analogous to the expression obtained in the case of stationary equilibria. It will be useful to decompose this term further: The total transfer of the young $T^y(s^{t+1})$ is in turn equal to $\tau(s^{t+1}) + \Delta q(s^{t+1}) + [\Delta k(s^{t+1}) - \Delta w(s^{t+1})]$, where $\tau(s^{t+1})$ is the direct transfer prescribed by the social security scheme, $\Delta q(s^{t+1})$ the transfer induced by the change in the price of land (i.e. it is the difference of or price of land with social security and price of land without), and $[\Delta k(s^{t+1}) - \Delta w(s^{t+1})]$ the transfer induced by the change in the stock of capital (i.e. changes in wage and savings). As a consequence we obtain

$$\mathbb{E}_{s^0} \left\{ \left(-u'(c^y(s^t, s_{t+1})) + \beta v'(c^o(s^t, s_{t+1}))\right) T^y \left(s^t, s_{t+1}\right) \right\} = D_d(t+1) + D_q(t+1) + D_k(t+1)$$

with

$$\begin{aligned}
D_d(t+1) &= \mathbb{E}_{s^0} \left\{ \left(-u'(c^y(s^t, s_{t+1}) + \beta v'(c^o(s^t, s_{t+1}))\right) \tau(s^{t+1}) \right\} \\
D_q(t+1) &= \mathbb{E}_{s^0} \left\{ \left(-u'(c^y(s^t, s_{t+1}) + \beta v'(c^o(s^t, s_{t+1})))\right) \Delta q(s^{t+1}) \right\} \\
D_k(t+1) &= \mathbb{E}_{s^0} \left\{ \left(-u'(c^y(s^t, s_{t+1})) + \beta v'(c^o(s^t, s_{t+1}))\right) (\Delta k(s^{t+1}) - \Delta w(s^{t+1}) \right\}
\end{aligned}$$

These are the three effects we discussed in Sections 4, 5 and 6. In our computations, the changes are not infinitesimal and we cannot compute $D(t)$ and $C(t)$ as $t \to \infty$. Nevertheless, it is instructive to report $D(t'+6)$ and $C(t'+6)$. It turns out that after 6 periods, the changes in these numbers are very small and reporting these number allows us to relate the experiments in this section to the previous results.

7.3 Introduction of social security

In this section, we consider the experiment where the economy starts without any social security system and consider the introduction of a small pay-as-you go system. We first consider the introduction of a defined benefits system and then discuss the case of defined contributions.

7.3.1 Defined benefits

We compute equilibrium for $\nu = 0$ and for $\nu = 0.01$. As explained above, to verify that the change is Pareto-improving we both compute the welfare changes for the next 6 generations as well as the decomposition described above. The following table shows the welfare effects as well as the direct and the crowding out effects as explained above. In order to make the table easier to read, all welfare changes are multiplied by 10^4.

(ζ, π)	total changes	D_k	D_l	D_d	crowding out
$(0, \pi^1)$	-5.5	0.58	4.57	-6.73	-3.78
$(1, \pi^1)$	0.18	-0.61	4.87	-4.03	-0.05
$(2, \pi^1)$	1.13	-0.56	4.05	-2.37	0.34
$(1, \pi^2)$	2.10	-1.29	3.29	-0.47	1.01
$(2, \pi^2)$	2.64	-0.87.	1.02	2.61	0.78
$(1, \pi^3)$	-5.1	0.92	6.01	-9.52	-2.77
$(2, \pi^3)$	-4.5	0.61	8.03	-11.99	-1.91

The cases where total changes are positive in fact constitute a Pareto improvement. In all of those cases, the current old as well as all generations in the transition gain. For example, for $(1, \pi_1)$ the current old gain 9.4, the next 6 generations gain $(0.2, 0.08, 0.14, 0.17, 0.18, 0.18)$. The results for the other cases are similar and not reported. There are several results which are worth commenting on.

Both for π^1 and π^2, there is a range of parameters generally considered realistic for which an introduction of social security is Pareto-improving. This is consistent with our earlier analysis as these are the cases where consumption when old is more volatile than consumption when young and they are positively correlated.

