Rent dissipation in repeated entry games some new results

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Abstract

Two player infinitely repeated entry games are revisited using a new Markov equilibrium concept. The idea is to have an incumbent face an entrant which may use hit and run strategies. Contrarily to most existing approaches, rent dissipation no longer necessarily holds. It will not when competition is tough in case of entry. Similarities and differences with previous approaches are analyzed. Several examples are discussed. *Journal of Economic Literature* Classification Numbers : C73, D43, L12.

1 Introduction[1]

This paper revisits the dynamics of potential competition in a natural monopoly context. In such situations an incumbent is repeatedly facing an entrant. The originality of the paper is to assume that the entrant uses hit and run strategies : if the incumbent makes a mistake the entrant comes in, makes some money and then leaves. This is modeled through appropriate Markov strategies.

Contrarily to most existing approaches,[2] rent dissipation need not necessarily prevail. Rent dissipation, which means that the stage rent of the incumbent goes to zero when the discount factor goes to one, has almost the status of a folk theorem in industrial organization (see among others Farrell, 1986, Fudenberg and Tirole, 1987 and Wilson, 1992). In traditional economics, the intuitive idea behind rent dissipation is a rational expectation argument. In Maskin and Tirole (1988) and in Ponssard (1991) a proper game analysis confirms the

[1] Previous versions of this paper have been presented at Core (Louvain), Cirano (Montreal), CMM (Santiago de Chile), Etape (Paris), Hec (Jouy en Josas), Idei (Toulouse), IHP (Paris) and SSE (Stockholm). I thank David Encaoua, Denis Gromb, Rim Lahmandi-Ayed, Rida Laraki, Jean François Laslier, Anne Perrot, Patrick Rey, David Sevy, Nicolas Vieille and the participants to the seminars for helpful comments and suggestions.

[2] See Boyer et al., 2003, for an recent contribution in which rent dissipation does not prevail. Their argument is based on capacity constraints and is unrelated to the current discussion.

rational expectation intuition. In our approach rent dissipation depends on the monopoly rent relative to the cost of entry : when this ratio is high rent dissipation occurs and when it is low it does not. Consequently, the tougher the competition in case of entry the more likely rent dissipation fails to hold.

These results provide some formal grounds to the critics of contestability theory (Baumol et al., 1982). Contestability, originally designed for the airline industry and then applied to many sectors such as telecoms, emphasizes the length of commitment (i.e., the discount factor) as the major strategic entry barrier irrespective of the other structural characteristics. Some empirical economists have challenged this view (Shepherd, 1984). More generally, Sutton (p. 35, 1991) has argued and documented that "a very sharp fall in price suffices to deter entry and maintain a monopoly outcome", this is very much in line with the theoretic results obtained in this paper.

In addition to rent dissipation, selection is also analyzed. Selection means that only the more efficient firm can remain a permanent incumbent (this notion was introduced in Gromb et al., 1997, in which finitely repeated asymmetric entry games were first analyzed). It is proved that selection will always hold if the asymmetry between the two firms is high enough. This has some bearing with the empirical results reported by Scherer (1992) and Gerovski (1995) in which it is shown that most entries occur after large innovations.

These results on rent dissipation and on selection may be conveniently summarized through a new and interesting taxonomy of situations. The situation may be qualified as :

excess-competition : if an incumbent were to choose to deter entry forever, it would have to dissipate all of its profits;

under-competition : a less efficient incumbent can deter entry forever and make stationary positive profits;

selection : a less efficient incumbent is not able to deter entry forever and make stationary positive profits, but a more efficient incumbent may.

The adopted Markov equilibrium concept extends the one used by Maskin and Tirole (1988).[3] The idea is to relax the stationarity assumptions which they impose on the strategies. It is motivated by the results obtained in Gromb, Ponssard and Sevy (1997).[4] There it was shown that when one firm is more efficient than the other, at equilibrium, the less efficient firm cannot remain a permanent incumbent in a long enough game. This suggests that the stationarity assumption is too strong and is the reason for the drawback of the MT approach pointed out in Lahmandi et al. (1996) namely, that the less efficient firm can remain a permanent incumbent and, moreover, that its stage rent is higher than what the more efficient firm would obtain were it the permanent incumbent. Yet, the GPS approach, when extrapolated to infinitely repeated games, has drawbacks of its own. The issue remained open.

The paper is organized as follows. Section 2 introduces the class of infinitely repeated entry games under study. Various solution concepts may be used to

[3] to be later refered to as the MT approach. Originally designed for entry games, this approach has been extended to any extensive game (Maskin and Tirole, 2001).

[4] to be later refered to as the GPS approach.

analyze these games. In section 3 the MT approach is recalled, the GPS approach to finitely repeated games is extended to infinitely repeated ones, and the new approach is defined. Similarities and differences among these approaches are emphasized. Section 4 studies the mathematical properties of the proposed solution concept : the given finite time horizon for incumbency of the hit and run firm is the key parameter. It is proved that when an equilibrium exists it is necessarily unique, that this firm would prefer to play in a game in which its given time as an incumbent is as long as possible but that when this given time exceeds some bound such an equilibrium may no longer exist. Whenever there is no such bound the situation is one of rent dissipation. A necessary and sufficient condition for such a bound to exist is obtained. In section 5, the taxonomy of situations illustrating cases of under or excess-competition is introduced, some general comparative static properties are derived and specific examples detailed. Section 6 concludes with open questions that should be studied to improve our understanding of the proposed solution concept.

2 The game Γ_∞

Various game forms may be used to capture the idea of short run commitments in a natural monopoly context. For convenience, our model is very similar to the one introduced in GPS. Denote Γ_∞ such an entry game. The results can be easily extended to the other game forms.

Denote the two firms player 1 and player 2. Let i be anyone of the two players and $j \neq i$ be the other one.

The game Γ_∞ is constructed from two one stage Stackelberg entry games, denoted $G(1)$ and $G(2)$.The game Γ_∞ consists of a sequence of such one stage games $(G(i^1), G(i^2), ..., G(i^k)....)$ in which any i^k may be either 1 or 2 depending on the actual moves selected along the way.

Each game $G(i)$ is played as follows :

step 1 : the first player to move is player i , it is called *the incumbent*, player i may either select a move x_i in $\Re$ or move *out*;

step 2a : if player i selects x_i at step 1 then player j may either select *in* (with respective payoffs for i and j : $d_i(x_i), -C_j(x_i)$) or *out* (with respective payoffs : $v_i(x_i), 0$) ;

step 2b : if player i selects *out* at step 1 then player j may either select a move x_j in $\Re$ or move *out*;

step 3 : if player i selects *out* at step 1 and player j selects x_j at step 2b then player i may either select *in* (with respective payoffs : $-C_i(x_j), d_j(x_j)$) or *out* (with respective payoffs : $0, v_j(x_j)$);

step 4 : if player i selects *out* at step 1 and player j selects *out* at step 2b then the game Γ_∞ stops (with respective payoffs : $0, 0$).

Moreover the actual play of $G(i)$ determines the incumbent in the next $G(i')$ to be played :

- player i' is again player i if this sequence is (x_i, out) or (out, x_j, in);
- player i' is player j if the sequence is (x_i, in) or (out, x_j, out).

The payoffs in Γ_∞ are the discounted sum of the stage payoffs (with attention on discount factors close to 1).

This completely specifies Γ_∞.

The functions v_i, C_i, d_i may be interpreted as stage monopoly profits, entry costs and exit payoffs respectively. For $i \in (1,2)$ it is assumed that v_i is strictly increasing, C_i strictly decreasing and d_i non positive.

For Γ_∞ to be of economic interest it is assumed that :

$$C_j^{-1}(0) > v_i^{-1}(0)$$

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The first inequality means that the static “limit price” of player i, $C_j^{-1}(0)$, should be higher than player i 's “average cost”, $v_i^{-1}(0)$, otherwise player i is definitely barred from entry (the situation is one of ”bockaded entry”). Similarily for player j.

