Admission, Tuition, and Financial Aid Policies in the Market for Higher Education*

Dennis Epple
Carnegie Mellon University and NBER

Richard Romano
University of Florida

Holger Sieg
Carnegie Mellon University and NBER

November 10, 2004

*We would also like to thank Joseph Altonji, Charles de Bartolome, Pat Bayer, Steve Berry, Richard Blundell, Martin Browning, Steven Coate, Steven Durlauf, David Figlio, James Heckman, Carolyn Levine, Dean Littlefield, Charles Manski, Erin Mansur, Robert Moffitt, Tom Nechyba, Ariel Pakes, Steve Stern, Chris Taber, Miguel Urquiola, Michael Waldman, and seminar participants at Brown University, the University of California in Berkeley, University College London, the University of Colorado, Cornell University, the University of Kentucky, Northwestern University, Ohio State University, Stanford University, the University of Toronto, the University of Virginia, the Brookings Conference on social interactions, the SITE Workshop on structural estimation, the Triangle Applied Micro Conference, the CAM workshop on characteristics models, the ERC conference in Chicago, the SED in New York, the NASM at UCLA, and the NBER public economics meeting. We would also like to thank the National Center for Education Statistics and Petersons for providing us with data used in this paper. Financial support for this research is provided by the National Science Foundation, the MacArthur Foundation, and the Alfred P. Sloan Foundation.

Abstract

In this paper, we present an equilibrium model of the market for higher education. Our model simultaneously predicts student selection into institutions of higher education, financial aid, educational expenditures, and educational outcomes. We show that the model gives rise to a strict hierarchy of colleges that differ by the educational quality provided to the students. We develop an algorithm to compute equilibria for these types of models. We also develop a new estimation procedure that exploits the observed variation in prices within colleges. Identification is based on variation in differences in fundamentals such as endowments and technology, and not on the observed variation in potentially endogenous characteristics of colleges. We estimate the structural parameters using data collected by the National Center for Education Statistics and aggregate data from Peterson's and the National Science Foundation. Our empirical findings suggest that our model explains observed admission and tuition policies reasonably well. The findings also suggest that the market for higher education is very competitive.

Keywords: higher education, peer effects, college competition, non-linear pricing, equilibrium analysis, empirical analysis.

JEL classification: I21, C33, D58

1 Introduction

Over the past several years, research has investigated normative and positive consequences of competition in primary, secondary and higher education, and the likely effects of policy changes including vouchers, public school choice, and changes in education financing.[1] Some of this research has relied on a set of general equilibrium models. Given the absence of large scale policy experiments, these models have been a primary tool to evaluate the impact of a variety of education reform measures. To date, the predictions of these models have been subjected to little formal empirical testing. This paper provides an integrated approach for estimation and inference based on this class of models. For this purpose, we focus on the market for higher education. Colleges and universities provide a promising environment for developing this approach because a variety of data sets collected by the National Center for Education Statistics and commercial companies such as Peterson's are available to researchers.

We present a general equilibrium model of the market for higher education that extends earlier work on competition in the market for primary and secondary education. In our model, colleges seek to maximize the quality of the educational experience provided to their students.[2] The quality of the educational experience depends on peer ability and income of the student body, and on instructional expenditures per student. If peer "quality" is an important component of college quality, students and their parents will seek out colleges where the student body offers high quality peers.[3] Likewise, colleges will attempt to attract

[1]Recent theoretical studies include Benabou (1996), Caucutt (2002), deBartolome (1990), Epple and Romano (1998, 2003), Fernandez and Rogerson (1998), Manski (1991), Nechyba (2000), and Rothschild and White (1995). Some recent empirical studies include Bergstrom, Rubinfeld, and Shapiro (1982), Carlton, Bamberger, and Epstein (1995), Downes and Greenstein (1996), Ferreyra (2003), Fuller, Manski, and Wise (1982), Hoxby (2000), and Rouse (1998).

[2]We refer to institutions of higher education as "colleges" having in mind inclusion of the undergraduate division of universities.

[3]There is a large, growing, and controversial literature on peer effects by social scientists. Methodological issues are discussed in Manski (1993), Moffitt (2001), and Brock and Durlauf (2001). Limiting discussion to recent research on peer effects in higher education, Sacerdote (2001) and Zimmerman (2000) finds peer effects between roommates on grade point averages. Betts and Morell (1999) find that high-school peer groups affect college grade point average. Arcidiacono and Nickolson (2000) find no peer effects among medical students. Dale and Krueger (1998) has mixed findings.

students who contribute to improving peer quality. In higher education, colleges have the latitude to choose tuition and admission policies to attempt to attract a high quality student body. Our model thus yields strong predictions about the hierarchy of colleges that emerges in equilibrium, the allocation of students by income and ability among colleges, and about the pricing policies that colleges adopt.[4] We complete the theoretical development with a discussion of existence of equilibrium.

We also provide a new approach for identifying and estimating this class of differentiated product models. Our approach differs significantly from the previous empirical literature on differentiated products.[5] First, our data allow us to impute costs. We can, therefore, estimate cost functions without relying on strong functional form assumptions or assumptions about the demand side of the model. Second, our model endogenously generates a distribution of student types among colleges, as well as a distribution of college characteristics, given colleges' posted maximum tuitions and financial endowments. Our estimation procedure differentiates between variation in fundamentals (such as college endowments) and variation in product characteristics. Identification is primarily based on variation in fundamentals. Observed (and potentially endogenous) characteristics such as expenditures and peer quality measures are treated as endogenous latent variables in estimation. Third, identification of demand-side parameters of the model largely depends on the observed variation in prices within colleges. In contrast, almost all previous empirical papers have relied solely on variation in prices among products, ignoring for all practical purposes issues related to price discrimination. One central part of the estimation strategy is to match financial aid policies observed in the data to those predicted by our equilibrium model. This approach requires us to characterize optimal pricing functions for each college and hence compute the equilibrium of the model at each step of the estimation algorithm. We develop a new maximum likelihood estimator that controls for the fact that some of the most im-

[4]An insightful overview of the college quality hierarchy and its determinants is provided by Winston (1999).

[5]Recent studies include Berry, Levinsohn, and Pakes (1995),Epple, Romer, and Sieg (2001), Nevo (2001), Petrin (2002), Bajari and Benkard (2002), Blundell, Browning, and Crawford. (2003), and Ekeland, Heckman, and Nesheim (2004).

portant student characteristics such as income and ability are likely to be measured with error in the data. The result is, we believe, the first instance in which an equilibrium model with endogenous product characteristics, sorting, and price discrimination along multiple dimensions has been estimated.

To implement the empirical analysis, we obtained and merged three databases uniting student-level and college-level data. These include a database that provide individual-level information for a sample of students and two databases, one proprietary and one public, that provide complementary college-level information about the universe of colleges and universities in the U.S.

Our findings suggest that the market for higher education is very competitive. Colleges at low and medium quality levels have close substitutes in equilibrium and thus a limited amount of market power. Admission policies are largely driven by the "effective marginal costs" of educating students of differing abilities and incomes. Colleges with high quality have slightly more market power since they do not face competition from higher-quality colleges. Hence, they can set tuitions above effective marginal costs and generate additional revenues that are used to enhance quality. A main component of college quality is, of course, instructional expenditures. But colleges at all quality levels spend a significant amount of resources on merit aid. This finding is consistent with our modelling approach that assumes that peer quality is increasing in the ability of the mean student body. We also find that colleges at all levels link tuition to student (household) income. Some of this pricing derives from the market power of each college. This allows colleges to extract additional revenues from students that are inframarginal consumers of a college. However, as noted above, our empirical findings suggest that market power of most colleges is limited. This suggests that pricing by income may be driven by other causes. We, therefore, explore alternative explanations that give rise to pricing by income. Our findings here indicate that colleges are concerned about their relative affluence of their students as compared to the larger population and thus tend to subsidize students from families with lower incomes.

The paper is organized as follows. Section 2 lays out the general equilibrium model and derives a set of college-level conditions that characterize the equilibrium allocation. Section 3 defines equilibrium, analyzes existence, and characterizes properties of market equilibrium. Identification and estimation are discussed in Section 4. Section 5 provides information about our data set, which is created by drawing together information from the National Center for Education Statistics, the National Science Foundation, and Peterson's. The empirical results are discussed in Section 6, and then used in some policy analysis. Section 7 presents the conclusions of the analysis and discusses future research.

2 A Theoretical Model of Higher Education

2.1 Preferences and Technologies

In this section, we develop our theoretical model of provision of undergraduate higher education. There is a continuum of potential students who differ with respect to their household income, y , and their ability level, b.

Assumption 1 *The joint distribution of income and ability $F(b, y)$ is continuous with support $S \subset R_+^2$ and joint density $f(b, y)$.*

There are J colleges and each student chooses among the subset set of the colleges to which the student has access (as described below) and an outside option.

Assumption 2 *The outside option denoted by 0 provides an exogenous quality q_0 with zero cost to the student and is available to any student.*

The quality of college $j = 1, 2, ..., J$ is given by

$$q_j \quad = \quad q(I_j,\ \theta_j,\ d_j) \tag{2.1}$$

where θ_j is the peer-student measure, equal to mean ability level in the student body, I_j is the expenditure per student in excess of minimal or "custodial costs," and d_j is a measure of the (relative) mean income of the student body.[6]

Assumption 3 *The quality function $q(\cdot)$ is a twice differentiable, increasing, quasi-concave function of its arguments.*

As noted in the Introduction, the data on financial aid suggest that college quality includes a term that measures the relative income of the college's students relative to the larger population from which students are drawn. Denote mean income of students in college j by μ_j^y, and mean income in the population of all students by μ_p^y. When $\mu_j^y > \mu_p^y$ (as holds empirically), a measure of the relative affluence of a college's students relative to the larger population is the ratio of mean income in the population to mean income in college j:

$$d_j = \frac{\mu_p^y}{\mu_j^y} \tag{2.2}$$

The main motivation for using mean income relative to population income as a measure in the quality index is empirical. Reduced form regressions reported in Epple, Romano, and Sieg (2003) suggest that colleges engage in a significant amount of price discrimination by income, i.e. they provide need-based aid. Some price discrimination by income arises in our model even if we do not introduce a measures of the relative income position. Since there are only finitely many differentiated colleges in equilibrium, they have a limited amount of market power over inframarginal students and can extract additional revenues from them. However, this type of price discrimination is not sufficient to explain the amount of pricing by income observed in the data. As shown below, the income measure used above implies the type of monotonic pricing by income that is prevalent in our data.[7]

A less obvious consequence of including this income measure in the quality index is that it leads to more income diversity within a college in equilibrium. This result may

[6] We henceforth omit ranges of subscripts and the like when obvious by context.

[7] The stylized fact that colleges can extract so much revenue from higher income households is clearly an empirical puzzle given many colleges competing for students. Reduced form analysis also confirms that this fact is robust to a number of changes in the specification of the reduced form model. For example, adding race or ethnicity to the model does not fundamentally alter this finding. More future research is needed to find other compelling explanations for this puzzle.

be surprising because d_j does not measure income diversity per se. The intuition for this finding is that colleges – especially the most selective ones – will penalize higher income students and subsidize lower income students. As consequence, the most selective schools will have lower mean income levels in equilibrium than in a model without the income penalty term.[8]

The college cost function is

$$C(k_j, I_j) = F \; + \; V(k_j) \; + k_j \; I_j \tag{2.3}$$

where k_j is the size of the college j's student body, and I_j is per student expenditure on quality enhancing inputs. Schooling costs include components that are independent of educational quality, the "custodial costs" mentioned above.

Assumption 4 *$V(k)$ is an increasing and convex function that is twice differentiable in k, i.e. $V', V'' > 0$*

Note that the average cost function, C/k, is U-shaped in k, and denote the efficient scale by k^*.