With the low depreciation shock, social security is bad because the old almost always consume more than the young. That is, marginal utility of the old is only very rarely above marginal utility when young, and even then the difference is small.

With the high depreciation shock, the crowding out effect actually is positive. While it is always true that $\mathbb{E}_0(L(s^t))$ is negative, the covariance between $L(s^t)$ and $u'(c^o(s^t))$ is likely to be negative as well: One can easily verify that the change in resources $L(s^{t+1})$ is approximately equal to $\Delta k(s^t) f_k(s^{t+1}) - \Delta k(s^{t+1})$. In states where the old are relatively poor and $u'(c^o)$ is large, $f_k(s^t)$ is small. If in addition, returns to capital and wages are positively correlated, $\Delta k(s^{t+1})$ tends to be a smaller negative number than $\Delta k(s^t)$ and the correlation is positive.

In all cases, the effect of the land price is positive and relatively large. The effect of capital is negative except for the case of negatively correlated shocks, π^3. However quantitatively this effect is relatively small and does not determine the overall welfare changes. For the sign of the welfare change across different specifications, the direct effect D_d is crucial. As we saw in Section 4, this effect can only be positive if consumption when old and when young are positively correlated and when consumption when old is sufficiently volatile. This explains why a welfare improvement is easiest to obtain for π^2 and why it is impossible for π^3. It also explains why the size of ζ is crucial for the welfare effects.

7.3.2 Defined contributions

We now consider the introduction of a defined contribution system. since equilibrium wages lie around 2.5-3 in most examples consider, to make the numbers comparable to those above, we choose the size of the system to be 0.35 percent, i.e. set $\nu = 0.0035$. The following table shows that it is slightly more difficult to obtain an improvement than it was in the case of defined benefits.

(ζ, π)	total changes	D_k	D_l	D_d	crowding out
$(0, \pi^1)$	-6.95	0.79	5.62	-8.46	-4.64
$(1, \pi^1)$	-0.40	-0.36	5.76	-5.53	-0.31
$(2, \pi^1)$	0.35	-0.30	4.96	-4.23	0.33
$(1, \pi^2)$	0.72	-0.68	5.30	-4.05	0.31
$(2, \pi^2)$	1.01	-0.42	4.12	-2.71	0.33
$(1, \pi^3)$	-2.99	0.34	5.10	-6.88	-1.42
$(2, \pi^3)$	-2.32	0.24	6.21	-8.13	-1.01

Just as above, in cases where the total changes are possible, the current old and all future generations gain from an introduction of social security. The results are largely analogous to the defined benefit case.

7.4 Sensitivity analysis

The choice of the preference parameters above was somewhat arbitrary. We discuss in this section how risk aversion and discounting affect the possibility of Pareto-improving social security. We consider $\sigma \in \{0.5, 2, 4\}$ and $\beta = \{0.44, 1\}$. These values cover the ranges considered realistic in the literature.

The analysis above showed that the higher the depreciation shock, the more likely it is to obtain a Pareto-improvement through a social security system. In the sensitivity analysis, given a specifications of preference parameters and probabilities we search for the smallest $\zeta \in \{0, 0.1, 0.2, \ldots, 2\}$ for which the introduction of social security constitutes a Pareto-improvement in the implied economy. If there is no improvement for $\zeta = 2$, we report "> 2". The following table shows the results for our different specifications of preference parameters.

Defined benefits

$\pi \backslash (\sigma, \beta)$	(0.5,1)	(2,1)	(4,1)	(0.5,0.44)	(2,0.44	(4,0.44)
π^1	> 2	1.0	0.5	> 2	1.4	0.7
π^2	1.9	0.5	0.3	> 2	0.8	0.4

Defined contributions

$\pi \backslash (\sigma, \beta)$	(0.5,1)	(2,1)	(4,1)	(0.5,0.44)	(2,0.44	(4,0.44)
π^1	> 2	1.3	0.6	> 2	1.6	0.8
π^2	> 2	0.7	0.4	> 2	0.9	0.5

The tables show that risk aversion always helps, more discounting slightly hurts. In line with what we found earlier, defined contributions is a bit more difficult, but it seems that for higher risk aversions the differences become very small.

Note in particular, that for a coefficient of relative risk aversion of 4 and π^2 an improvement in possible in an economy with only very modest depreciation shocks, for $\delta = \{0.6, 1.2\}$.