For mathematical convenience it will be further assumed that the functions v_i and C_i have derivatives and that these derivatives are uniformly bounded away from zero and from infinity. It is believed that this technical assumption may be relaxed without affecting the results.

Two classes of games shall be of special interest.

The class of *symmetric* entry games in which $v_1 = v_2$ and $C_1 = C_2$. The economic issue is *rent dissipation* : at equilibrium, does the stage payoff of the incumbent $v_i(x_i)$ converges to zero when the discount factor goes to one ?

The class of *asymmetric* entry games in which $v_1 = v_2 + \Delta f$ in which Δf is a constant, and $C_1 = C_2$. The economic issue is *selection* : suppose $i^1 = 2$, then is it true that for all $\Delta f > 0$, at equilibrium, player 1 is the long run incumbent i.e., for all $k \geq k^*$, $i^k = 1$, whatever the discount rate close enough to one ?

3 Markov equilibrium concepts

This section compares three refinements of Markov equilibrium. In all these refinements, the state variable may depend on time but not on the past moves played by the players. Varying the way the state variable depends on time, one defines the MT approach, the GPS extension to infinitely repeated games, and the new approach proposed in this paper.

Let

- τ be the time sequence with $\tau \in \{1, 2 ... \infty\}$;
- θ be the state variable, $\theta = (i, \tau)$ which means that Γ_∞ is at time τ and that the players are currently playing game $G(i)$;
- (z_1^θ, z_2^θ) stand for the decisions in $G(i)$ given θ and (Z_1, Z_2) stand for the strategies ;

- $G_1^\theta(z_1^\theta, z_2^\theta)$ and $G_2^\theta(z_1^\theta, z_2^\theta)$ stand for the respective stage payoffs in $G(i)$ given θ and (z_1^θ, z_2^θ) ;
- $\theta^+(z_1^\theta, z_2^\theta) = (i^+, \tau+1)$ be the next stage state variable given θ and (z_1^θ, z_2^θ) where i^+ is defined according to the rules of Γ_∞;
- δ be the discount rate ;
- (Y_1, Y_2) be a Markov equilibrium when it exists ;
- $\pi_1(Y_1, Y_2 \mid \theta)$ and $\pi_2(Y_1, Y_2 \mid \theta)$ refer to the players discounted equilibrium payoffs given an initial state θ and an equilibrium (Y_1, Y_2).

Definition 1 *Markov equilibria*[5] *are such that :*

$$\pi_1(Y_1, Y_2 \mid \theta) = Max_{z_1^\theta}\left[G_1^\theta(z_1^\theta, y_2^\theta) + \delta\pi_1\left(Y_1, Y_2 \mid \theta^+(z_1^\theta, y_2^\theta)\right)\right]$$

$$\pi_2(Y_1, Y_2 \mid \theta) = Max_{z_2^\theta}\left[G_2^\theta(y_1^\theta, z_2^\theta) + \delta\pi_2\left(Y_1, Y_2 \mid \theta^+(y_1^\theta, z_2^\theta)\right)\right]$$

The first refinement to be considered consists in assuming that θ does not depend on τ ; then the strategies of the players depend only on who is the current incumbent. Provided that δ is sufficiently close to one, the following proposition holds (Maskin and Tirole, 1988)[6]:

Proposition 2 *Let y_1 and y_2 be the solution, proved to exist and to be unique, of the following system of two equations :*[7]
for all $i \neq j \in \{1, 2\}$ $\qquad -C_i(y_j) + \delta v_i(y_i)/(1-\delta) = 0$
then the following moves in $G(i)$ generate a Markov equilibrium :
player i plays $x_i = y_i$
player j plays out if and only if $x_i \leq y_i$.

This Markov equilibrium satisfies rent dissipation ($\lim_{\delta\to 1} v_i(y_i) = 0$) but not selection (suppose $i^1 = 2$, for all k and all δ close to one we have $i^k = 2$).

A second refinement of Markov equilibria is obtained as a direct extension of GPS for finite games. Denote Γ_k a finite entry game where k here refers to the maximal number of times the stage games $G(i)$ may be repeated. The notation $\Gamma_k(1)$ and $\Gamma_k(2)$ makes precise who is the initial incumbent in Γ_k. Observe that any such game $\Gamma_k(i^1)$ is a finite perfect information game so that it has a unique perfect equilibrium. Provided that δ is sufficiently close to one, the following proposition holds for the case of asymmetric games (Gromb et al., 1997).

[5] This system of equations is known as the Shapley recursive equations for recursive games (Shapley, 1953)

[6] Maskin and Tirole's analysis is carried out on a specific Cournot model but their Markov approach can be directly applied to the class of entry games defined in this paper. While they identify other Markow equilibria, the focus in their paper is on the one characterized by proposition 2.

[7] In the following three propositions, we should more precisely write $y_i(\delta)$, the dependency of y_i on the discount factor δ is left out to simplify the notations.

Proposition 3 *For all $\Delta f > 0$ (i.e. player 1 is more efficient than player 2) let an integer N and two sequences y_1^t and y_2^t for $\tau \in \{0, 1, 2...N\}$ be the solution, proved to exist and be unique, of the following system of equations :*

for all $i \neq j \in \{1, 2\}$

$-C_i(y_j^N) = 0$

$-C_i(y_j^\tau) + \sum_{\tau'=\tau+1}^{\tau'=N} \delta^{\tau'-\tau} v_i(y_i^{\tau'}) = 0$ *for* $\tau \in \{0, 1, 2...N-1\}$

and

$\sum_{\tau'=\tau}^{\tau'=N} \delta^{\tau'-\tau} v_i(y_i^{\tau'}) \geq 0$ *for* $\tau \in \{1, 2...N\}$

but

$\sum_{\tau=0}^{\tau=N} \delta^{\tau} v_i(y_i^{\tau}) < 0.$

Then the unique perfect equilibrium in $\Gamma_k(2)$ is such that : [8]

for $k \leq N$ player i plays $x_i = y_i^{N-k+1}$ and player j plays out if and only if $x_i \leq y_i^{N-k+1}$

but for $k = N + 1$ player 2 plays out, then player 1 plays $x_1 = y_1^0$ and player 2 plays out if and only if $x_1 \leq y_1^0$.

This proposition allows to construct the equilibrium of any game $\Gamma_k(i^1)$ whatever its length $k > N + 1$ using a backward induction argument. Indeed, since the initial equilibrium move for player 2 in $\Gamma_{N+1}(2)$ is *out* then in terms of payoffs, we have $\Gamma_{N+1}(2) \equiv \Gamma_{N+1}(1)$ with a zero equilibrium payoff for player 2, so that, $\Gamma_{N+2}(2) \equiv \Gamma_1(2)$ while $\Gamma_{N+2}(1) \equiv \Gamma_1(1)+\Gamma_{N+1}(1)$. Let $k = (N+1)q+r$ with $r < N + 1$, then $\Gamma_k(i^1)$ will be played as a game $\Gamma_r(i^1)$ followed by q times $\Gamma_{N+1}(1)$ games. Since $\Gamma_k(i^1)$ is a perfect information game the unique equilibrium moves at a given stage do not depend on the past moves.

This implies that selection is satisfied in $\Gamma_k(2)$ provided that k is large enough. Rent dissipation is also satisfied since it can be shown that $\lim_{\Delta f \to 0} N(\Delta f) = \infty$ and that $\lim_{\Delta f \to 0} v_i(y_i^1(\Delta f)) = 0$.

It is possible to extend the equilibrium in $\Gamma_k(i^1)$ into a Markov equilibrium in $\Gamma_\infty(i^1)$. The direct way to proceed consists in assuming that the state variable θ depends on $\tau \bmod N + 1$. Let $\bar{\tau} = \tau \bmod N + 1$. At $\bar{\tau}$ the players will play $\Gamma_{N+1-\bar{\tau}}$. If $\bar{\tau} = 0$, player 2's first move is to move *out*.