Substantial financial aid to many undergraduates in the form of grants, loans, and work-study funding is provided by the federal government and to a lesser extent by other entities that are also independent of the student's college. We refer to such aid as non-institutional aid. Let p_j denote the tuition at college j. We presume that the value of non-institutional aid to the student at college j, denoted a_j, can be written as:

$$a_j \quad = \quad a(b, y, p_j) \tag{2.4}$$

[8]Concern for socio-economic diversity of student bodies is cited by Harvard president Lawrence Summers as motivating Harvard's recent announcement that no tuition will be charged to students from households with less than $40,000 in annual income. "When only 10 percent of the students at elite higher education come from the lower half of the income distribution, we are not doing enough." (NYT, Feb 29, 2004, p.14.)

Much of this aid is based on the federal government's calculation of the family's ability to pay. Such aid is need based, implying it depends on income and tuition of the college attended. Some non-institutional aid seems to be meritorious, so we will allow it to depend on ability as well. More aid is also given to students attending more expensive colleges. We, therefore, assume that

Assumption 5 *$a(\cdot)$ is increasing in b and p_j and decreasing in y.*

We assume that the decision to attend college is made by the student's household. Household utility depends on numeraire consumption, educational quality, and the student's ability. Household utility from attendance at college j is given by the following (conditional) utility function:

$$U(y - p_j + a_j, q_j, b) \tag{2.5}$$

where numeraire consumption is $x = y - p_j + a_j$.[9] We assume that U satisfies standard regularity conditions. In addition, we assume that the utility function satisfies the following single-crossing conditions:

Assumption 6 *The utility function satisfies:*

$$\frac{\partial \left(\frac{\partial U / \partial q}{\partial U / \partial x} \right)}{\partial x} > 0 \tag{2.6}$$

$$\frac{\partial \left(\frac{\partial U / \partial q}{\partial U / \partial x} \right)}{\partial b} \geq 0 \tag{2.7}$$

[9]We assume that households and colleges share the same college quality function. It is possible, however, that the factors affecting quality are weighted differently by households than by colleges. Some implications of such divergence of objectives with respect to racial/ethnic diversity are explored in Epple, Romano, and Sieg (2002). Generalizing that analysis and exploring identification and estimation in the framework developed here are promising avenues for future research.

Thus the demand for college quality is increasing in income and non-decreasing in ability.[10] One interpretation of the utility function has utility increasing in the numeraire good and educational attainment of household's student, $A(q, b)$, with A increasing in its arguments. This interpretation is consistent with the utility function that we estimate below.

Households choose among colleges or no college, taking as given college qualities and their tuition and admission policies.

2.2 The Decision Problem of a College

Colleges are assumed to maximize quality.[11] Their choices must satisfy a profit constraint, with revenue equal to the sum of all tuition from students plus other college earnings, the latter denoted R_j. We refer to R_j as college j's endowment earnings, but it includes also non-tuition revenues like state subsidies.[12] The number of colleges and their endowments are taken as exogenous. We do not allow for entry and restrict attention to cases where all J colleges can cover their costs. While colleges will condition tuition on student characteristics, we presume that college j charges a maximum tuition denoted p_j^m. We do not have an explicit theory to explain or determine the magnitude of p_j^m, so we treat it as exogenous. Our motivation for introducing these price caps is empirical. We interpret the price cap as the college's posted tuition, with lower tuition framed as financial aide, a scholarship, or, perhaps, a fellowship. To derive certain properties of the equilibrium allocation, we require that:

Assumption 7 $0 \leq R_1 < ... < R_J < \infty$ *and* $0 < p_1^m < ... < p_J^m < \infty$.

[10] Assumption (2.7) can reasonably be challenged. If it fails to hold, then the sorting pattern our model predicts might be disrupted. As detailed below, our empirical specification of the model treats the boundary case of equation (2.7) which has zero ability elasticity of demand for college quality. The empirical results generally support this utility specification, but it is of interest to examine alternatives in future research.

[11] See Epple and Romano (1998, 2003) for an analysis of profit maximization by (secondary) schools in a related model. Another preliminary analysis of achievement maximization, using the college mean of the $A(\cdot)$ function discussed above, demonstrates that it leads to predictions that are similar to the ones implied by quality maximization.

[12] An interesting extension that we do not pursue in this paper would be to endogenize endowment income.

To avoid the problem that a college may operate with infinite per student expenditure levels, we also need to assume that:

Assumption 8 $F > \max_j R_j = R_J$

We observe in the data that a fraction of students in each college pay the posted tuition (price cap). Price caps are thus an important feature of currently used financial aid and pricing policies. Introducing price caps thus seems to be necessary if the objective is empirical analysis. As discussed below, price caps somewhat alter the predictions of the model. One important implication of the price cap is that the more selective colleges will define minimum ability thresholds that students have to satisfy to be admitted (see Figure 1). In this model we do not differentiate between administrators (agents) that operate the college and trustees (principals) that set objectives for the college. We thus assume that there are no informational asymmetries and administrators implement policies preferred by the trustees. In a more general model, there may be some agency problems. Trustees may, for example, be concerned that a significant fraction of high-income-low-ability students may harm the reputation of the college. Administrators may have some private incentives to admit these students (e.g., to associate with very wealthy households). A binding price cap, that limits the ability of administrators to admit these type of students, may then result in equilibrium. An interesting extension of our model, that we do not explore in this paper, would be to formalize these ideas.[13] For the rest of the analysis, we treat price caps as predetermined while acknowledging that this is a limitation of the analysis.

Colleges take type (b, y)'s alternative utility as given when maximizing quality. Let $r_j = r_j(y + a_j, q_j, b)$ denote a student's reservation price for attending a college of quality q_j. That is, r_j, satisfies:

$$U(y - r_j + a_j, q_j, b) \quad = \quad U_j^A(b, y) \tag{2.8}$$

[13]In a dynamic model, price caps may also arise because colleges may charge lower tuition rates since they expect to receive donations from wealthy alumni in the future.

where $U_j^A(b, y)$ denotes the beliefs that college j holds about the maximum alternative utility that type (b, y) can attain if the household does not attend college j. Equilibrium requires that these beliefs are consistent with optimization by other colleges as detailed below. Utility-taking implies that a college expects to attract as many type (b, y) students as they would like if the college's tuition equals the type's reservation price. Given the college's objective function, it will be optimal for j to set tuition p_j such that:

$$p_j = r_j \tag{2.9}$$

if r_j is below p_j^m. Colleges always want to maximize revenue for a given student body since this permits maximal instructional expenditures and thus quality.

Let $\alpha_j(y, b) \in [0, 1]$ denote the admission function that indicates the proportion of type (b, y) in the population that college j admits. Each college solves the following problem:

$$\max_{\alpha_j(b,y), p_j(b,y), k_j, \theta_j, \mu_j^y, d_j} q(\theta_j, I_j,\ d_j) \tag{2.10}$$

subject to:

- Identity Constraints:

$$\begin{aligned} k_j &= \int\int_S \alpha_j(y, b)\ f(y, b)\ dy\ db & (2.11) \\ \theta_j &= \frac{1}{k_j} \int\int_S b\ \alpha_j(y, b)\ f(y, b)\ dy\ db & (2.12) \\ \mu_j^y &= \frac{1}{k_j} \int\int_S y\ \alpha_j(y, b)\ f(y, b)\ dy\ db & (2.13) \\ d_j &= \frac{\mu_p^y}{\mu_j^y} & (2.14) \end{aligned}$$

- Budget Constraint:

$$\begin{aligned} F\ +\ V(k_j)\ +\ k_j\ I_j &= T_j\ +\ R_j & (2.15) \\ T_j &= \int\int_S p_j(y, b, q_j(\cdot))\ \alpha_j(y, b)\ f(b, y)\ db\ dy & (2.16) \end{aligned}$$

- Feasibility Constraints:

$$\begin{aligned} \alpha_j(y,b) &\in [0,1] \;\; \forall\,(b,y) && (2.17)\\ p_j(b,y) &\le p_j^m \quad \forall\,(b,y) && (2.18)\end{aligned}$$

- Reservation Utility Constraint:

$$U(y - p_j(b,y) + a_j, q_j, b) \;\; \geq \;\; U_j^A(b,y) \quad \forall\,(b,y) \tag{2.19}$$

Each college thus ignores the fact that changing its own policies alter the utilities attainable at other colleges.

The first-order conditions for the optimal admission and tuition policy may be written:[14]

$$\alpha_j(b,y) \begin{pmatrix} =1 \\ \in [0,1] \\ =0 \end{pmatrix} \;\; \text{as} \;\; p_j \begin{pmatrix} > \\ = \\ < \end{pmatrix} \;\; \text{EMC}_j(b,y) \tag{2.20}$$

where

$$\begin{aligned} \text{EMC}_j(b,y) &= V'(k_j) \; + \; I_j \; + \; \frac{\partial q_j/\partial\theta}{\partial q_j/\partial I}\,(\theta_j - b) \; + \; \frac{\partial q_j/\partial \mu_j^y}{\partial q_j/\partial I}\,(\mu_j^y - y) && (2.21)\\ p_j(b,y) &= \min\left[r_j(b,y),\; p_j^m\right] && (2.22)\end{aligned}$$

and we use the more compact notation $r_j(b,y)$ for the reservation price function. Equation (2.21) defines the "effective marginal costs (EMC)" of admitting a student of type (b,y) to college j. EMC is the sum of the marginal resource cost of educating the student, the cost

[14] Beside the constraints, the remaining first-order condition regards the choice of I_j. Letting $\lambda > 0$ denote the multiplier on the revenue constraint (2.15), this condition may be written:

$$\int\!\!\int_S \frac{\partial p_j}{\partial q_j}\frac{\partial q_j}{\partial I_j}\;\alpha\; f\; db\, dy \;\; - k_j = -\frac{\partial q_j/\partial I_j}{\lambda}$$

This result establishes that college j elects to spend more than the Samuelsonian amount on educational inputs, because college j values quality per se. The first-order conditions are derived in an appendix that is available from the authors.

of maintaining quality due to the student's peer-ability externality, and the analogous cost associated with the relative income measure. The ability-related peer effect is captured by the third term on the right-hand side in (2.21), which equals the peer-ability measure change from admitting a student of ability b, multiplied by the expenditure change that maintains quality. Note that this term is negative for students with ability above the college's mean, and EMC_j itself can be negative. The peer income effect is captured by the last term in (2.21), which equals the measure change from admitting a student of income y, multiplied by the expenditure change that maintains quality. Note that this term is positive for students with income above the college's mean, assuming the mean college income is higher than the mean income in the population. The optimal tuition to student (b, y) is given in equation (2.22). From (2.20), students whose optimal tuition exceeds EMC_j permit quality increases and are all admitted, and the reverse for students who cannot be charged a tuition that covers their EMC_j. The college is willing to admit any proportion of students who can be charged at most their EMC.

3 Equilibrium

3.1 Definition of Market Equilibrium

To define a market equilibrium, it is necessary to determine the equilibrium reservation utility function of each college and to consider strategic interaction between colleges. We define equilibrium as a competitive equilibrium assuming that students behave as price takers and colleges as utility takers. The assumption of utility taking is a generalization of price taking that has been utilized in the competitive club goods literature.[15] In equilibrium, the reservation price functions of each college and the beliefs that students hold about the set of colleges that would be willing to admit them if they did not attend their chosen college must be consistent with the actions of the other colleges. We refer to the set of colleges willing to admit a student in equilibrium along with the outside option as the effective choice

[15] See, for example, Gilles and Scotchmer (1997) and Ellickson, Grodal, Scotchmer, and Zame (1999).

set of a student and denote it $\tilde{J}(b, y)$. More formally, it is defined as the set of colleges that are willing to admit that student at effective marginal costs, along with the outside option:

$$\tilde{J}(b, y) \quad = \quad \{j | \, EMC_j(b, y) \leq p_j^m\} \cup \{0\} \tag{3.1}$$

The FOC's of the college's decision problem imply that any college is willing to admit a student at tuition equal to effective marginal costs as long as admissible tuition exceeds effective marginal costs. Knowing the colleges in the effective choice set allows us to characterize reservation utilities and impose the equilibrium restriction that reservation utility functions must be consistent with optimal choices. A competitive market equilibrium for this economy can then be defined as follows:

Definition 1 *An economy E of our model consists of an outside option with quality q_0; a set of colleges $\{1, ..., J\}$, a vector of endowment incomes $(R_1, ..., R_J)$ and price caps $(p_1^m, ..., p_J^m)$, a set of non-institutional financial aid functions $(a_1(\cdot), ..., a_J(\cdot))$ and a cost function $C(k, I)$; a continuous distribution F of household types (b, y) with support S, and an utility function $U(y - p + a, q, b)$.*

A competitive utility-taking equilibrium for this economy E consists of a set of admission and pricing functions, $\alpha_j(b, y)$ and $p_j(b, y)$, a reservation utility function $U_j^A(b, y)$ for each college, a vector of college characteristics $(k_j, \theta_j, \mu_j^y, I_j)$ for each college; an allocation of students into the J colleges and no college such that:

1. *Every student (b, y) attends a preferred college j in his or her effective choice set or chooses the outside option.*

2. *Each college chooses its size, peer quality, relative income position, expenditures, as well as admission and tuition policies to maximize quality, taking as given its endowment, price cap, and its reservation utility function.*

3. *Beliefs about reservation utilities are consistent with optimal choices, i.e. for each*

household type (b,y) and each college j:

$$U_j^A(b,y) \quad = \quad \max\left[U(y,q_0,b), \max_{i\neq j, i\in\tilde{J}(b,y)} (U(y-EMC_i+a_i,q_i,b))\right] \tag{3.2}$$

Thus the maximum alternative utility is the maximum over the outside option and the next best college alternative taking qualities, tuitions, and access as given.