For π^3 an improvement was not possible in any of the cases considered. For the case of low risk aversion this is somewhat surprising, since our previous analysis suggests that for the case of defined contributions, a negative correlation between consumption when old and consumption when young should help. However, it still turns out that the direct effect is significantly negative. In fact, for $\zeta = 1$, we obtain $D_l = 10.07$, $D_k = 0.40$ and $D_d = -13.05$. As before, the direct effect is crucial for the overall welfare effect.

Lastly, we perform a sensitivity analysis with respect to the variance of the tfp shock, ξ. We consider $\xi_1 = \xi_2 = 0.08$, $\xi_3 = \xi_4 = 0.92$. Both with defined benefits and with defined contribution, it is a bit more difficult to obtain an improvements, but the differences are quantitatively quite small. For defined benefits, $\sigma = 2$, $\beta = 1$ and π^2 and improvement is obtained for all $\zeta > 0.7$. The results for other specifications are very similar.

8 Pareto improving reform of social security

The actual size of the system in the US lies around 12 percent. By most accounts this is viewed as too large a system. However, it seems that for some of our parameterizations above, this is actually the 'right size'. In this section, we focus on the benchmark case $\sigma = 2$, $\beta = 1$.

8.1 Optimal size of the system

Suppose the economy starts with a positive social security system. Under which conditions is it Pareto-improving to increase the size of the system? We search for all $\nu \in \{0, 0.01, 0.02, \ldots, 1\}$ for the largest initial system for which an increase in the social security tax is Pareto-improving. To make the numbers comparable, we report for the case of defined benefits the size of the system as a percentage of average wages. The following table shows the size of the system as tax-payments as a percentage of average wages at which an increase of taxes is still slightly Pareto-improving, but for which an increase by 0.005 makes future generations worse off.

	π^1, $\zeta = 2$	π^2, $\zeta = 1$	π^2, $\zeta = 2$
Defined benefit	0.11	0.12	0.21
Defined contributions	0.08	0.09	0.19

There is still a variety of specifications for which a social security system that is roughly the size of the current US system seems about optimal.

8.2 Risk-sharing reforms

As argued above, a large defined contributions system as observed in many industrial countries cannot be justified in this model. The question then arises if a reform of such a system can lead to a Pareto-improvement. It is clear that this is only possible if the transfer to the old is increased in at least one state, given that it should be decreased in others. Given our theoretical results in Section 2 above, a candidate reform would be to decrease the transfers in states where the old are relatively rich while increasing it (to compensate the current old) in states where the old are relatively poor.

We now consider the case $\zeta = 1$, $\pi = \pi^2$ and $\nu = 0.10$, i.e. given the specification of the economy, the size of the system is sightly too large. In this situation, quantitatively large welfare gains can be obtained by changing the system considerably. If the system is reformed to $\tau(s^t) = 0$ for $s_t = 2$ and $\tau(s^t) = 0.2w(s^t)$ for $s_t = 3$, i.e. if for the bad return state, the payroll tax is increased to 20 percent while in the good return state it is decreased to zero, the welfare gains for the current old and all future generations *in percent* are given by

$$(2.9, 2.6, 2.6, 3.1, 3.2, 3.2).$$

That is, the representative future generation gains 3.2 percent in in wealth equivalence, the current old, although they paid the full tax of 10 percent when young, gain in expected value 2.9 percent through the change in the risk-sharing characteristic of the system.

9 Conclusion

The idea that a PAYGO social security system can lead to enhanced intergenerational risk sharing has been formalized in various papers (see the literature review in the introduction). However, it is also well known that in a general equilibrium model social security crowds out private savings and thus capital formation, and therefore leads to lower wages for future generations. In most quantitative studies this second effect seems to overcompensate any beneficial effects of enhanced risk-sharing.

We show that the presence of durable land as an additional factor of production mitigates the crowding out effect and that intergenerational risk-sharing provides a normative justification of a PAYGO social security system even if one takes into account the effects on the capital stock. It is crucial to note that in our framework social security is only desirable under an ex-ante welfare criterion. Under an interim criterion the presence of land tends to decrease the scope for social security because it provides an important tool for self-insurance.