This refinement of Markov equilibria satisfies both rent dissipation and selection but is not appealing. Why would the two players "agree" on the sequencing of $\Gamma_\infty(i)$ into finite games, the length of which should precisely be $N+1$? After many stages why should the origin of the game still matter ? This is certainly not what economists mean by non collusion in an infinite horizon.

The third Markov equilibrium refinement to be further considered is constructed along the following ideas.[9] Take as a working assumption that one of the player cannot remain as an incumbent indefinitely. Call S this player (for *short term* player) and denote L the other one (for *long term* player). Denote by n the number of stages that S may stay as an incumbent. Four conditions come to mind.

[8] For simplicity only the equilibrium path is described, the best response after non equilibrium moves is easily completed.

[9] These ideas go back to Louvert, 1998.

Condition 1 (*backward induction*) : as long as S 's moves are reasonable, L should be patient enough to wait until S moves *out*;

Condition 2 (*recursivity*) : L should permanently deter S from moving *in* i.e., the entry cost generated by L 's move as an incumbent should be equal to the total rent S can get through his one shot hit and run strategy;

Condition 3 (*self-enforceability*) : at the end of his stay S should indeed find that his best interest is to move *out*;

Condition 4 (*optimality of the duration n of the hit and run strategy*) : n should be selected to S 's best interest so as to maximize the total rent of his one shot strategy.

It is believed that these conditions capture well the underlying ideas of the hit and run strategy as described in Baumol et al. (1982). The only difference we can think of is that in our approach L is not constrained to stand still while S gets his profit, but it is in his best interest to do so. Our approach should gain more acceptance among economists.

These ideas are now formalized. It will be convenient to keep condition (4) aside and define our Markov solution concept for any exogenously given n, then look for the best n from S 's point of view.

The new state variable and the associated transition rules are now defined. Let

- t be a relative time variable with $t \in \{0, 1, 2...n\}$;
- θ' be the new state variable, the possible values of which are $\theta' = L$ or $\theta' = (S, t)$ with $t \in \{0, 1, 2...n\}$; θ' specifies the incumbent in the current stage game $G(i)$ and the relative time variable t if this incumbent is S ;
- again, $(z_1^{\theta'}, z_2^{\theta'})$ stand for the decisions in $G(i)$ given θ' and (Z_1, Z_2) stand for the strategies, and $G_1^{\theta'}(z_1^{\theta'}, z_2^{\theta'})$ and $G_2^{\theta'}(z_1^{\theta'}, z_2^{\theta'})$ stand for the respective stage payoffs in $G(i)$ given θ' and $(z_1^{\theta'}, z_2^{\theta'})$;
- i^+ be the next stage incumbent given θ', $(z_1^{\theta'}, z_2^{\theta'})$ and the rules of Γ_∞ ;
- the transition rules to get θ'^+, the next stage state variable given θ' and i^+, be such that :
  - if $\theta' = L$ and $i^+ = L$ then $\theta'^+ = L$
  - if $\theta' = L$ and $i^+ = S$ then $\theta'^+ = (S, 1)$
  - if $\theta' = (S, t)$ and $i^+ = L$ then $\theta'^+ = L$
  - if $\theta' = (S, t)$ and $i^+ = S$ then $\theta'^+ = (S, t + 1 \bmod n + 1)$.

This definition of the state variable is such that when L is the (unchallenged) incumbent tomorrow is like today but, when S is, both players know that S 's incumbency time will eventually come to an end.

For a given value of the parameter n, the following proposition gives sufficient conditions to obtain such a Markov equilibrium in Γ_∞. Existence and uniqueness will be addressed later. In the remaining part of this paper, when it exists, such an equilibrium will be denoted a SME of Γ_∞ (for *selected Markov equilibrium).*

Proposition 4 *Let y_L and y_S^t for $t \in \{0, 1.2...n\}$ be a solution (if it exists) of the following system of equations :*

$-C_L(y_S^t) + v_L(y_L)\delta(1-\delta^{n-t})/(1-\delta) = 0 \qquad$ *for* $t \in \{0, 1, 2...n\}$

$-C_S(y_L) + \Sigma_{t=1}^{t=n}\delta^t v_S(y_S^t) = 0$

and

$v_L(y_L) \geq 0$

$\sum_{t'=t}^{t'=n} \delta^{t'-t} v_S(y_S^{t'}) \geq 0 \qquad$ *for* $t \in \{1, 2...n\}$

but

$\sum_{t'=0}^{t'=n} \delta^{t'} v_S(y_S^{t'}) < 0$

Then the following moves in Γ_∞ are a Markov equilibrium :

- if $\theta' = L$ player L plays $x_L = y_L$ and player S plays out iff $x_L \leq y_L$

(in case player L moves out then player S also moves out[10]);

- if $\theta' = (S, 0)$ player S plays out then player L plays $x_L = y_L$ and player S plays out iff $x_L \leq y_L$

(in case player S plays x_S player L moves out iff $x_S \leq y_S^0$);

- if $\theta' = (S, t)$ with $t \neq 0$ player S plays $x_S = y_S^t$ and player L plays out iff $x_S \leq y_S^t$

(in case player S moves out then player L plays $x_L = y_L$ and player S plays out iff $x_L \leq y_L$).

Proof. Let (Z_L, Z_S) be the Markov strategies as defined in the proposition. The associated equilibrium paths are as follows :

- if $\theta' = L$ the path is : $((y_L, out), (y_L, out), ...)$;
- if $\theta' = (S, 0)$ the path is : $((out, y_L, out), (y_L, out), (y_L, out), ...)$;
- if $\theta' = (S, t)$ with $t \neq 0$ the path is : $((y_S^t, out), (y_S^{t+1}, out), ..., (y_S^n, out), (out, y_L, out), (y_L, out), (y_L, out), ...)$;

The corresponding non negative discounted payoffs are easily computed as follows :

$\pi_L(Z_L, Z_S \mid \theta' = L) = \pi_L(Z_L, Z_S \mid \theta' = (S, 0)) = v_L(y_L)/(1-\delta)$

$\pi_L(Z_L, Z_S \mid \theta' = (S, t)) = \delta^{n+1-t}\pi_L(Z_L, Z_S \mid \theta' = L)$

$= v_L(y_L)\delta^{n+1-t}/(1-\delta)$ for $t \in \{1, 2...n\}$

$\pi_S(Z_L, Z_S \mid \theta' = (S, t)) = \Sigma_{t'=t}^{t'=n}\delta^{t'-t}v_S(y_S^{t'}) \qquad$ for $t \in \{1, 2...n\}$

$\pi_S(Z_L, Z_S \mid \theta' = (S, 0)) = \pi_S(Z_L, Z_S \mid \theta' = L) = 0$

Part I : Z_L is a best response to Z_S.

Let $\theta' = L$

- if player L selects $x_L \leq y_L$ player S moves *out* so that $\theta'^{+} = L$, then player L discounted payoff is $v_L(x_L) + \delta\pi_L(Z_L, Z_S \mid \theta' = L)$

which is maximized at $x_L = y_L$ to be $\pi_L(Z_L, Z_S \mid \theta' = L)$;

- if player L selects $x_L > y_L$ player S moves *in* so that $\theta'^{+1} = (S, 1)$, then player L discounted payoff is $d_L(x_L) + \delta\pi_L(Z_L, Z_S \mid \theta' = (S, 1)) = d_L(x_L) + \delta^{n+1}\pi_L(Z_L, Z_S \mid \theta' = L) < \pi_L(Z_L, Z_S \mid \theta' = L)$ since $d_L(x_L) \leq 0$ and $\delta < 1$;

- if player L moves *out* player S also moves *out* and the game ends with zero payoff for player L;

[10] To have a perfect Markov equilibrium, it may be necessary to let player S have a "free ride" if player L makes the mistake of moving *out*. This detail is left out for simplicity.