4. *Each student attends at most one college, i.e, markets for the J colleges clear:*

$$\sum_{j=1}^{J} \alpha_j(b,y) \leq 1 \;\forall\; (b,y) \;\; \textit{for an optimal}\, \alpha_j(\cdot) \tag{3.3}$$

where types for whom the inequality is strict are attending no college.

Several issues regarding the definition of equilibrium warrant elaboration. To confirm that the beliefs in (3.2) are correct, first suppose that higher utility were attainable. This would imply $p_i < EMC_i$ at some college i $\neq$ j willing to admit the student, contradicting the optimal admission policy of college i (see (2.20). Further, student (b,y) will obtain at least the utility specified in (3.2). If not, then either the outside option is unavailable or $r_i > EMC_i$ at some college i $\neq$ j in $\tilde{J}(b,y)$. The former is obviously a contradiction and the latter would imply $\alpha_i > 0$ is an optimal choice for college i, also a contradiction.

Generally, students attending college j will fall into two groups. One group will obtain utility equal to $U_j^A(b,y)$, paying tuition equal to r$_j$. If a college i $\neq$ j in $\tilde{J}(b,y)$ is their best alternative (rather than the outside option), then $r_i = EMC_i$ and any $\alpha_i(b,y) \in [0,1]$ is optimal for college i, implying college i is willing to admit the student at the margin. The second group attending college j obtains higher utility than $U_j^A(b,y)$, paying p_j^m with $r_j > p_j^m$. For these students, $r_i < EMC_i$ at other colleges, but the price cap prevents college j from increasing tuition.[16]

[16]Colleges would be willing to admit these students at $p_i = EMC_i$, however, so alternative utility to college j continues to be as in (3.2). If one is uncomfortable with the latter assumption about potential access to other colleges, note that any specification of alternative utility that is lower is consistent with equilibrium price since this constraint (i.e., (2.19)) is not binding on college j anyway.

Related to the latter discussion, let $B_j = \{(b, y)|$ student (b, y) has access to college $j\}$ denote the admission set of student types; these are students for whom $\alpha_j(b, y) > 0$ is *an* optimal choice for college j. Let $A_j = \{(b, y)|$ student (b, y) attends college $j\}$ denote the attendance set for student types; these are the students for whom either $\alpha_j(b, y) > 0$ is uniquely optimal for j or for whom any $\alpha_j(b, y) \in [0, 1]$ is optimal and some of these types matriculate. Observe that $A_j \subseteq B_j$. Referring to the first-order condition (2.20), for students such that $p_j(b, y) > EMC_j(b, y)$, $\alpha_j(b, y) = 1$ is uniquely optimal and all such students must attend college j in equilibrium. For students such that $p_j(b, y) = EMC_j(b, y)$, any $\alpha_j \in [0, 1]$ is optimal for college j. Such students are then admitted to college j, but may not attend college j in equilibrium (i.e., as long as (3.3) can be satisfied for some optimal α_j).[17]

We also contrast our competitive specification with an alternative approach that defines equilibrium as a non-cooperative Nash equilibrium. If we modeled equilibrium as Nash in tuition policies (where no admission would correspond to a prohibitive tuition), then each college would take account of the effects of varying its own tuition policy and thus student body on the student bodies and thus qualities–via the within-college ability and income externalities–on other colleges. Utility taking implies that colleges take *both* tuition and qualities of other colleges as given. Hence colleges do not fully anticipate the strategic impacts of their own choices in our model. Our specification yields a simpler reservation price function, that would become a complex function of every college's quality in the Nash-specification. Thus we avoid considerable complication while abstracting from a strategic element that may be relevant in reality.

[17] To be clear, the relevant $\alpha_j(b, y)$ in (3.3) is that corresponding to the measure in attendance. We are avoiding more notation. If one would like, let $\alpha_j^A(b, y)$ denote the proportion of type (b, y) attending college j, and let $\alpha_j^B(b, y)$ denote the proportion of type (b, y) admitted to college j. Then both must be optimal for j. $\alpha_j^B(b, y) > 0$ defines access to college j in (3.2) and $\alpha_j^A(b, y)$ enters the market clearing condition in (3.3).

3.2 Properties of Equilibrium

Before we discuss existence of equilibrium, we discuss necessary properties of equilibria to illustrate some of the features of our model.

Proposition 1 *A market equilibrium satisfies the following four conditions:*

1. *There is a strict quality hierarchy of colleges in equilibrium. The hierarchy follows the endowment ranking.*

2. *Generically in the parameter space of utility and non-institutional aid functions, for $(b, y) \in A_j$ and all j, $\alpha(b, y) = 1$ for almost all students. The type space S is partitioned into colleges and the no-college option.*

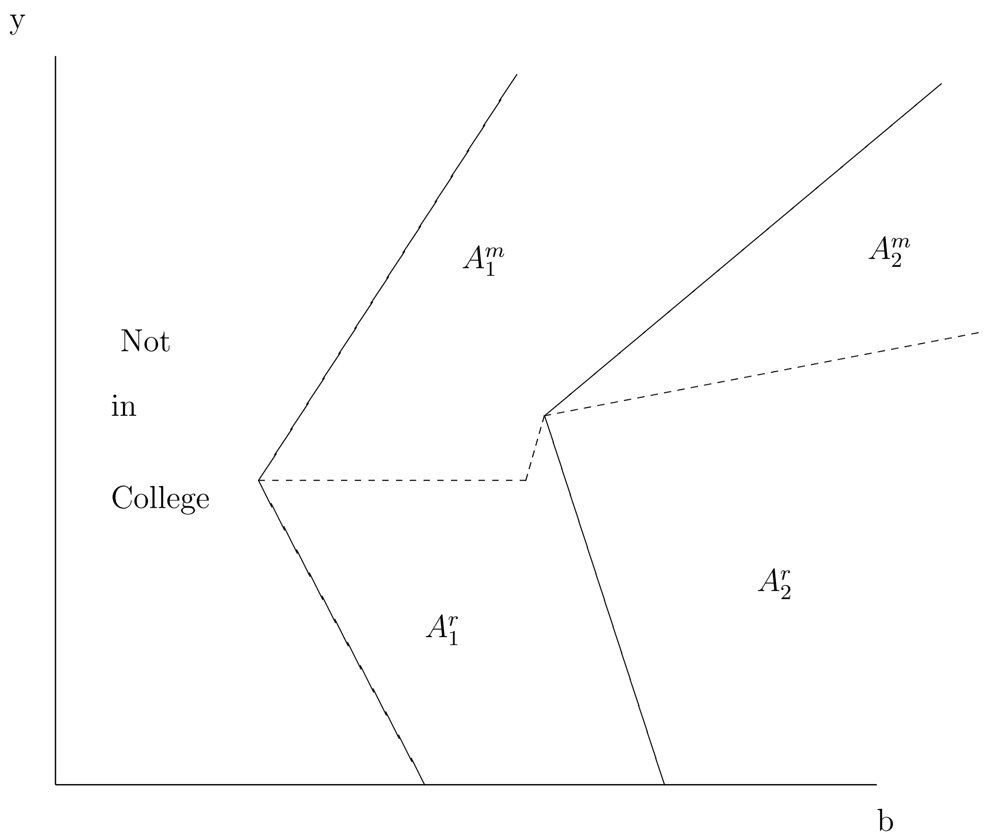
3. *There exists a locus $b_j(y)$ for each college that defines the minimum ability that a student with income y must have to attend college j. This threshold function is implicitly defined by the minimum b satisfying $EMC_j(b, y) = p_j(b, y)$ if there exists such a pair $(b, y) \in S$. Otherwise $b_j(y)$ equals the minimum b in S for given y.*

4. *Choosing among the set of colleges and no college, almost every student-type (b, y) attends the college or no college that would maximize utility if $p_j = EMC_j(b, y)$ for all $j \in \tilde{J}(b, y)$. College pricing in excess of EMC is to take away consumer surplus (constrained by the price cap for some students).*

A proof of Proposition 1 is in the appendix.

Figure 1 shows an example of how the type space is partitioned into colleges, assuming just two colleges. The example assumes that non-institutional aid is independent of ability as we find empirically. The tipped-L solid lines, or "boundary loci," separate types into colleges and no college. Those to the right of the right-most boundary locus attend the higher-quality college 2. Those between the boundary loci attend college 1, with the rest not attending college. The upward-sloping part of each boundary locus satisfies $p_i^m = EMC_i(b, y) \leq r_i(b, y)$, and the downward-sloping part satisfies $r_i(b, y) = EMC_i(b, y) \leq p_i^m$, where i is the number of the college to the right of the locus. The value of r_i depends on the student's type, including the options available to the student.

Figure 1: An Example of College Admission Spaces



This figure illustrates the boundaries of college attendance sets.

Consider students attending college 2 in this equilibrium. They all have access to college 1 at $EMC_1(b, y)$, since this is below p_1^m for all these students.[18] Their reservation price for college 2 is then determined by the utility they could obtain at college 1 at tuition $EMC_1(b, y)$. All students in college 2 not on the downward-sloping part of the boundary locus would strictly prefer college 2 at tuition equal to $EMC_2(b, y)$. Let $A_j^r = \{(b, y) \mid p_j(b, y) = r_j(b, y)\}$ and $A_j^m = A_j \setminus A_j^r$. College 2 charges all those in A_2^r their reservation price, which is below p_2^m, taking away their consumer surplus relative to

[18] To confirm the statement in the text, use that the upward-sloping line of the left-most boundary locus satisfies $p_1^m = EMC_1(b, y)$ and $EMC_1(b, y)$ is decreasing in b.

attending college 1. Those in A_2^m have reservation price exceeding both $EMC_2(b, y)$ and p_2^m, so college 2 can "only" charge them p_2^m leaving them some surplus. Those in college 1 may or may not have access to college 2 (depending respectively on whether $EMC_2(b, y)$ is less than or greater than p_2^m), and those with access to college 2 may have it or no college as their preferred option. This somewhat complicates the determination of prices, but it is nevertheless straightforward to make the calculations. Note that students near their left-side boundary locus will pay tuition close to their $EMC(b, y)$ at their college, as will also students near the downward sloping portion of the right-side boundary locus (i.e., the latter being students who have access to the next best college.) Hence, the presence of a relatively large number of colleges will squeeze attendance sets and lead to relatively competitive outcomes with tuition close to EMC for most students.

3.3 Existence and Computation of Equilibrium

It should be clear from the discussion so far that there is no hope to solve for equilibrium analytically. A college's optimum will necessarily be on a boundary: prices will be set to equal reservation prices or the price cap, and attendance of each type will satisfy $\alpha_i = 1$ for some i (except on a set with measure zero). Hence we need to rely on numerical techniques to compute equilibria which is also quite challenging. To compute equilibrium allocations, it is useful to focus on a slightly broader class of allocations that satisfy the following four conditions: a) market clearing; b) utility maximization; c) consistency of reservation utilities; and d) the FOC's of the colleges' optimization problems. Any equilibrium must satisfy these conditions. But an allocation that satisfies these conditions may not be an equilibrium because it may fail to satisfy global optimality of college choices. For a lack of a better term, we will refer to these allocations as "local equilibria."