Appendix

Details on computations

In Sections 7 and 8, we seek to find an admissible range for the capital stock $\Theta \subset \mathbb{R}_{++}$ as well as functions from the current shock and the beginning of period capital-stock to land-prices and investments, $\rho_q : \Theta \times \mathcal{S} \to \mathbb{R}_+$, $\rho_k : \Theta \times \mathcal{S} \to \Theta$ such that for all shocks $\bar{s} \in \mathcal{S}$ and all $k_- \in \Theta$ the following inequalities hold for small $\epsilon \geq 0$

$$\| - 1 + \beta \sum_{s \in \mathcal{S}} \pi_s f_k(\rho_k(k, \bar{s}), 1, 1; s) \frac{v'(c_s^o)}{u'(c^y)} \| < \epsilon$$

$$\| - \rho_q(k, \bar{s}) + \beta \sum_{s \in \mathcal{S}} \pi_s (\rho_q(\rho_k(k, \bar{s}), s) + f_b(\rho_k(k, \bar{s}), 1, 1; s)) \frac{v'(c_s^0)}{u'(c^y)} \| \quad < \quad \epsilon,$$

where $c^y = f_l(k, 1, 1; \bar{s}) - \rho_k(k, \bar{s}) - \rho_q(k, \bar{s})$ and $c_s^o = \rho_k(k, \bar{s}) f_k(\rho_k(k, \bar{s}), 1, 1; s) + \rho_q(\rho_k(k, \bar{s}), s)$, for all $s \in \mathcal{S}$.

We use a collocation algorithm as described for example in Krueger and Kubler (2004) to approximate these functions numerically. For this, we write ρ_k and ρ_q as cubic splines (i.e. piece-wise cubic polynomials) with 200 collocation points. We solve for the unknown spline coefficients using time-iteration, i.e. given an approximation for ρ_k and ρ_q tomorrow, ρ_k^N and ρ_q^N, we solve for optimal choices and prices today on a grid of 200 points and interpolate the solution to obtain new functions ρ_k^{N+1} and ρ_q^{N+1}. This procedure is repeated until for some $\bar{N}$,

$$\|\rho_q^{\bar{N}} - \rho_q^{\bar{N}-1}\|_\infty + \|\rho_k^{\bar{N}} - \rho_k^{\bar{N}-1}\|_\infty < 10^{-10}.$$

With the candidate function $\rho_q^{\bar{N}}$ and $\rho_k^{\bar{N}}$, we determine the error in the above system of equations. If $\epsilon < 10^{-5}$, we accept this as an approximate solution and report equilibrium prices and welfares for this approximation.

Welfare computations

We suppose that social security is introduced, unanticipated, at some time t and all 4 possible shocks $1, ..., 4$, i.e. at nodes $(s^{t-1}, 1), ..., (s^{t-1}, 4)$. Let $\tilde{c}^y, \tilde{c}^o$ denote equilibrium consumptions in the economy with social security and $\tilde{c}^y, \tilde{c}^o$ consumptions in the economy without social security. Given our assumption of CRRA utility, the welfare changes of the initial old are then given by $\left(\frac{\sum_{s \in \mathcal{S}} \pi_s (\tilde{c}^o(s^{t-1}, s))^{1-\sigma}}{\sum_{s \in \mathcal{S}} \pi_s (c^0(s^{t-1}, s))^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} - 1$. Welfare changes for a generation born T generations after introduction of social security $T = 0, 1, ...$ are given by

$$\left(\frac{\sum_{s^{t+T} \succ s^{t-1}} \pi(s^{t+T}|s^{t-1})((\tilde{c}^y(s^{t+T}))^{1-\sigma} + \beta \sum_{s \in \mathcal{S}} \pi_s (\tilde{c}^o(s^{t+T}, s))^{1-\sigma})}{\sum_{s^{t+T} \succ s^{t-1}} \pi(s^{t+T}|s^{t-1})((c^y(s^{t+T}))^{1-\sigma} + \beta \sum_{s \in \mathcal{S}} \pi_s (c^o(s^{t+T}, s))^{1-\sigma})}\right)^{\frac{1}{1-\sigma}} - 1$$

Proofs

Proof of Proposition 5. In Proposition 2 we showed that, under the same condition of this Proposition, the following transfer scheme is always welfare improving, at an autarkic equilibrium: $\tau_{\hat{s}} > 0$, $\tau_s = 0$ for all $s \neq \hat{s}$. We will show that, in the presence of land, it is always possible to design an ideal social security system for which the set of total transfers is exactly the same (and hence will be Pareto improving).