- this shows that $x_L = y_L$ is a best response.

Let $\theta' = (S, t)$ for $t \in \{0, 1, 2...n\}$

- if player S selects $x_S \leq y_S^t$ then by moving *out* player L gets $\pi_L(Z_L, Z_S \mid \theta' = (S,t)) = \delta^{n+1-t}\pi_L(Z_L, Z_S \mid \theta' = L)$ while by moving *in* he gets $-C_L(x_S) + \delta\pi_L(Z_L, Z_S \mid \theta' = L)$

since $-C_L(x_S) \leq -C_L(y_S^t)$ we have :

$-C_L(x_S) + \delta\pi_L(Z_L, Z_S \mid \theta' = L) \leq -C_L(y_S^t) + \delta\pi_L(Z_L, Z_S \mid \theta' = L)$

by construction $-C_L(y_S^t) = -\Sigma_{t=1}^{t=n}\delta^t v_S(y_S^t)$ while $\pi_L(Z_L, Z_S \mid \theta' = L) = \delta v_L(y_L)/(1-\delta)$ so that

$-C_L(y_S^t) + \delta\pi_L(Z_L, Z_S \mid \theta' = L) = \delta^{n+1-t}\pi_L(Z_L, Z_S \mid \theta' = L)$

hence player L is indifferent and to move *out* is a best response;

- if player S select $x_S > y_S^t$ the same argument but with $-C_L(x_S) > -C_L(y_S^t)$ shows that player L best response is to move *in*;

- if player S moves *out* then, using the argument given for $\theta' = L$ it is clear that player L should play $x_L = y_L$.

Part II : Z_S is a best response to Z_L.

Let $\theta' = L$

- if player L selects $x_L \leq y_L$ by moving *out* player S gets $\pi_S(Z_L, Z_S \mid \theta' = L) = 0$, while if he moves *in* he gets $-C_S(x_L) + \delta\pi_S(Z_L, Z_S \mid \theta' = (S,1))$. Since $-C_S(x_L) \leq -C_S(y_L)$ we have

$-C_S(x_L) + \delta\pi_S(Z_L, Z_S \mid \theta' = (S,1)) \leq -C_S(y_L) + \delta\pi_S(Z_L, Z_S \mid \theta' = (S,1))$

by construction $-C_S(y_L) =$ -$\Sigma_{t=1}^{t=n}\delta^t v_S(y_S^t)$ while $\pi_S(Z_L, Z_S \mid \theta' = (S,1)) = \Sigma_{t'=1}^{t'=n}\delta^{t'-1}v_S(y_S^{t'})$

so that

$-C_S(y_L) + \delta\pi_S(Z_L, Z_S \mid \theta' = (S,1)) = 0$

hence player S is indifferent and to move *out* is a best response;

- if player L select $x_L > y_L$ the same argument but with $-C_S(x_L) > -C_S(y_L)$ shows that player S best response is to move *in*;

- if player L selects *out* and player S moves *out* the game ends and player S gets 0, while if he selects $y_S^0 \leq x_S$, by the Markov assumption player L moves *in* and player S discounted payoff is $d_S(x_S) + 0 \leq 0$, finally if player S selects $x_S < y_S^0$ player L will move *out* but in that case player S discounted payoff is strictly negative since $\Sigma_{t'=0}^{t'=n}\delta^{t'}v_S(y_S^{t'}) < 0$, player S best response is to move *out*.

Let $\theta' = (S, t)$ for $t \in \{0, 1, 2...n\}$

- if player S selects moving *out* or $x_S > y_S^t$ (in which case player L moves *in*) he gets 0;

- if player S selects $x_S \leq y_S^t$ player L plays *out* and player S discounted payoff is $v_S(x_S) + \delta\pi_S(Z_L, Z_S \mid \theta' = (S, t+1 \text{ modulo } n+1))$

which is maximized at $x_S = y_S^t$ to be $\sum_{t'=t}^{t'=n}\delta^{t'-t}v_S(y_S^{t'})$; by construction this expression is non negative if t is different from 0, and non positive if $t = 0$, consequently player S best response is $x_S = y_S^t$ for $t \in \{1, 2...n\}$ but *out* for $t = 0$. ■

In such an equilibrium player L is the only permanent incumbent and plays $x_L = y_L$. If he makes the mistake of playing $x_L > y_L$, then player S moves *in*, gets some money and moves *out* after n stages. Player L is patient enough to wait until player S moves *out* and it is player $S's$ best interest to move *out* after n stages of incumbency.

4 Properties of SME's for a given n

This section addresses the mathematical properties of SME's for a given n. Some preliminary comments are in order.

Our attention is focused on the case δ close to 1. It will be proved that if the system of equations given in Proposition 4 has a solution it is necessarily unique. For mathematical simplicity, the proof is made on the limit system when δ goes to 1. The same arguments would apply to the system when δ is close to 1. Then, because this system is continuous in δ its unique solution when δ goes to 1 converges to the unique solution of the limit system. It is the properties of this solution which we study further.

The limit system is easily derived, it is denoted Σ_1.

Lemma 5 *When δ goes to 1 the limit system Σ_1 of the system defined in Proposition 4 is such that :*

for $t \in \{0, 1, 2...n\}$ $\quad -C_L(y_S^t) + (n-t)v_L(y_L) = 0 \quad (1)$

$-C_S(y_L) + \Sigma_{t=1}^{t=n} v_S(y_S^t) = 0 \quad (2)$

and

$v_L(y_L) \geq 0 \quad (3)$

for $t \in \{1, 2...n\}$ $\quad \Sigma_{t'=t}^{t'=n} v_S(y_S^{t'}) \geq 0 \quad (4)$

but

$\Sigma_{t'=0}^{t'=n} v_S(y_S^{t'}) < 0 \quad (5)$

Some further notation is now introduced. Recall that n is a parameter that specifies how long player S may stay as an incumbent. To avoid ambiguity it may be useful to index by n the game as Γ_∞^n and a SME in Γ_∞^n as $y_L(n)$ and $y_S^t(n)$ for $t \in \{0, 1.2...n\}$. In Γ_∞^n , the *stage rent* for player L refers to $v_L(y_L(n))$ and the *total rent* for player S refers to $\Sigma_{t'=1}^{t'=n} v_S(y_S^{t'}(n))$.

We will prove that if there exist respective SME's for some n and some m with $m > n$ then player S total rent is higher in Γ_∞^m than in Γ_∞^n. A necessary and sufficient condition will be provided for the existence of a SME in Γ_∞^n for arbitrarily large values of n. It will be shown that this existence implies that the stage rent of player L goes to zero as n goes to infinity.

Theorem 6 *: Γ_∞^n admits at most one SME for large enough n.*

Proof. [11] The proof goes as follows. Firstly prove that conditions (1-2-3-4) of Σ_1 have a unique solution. Secondly, check whether condition (5) is satisfied : if it is, the unique solution of Σ_1 is obtained, if it is not Σ_1 has no solution.

To prove the first part, for all $x_L \in \left[v_L^{-1}(0), C_S^{-1}(0)\right]$, define the function $W(x_L) = C_S(x_L) - \Sigma_1^n v_S(x_S^t)$ in which the sequence (x_S^t) is derived from x_L through (1) that is,

$$-C_L(x_S^t) + (n-t)v_L(x_L) = 0 \qquad \text{for } t \in \{0, 1, 2...n\}$$

then, show that $W(x_L)$ is negative (step 1) then positive (step 2) and that its derivashort termtive is strictly positive (step 3) so that there is a unique solution to the equation $W(x_L) = 0$. Indeed :

Step 1 : if $x_L = v_L^{-1}(0)$ then $W(x_L) < 0$

In that case $x_S^t = C_L^{-1}(0)$ for all t so that $W\left(v_L^{-1}(0)\right) = C_S\left(v_L^{-1}(0)\right) - nv_S\left(C_L^{-1}(0)\right)$

and by assumption $v_S\left(C_L^{-1}(0)\right) > 0$ so that for n large enough $W\left(v_L^{-1}(0)\right) < 0$.