It is not hard to see that a local equilibrium for our model is fully characterized by a vector of college sizes, peer ability and income measures, instructional expenditures for each college, thus yielding $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$. The value of k_0 is given by $1 - \sum_{j=1}^J k_j$. Finding a local equilibrium for this model can be viewed as a classical fixed-point problem of an equilibrium correspondence that maps $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$ into $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$. Such a mapping exists.[19] Moreover, we can show that the mapping used in this analysis has a fixed point under suitable regularity conditions. Finally, this mapping can be used to design an algorithm that can be used to compute equilibria numerically. To implement this algorithm, we need to evaluate integrals numerically using simulation techniques. We compute fixed points using an algorithm that finds a root to a nonlinear system of equations. In our computations we use Broyden's method. Once we have found local equilibria, we only need to verify ex post that college decisions satisfy global optimality. Verifying global optimality is a straight-forward exercise, but computationally costly, since we need to consider deviations in four strategic variables for eachof the J colleges.[20] Our computational analysis suggests that allocations computed as outlined above satisfy global optimality.

4 Identification and Estimation

4.1 The Supply Side

First, we consider the problem of identifying and estimating the parameters of the custodial cost function. We observe $\{T_j, R_j, I_j, k_j\}_{j=1}^J$ for a large sample of colleges with size J. The budget constraint (2.15) then implies that we can impute custodial costs $C_j^c = T_j + R_j - k_j I_j$. Assuming that custodial costs are quadratic in k_j, we obtain:

$$C_j^c = F + c_1 k_j + c_2 k_j^2 + \epsilon_j \tag{4.1}$$

where ϵ_j denotes an error term. To achieve identification, we can assume that the left hand side cost variable in equation (4.1) is measured with error (for example due to measurement error in expenditures.) In that case it is reasonable to assume that the measurement error ϵ_j is uncorrelated with college size, i.e. $E[\epsilon_j | k_j] = 0$. Under these identifying assumptions,

[19] An outline of the mapping is reported in Appendix B. A detailed discussion of this mapping and its properties is available upon request from the authors.

[20] See Appendix C for an outline of the algorithm that we use to check for global optimality.

we can then estimate the parameters of the cost function using OLS. (Of course, we do not need to use a quadratic functional form, we can be fully non-parametric at this stage of the analysis.)

Alternatively, we can follow more recent practice in the industrial organization literature and interpret the error term in the cost function (4.1) as an unobserved shock to the cost function. In that case, the assumption that $E[\epsilon_j|k_j] = 0$ is likely to be violated because differences in costs across colleges will affect their behavior. We may, therefore, expect that ϵ_j and k_j are correlated, as discussed for, for example, in Berry (1994). In that case, we need to find instruments for k_j to estimate the parameters of the cost function. Based on the structure of the model, functions of the rank of endowment income may serve as valid instruments.

The main problem associated with introducing unobserved elements to the cost function is that we need to alter the underlying equilibrium model, i.e. we need to account for idiosyncratic differences in costs among colleges. In the model outlined in the previous section, we have assumed that all colleges have identical cost functions. While allowing for heterogeneity in cost functions is promising, we do not explore it in detaik in this paper. However, we explore IV estimation in Section 5 as part of a sensitivity analysis.

4.2 The Demand Side

We assume that the joint distribution of log-income and ability among students who attend college is bivariate normal:

$$\begin{bmatrix} \ln(y) \\ b \end{bmatrix} \sim N \left[\begin{pmatrix} \mu_{\ln(y)} \\ \mu_b \end{pmatrix}, \begin{pmatrix} \sigma^2_{\ln(y)} & \rho\sigma_{\ln(y)}\sigma_b \\ \rho\sigma_{\ln(y)}\sigma_b & \sigma^2_b \end{pmatrix} \right] \tag{4.2}$$

We use a Cobb-Douglas specification for q_j:

$$q_j = I_j^{\omega} \theta_j^{\gamma}\, d_j^{\psi} \quad \omega,\ \gamma,\ \psi > 0 \tag{4.3}$$

Household preferences are also Cobb-Douglas:

$$U(y - p_j + a_j, q_j, b) \quad = \quad (y - p_j + a_j)\; q_j \; b^{\beta} \tag{4.4}$$

Note that this specification implies that the own-ability elasticity of demand for quality is zero, so ability sorting that arises in equilibrium is driven by pricing of the ability externality.

Our approach to estimation utilizes both college-level and student-level data. We have detailed data for each college. In addition to the variables mentioned above, we also observe price caps, p_j^m. We assume that price caps are measured without error. Moreover, for a subset of colleges in our data set, we also observe a rich cross-sectional sample of students of size N. For each student, we observe an income and ability measure. Since we will assume that income and ability are measured with error, let us denote these observed measures with $\tilde{y}$ and $\tilde{b}$. We also observe the price paid by the student, denoted by $\tilde{p}$, at the college that is attended.

One of the objectives of this analysis is to evaluate the quality of the goodness of fit of our equilibrium model. Thus we want to impose all restrictions that arise from equilibrium in our estimation procedure. We can compute equilibria for (reasonably aggregated versions of) our model for each parameter vector. Given the structure of our model, the probability of observing an individual with characteristics (b, y) in college j is given by:

$$f_j(b, y) = \begin{cases} f(b, y)\;/k_j & \text{if } \; (b, y) \in A_j \\ 0 & \text{otherwise} \end{cases} \tag{4.5}$$

where $f(\cdot)$ is the joint density function of (b, y). As above, let $p_j(b, y)$ denote the price predicted by our model, which is a deterministic function of (b, y).[21] Assuming the difference between the predicted price $p_j(b, y)$ and the observed price $\tilde{p}$ is independent of (b, y), then

[21] We suppress q_j as a an argument of $p_j(\cdot)$ henceforth.

the joint likelihood of $(\tilde{p}, b, y)$ is given by:

$$f_j(\tilde{p}, b, y) = \begin{cases} g(\tilde{p} - p_j(b, y))\ f(b, y)\ /\ k_j & \text{if } (b, y) \in A_j \\ 0 & \text{otherwise} \end{cases} \tag{4.6}$$

where $g(\cdot)$ is the density of the measurement error for price.

This likelihood function assigns zero probability to observations $(b, y) \notin A_j$. Hence the likelihood function for any particular sample will not be well-defined. To obtain non-degenerate choice probabilities, we assume that income and ability are also measured with error.[22] Let b (y) denote unobserved ability (income), and $\tilde{b}$ $(\tilde{y})$ the observed ability (income) which includes measurement error. Let $h_b(\tilde{b}|b)$ and $h_y(\tilde{y}|y)$ be the corresponding density functions. The probability of observing $(\tilde{p}, \tilde{b}, \tilde{y})$ in college j is then given by:

$$\begin{aligned} f_j(\tilde{p}, \tilde{b}, \tilde{y}) &= \int_{A_j} f_j(\tilde{p}, \tilde{b}, b, y, \tilde{y})\ db\ dy \\ &= \int_{A_j} g(\tilde{p} - p_j(b, y))\ h_b(\tilde{b}|b)\ h_y(\tilde{y}|y)\ f(b, y)\ /\ k_j\ db\ dy \end{aligned} \tag{4.7}$$

We assume that measurement errors in log income, ability, and price are additive, distributed normally, and drawn independently with standard deviations $(\sigma^e_{\ln(y)}, \sigma^e_b, \sigma^e_p)$ respectively.

The integral on the right-hand side of equation (4.7) is evaluated numerically.[23] The log-likelihood function for a sample of N students is then simply given by

$$\begin{aligned} &L(\omega, \gamma, \psi, \mu_{\ln y}, \sigma_{\ln y}, \rho, \mu_b, \sigma_b, \sigma^e_{\ln(y)}, \sigma^e_b, \sigma^e_p) \\ &= \sum_{n=1}^{N} \sum_{j=1}^{J} w_n\ d_{jn}\ 1\{f_{jn} > l_N\}\ \ln(f_{jn}(\tilde{p}_n, \tilde{b}_n, \tilde{y}_n)) \end{aligned} \tag{4.8}$$

where w_n is the weight associated with observation n and d_{jn} is equal to 1 if individual n attends college j and zero otherwise. Weights are necessary to reflect the sampling design

[22] Our approach is thus similar in spirit to work on kinked budget constraints by Hausman (1985).

[23] For a discussion of simulation in estimation, see, for example, Pakes and Pollard (1989), McFadden (1989) and Gourieroux and Monfort (1993).

of the NCES data set used in the analysis. The likelihood function above also implies that we use a trimmed estimator to avoid having results driven by outliers. $1\{f_{jn} > l_N\}$ is an indicator function and l_N is a sequence of trimming parameters.

The parameters of the likelihood function can be decomposed into the structural parameters of the equilibrium model and the parameters of the distributions of the measurement errors. Maximization of the likelihood function is computationally intensive since we need to solve for the equilibrium of the model for each evaluation of the likelihood function. The estimation procedure consists of an outer loop, that searches over the parameter space; and an inner loop that computes the equilibrium, boundary loci and the choice probabilities, and evaluates the likelihood function for each parameter vector.

4.3 Discussion

It is fairly intuitive that the parameters of the utility function are identified from the observed college choices and the variation of prices within and across colleges. One technical difficulty arises because income and ability are assumed to be measured with error. This induces an error-in-variables problem into the analysis. However, the properties of the ecoometric model allow us to decompose the distribution of observed income and ability into the latent distribution of income and ability and the distribution that reflects measurement errors. The variances of income and ability determine the distribution of households by income and ability in equilibrium of the model. The variances of the error components do not affect equilibrium. The variances of the measurement errors are thus ultimately identified from variation of choices holding observed income and ability fixed. If there is much variation in choices at observed income and ability levels, we will need large variances in measurement error to reconcile our model with the observed distribution of students across colleges. Hence we can estimate the variances of the measurement errors and correct for the bias in the pricing and admission equations to obtain a consistent estimator for the parameters of the utility function.[24]

[24] A formal analysis of identification is available upon request from the authors.

It is also useful to compare our approach for demand side estimation to other commonly used approaches in the literature on differentiated products. There are many similarities, but there are, at least, four important differences. First, our specification of the model does not contain additively separable idiosyncratic errors (as typically assumed in random utility models). While we could add these error terms to our specification of utility in (4.4), it would significantly alter the structure of model. The resulting model would more closely correspond to a model developed by Anderson and de Palma (1988) that has different equilibrium properties. We thus cannot add additively separable errors to our utility function in estimation without fundamentally altering the equilibrium properties of the model. To generate meaningful choice probabilities, we therefore do not rely on additively separable error terms in the utility function, but introduce measurement error in income and ability. Our equilibrium model gives rise to deterministic admission functions. Integrating out the measurement error distribution yields well-behaved choice probabilities.

Second, most popular aggregate discrete choice estimators such as Berry et al. (1995) or Epple et al. (2001) do not rely on full solution algorithms, i.e. these studies do not compute equilibria of the underlying model to estimate the parameters of the model. Such an approach has some obvious advantages, but it is hard to see how it can be applied in our context. Much of the identification in our model arises from the observed variation of prices within colleges. To exploit this variation in estimation, we needs to characterize optimal pricing functions and hence compute equilibria. Computing equilibria in estimation is of course costly, and restricts our ability to model the variation in the observed choice sets. We discuss these issues in detail in the next section.