To make the notation a bit simpler we will assume that for all other $s \neq \hat{s}$, $\beta v'(c_s^o) < 1$ (it should become clear that this is an innocuous assumption).

It is immediate to see that we can always find some $\delta < 1$ such that

$$\delta(1 - \beta \sum_{s \in \mathcal{S}} v'(c_s^o)) + \pi_{\hat{s}}(1-\delta)\beta v''(c_{\hat{s}}^o)(q + d_{\hat{s}}) = 0.$$

Consider then a stationary social security system characterized by $\tau_{\hat{s}} = 1$ and $\tau_s = \delta$ for all $s \neq \hat{s}$. By substituting the previous expression in (20), we find that this system will induce the following total transfers across generations:

$$T_s = \tau_s + \frac{dq}{d\nu} = \frac{\delta(1 - \beta \sum_{s \in \mathcal{S}} \pi_s v'(c_s^o)) + \pi_{\hat{s}}(\delta - 1)\beta v''(c_{\hat{s}}^o)(q + d_{\hat{s}})}{1 - \beta \sum_{s \in \mathcal{S}} \pi_s \left[v'(c_s^o) + (q + d_s)v''(c_s^o)\right]}, \text{ for } s \neq \hat{s}.$$

which, given the above specification of δ, equals zero, and

$$T_{\hat{s}} = \tau_{\hat{s}} + \frac{dq}{d\nu} = \frac{(1 - \beta \sum_{s \in \mathcal{S}} \pi_s v'(c_s^o)) + \sum_{s \neq \hat{s}} \pi_s(\delta - 1)\beta v''(c_s^o)(q + d_s)}{1 - \beta \sum_{s \in \mathcal{S}} \pi_s \left[v'(c_s^o) + (q + d_s)v''(c_s^o)\right]} > 0$$

where the sign follows from the conditional optimality of the equilibrium and the fact that $\delta < 1$. Under the condition $\beta v'(c_{\hat{s}}^o) > 1$, this policy will clearly improve future generations, as well as the initial old, i.e. satisfy (22) and (21). ■

Proof of Proposition 6. If τ_s is constant across all shocks s, since the price effect $\frac{dq}{d\nu}$ is also, as we saw, independent of s, so will be the total transfer induced by the policy. From (20) we see in fact that, when $\tau_s = \tau$ for all s:

$$T_s = \tau_s + \frac{dq}{d\nu} = \frac{\tau(1 - \beta \sum_{s \in \mathcal{S}} \pi_s v'(c_s^o))}{1 - \beta \sum_{s \in \mathcal{S}} \pi_s \left[v'(c_s^o) + (q + d_s)v''(c_s^o)\right]},$$

which is constant across s. Since in Proposition 3 we showed that, when agents have linear - concave preferences, a deterministic transfer from the young to the old can never be welfare improving, this implies that a defined benefits system can never be welfare improving in this case.

In the case of a defined contributions system, on the other hand, the total transfer induced by the policy is

$$T_s = e_s^y + \frac{dq}{d\nu} = e_s^y + \frac{\beta \sum_{s \in \mathcal{S}} \pi_s e_s^y (q + d_s) v''(e_s^o + d_s + q)}{1 - \beta \sum_{s \in \mathcal{S}} \pi_s \left[v'(e_s^o + q + d_s) + (q + d_s)v''(e_s^o + q + d_s)\right]}.$$

Since the second term is always negative and independent of s, we can say the total effect of a defined contributions system with land is analogous to that of a combination of a defined contributions and a negative defined benefits scheme under autarky. By comparing Equations (8) and (1), we see that the latter has always (at least with linear - concave preferences) a positive effect on the welfare of the representative generation. Hence the conditions (on the parameters of the economy) under which a defined contributions system is Pareto improving are weaker than the ones we found under autarky. ■

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