Step 2 : if $x_L = C_S^{-1}(0)$ then $W(x_L) > 0$

Since C_L is strictly decreasing, the sequence (x_S^t) is a strictly increasing sequence bounded by $C_L^{-1}(0)$. Since v_S is strictly increasing this implies that $\Sigma_1^n v_S(x_S^t)$ is certainly negative for n large enough so that $W\left(C_S^{-1}(0)\right) = -\Sigma_1^n v_S(x_S^t)$ is certainly positive.

Step 3 : $\frac{dW}{dx_L} > 0$

We have

$$\frac{dW}{dx_L} = \frac{dC_S}{dx_L} - \Sigma_{t=1}^{t=n}\left(\frac{dv_S}{dx_S^t} \cdot \frac{dx_S^t}{dx_L}\right)$$

Using (1) we get :

$$\frac{dx_S^t}{dx_L} = (n-t)\frac{dv_L}{dx_L} / \frac{dC_L}{dx_S^t}$$

By substitution it follows that :

$$\frac{dW}{dx_L} = \frac{dC_S}{dx_L} - \frac{dv_L}{dx_L}\Sigma_{t=1}^{t=n}\left((n-t)\frac{dv_S}{dx_S^t} / \frac{dC_L}{dx_S^t}\right)$$

By assumption $-\frac{dv_S}{dx_S^t} / \frac{dC_L}{dx_S^t}$ is uniformly bounded away from zero by ε so that

$$\frac{dW}{dx_L} \geq \frac{dC_S}{dx_L} + \frac{dv_L}{dx_L}\frac{n(n-1)}{2}\varepsilon$$

[11] I am indebted to Rida Laraki for providing the argument for this proof.

Since $\dfrac{dv_L}{dx_L}$ is bounded away from zero and since $\dfrac{dC_S}{dx_L}$ is bounded away from $-\infty$ we certainly have $\dfrac{dW}{dx_L} > 0$ for n large enough.

Hence for a given n large enough there is a unique solution to $W(x_L) = 0$ that is to (2). This solution is in $\left]v_L^{-1}(0), C_S^{-1}(0)\right[$ so that (3) is also satisfied. Denote $\hat{y}_L$ this solution and $(\hat{y}_S^t)$ for $t \in \{0, 1, 2...n\}$ the associated sequence obtained through (1). Observe that (4) is satisfied as well : since v_S is increasing the function $\Sigma_{t'=t}^{t'=n} v_S(\hat{y}_S^{t'})$ is bell shaped with respect to t so for all t we have :

$$\Sigma_{t'=t}^{t'=n} v_S(\hat{y}_S^{t'}) \geq Min(\Sigma_1^n v_S(\hat{y}_S^t), v_S(\hat{y}_S^n)) = Min(C_S(\hat{y}_L), v(C_S^{-1}(0)) > 0$$

because $\hat{y}_L < C_S^{-1}(0)$ implies $C_S(\hat{y}_L) > 0$ while $\hat{y}_S^n = v(C_S^{-1}(0) > 0$ by construction.

It is now a simple matter to check whether (5) holds or not. If it does a complete solution to Σ_1 is obtained, if it does not there cannot be a solution for that value of n since conditions (1) through (4) have a unique solution.

Theorem 7 *If there exists a SME respectively in Γ_∞^m and in Γ_∞^n with $m > n$ then $y_L(m) \leq y_L(n)$ so that player S total rent is higher in Γ_∞^m than in Γ_∞^n.*

■

Proof. Suppose $y_L(m) > y_L(n)$ then $v_L(y_L(m)) > v_L(y_L(n))$. Since C_L is strictly decreasing this implies for all $t \in \{0, 1, 2...n\}$:

$$y_S^{m-t}(m) < y_S^{n-t}(n)$$

so that

$$\Sigma_{t=n}^{t=0} v_S(y_S^{m-t}(m)) < \Sigma_{t=n}^{t=0} v_S(y_S^{n-t}(n))$$

For $t \in \{n+1, ..m\}$ we still have $y_S^{m-t}(m) < y_S^0(n)$ and, because of (5) we also certainly have $v_S(y_S^0(n)) < 0$ then

$$\Sigma_{t=m-1}^{t=0} v_S(y_S^{m-t}(m)) \leq \Sigma_{t=n}^{t=0} v_S(y_S^{m-t}(m))$$

Then

$$\Sigma_{t=m-1}^{t=0} v_S(y_S^{m-t}(m)) < \Sigma_{t=n}^{t=0} v_S(y_S^{n-t}(n))$$

By construction the left hand side should be greater or equal to zero while the right hand side should be strictly negative thus a contradiction. ■

Theorem 8 *Suppose there is a SME in Γ_∞^n for all values of n then :*

$$\lim_{n\to\infty} v_L(y_L(n)) = 0$$

$$\lim_{n\to\infty} \Sigma_1^n v_S(y_S^t(n)) = C_S(v_L^{-1}(0))$$

$$\lim_{n\to\infty} y_S^1(n) = x^*$$

in which x^* *is uniquely defined as :*

$$\int_{x^*}^{C_L^{-1}(0)} v_S(x)\frac{dC_L}{dx}(x)dx = 0$$

Proof. Parts 1 and 2 of the theorem are easily proved.

Indeed, suppose $v_L(y_L(n)) \geq \varepsilon > 0$ for all n, then using (1) the sequence $(y_S^t(n))$ is a strictly increasing sequence defined backwards from $y_S^n(n) = C_L^{-1}(0)$ so (4) cannot hold for large n hence $\lim_{n\to\infty} v_L(y_L(n)) = 0$. Then Part 2 follows from (2).

Part 3.

First of all, given that $\frac{dC_L}{dx}$ is bounded away from infinity and from zero and that $v_S(x)$ is bounded away from zero, there exists a unique $x^* < C_L^{-1}(0)$ such that

$$\int_{x^*}^{C_L^{-1}(0)} v_S(x)\frac{dC_L}{dx}(x)dx = 0$$

For all $x \leq C_L^{-1}(0)$ define $F(x) = \int_x^{C_L^{-1}(0)} v_S(u)\frac{dC_L}{du}(u)du$, the function F is such that $F(x) > 0$ iff $x < x^*$.

We now show the convergence of $y_S^1(n)$ to x^*.

Using (1) and (2) we get :

$$\begin{aligned} C_S(y_L(n))v_L(y_L(n)) &= \Sigma_{t=1}^{t=n} v_S(y_S^t(n))v_L(y_L(n)) \\ &= \Sigma_{t=1}^{t=n} v_S(y_S^t(n))\left[C_L\left(y_S^{t-1}(n)\right) - C_L\left(\ y_S^t(n)\right)\right] \end{aligned}$$

When $v_L(y_L(n)$ is small this non negative expression is close to $F(y_S^1(n))$.[12] This proves that $y_S^1(n)$ cannot be far below x^*. Using (2) and (5) for the two sequences n and $n+1$, it is clear that $y_S^1(n+1)$ and $y_S^1(n)$ cannot be far appart either. More precisely :

$$\left|y_S^1(n+1) - y_S^1(n)\right| \leq -Min(\frac{dC_L}{dx}(y_S^1(n)), \frac{dC_L}{dx}(y_S^0(n)))v_L(y_L(n))$$

Since $\frac{dC_L}{dx}$ is bounded away from infinity, $\lim_{n\to\infty}\left|y_S^1(n+1) - y_S^1(n)\right| = 0$, this is enough to prove that $y_S^1(n)$ converges to some limit and this limit can only

[12] **Proof.** Make the change of variable from x_S to $u = C_L(x_S)$. As t goes from 1 to n, x_S increases from $y_S^1(n)$ to $y_S^n(n)$ and u from $u^1(n) = C_L\left(y_S^1(n)\right)$ to $u^n(n) = C_L\left(y_S^n(n)\right) = 0$ but $u^{t-1}(n) - u^t(n)$ remains t independant and equals $v_L(y_L(n))$, let $\Delta u(n) = v_L(y_L(n))$.