Third, and perhaps most importantly, our approach to identification differs from state-of-the-art differentiated product models in the following fundamental way. Previous studies typically treat product specific characteristics as exogenous. Endogenous product characteristics are inherent in our framework in which key "product" characteristics such as peer quality and expenditures are determined by choices of students and colleges. Thus, in our approach, we do not exploit the variation in observed characteristics. Instead we exploit variation in fundamental differences in endowments and price caps. In estimation, we compute the equilibrium values of I_j, θ_j, and μ_j^y and thus treat characteristics such as instructional expenditures and peer quality measures as latent variables. Much of the recent literature on estimating differentiated products revolves around dealing with unobserved product characteristics. In our approach, it is irrelevant whether a product characteristic is observed or not. All product characteristics are treated as latent variables in estimation.[25]

Finally, most recent papers in industrial organization treat costs as latent variables and identify the parameters of the marginal costs function from the the first-order condition that prices, mark-ups, and marginal costs need to satisfy in equilibrium. This approach ultimately rests on functional form assumptions. Moreover fixed costs are typically not identified. In our data set total costs can be imputed with error from college level data. Identification thus exploits variation in observed costs and college sizes within the population of colleges.

One additional complication arises in our approach because we cannot compute equilibria of our model at the disaggregate level observed in our data. If we were able to compute equilibria for a large set of colleges, we would use the estimates of the college level cost function based on equation (4.1). Instead we construct an aggregated cost function that preserves the main features of the disaggregate cost function, as discussed in detail in the next section.

In summary, the approach used in this study to identify the parameters of the model differs significantly from previous approaches in the literature on differentiated products. We combine a rich cross-sectional data set of consumer choices with aggregate data on college characteristics. In contrast to previous studies, our aggregate data set allows us to impute total costs. Our micro data set contains variation in prices for each product. We have shown how to exploit this additional source of variation in the data to estimate the model. Moreover, our strategy for identification does not exploit the variation in (potentially endogenous) product characteristics. Instead we focus on the variation in more fundamental

[25] Most of the recently proposed estimators that deal with unobserved product characteristics typically rely on large J asymptotics. See for example Berry, Linton, and Pakes (2002). In contrast, our estimator of the demand side parameters only requires that N, the number of students in our sample, be large.

differences characterizing initial endowments and price caps across colleges. We assume that there are no unobserved differences in cost functions among colleges.[26] To exploit fully this feature of our approach, it would be desirable to estimate a model with a richer choice set allowing for unobserved differences in initial endowments or technology. We view this as important future research.

5 Data

5.1 College Data

We have assembled a comprehensive data base for public and private colleges and universities in the U.S. Peterson's conducts a survey of all colleges and universities, obtaining information on faculty resources, financial aid, the distribution of standardized test scores, and a host of other variables. We have supplemented this data set with information on educational expenditures and endowments from the NSF Web accessible Computer-Aided Science Policy Analysis and Research (WebCASPAR) database. The Peterson's database contains a total of 1868 four-year colleges and universities within the United States. We view our model as being better suited to characterizing private than public institutions. Public universities and colleges are, therefore, not included in our sample.[27] We also do not consider private colleges that are highly specialized, do not have a regular accreditation, have price caps of less than $6,000, or have missing values for key variables that are the focus of our analysis. This leaves us with a sample of 768 private universities and colleges.

Table 1 provides some descriptive statistics for the 768 colleges in our sample. In our model we differentiate between exogenous differences in fundamentals such as price caps and endowments, and endogenous differences in observed characteristics such as expenditures

[26] One of the advantages of the approach in Berry et al. (1995) is that is allows for unobserved differences in marginal costs in a simpler supply model with exogenous characteristics and no price discrimination.

[27] Given the presence of a substantial number of selective public institutions, this is an important simplification. Modifying our theoretical framework to reflect objectives and constraints of public institutions is an important task for future research. First, however, it seems prudent to investigate how well the framework we have developed is able to capture the admission and pricing decisions of private institutions.

Table 1: Descriptive Statistics: Private Colleges

Variable	Mean	Std Dev	Minimum	Maximum
Enrollment	1951.69	1935.26	66.00	19737.00
Market share	0.001302	0.0012911	0.000044	0.013167
Price cap	12226.89	3736.33	6000.00	22000.00
Endowment income	357.13	865.31	0.00	11125.88
Expenditures	4924.22	2309.43	847.24	17749.94
Imputed custodial costs	2894.64	1673.00	15.76	9069.95

Sample Size is 768

and peer quality measures. We find that there is large variation in endowment and price caps, as well as in observed characteristics.

One of the advantages of our data is that we can impute (custodial) costs by using the budget identity of our model. Table 1 shows that average custodial cost are approximately \$2894 with a standard deviation of \$1673 (in 1995 dollars). Based on the imputed costs, we can estimate cost functions for the colleges in our sample. First we estimate a simple quadratic cost function using OLS. We then consider alternative restricted least squares estimators which impose cost-minimizing school sizes of .0025 and .003 respectively. We find that these restricted estimators typically perform better than the unrestricted estimators. As part of our sensitivity analysis, we also consider simple IV and GLS estimators. Price caps and endowments are used as instruments for college size. Table 2 summarizes the main results.[28] We find that the parameter estimates are reasonably robust among all specifications. Given a quadratic cost function, we can define the optimal school size k^*as

[28]The results reported in Table 2 are based on the subsample for which imputed costs are strictly positive. Using the full sample lowers the cost estimates and increases the estimated standard errors. We also rescaled the Peterson's financial aid variable to make it comparable to our financial aid estimates based on the NPSAS sample. The main qualitative results about the shape of the average cost function are not sensitive to this adjustment.

Table 2: Average Custodial Cost Function

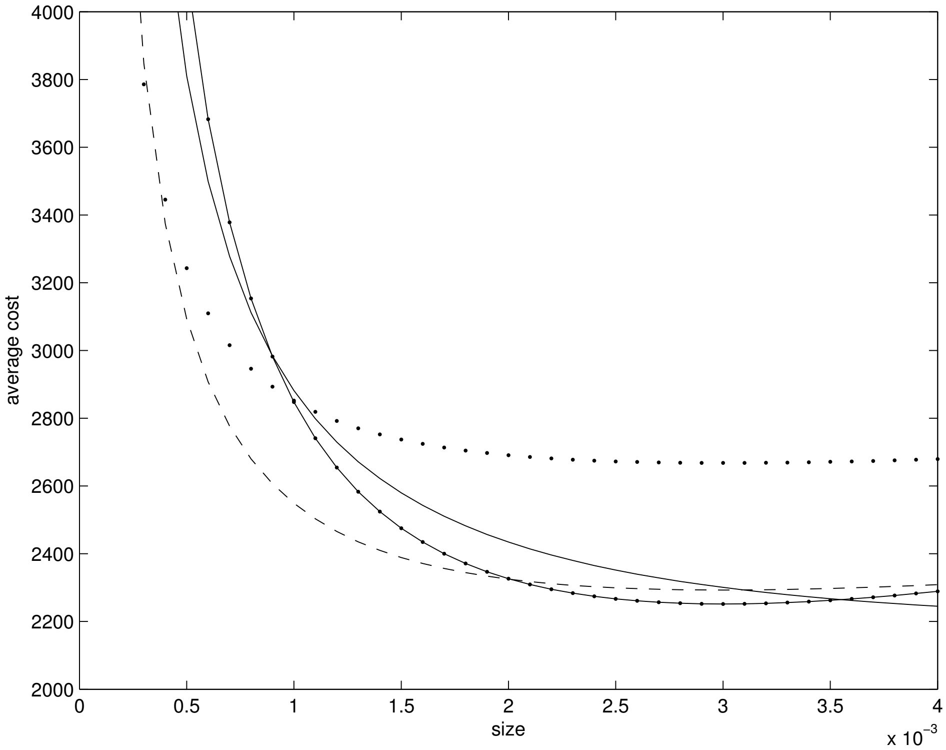
Parameter	Estimate	Std Error	t Value	$Pr > \|t\|$
	Model 1: OLS			
F	0.940	0.194	4.83	<.0001
c_1	1919	178	10.74	<.0001
c_2	22783	21716	1.05	0.2945
	Model 2: OLS: $F = [k^*]^2 c_2$ and $k^* = 0.003$			
c_1	1905	181	10.49	<.0001
c_2	64001	20069	3.19	0.0015
	Model 3: OLS: $F = [k^*]^2 c_2$ and $k^* = 0.0025$			
c_1	2242	172	13.01	<.0001
c_2	28602	22192	1.29	0.1979
	Model 4: IV: $F = [k^*]^2 c_2$ and $k^* = 0.003$			
c_1	1358	635	2.14	0.0330
c_2	148968	77015	1.93	0.0535
	Model 5: $F = [k^*]^2 c_2$ and $k^* = 0.003$			
c_1	2392	258	9.24	<.0001
c_2	45991	22343	2.06	0.0401
First stage of the IV regression: $R^2 = 0.089$.				

well as average and marginal costs as follows:

$$
\begin{aligned}
k^* &= (F/c_2)^{0.5} \\
AC &= C(k)/k = F/k + c_1 + c_2 k \\
MC &= C'(k) = c_1 + 2\, c_2 k
\end{aligned}
$$

Figure 2 plots the average cost curves implied by the estimates of four of the models. The figure illustrates that the estimated average cost functions are initially quite steep. But once college size is larger than 0.002 the curve flattens out considerably.

Figure 2: Average Cost Functions



Notation: Model 1 '—' , Model 2 '- -' , Model 4 '.- ', Model 5 '.'.

5.2 Market Structure and Aggregation

The college level cost function can, therefore, be estimated under alternative identifying assumptions. Identification and estimation of the remaining parameters of the model requires us to solve for the equilibrium of the model. There are approximately 768 colleges and universities in our data set. Solving our model for such a large value of J poses major computational challenges that we have not yet solved. By aggregating colleges with similar observed characteristics, we also abstract from a number of idiosyncratic factors such as regional preferences that may be important at a disaggregate level, but are likely to be less important at a more aggregate level. We first rank the 768 private colleges by their price caps. We then define six groups of colleges using the following aggregation rule:

$$\begin{array}{lrcl} \text{group 1} & 6000 \leq & p_j^m & \leq 11500 \\ \text{group 2} & 11500 < & p_j^m & \leq 13500 \\ \text{group 3} & 13500 < & p_j^m & \leq 15000 \\ \text{group 4} & 15000 < & p_j^m & \leq 18000 \\ \text{group 5} & 18000 < & p_j^m & \leq 20000 \\ \text{group 6} & 20000 < & p_j^m & \end{array}$$

This aggregation makes sure that schools within each group have similar price caps, which is important for defining financial aid.

Since we aggregate colleges into college types, we need to construct an aggregate cost function. To illustrate the basic idea, suppose there are n colleges in a group. Treating colleges within a group as identical permits us to derive the cost function for the group from the estimated parameters. Let $K = nk$ be the population of students served by the group of colleges. Then the cost function for the group can be written as a function of the population served:

$$\begin{aligned} C(K) &= n\, c(k) \\ &= n\, F \;+\; c_1\, n\, k \;+\; c_2\, n\, k^2 \end{aligned} \tag{5.1}$$

$$= \bar{F} + C_1 K + C_2 K^2$$

where $\bar{F} = nF$, $C_1 = c_1$, and $C_2 = \frac{c_2}{n}$. Following the preceding logic, we treat the cost function of a college group as quadratic, with the fixed cost component is proportional to the fixed costs of an individual college. To illustrate this approach consider the IV parameter estimates reported in Table 2. For these estimates, we obtain for $n = 100$, $\bar{F} = 134.10$, $C_1 = 1358$, and $C_2 = 1490$, and $k^* = 0.30$. In the next section, we also explore how sensitive the estimates of the remaining parameters of the model are to the choice of the cost function.

5.3 Admissions and Financial Aid

Our primary data source is the National Post-secondary Student Aid Study (NPSAS) obtained from the National Center for Education Statistics (NCES). The NPSAS contains extensive information for a sample of students. Of particular relevance for our work, the NPSAS contains the student's performance on standardized tests (either SAT or ACT), income of the student's family, and information about the financial aid received by the student. We have secured from the NCES a restricted-use version of the NPSAS that contains student-level data for 1995-96 and links each student in the sample to the college the student attended in academic year 1995-96. We study four-year private colleges and universities. For a given wave of the NPSAS survey, the NCES chooses a set of colleges and universities. It then selects a sample of students from within each of those institutions. Our sample consists of 1755 incoming freshman students.[29] We use SAT score as our measure of student ability, making the standard conversion for those taking the ACT. The mean SAT score is 1054 with a standard deviation of 196. Mean income is 60080 with a standard deviation of 39,455. The correlation between income and SAT score is 0.24. We use these estimates as parameters of the distribution of measured income and ability. We then estimate the model allowing for measurement error in both income and ability.