We may then write

$$v_L(y_L(n))\Sigma_{t=1}^{t=n} v_S(y_S^t(n)) = \Sigma_{t=1}^{t=n} v_S(C_L^{-1}(u^t(n)))\Delta u(n)$$

For large values of n we have

$$\Sigma_{t=1}^{t=n} v_S(C_L^{-1}(u^t(n)))\Delta u(n) \approx \int_{u^1(n)}^0 v_S(C_L^{-1}(u))du = \int_{y_S^1(n)}^{C_L^{-1}(0)} v_S(x)\frac{dC_L}{dx}(x)dx.$$

■

be x^* since $\lim_{n\to\infty} C_S(y_L(n))v_L(y_L(n)) = \lim_{n\to\infty} C_S(y_L(n)) \lim_{n\to\infty} v_L(y_L(n)) = C_S(0).0 = 0$. ■

Corollary 9 *If there is a SME in* Γ_∞^n *for arbitrarily large values of* n *it is necessary that :*

$$v_S(x^*) + C_S(v_L^{-1}(0)) \leq 0$$

and if

$$v_S(x^*) + C_S(v_L^{-1}(0)) < 0$$

there is a SME for arbitrarily large values of n *.*

Proof. Consider the first part. Using (5), for all n we have $v_S(y_S^0(n)) + C_S(y_L(n)) < 0$ so that at the limit we certainly have $v_S(x^*) + C_S(v_L^{-1}(0)) \leq 0$.

As for the second part the above theorem in fact proves that in the construction of theorem 6 for n large enough $\hat{y}_S^1$ converges to x^* as $\hat{y}_L$ goes to $v_L^{-1}(0)$; since $v_S(x^*) + C_S(v_L^{-1}(0)) < 0$ it must be that (5) will be satisfied and a SME is obtained. ■

5 Economic analysis of SME's

Recall the main results. If Γ_∞^n has a SME it is unique. If both Γ_∞^n and Γ_∞^m have a SME with $m > n$ player S 's total rent is higher in Γ_∞^m than in Γ_∞^n. The application of condition (4) is straighforward. Player S should select as the duration for his hit and run strategy the largest integer for which Γ_∞^n has a SME or any arbitrarily large integer if Γ_∞^n has a SME for arbitrarily large n. Moreover if there is a SME in Γ_∞^n for arbitrarily large values of n player L 's stage rent goes to 0 as n goes to infinity.

In the remaining part of this paper n is selected according to condition (4).This defines the solution of the game Γ_∞, the economic properties of which we want to now discuss.

We now come back to symmetric and asymmetric entry games. Recall that a symmetric game is such that $v_1 = v_2$ and $C_1 = C_2$ and that an asymmetric game may be derived from a symmetric one such that $v_1 = v_2 + \Delta f$, in which Δf is a constant, and $C_1 = C_2$. If $\Delta f \geq 0$ player 1 is said to be *strong* and player 2 *weak,* and vice et versa.

Consider first the issue of rent dissipation in symmetric games. Theorem 8 proves that if there is rent dissipation with SME in Γ_∞ , this solution is consistent with both HT and GPS : the relative time horizon for S can be extended to infinity and the stage rent $v(y_L)$ goes to zero while the total rent $\Sigma_1^n v(y_S^t(n))$ goes to $C(v^{-1}(0))$. Yet, the major difference with SME lies in corrollary 9 : there may not always exist an equilibrium in Γ_∞ for arbitrarily large values of n. With SME rent dissipation is no longer a generic property but need be studied case by case.

This result is contrary to the long tradition of industrial economics. The corresponding analysis of potential competition should be revisited : the issue of short time commitment should decrease and more attention should be given to other structural characteristics of the situation.

We proceed as follows. Firstly, a taxonomy of competitive situations is used to interpret the results.[13] Secondly, examples of entry games are introduced to show that all cases in the typology may indeed be related to meaningful economic situations.

5.1 A taxonomy of competitive situations and some preliminary properties

Definition 10 *A competitive situation is said to be one of :*

under-competition *: if $L = weak$ and $S = strong$ there exists n^* such that Γ_∞^n has no SME for all $n > n^*$*

(a less efficient incumbent can deter entry forever and make stationary positive profits);

selection *: if $L = weak$ and $S = strong$ there always exist SME's for all n large enough*

but if $L = strong$ and $S = weak$ there exists n^* such that Γ_∞^n has no SME for all $n > n^*$

(a less efficient incumbent is not able to deter entry forever and make stationary positive profits, but a more efficient incumbent may);

excess-competition *: whether $L = weak$ or $L = strong$ there always exist SME's for all n large enough*

(if an incumbent were to choose to deter entry forever, it would have to dissipate all of its profits).

Only *excess-competition* occurs with HT. Only *excess-competition* or *selection* may occur with GPS. All three cases may occur with SME.

Under SME two simple results can be proved to relate a given competitive situation to one of the three cases identified. Firstly consider the special case in which v and C are linear functions.

Proposition 11 *If v and C are linear functions then rent dissipation prevails if $v_S(C_L^{-1}(0)) > C_S(v_L^{-1}(0))$ while no rent dissipation prevails if $v_S(C_L^{-1}(0)) < C_S(v_L^{-1}(0))$.*

Proof. The equation that defines x^* is $\int_{x^*}^{C^{-1}(0)} v(x)\frac{dC}{dx}(x)dx = 0$. Because of the linearity of the v and C functions, x^* is the symmetric of $C^{-1}(0)$ with respect to $v_S^{-1}(0)$ so that $v_S(x^*) = -v_S(C_L^{-1}(0))$. Then the result follows from corrolary 9. ■

This proposition is quite useful to classify situations in this special case. Qualitatively speaking, it says that the entry cost should be high enough with respect to the monopoly profit for rent dissipation to occur. This simple interpretation may then be related to structural characteristics of the economic situation at hand.

[13] This taxonomy greatly benefited from the comments of an anonymous referee.

Secondly, a general comparative statics result always (not only in the linear case) holds. If there is *excess-competition* in a symmetric game then in the associated asymmetric game in which the long-term player is more and more efficient (increasing Δf), at some point his rent will become strictly positive. Conversely, if there is *under-competition* in a symmetric game then in the associated asymmetric game in which the long-term player is less and less efficient (decreasing Δf), at some point his rent will be fully dissipated. This means that the comparative statics of SME with respect to the relative efficiency of the two players goes in the expected direction contrarily to HT.

Proposition 12 *Take $v_L = v + \Delta f$, $v_S = v$ and $C_L = C_S = C$. Denote $H(\Delta f) = v_S(x^*) + C_S(v_L^{-1}(0))$ then :*

- if $H(0) < 0$ there exists some $\Delta F > 0$ such that for all $\Delta f > \Delta F$ we have $H(\Delta f) > 0$;

- if $H(0) > 0$ then there exists some $\Delta F' < 0$ such that for all $\Delta f < \Delta F'$ we have $H(\Delta f) < 0$.

Proof. Consider the first statement, in $H(\Delta f) = v_S(x^*) + C_S(v_L^{-1}(0))$ observe that the first term does not depend on Δf since x^* is defined from the equation $\int_{x^*}^{C_L^{-1}(0)} v_S(x) \frac{dC_L}{dx}(x)dx = 0$ is independant of Δf. As for the second term we certainly have $v_L^{-1}(0)$ strictly decreasing in Δf so that, since C_S is a decreasing function and since we are assuming that its derivative is uniformly bounded away from zero this provides the result. Reversing the argument we obtain the second statement. ■

5.2 Economic models of potential competition

Two economic models of entry are analysed. In both models *under-competition* prevails when competition is tough in case of entry.

5.3 Competition through durable capital

Consider the profit and entry cost functions defined as follows :

$$v(x) = \pi^m x - f$$

$$C(x) = (\pi^m - \pi^d)(H - x)$$

Define symmetric and asymmetric games from these v and C functions.

This model may be interpreted as a reformulation[14] of the Eaton and Lipsey model (1980). There firms compete through plants which become obsolete after H units of time. The strategic decision for the incumbent consists in early replacement of its own plant. By doing so the incumbent prevent an entrant

[14] See Gromb et al (1997) for details on this reformulation.

to preempt its market. Each time it sets a new plant, a firm incurs a fixed cost f. Operating costs are assumed to be zero. If both firms are in the market simultaneously, their duopoly flow of revenue is π^d per unit of time. If only one is present, its flow of revenue is π^m. For consistency we certainly have $\pi^m > 2\pi^d$ and $\pi^m H \geq f$ so that $0 \leq v^{-1}(0) = f/\pi^m \leq H$. For simplicity also assume that $\pi^d H \leq f$. Note that Δf may be interpreted as the difference in the fixed costs of the two firms.

In our framework a new stage game occurs each time a plant is set up, then $G(i)$ is such that :

- player i is the incumbent, player i may either select a move x_i with $0 \leq x_i \leq H = C^{-1}(0)$, i.e. he sets at what time he will replace his plant, or if he will move *out* that is,. he will decide not to set a new plant ;

- if player i selects x_i then player j may select *in*, i.e. player j will preempt player i by setting up his own plant exactly before the selected time x_i (with respective payoffs for i and j : $v_i(x_i), -C_j(x_i)$; indeed player i 's plant will have operated only for x_i units of time while player j 's plant will be operating under a duopoly situation for the remaining life time of player i 's plant that is $H - x_i$ units of time ; this entry cost is the opportunity cost that need be substracted from player j 's next stage profit when he will be playing as the incumbent); or player j may select *out* (with self explanatory payoffs $v_i(x_i), 0)$;

- etc.

Beware that the plant life time goes with the moves within a stage game $G(i)$ while the discrete time in Γ_∞ goes with the setting of new plants. Note that the exit payoff is $-d_i(x_i) = v_i(x_i)$ so that it is non positive for all x_i since $\pi^d H \leq f$.

Corollary 13 *The game of competition through durable capital has a SME for arbitrary large values of n (i.e. there is rent dissipation for the long-term incumbent) :*

for $L = strong$ and $S = weak$, iff $\quad \Delta f < (H\pi^m - f)\pi^d/(\pi^m - \pi^d)$;

for $L = weak$ and $S = strong$, iff $\quad -(H\pi^m - f)\pi^d/\pi^m < \Delta f$.

Proof. Consider the first case that is, $v_L(x_L) = \pi^m x_L - f + \Delta f$ and $v_S(x_S) = \pi^m x_S - f$ and $C_1 = C_2 = C(x) = (\pi^m - \pi^d)(H - x)$.

Proposition 11 may be used. We have $v_L^{-1}(0) = (f - \Delta f)/\ \pi^m$ and $C^{-1}(0) = H$.

Then $-v_S(C^{-1}(0)) + C(v_L^{-1}(0)) = f - H\pi^m + (\pi^m - \pi^d)(H\pi^m - f)/\pi^m + \Delta f(\pi^m - \pi^d)/\pi^m = -\pi^d(H\pi^m - f)/\pi^m + \Delta f(\pi^m - \pi^d)/\pi^m$

which is indeed negative iff $\Delta f < (H\pi^m - f)\pi^d/(\pi^m - \pi^d)$.

The second case that is, $v_L = \pi^m x - f$ and $v_S = \pi^m x_i - f + \Delta f$ and C unchanged, is obtained through similar calculations. ■

Fix π^m, H and f and let π^d and Δf vary. Depending on the values of $(\pi^d, \Delta f)$ the type of competition lies in different zones that may be depicted in a diagram (see figure 1). This illustrates the taxonomy. The corresponding zones are interpreted as follows:

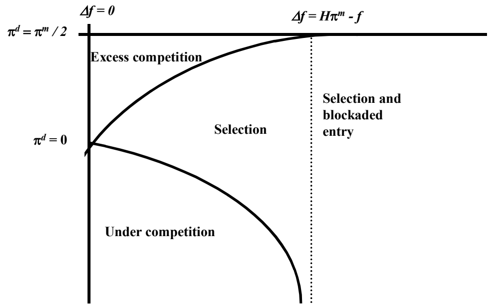

Figure 1: The taxonomy for the Eaton and Lipsey model

- *selection* is obtained whatever π_d as long as Δf is large enough ; observe that for $\Delta f > H\pi^m - f$ the least efficient firm is barred from the market anyway (blockaded entry);

- *excess-competition* prevails for $\pi_d > 0$ and small enough Δf ;

- *under-competition* prevails for $\pi_d < 0$ and small enough Δf.

Rent dissipation does not always prevail. The toughness of competition in the case of entry is the structural factor that matters : it directly determines the ratio of the entry cost relative to the monopoly profit. With a negative duopoly revenue the incumbent is able to secure a positive stage rent.

With GPS, selection always occurs as soon as $\Delta f > 0$. With SME it only occurs with Δf large enough (except at the singular point $\pi^d = 0$).

These results seem to make economic sense.

5.4 Competition through short run price commitments

This form of competition is illustrative of the very notion of contestability (Baumol et al 1982). It has been formalized as an entry game in Ponssard (1991). Generally speaking, the stage game may be described as follows :

- firm i is the incumbent and sets a price p_i ;

- firm j decides to enter or not, if it does enter it sets a price $p_j = p_i - \varepsilon$ in which ε is a strictly positive function which may depend on p_i according to the specific formulation of the entry game under study;[15]

[15] Ordinarily the revenue function $R(p)$ has a maximum (at the unconstrained monopoly price); we assume that ε is small enough so that $C^{-1}(0)$ be less than this maximum so that v is indeed increasing on the relevant range of analysis. Still our construction may be extended if this were not the case.

- if firm j did not enter, firm i payoff is $R(p_i) - f$ in which R is the monopoly revenue function with $R' > 0$ and $R'' < 0$ and f a fixed cost incurred in case of production ;

- if firm j did enter, firm j payoff is $R(p_i - \varepsilon) - f$.

The symmetric entry game is thus defined with

$$\begin{aligned} v(p) &= R(p) - f \\ C(p) &= -(R(p - \varepsilon) - f\ ) \end{aligned}$$

In this framework the notion of toughness of price competition can be captured by letting ε go to zero. In the symmetric case, with HT or GPS, such is a situation is as usual one of *excess-competition* : the incumbent price is forced to average cost i.e. to the price p_{ac} such that $R(p_{ac}) - f = 0$.

Ordinarily, such is not the case with SME : this situation is more typically one of *under-competition.* Consider the simple case of constant switching cost that is, $\varepsilon = cst$.

Proposition 14 *In an entry game with short run price commitments and constant switching costs, if this switching cost is close to zero, the situation is one of under-competition.*

Proof. Since ε is close to zero we may use linear approximations of the v and C functions around the value $p = p_{ac}$. Denote by p_L the value of p such that $C(p_L) = 0$. We certainly have p_L close to p_{ac} . According to proposition 11 the ratio $v(p_L)/C(p_{ac})$ relative to 1 characterizes the situation. We may write $v(p_L)/C(p_{ac}) = -(v(p_L) - v(p_{ac})/(p_L - p_{ac}))/(C(p_{ac}) - C(p_L))/(p_{ac} - p_L))$ so that $v(p_L)/C(p_{ac})$ is close to $-v'(p_{ac})/C'(p_{ac}) = R'(p_{ac})/R'(p_{ac} - \varepsilon) < 1$ since $R'' < 0$. ■

Consider now the specific price model in Ponssard (1991), there ε is not constant. Still the situation remains one of *under-competition.* Yet, whenever ε is not close zero, we may have *excess-competition.* Again, how tough price competition is is the key factor.