[29] In selecting our sample of students we deleted observations for students with athletic scholarships, since their criteria for admission may not conform to the spirit of our analysis.

Table 3: Descriptive Statistics by College Type

college number	number of students	market share	price cap	mean income	mean SAT score	input	inst. aid	other aid
1	377	0.284	9101	51267	957	3801	2564	4018
2	327	0.181	12692	57692	1011	5078	4803	3962
3	345	0.183	14423	62764	1060	6484	4964	4248
4	263	0.140	16175	64005	1076	6836	6972	4083
5	277	0.128	19074	67816	1132	9423	8651	4576
6	166	0.065	21820	73816	1237	10404	7898	5465

Expenditures and endowment data are from the NSF WebCASPAR database.

All remaining variables are from the NCES.

We then match each student in the NPSAS to one of the six groups defined above. Table 3 reports the weighted means for the six college types in our analysis.[30] The average price caps range from $9,101 in the lowest ranked college group to $21,820 in the highest ranked group. We find that there is a hierarchy in mean income that follows the ranking among colleges. Mean income ranges from $51,267 in the lowest ranked college to $73,816 in the highest college. The same hierarchy holds for tuition, institutional aid per student, expenditure per student, and endowment per student. Students receive substantial amounts of financial aid from the institution they attend. Mean institutional financial aid ranges from $2,564 to $8651.

Finally, we need a simple representation of the noninstitutional aid formula. Approximating this formula is challenging, however, because noninstitutional aid comes as federal aid, state aid, or private aid, and aid from each source comes in a variety of forms. Noninstitutional financial aid takes the form of grants, loans, work-study aid, and other forms.

[30]All empirical results reported in this paper use weights provided by the NPSAS to account for the sampling design used by the NCES.

Federal grants include PELL grants, supplemental educational opportunity grants and other grants and fellowships. Federal loans include Perkins and Stafford loans and loans through the Public Health Service. Work-study aid reflects aid under the Federal Work Study Program. Other federal grants include Byrd scholarships for undergraduates, and Bureau of Indian Affairs scholarships. There is also a multitude of state aid programs, such as the State Student Incentive Grants and state loans. Individuals may also receive private grants or loans that are not tied to a particular institution.

The NPSAS data includes a number of measures of noninstitutional aid received by the sampled students. One measure of noninstitutional aid, a_n, can be defined as a weighted sum of grants, g_n, loans, l_n, work-study aid, w_n, and other forms, o_n: $a_n = g_n \ + \ o_n \ + \ 0.25 \ l_n \ + \ 0.5 \ w_n$. The weights used in this formula are somewhat arbitrary, but are similar to ones used in the literature (Clotfelter, Ehrenberg, Getz, and Siegfried, 1991). We are interested in approximating a noninstitutional aid formula that expresses financial aid as a function of student and college characteristics. Most of the non-institutional aid is need-based. We would therefore expect that need-based non-institutional aid is primarily a function of income and tuition. We find that the vast majority of students in our sample receives some type of non-institutional financial aid. The fraction of students in each rank of colleges receiving positive amounts ranges from 0.9 to 0.95. Mean financial aid amounts are similar across college types as reported in Table 1. The mean aid ranges form \$3,490 to \$4,917.

To get some additional insights into the relationship between non-institutional aid and household characteristics, we estimated a number of reduced form models. Given that the vast majority of students receive positive aid, we use a simple regression framework. We pool the data for all students in our sample, capturing cross-college differences in non-institutional aid by inclusion of price caps as a variable. We then obtain the following non-institutional aid function: $a_{jn} \ = \ 3978 \ - \ 0.024 \ y_n \ + \ 0.121 \ p_j^m$. This function then yields federal financial aid payments that approximate those observed in the data. We found that merit aid was not significant.[31]

6 Empirical Results

6.1 Parameter Estimates

We estimate the remaining parameters of the model using full information MLE. We explored a number of different specifications of our model. Table 4 reports the parameters estimates and estimated standard errors for four versions of the model.[32] The first three specifications differ by the specification of the cost function. The results are shown in columns I through III of Table 4. In column IV we explore how sensitive our estimates are to the choice of the price caps. The specification of the cost function in column IV is the same as the one in column II, but assumes that price caps for all colleges are $1000 lower than the sample means used in specifications I through III.

We find that the estimates for γ, which measures the effect of peer ability in quality, are positive and range between 0.59 and 0.61. This finding supports our model's prediction that individuals sort based on the perceived quality of their peer group. We find that the estimates for ψ, the coefficient measuring the effect of relative income, are positive, ranging between 0.09 and 0.139. To attract students from lower-income backgrounds, colleges give financial aid that is inversely related to income as detailed below. The estimates of ω, the coefficient determining the demand for instructional expenditures, fall between 0.065 and

[31] In principle, we could also estimate the parameters of the federal aid formula simultaneously with the other parameters of the model. Given the various sources of non-institutional aid and the inherent complexities of allocating such aid, any formula used in empirical analysis will be a rough approximation of the procedures used in practice. The additional computational complexity of estimating three more parameters therefore does not seem to be justified.

[32] The estimated standard errors of the parameters are small, and this warrants comment. First, we are only estimating a small number of parameters using a sample of 1755 observations. Second, the asymptotic approximation used to compute standard errors could be a bad approximation of the finite sample distribution of the estimator.We also computed confidence intervals by inverting likelihood ratio tests. The results were not markedly different. Alternatively, one could invert Wald tests. However, it is well-known that Wald statistics are not invariant to scaling. Phillips and Park. (1988) suggest using Edgeworth expansions to compute better approximations of the small sample distribution. Finally, bootstrap techniques, which are frequently used to estimate standard errors, are computationally infeasible in our application. Given these limitation we cannot rule out the possibility that our standard errors are understated.

Table 4: Demand Side Parameter Estimates

	I	II	III	IV
Measurement Error Parameters				
$\sigma^e_{ln(y)}$	0.4500	0.4007	0.3981	0.4420
	(0.0007)	(0.0005)	(0.0007)	(0.0005)
σ^e_b	146.35	151.23	147.31	150.63
	(0.22)	(0.22)	(0.23)	(0.22)
σ^e_p	3480.90	3103.85	2932.08	3464.22
	(5.66)	(4.96)	(5.11)	(5.31)
Utility Function Parameters:				
γ	0.6161	0.6061	0.6038	0.5984
	(0.0009)	(0.0009)	(0.0009)	(0.0009)
ω	0.0801	0.0826	0.0855	0.0656
	(0.0001)	(0.0002)	(0.0001)	(0.0002)
ψ	0.0901	0.1296	0.1392	0.1222
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Likelihood function	30803	30749	30758	30713
Cost Function Parameters:				
I: $c_1 = 1358$, $c_2 = 2420$, $F = 134.10$, $k^* = 0.2354$.				
II: $c_1 = 1386$, $c_2 = 1920$, $F = 124.10$, $k^* = 0.2542$.				
III: $c_1 = 1358$, $c_2 = 1420$, $F = 124.10$, $k^* = 0.2956$.				
IV: $c_1 = 1386$, $c_2 = 1920$, $F = 124.10$, $k^* = 0.2542$. Price caps minus 1000.				
Estimated standard errors are given in parenthesis.				

0.085. Instructional expenditures are a substantial component of college quality, but with markedly lower elasticity than the peer measure.

The estimates in Table 4 also suggest that both income and ability are measured with error. The estimated standard deviation of observed ability (log-income) is 195 (0.605). The point estimates for the standard deviation of measurement error in the logarithm of income range between 0.398 and 0.450. Similarly, the point estimates for the standard deviation of the measurement error in ability are between 146 and 151. These estimates imply that there is a significant amount of variation in the data that is not explained by our model. Undoubtedly income is measured with error. Colleges often use measures of ability to determine admission and financial aid policies that are more comprehensive than simple SAT scores. Nevertheless, our estimates of the measurement components are large, suggesting that our simple equilibrium model may not capture some important aspects of admission and pricing.

6.2 Admission and Aid Policies in Equilibrium

The estimates reported in Table 4 correspond to equilibria in the market for higher education. Table 5 summarizes some of the key features of the equilibrium predicted by the estimates reported in Column II of Table 4. Recall that the variance of latent income (ability) is significantly less than the variance of observed income (ability). As a consequence the ability and income levels reported in Table 5 are not directly comparable to the ones in Table 3. Nevertheless, we find that our model replicates the main qualitative features of sorting by income and ability well. Our model also yield predictions for inputs per student although we treat inputs as latent variables in estimation. In equilibrium, we find that educational inputs per student range between 5357 and 9293. Our model thus explains the inputs for most schools well, except for the lowest ranked school.

We now turn our attention to financial aid policies. As explained in the previous section, the equilibrium of our model reflects noninstitutional financial aid received by students. Table 3 suggests that individuals in our sample receive on average \$4201 in noninstitutional

Table 5: Equilibrium

school	price cap	endowment income	school size	fraction at cap	mean income	mean SAT score	inputs	inst. aid	other aid
1	9101	86	0.289	0.293	58504	1001	5357	1458	3675
2	12692	204	0.200	0.000	53967	1040	6169	4335	4218
3	14423	301	0.183	0.000	58868	1062	6870	5439	4310
4	16175	493	0.153	0.000	62592	1083	7476	6700	4433
5	19074	751	0.121	0.000	66305	1107	8170	9010	4694
6	21820	2053	0.054	0.001	74132	1141	9293	10787	4839

aid. Table 5 shows that our model matches these levels of noninstitutional aid closely. The predicted mean of the distribution of noninstitutional aid is close to the observed mean in each college type.

A challenging part of our analysis is to explain institutional financial aid policies. We observe in the data that a large fraction of students receives quite substantial amounts of institutional aid. We would like to know whether our model can replicate these often generous financial packages. A comparison between Table 5 and Table 3 shows that our model explains observed financial aid well on average.

To get additional insights into the nature of price discrimination predicted by our model, we compute the shadow prices for ability and income. Recall that equation (2.21) implies that the shadow price for ability is given by $\frac{\partial q_j/\partial\theta}{\partial q_j/\partial I}$. Similarly, the shadow price for income is $\frac{\partial q_j/\partial\mu_j^y}{\partial q_j/\partial I}$. Table 5 reports the shadow prices for each college. We find that the shadow price for income ranges between -0.144 to -0.197. The estimated model predicts a \$10,000 increase in household income raises tuition to students at the margin of switching colleges by \$1440 in the lowest ranked college and \$1970 in the highest ranked college. Our model thus predicts that colleges will offer less financial aid to higher income students. Our model also predicts quite significant financial aid for higher ability students. The shadow price for ability ranges between 39.25 and 59.77. The estimated model predicts 100 SAT points lowers tuition to students at the margin of switching colleges by \$3925 and \$5977 at the bottom and top ends of the college hierarchy respectively.

Table 6: Shadow Prices for Ability and Income

college	shadow price of ability	shadow price of income
1	39.25	-0.144
2	43.50	-0.179
3	47.43	-0.183
4	50.64	-0.187
5	54.13	-0.193
6	59.77	-0.197

The ascension along the college quality hierarchy of the shadow values of ability obviously implies ability is valued at the margin more highly in better colleges, but also is sufficient for "ability stratification." As ability rises for given income, students that switch colleges will always attend higher quality colleges. This is straightforward to show using Assumption 6 on the utility function and that the allocation is consistent with EMC-pricing (i.e., Proposition 1.4). The ascension in absolute value of the shadow prices on income implies lower income is valued more highly at the margin as one moves up the college quality hierarchy. The latter effects on the EMC's are more than offset by the unitary income elasticity of demand for quality in (4.3), implying the boundaries of attendance sets are downward sloping where the price caps are not binding as in Figure 1.

These predictions are qualitatively in line with reduced form estimates. The magnitudes of the marginal effects of income and ability on financial aid are, however, larger than those found in reduced form studies. There are at least two explanations for our estimates. First, reduced form estimates often control for multiple measures of ability such as GPA in college and high college. Second and perhaps more importantly, ability is most likely measured with error, which biases reduced form estimates towards zero.