Let ω be a parameter to be interpreted as a price cross elasticity. The demand D_i to firm i is defined as follows :

$$\begin{array}{ll} 0 \le p_i \le p_j - (1 - p_j)/\omega & D_i(p_i, p_j) = D_i^m(p_i) = 1 - p_i \\ p_j - (1 - p_j)/\omega \le p_i & D_i(p_i, p_j) = D_i^d(p_i, p_j) \\ p_i \le p_j + (1 - p_j)/(1 + \omega) & = (1 + \omega)(1 - p_i + \omega(p_j - p_i))/(1 + 2\omega) \\ p_j + (1 - p_j)/(1 + \omega) \le p_i & D_i(p_i, p_j) = 0 \end{array}$$

The demand function is piece wise linear with a kink when the regime changes from a monopoly to a duopoly one. The higher ω the smaller the duopoly range in which $D_i = D_i^d$ and the tougher the price competition (ε close to zero corresponds to large ω).

Firm i incurs a stage fixed cost f if and only if it produces. There is no variable cost. Consequently player i's profit as a function of the prices (p_i, p_j) writes :

if $D_i(p_i, p_j) > 0$ $\quad \pi_i(p_i, p_j) = D_i(p_i, p_j).p_i - f$
if $D_i(p_i, p_j) = 0$ $\quad \pi_i(p_i, p_j) = 0$

The stage game $G(i)$ is :

- player i is the incumbent, player i may either select a move p_i with $0 \leq p_i \leq 1$, or move *out* i.e., he proposes no price ;

- if player i selects p_i then player j may select *in*, in this case this means that he sets a price p_j which is his best response to p_i in terms of revenue that is, *without taking account of his fixed cost* ; denote $p_j(p_i)$ this best response, then the players respective payoffs are $(\pi_i(p_i, p_j(p_i)), \pi_j(p_i, p_j(p_i)))$; or he may select *out* with respective payoffs $(D_i^m(p_i).p_i - f, 0)$;

- etc.

The exit payoff which is $d_i(p_i) = \pi_i(p_i, p_j(p_i))$ is certainly non positive for all relevant p_i.

Summing up the monopoly profit function v is such that :

$$v(p) = D^m(p).p - f = (1 - p)p - f$$

and the entry cost C is defined as[16]

$$C(p) = -Max_{p_j}(D_j(p, p_j).p_j) + f$$

Our attention will be limited to symmetric games. We want to investigate whether there is *under* or *excess-competition* depending on the two parameters ω and f. This depends on the sign of $v(p^*) + C(v^{-1}(0))$ in which p^* is defined by

$$\int_{p^*}^{C^{-1}(0)} v(p)\frac{dC}{dp}(p)dp = 0$$

Observe that the function C is not differentiable when the regime goes from the monopoly one to the duopoly one, but this is for only one point and has no bearing for the integral. Still we cannot obtain p^* from an analytical formula and a numerical analysis has to be made to characterize the competitive situation.[17] Figure 2 summarizes the results. In this table, UC refers to *under-competition*, EC refers to *excess-competition* .

The positions of UC and EC in the table clearly suggests a pattern. As stated, the situation is always one of *under-competition* when w is large (i.e. ε close to zero). The higher f the lower w to obtain *under-competition*. Such a context may be qualified as tough since the profit to be made is structurally low because of f and the price competition is high because of w. On the opposite side the case of *excess-competition* corresponds to simultaneous low values of f and w, a context of less intense competition for the reverse reasons. This confirms the interpretation obtains with the previous example : structural factors related to the degree of toughness of competition do impact potential competition, the

[16] This makes the calculation of $C^{-1}(0)$ somewhat cumbersome.

[17] The corresponding calculations may be obtained from the author upon request.

f							
0,225	UC	UC	UC	UC	UC	UC	UC
0,200	UC	UC	UC	UC	UC	UC	UC
0,175	EC	UC	UC	UC	UC	UC	UC
0,150	EC	EC	EC	UC	UC	UC	UC
0,125	EC	EC	EC	EC	EC	UC	UC
0,100	EC	EC	EC	EC	EC	EC	UC
w	4	6	8	10	12	14	20

Figure 2: The taxonomy for the price model

tougher the competition, the more likely it is that the incumbent may secure a positive rent.

6 Discussion

The major result of this paper is that under the SME approach, the celebrated rent dissipation result no longer holds systematically. In the analysis of the economics of potential competition the issue of short-term commitments no longer dominates the other economic characteristics of the game. Indeed, the tougher the competition in case of entry, the more likely rent dissipation does not hold.

Admittedly, these new results are obtained through a Markov equilibrium concept which may seem technically complex. But the underlying ideas, such as hit and run strategies, are intuitively simple and the results are derived under fairly general assumptions. Furthermore, this concept is directly in line with previous works which harbor clear drawbacks.

Still, to be more satisfactory the SME approach requires some further work. In this last section we point out some open questions.

Question 1 : There may be other Markov equilibria which are consistent with the proposed definition of the state variable. Full characterization would be helpful. This has been obtained with the MT approach.

Question 2 : In corollary 9 it is suspected that the condition $v_S(x^*) + C_S(v_L^{-1}(0)) \leq 0$ is not only necessary but also sufficient.

Question 3 : For the proposed taxonomy, it should be true that in case of *under-competition*, when a weak player may remain with a strictly positive rent indefinitely, a strong player may also. This is more difficult than it may seem and does not follow from proposition 12.

Question 4 :[18] A more interesting issue concerns the fact that our assignment of player 1 and player 2 to the respective L or S positions may be considered as arbitrary. A more formal approach might be to have a preliminary stage at which

[18] This question greatly benefited from the comments of the anonymous referee who suggested the taxonomy.

each player decides how long he could stay, say n_1 for player 1 and n_2 for player 2. The choices n_1 and n_2 are then revealed and an infinitly repeated game $\Gamma_\infty^{n_1,n_2}$ is played in which each player can only use hit and run strategies according to the number of stages decided at the preliminary stage. Using a Markov formalization similar to the present one, it is suspected that the equivalent of theorem 7 holds (i.e., given n_i the best response $n_j(n_i)$ is the highest n_j for which the entry game $\Gamma_\infty^{n_1,n_2}$ has an equilibrium). If this were indeed the case, our taxonomy could be stated as follows :

- *selection* : only the strong player would select to stay infinitely (the limit equilibria in $\Gamma_\infty^{n_1,n_2}$ with respect to large values of (n_1, n_2) would have $n_1^* = \infty, n_2^* < \infty$, where player 1 is the strong player) ;

- *under-competition* : either player could select to stay infinitely but if one does, the other would not wish to, the preliminary game would be similar to a battle of the sexes game (there would be two limit equilibria $\Gamma_\infty^{n_1,n_2}$ with $n_1^* = \infty, n_2^* < \infty$ and $n_1^* < \infty, n_2^* = \infty$) ;

- *excess-competition* : either player would select to stay infinitely whatever the other one does, the preliminary game would be similar to a prisonner dilemma game (formally $\Gamma_\infty^{n_1,n_2}$ would have no limit equilibrium with respect to large values of (n_1, n_2), the best response $n_1(n_2)$ being ∞ and vice versa, while both equilibrium payoffs in a game $\Gamma_\infty^{n_1,n_2}$ would decrease as (n_1, n_2) increases).

Answers to these questions would provide a better understanding of the SME concept and make the MT concept appear as the limit of SME in case of excess-competition.

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