We compute a measure of market power by calculating the ratio of each college's tuition revenue to the revenue it would obtain if all students were charged effective marginal cost. We find that this measure of market power ranges from 1.001 in the lowest ranked college to 1.01 in the highest ranked college. We thus conclude that colleges have negligible market power. This finding also explains why we predict almost no students paying the price cap in the top five tiers of colleges (see Table 5). Excepting those in the bottom tier of colleges, students have access to the next lower-tier college as a close substitute at tuition equal to effective marginal cost there. This alternative depresses their reservation price at the college they attend, keeping it below the price cap.

6.3 Policy Analysis

Finally, we can use our model to perform policy experiments and evaluate education reform measures. There are a number of interesting policy questions that can be analyzed in this framework. Under the current non-institutional aid system, a large fraction of aid is given to families that are reasonably well off. Our estimates of the non-institutional aid formula suggests that families with income above the sample median receive a significant amount of non-institutional aid. An interesting policy simulation considers the effects of setting strict limits for the eligibility for non-institutional financial aid programs.[33].

The effects of increasing federal aid to low-income students are a central concern of policy. The objective of such policies is to increase college attendance by low-income students. It is sometimes alleged, however, that such aid largely has the effect of bidding up college tuitions, offsetting the intended improvement in college access for low-income students. Alternatively, might such aid get passed through to students in the form of increased grants

[33] Keane and Wolpin (1997) analyze the impact of a college tuition subsidy on school attainment and inequality. They find that such a policy had small effects because of unobserved heterogeneity in skill endowments. Carneiro, Hansen, and Heckman (2003) also consider a full subsidy to college tuition. They report that such a policy would primarily affect people at the top end of the high school earnings distribution. Neither paper considers the type of equilibrium effects considered in this study.

from colleges as they compete for students, again with minimal effect on improving college attendance among the poor, but for a quite different reason? Sorting out these three possible effects of government aid policies cannot be accomplished without recourse to an equilibrium framework.

To gain additional insights into the properties of the model and to measure the potential impact of changes in non-institutional aid policies, we consider the following policy experiment. We change the non-institutional aid policies such that households that are above a given threshold (which we set at one half standard deviation above mean income, $79,875) are ineligible for aid. We then adjust the level of aid to households below the threshold to keep average aid approximately constant. The following modified aid formula implements this new policy: $a_{jn} = 1\{y_n \leq 79875\}(4028 \; - \; 0.024 \; y_n \; + \; 0.121 \; p_j^m)$. Table 7 summarizes our main findings from this policy experiment.

Table 7: Equilibrium after the Policy Change

school	price cap	endowment income	school size	fraction at cap	mean income	mean SAT score	inputs	inst. aid	other aid
1	9101	90	0.277	0.270	61715	1005	5444	1381	3124
2	12692	205	0.198	0.000	52606	1036	5925	4579	4301
3	14423	298	0.184	0.000	60041	1063	6837	5471	4057
4	16175	485	0.155	0.000	60801	1078	7158	7019	4504
5	19074	735	0.124	0.000	63132	1102	7701	9480	4821
6	21820	1821	0.061	0.000	69004	1135	8543	11553	4893

Our simulations suggest that this policy change would lead to changes in the allocation of households among colleges and related behavioral adjustments of colleges. Comparing Tables 5 and 7, we see that with noninstitutional aid redistributed from richer to poorer students, the income effect causes some of the poorer students to attend higher-quality colleges and the reverse for some of the richer students.[34] Hence, mean income increases in the lower-quality colleges and decreases in the higher-quality colleges. Educational inputs per student move in the same direction as mean income and per student institutional aid moves in the opposite direction. Mean SAT is essentially unaffected.

There are a number of other policy questions that could be analyzed with our model. For example, the model could be used to investigate the effects of reducing the amount of federal aid given directly to colleges while increasing financial aid to students. Similarly, the model could be used to study the effects of targeting aid to lower quality colleges in order to offset a part of the existing differences in financial endowment. Still another policy issue that could be studied is re-orientation of non-institutional aid to place greater weight on merit.

7 Conclusions

We have developed an equilibrium model of the market for higher education. We have shown that this model has strong predictions regarding the sorting of students by income and ability among private colleges and the resulting financial aid policies. Our approach for identification and estimation accounts for the fact that important product characteristics are likely to be endogenous and that student characteristics are measured with error. The techniques developed in this paper allow for price discrimination and thus exploit an important source of variation in the data. We have estimated the structural parameters of the model using a combination of student-level and college-level data. The findings suggest that our equilibrium model replicates many of the empirical regularities observed in our data reasonably well.

One of the limitations of this framework has been that we needed to aggregate the choice set and estimate a model of college types. Given the current computational constraints, we focused on a model with only six types. To exploit fully the inherent advantages of

[34]We hold constant total endowment earnings of each college, so the change in per student endowment incomes between Table 5 and 7 reflect size changes.

the techniques developed in this paper, one would like to consider models with a richer choice set. There are two possible approaches. First, we can simplify the model, ignore price caps, and restrict pricing to be equal to effective marginal costs. Based on our current knowledge, it should be possible to estimate such a model with a much larger choice set given the current set of personal computers. Alternatively, one could explore the use of parallel computing techniques as suggested, for example, by Ferreyra (2003).

We view the methods developed in this paper and our main empirical results as promising for future research. An interesting extension would be to control for additional sources of observed heterogeneity. An extension of our model that controls for minority status is feasible. Epple, Romano, and Sieg (2002) discuss different strategies of incorporating minority status into a similar equilibrium model. Equilibrium of the model then depends on whether and how racial diversity measures enter the objective functions of colleges and the different types of households. Another focus of future research should be to analyze jointly the markets for public and private education. The benefits from modelling private and public colleges in equilibrium would be substantial from the perspective of policy analysis. It is not particularly difficult to include a public college sector as an outside option into our model. However, devising an equilibrium model that can explain the observed sorting of students among a set of public and private colleges is more complicated and will require significant modifications of the theoretical model used in this study.

References

Anderson, S. and de Palma, A. (1988). Spatial Price Discrimination with Heterogeneous Products. *Review of Economic Studies*, *55*, 573–592.

Arcidiacono, P. and Nickolson, S. (2000). The Mirage of Peer Effects: Evidence from U.S. Medical Schools. Working Paper.

Bajari, P. and Benkard, L. (2002). Demand Estimation With Heterogenous Consumers and Unobserved Product Characteristics: A Hedonic Approach. Working Paper.

Benabou, R. (1996). Heterogeneity, Stratification and Growth: Macroeconomic Effects of Community Structure. *American Economic Review*, *86*, 584–609.

Bergstrom, T., Rubinfeld, D., and Shapiro, P. (1982). Micro-Based Estimates of Demand Functions for Local School Expenditures. *Econometrica*, *50*, 1183–1205.

Berry, S. (1994). Estimating Discrete-Choice Models of Product Differentiation. *Rand Journal of Economics*, *25(2)*, 242–262.

Berry, S., Levinsohn, J., and Pakes, A. (1995). Automobile Prices in Market Equilibrium. *Econometrica*, *63 (4)*, 841–890.

Berry, S., Linton, O., and Pakes, A. (2002). Limit Theorems For Estimating Parameters of Differentiated Product Demand Systems. Working Paper.

Betts, J. and Morell, D. (1999). The Determinants of Undergraduate GPA: The Relative Importance of Family Background, High School Resources, and Peer Group Effects. *Journal of Human Resources*, *107*, 797–817.

Blundell, R., Browning, M., and Crawford., I. (2003). Nonparametric Engel Curves and Revealed Preferences. *Econometrica*, *71*, 205–240.

Brock, W. and Durlauf, S. (2001). Discrete Choice with Social Interactions. *Review of Economic Studies*, *68 (2)*, 235–260.

Carlton, D., Bamberger, G., and Epstein, R. (1995). Antitrust and Higher Education: Was There a Conspiracy to Restrict Financial Aid?. *Rand Journal of Economics*, *26*, 131–147.

Carneiro, P., Hansen, K., and Heckman, J. (2003). Estimating Distributions of Treatment Effects with an Application to the Returns of Schooling and Measurement of the Effects of Uncertainty on College Choice. *International Economic Review*, *44(2)*, 361–422.

Caucutt, E. (2002). Educational Policy When There Are Peer Group Effects - Size Matters. *International Economic Review*, *43*, 195–222.

Clotfelter, C., Ehrenberg, R., Getz, M., and Siegfried, J. (1991). *Economic Challenges in Higher Education*. University of Chicago Press. Chicago.

Dale, S. and Krueger, A. (1998). Estimating the Payoff to Attending a More Selective College: An Application of Selection on Observables and Unobservables. Working Paper.

deBartolome, C. (1990). Equilibrium and Inefficiency in a Community Model with Peer Group Effects. *Journal of Political Economy*, *98(1)*, 110–133.

Downes, T. and Greenstein, S. (1996). Understaning the Supply of Non-Profits: Modeling the Location of private Schools. *RAND Journal of Economcs*, *27(2)*, 365–390.

Ekeland, I., Heckman, J., and Nesheim, L. (2004). Identification and Estimation of Hedonic Models. *Journal of Political Economy*, *112(1)*, S60–S101.

Ellickson, B., Grodal, B., Scotchmer, S., and Zame, W. (1999). Clubs and the Market. *Econometrica*, *67 (5)*, 1183–1217.

Epple, D. and Romano, R. (1998). Competition Between Private and Public Schools, Vouchers and Peer Group Effects. *American Economic Review*, *88*, 33–63.

Epple, D. and Romano, R. (2003). Educational Vouchers and Cream Skimming. Working Paper.

Epple, D., Romano, R., and Sieg, H. (2002). On the Demographic Composition of Colleges and Universities in Market Equilibrium. *American Economic Review: Papers and Proceedings*, *92 (2)*, 310–314.

Epple, D., Romano, R., and Sieg, H. (2003). Peer Effects, Financial Aid, and Selection of Students into Colleges. *Journal of Applied Econometrics*, *18 (5)*, 501–525.

Epple, D., Romer, T., and Sieg, H. (2001). Interjurisdictional Sorting and Majority Rule: An Empirical Analysis. *Econometrica*, *69*, 1437–1465.

Fernandez, R. and Rogerson, R. (1998). Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform. *American Economic Review*, *88(4)*, 813–833.

Ferreyra, M. (2003). Estimating the Effects of Private School Vouchers in Multi-District Economies. Working Paper.

Fuller, W., Manski, C., and Wise, D. (1982). New Evidence on the Economic Determinants of Postsecondary Schooling Choices. *Journal of Human Resources*, *17*, 477–498.

Gilles, R. and Scotchmer, S. (1997). Decentralization in Replicated Club Economies with Multiple Private Goods. *Journal of Economic Theory*, *72*, 363–387.

Gourieroux, C. and Monfort, A. (1993). Simulation Based Inference. *Journal of Econometrics*, *59*, 5–33.

Hausman, J. (1985). The Econometrics of Nonlinear Budget Sets. *Econometrica*, *53(6)*, 1255–1282.

Hoxby, C. (2000). Does Competition Among Public Schools Benefit Students and Taxpayers? Evidence from natural Variation in School Districting. *American Economic Review*, *90(5)*, 1209–1238.

Keane, M. and Wolpin, K. (1997). The Carrer Decisions of Young Men. *Journal of Political Economy*, *105(3)*, 473–523.

Manski, C. (1991). Educational Choice (Vouchers) and Social Mobility. *Economics of Education Review*, *11 (4)*, 351–369.

Manski, C. (1993). Identification of Endogenous Social Effects: The Reflection Problem. *Review of Economic Studies*, *60*, 531–542.

McFadden, D. (1989). A Method of Simulated Moments for Estimation of Discrete Response Models without Numerical Integration. *Econometrica*, *57 (5)*, 995–1027.

Moffitt, R. (2001). Policy Interventions, Low-level Equilibria, and Social Interactions. In *Social Dynamics*. MIT Press.

Nechyba, T. (2000). Mobility, Targeting and Private School Vouchers. *American Economic Review*, *90*, 130–146.

Nevo, A. (2001). Measuring Market Power in the Ready-to-Eat Cereal Industry. *Econometrica*, *69(2)*, 307–342.

Pakes, A. and Pollard, D. (1989). Simulation and the Asymptotics of Optimization Estimators. *Econometrica*, *57*, 1027–1058.

Petrin, A. (2002). Quantifying the Benefits of New Products: The Case of the Minivan. *Journal of Political Economy*, *110(4)*, 705–729.

Phillips, P. and Park., J. (1988). On the Formulation of Wald Tests of Nonlinear Restrictions. *Econometrica*, *56 (5)*, 1065–1083.

Rothschild, M. and White, L. (1995). The Analytics of the Pricing of Higher Education and Other Services in Which the Customers are Inputs. *Journal of Political Economy*, *103*, 573–623.

Rouse, C. (1998). Private School Vouchers and Student Achievement: An Evaluation of the Milwaukee Parental Choice Program. *Quarterly Journal of Economics*, *113*, 553–602.

Sacerdote, B. (2001). Peer Effects with Random Assignment: Results from Dartmouth Roommates. *Quarterly Journal of Economics*, *116(2)*, 681–704.

Winston, G. (1999). Subsidies, Hierarchies, and Peers: The Awkward Economics of Higher Education. *Journal of Economic Perspectives*, *13*, 13–36.

Zimmerman, D. (2000). Peer Effects in Private and Public Schools: A Cross Country Empirical Analysis. Working Paper.

A Proof of Proposition 1

1. Suppose that $R_i > R_j$. We show a contradiction to either $q_i = q_j$ or $q_i < q_j$. In either of the latter conjectured equilibria, let college i follow the admission and tuition policies of college j and spend on inputs so as to have a balanced budget (which is feasible by Assumption 7). College i would attract the same student body as college j since i would be spending more on inputs than j and thus be of higher quality. Hence, we have a contradiction to quality maximization by college i in the conjectured equilibrium.

2. For students of type (b, y) attending both college j and college $i \neq j$ in equilibrium, it must be that $\alpha_i > 0$, $\alpha_j > 0$, and $\alpha_i + \alpha_j \leq 1$, the latter implied by market clearance. From the first-order condition (2.20), then $p_i = EMC_i$ and $p_j = EMC_j$. Utility maximization implies: $U(y + a_j - EMC_j, q_j, b) = U(y + a_i - EMC_i, q_i, b)$ for these students. Types (b, y) satisfying the latter equality have measure zero for generic utility and non-institutional aid functions. To confirm this, assume for simplicity that $a_i = a_j = 0$, and differentiate the indifference condition with respect to y and b:

$$\left[\frac{\partial U^j}{\partial x}\left(1+\frac{\partial q_j/\partial \mu_j^y}{\partial q_j/\partial I_j}\right)-\frac{\partial U^i}{\partial x}\left(1+\frac{\partial q_i/\partial \mu_i^y}{\partial q_i/\partial I_i}\right)\right] dy \quad = \\ \left[\frac{\partial U^i}{\partial x}\left(\frac{\partial q_i/\partial \theta_i}{\partial q_i/\partial I_i}\right)-\frac{\partial U^j}{\partial x}\left(\frac{\partial q_j/\partial \theta_j}{\partial q_j/\partial I_j}\right)+\frac{\partial U^i}{\partial b}-\frac{\partial U^j}{\partial b}\right] db,$$

where the superscripts on U indicate at what values the function is evaluate. Unless both bracketed terms vanish for a positive measure of types (b, y), the result follows. Since $q_i \neq q_j$ by Proposition 1.1, Assumption 6 implies that $\frac{\partial U^j}{\partial x} \neq \frac{\partial U^i}{\partial x}$. Moreover, the shadow price on diversity (i.e. the other terms in the brackets on the left-hand side of the indifference condition) are determined independently of $\frac{\partial U^j}{\partial x}$ and $\frac{\partial U^i}{\partial x}$. Hence the bracket term on the left-hand side will not vanish generically. The analogous argument applies among those (b, y) types indifferent between a college and the outside option. Introducing non-institutional aid functions does not alter the argument unless the aid functions are contrived to make the bracketed terms vanished. hence, almost every student (b, y) attends the same college or no college.

3. For all types attending college j, $p_j(b, y) \geq EMC_j(b, y)$, by the admission criterion (2.20). To show a contradiction, suppose that $p_j(b, y) > EMC_j(b, y)$ for student of minimum ability b' for given y. Since both $p_j(b, y)$ and $EMC_j(b, y)$ are continuous in b, there exists types with $b < b'$ and $p_j(b, y) > EMC_j(b, y)$. For these types $\alpha_j(b, y) = 1$, in equilibrium which would contradict market clearance since their attendance elsewhere implies $\alpha_j(b, y) + \alpha_i(b, y) > 1$ for some i.

4. Suppose not. Then there exists a set with positive measure of student types Z attending college i (or choosing the outside option as discussed below) such that:

$$U(y + a_i - EMC_i, q_i, b) \quad < \quad U(y + a_j - EMC_j, q_j, b)$$

   for all students in Z and some college $j \neq i$ in $\hat{J}(b, y)$ (or, in case of the outside option on the right-hand side, having $j = 0 = EMC_j = a_j$). Fix $(b, y) \in Z$. Since $p_i(b, y) \geq EMC_i$,

$$U(y + a_i - p_i, q_i, b) \ \leq \ U(y + a_i - EMC_i, q_i, b) \quad < \quad U(y + a_j - EMC_j, q_j, b).$$

   This implies that $r_j(b, y) > EMC_j(b, y)$ and since $p_j^m \geq EMC_j(b, y)$, that $p_j(b, y) \geq EMC_j(b, y)$. If $p_j(b, y) > EMC_j(b, y)$, then $\alpha_j(b, y) = 1$ contradicting market clearance (since $\alpha_i(b, y) > 0$.) If $p_j(b, y) = EMC_j(b, y)$, then utility maximization is violated since all students in Z strictly prefer j. Set $EMC_i \equiv a_i \equiv 0$ and $q_i = q_0$ if i is the outside option, and the proof goes through. (If j is the outside option,then the utility maximization contradiction necessarily arises.)

B A Sketch of the Equilibrium Mapping

In this section we sketch the construction an equilibrium-correspondence which maps $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$ into $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$. Let $x' = T(x)$ denote the equilibrium mapping where $x \in X \subset R^{4J}$. Define $X = [0,1]^J \times [0, I_{max}]^J \times [0, \theta_{max}]^J \times [0, \mu^y_{max}]^J$. We can show that the underlying set X, on which we define the local equilibrium mapping, is a non-empty, compact, and convex subset of R^{4J}.

We assume that quality of the outside option, q_0, and hence the utility $U_0(y, q_0, b)$ is fixed. The number of household choosing the outside option is given by $k_0 = 1 - \sum_{j=1}^{J} k_j$.

To define the equilibrium mapping, we partition the underlying space X into two sets. The first contains the points at which the measure of households (b, y) for which the set of colleges that are the best choice has more than one element is zero. This set is denoted by X^{ni}. The second set of points are the complement of that set, denoted by X^i. We can proof that for $x \in X^i$, there exist at least two colleges i and j which are identical in k, I, θ, and μ^y.

For any point $x \in X^{ni}$, we define a mapping, $x' = T(x)$ which maps X^{ni} into X. Denote with $k' = T_k(x)$ the projection of $T(x)$ on k. Similarly define the projections $I' = T_I(x)$, $\theta' = T_\theta(x)$ and $\mu^{y'} = T_{\mu^y}(x)$. The local equilibrium mapping is constructed as follows:

1. Given x, we have θ_j, I_j, k_j and μ^y_j for each college, including k_0 for the outside option.
2. Attendance is as if $p_j = EMC_j(b, y)$, for the relevant choice set of colleges.
3. The choice set of a household (b, y) is given by all colleges such that: $EMC_j(b, y) \leq p^m_j$.
4. For each college in choice set compute utility:
   $U_j \; = \; U(y - EMC_j + a_j(EMC_j),\; q_j,\; b)$.
5. Rank colleges by utility and determine first best (fb) and second best (sb) choices for (b, y) and compute attendance sets for each college.
6. Given the attendance sets of each college, compute θ', k', $\mu^{y'}$, and k'_0
7. Compute utility of best alternative (second best choice). If the second best is another college, the reservation utility is given by:
   $U^A(b, y) \; = \; U(y - EMC_{sb} + a_{sb}(EMC_{sb}),\; q_{sb},\; b)$.
   If the second best is the outside option, $U^A(b, y) = U(b, y, q_0)$.

8. Compute reservations price, r_{fb}:

   $U(y - r_{fb} + a_{fb}(r_{fb}),\ q_{fb},\ b) \ = \ U^A(b, y).$

9. Compute price by comparing reservation price with price cap:

   $p_{fb}(b, y) \ = \ \min\{r_{fb}, p^m_{fb}\}.$

10. Compute tuition revenues by integrating reservation prices over attendance sets. Compute I'_j for each college using the budget identity. If $I'_j < 0$, set $I'_j = 0$

This mapping is well defined since the set of households that are indifferent between their first and second best choice has measure zero. In that case, we can arbitrarily assign the indifferent households to one of the two colleges. This will not affect the mapping since it is invariant to changes on a set with measure zero. Thus we have that for each $x \in X^{ni}$, $T(x)$ is a function.

For points $x \in X^i$, there exists a set of households with positive measure that will be indifferent between the first best and the second best. To keep the notation simple, consider the case in which there are two college i and j for which there exists a set of households

$$I_{i,j} = \{(b, y)|\ V_i(b, y|x) = V_j(b, y|x) > \max_{k \neq i,j} V_k(b, y|x)\} \tag{B.1}$$

with $P(I_{i,j}) > 0$ where $V_i(b, y|x)$ is utility for type b, y in college i given x. Households that are indifferent can be arbitrarily assigned to any of the colleges in the household's preferred set. Thus any set of admission policies $\alpha_i(b, y|x)$ and $\alpha_j(b, y|x)$ that satisfies:

$$\alpha_i(b, y|x) \ + \ \alpha_j(b, y|x) \quad = \quad 1 \quad \forall (b, y) \in I_{i,j} \tag{B.2}$$

is feasible and satisfies local equilibrium requirements. Each feasible assignment rule will thus yield a value for $T(x)$. As a consequence, we have for each $x \in X^i$, $T(x)$ is a correspondence.

A fixed point of this mapping will be an allocation that satisfies a) market clearing; b) utility maximization; and c) the FOC's of the college optimization problems. To prove existence, we need to show that, under suitable regularity conditions, the underlying set X is a non-empty compact and convex subset of R^{4J}, and T is a compact and convex-valued upper hemi-continuous correspondence from X into X. Existence of a fixed point then follows from Kakutani's Fixed Point Theorem.

C Global Optimality

To check whether our local equilibrium is in fact an equilibrium, it does not seem to be possible to check all possible deviations. Instead we use the following algorithm.

1. Consider a possible deviation by school j:
2. Hold the choices $(k_i, \theta_i, I_i, \mu_i^y)$ for $i \neq j$ at the levels of the old local equilibrium.
3. That defines the reservation utility function of all students that may want to attend j.
4. Consider a quality improvement for school j, i.e. values $\tilde{k}_j, \tilde{\theta}_j, \tilde{I}_j, \tilde{\mu}_j^y$ such that $\tilde{q}_j > q_j$
5. Using $\tilde{k}_j, \tilde{\theta}_j, \tilde{I}_j, \tilde{\mu}_j^y$ as starting values, solve the system of 4 equation for school j, i.e. try to find a new local equilibrium for school j
6. There are three possible outcomes:
   a) The algorithm will converge to our old local equilibrium.
   b) The algorithm will fail to converge from the new starting values.
   c) The algorithm will converge to a new local equilibrium and thus indicate failure of global optimality.

Finally, by randomly perturbing the quality improvements in step 4, we trace out alternative local equilibria. Performing a substantial number of such experiments, we found that large deviations tended to provide the second outcome while more modest deviations produced the first outcome. In no case was there a convergence to a new local equilibrium. This suggests that colleges are at the global optimum, and hence the allocation computed satisfies all conditions of equilibrium.