Comparative Advertising

Simon P. Anderson[*] and Régis Renault[†]

This version August 2005.
PRELIMINARY: COMMENTS WELCOME

Abstract

A firm introduces a product and consumers do not know how much they value it, while they know how they value a rival's product. If the product is low quality, the firm will advertise detailed product information that enables consumers to determine their matches: it will not if it is high quality. By contrast, the rival would like to advertise match information about the unknown product only if it is high quality. Such "comparative" advertising may improve welfare if qualities are similar, although it may deter desirable entry of high quality products. We extend the model to allow consumers to not know quality unless it is advertised (consumers know there is a positive probability that neither firm knows quality: this circumvents quality unravelling) Then a high quality firm will advertise quality, and the rival will advertise match information. The reverse happens for a low quality entrant. The policy implications for comparative advertising are discussed.

Keywords: persuasion game, advertising, search, content analysis, information, quality, comparative advertising

JEL Classification: D42 L15 M37

Acknowledgement 1 *We gratefully acknowledge travel funding from the CNRS and NSF under grants INT-9815703 and GA10273, and research funding under grant SES-0137001. We thank various seminar participants and IDEI (University of Toulouse I), the University of Montpellier and Melbourne Business School for their hospitality.*

[*]Department of Economics, University of Virginia, PO Box 400182, Charlottesville VA 22904-4128, USA. sa9w@virginia.edu

[†]ThEMA, Université de Cergy-Pontoise, 33 Bd. du Port, 95011, Cergy Cedex, FRANCE and Institut Universitaire de France. regis.renault@eco.u-cergy.fr

1 Introduction

Until the late 1990s, mentioning a competitor's brand in an ad was illegal in many EU countries. This situation was ended by a 1997 EU directive that made "comparative advertising" legal subject to the restriction that it should not be misleading. This brought the European approach closer to that of the FTC in the US. The rationale for such a favorable attitude towards "comparative advertising" on the part of competition authorities is that it improves the consumers' information about available products and prices (see Barigozzi and Peitz, 2005, for details). This raises a number of questions for the economic analysis of informative advertising. What is the scope of such a practice to the extent that it involves disclosure of information that the product's supplier would choose no to reveal: what are situation where a firm chooses to conceal information that competitors would then reveal through comparative ads? Regarding welfare, the benefit to consumers resulting from improved information may be mitigated by a welfare loss for competitors who are presumably hurt by comparative advertising pertaining to their own product? There may also be an undesirable impact on firm pricing especially if comparative advertising pertains to product attributes rather than prices. Here we consider a game between rival firms and their incentives to provide information.

We assume that there are two firms. One firm's product is known to consumers. Specifically, consumers know the characteristics of that firm's product, but do not know those of the other firm's product. This could be because one product has been around long enough that consumers are aware of its specifications. The setting could also be interpreted as a situation where one firm that is advertising its product's attributes for some exogenous reasons decides on whether or not to provide information about its competitor's product when comparative advertising is allowed. The firm producing the known product is fully aware of the other product's attributes, but consumers are not, although they have (correct) priors about their evaluations of it. They may aquire the missing information through search in setting that is similar to that used in Wolinsky (1986) aand Anderson and Renault (1999) and (2000). If comparative advertising is not permitted, then it is solely up to the firm with the unknown product what to communicate about its good. If comparative advertising is permitted, then the other firm can also inform consumers about some product attributes that perhaps its rival product does not wish to communicate. The analysis also treats the welfare economics of comparative advertising.

In much of the literature on informative advertising, sending ads is costly and it is the cost of advertising that limits the information transmission by firms (see Bagwell, 2002, and references therein). In Anderson and Renault (2005) we explore the possibility that a firm might choose to provide limited information about its product attributes even if advertising has no cost. This result is a starting point for the present paper because it identifies situations where a firm is hurt by information disclosure about its own product, so that there might be some incentives for competitors to provide that information through comparative ads. The paper is novel in several dimensions. First, by looking at the content of advertising, it allows a description of comparative advertising. Second, it considers equilibrium advertising content choice in a competitive setting whereas Anderson and Renault (2005) assumes a monopoly situation. The asymmetry, which is presumably the germane case in most market situations, is crystallized in asymmetric information of consumers.

We first consider the disclosure of horizontally differentiated attributes that are valued differently by different consumers, assuming that the unknown product's quality is known. We find that the seller of the unknown product would choose to disclose horizontal attributes if the product's quality is low whereas a high quality firm would not. If comparative advertising is allowed, then the competing firm will disclose the horizontal attributes of the unknown product when the unknown product's quality is high enough that the firm selling it would not disclose such information. Then we allow for disclosure of the unknown product's quality and find that it is disclosed by the unknown firm only if it is high enough whereas the competing firm will disclose it if comparative advertising is allowed and quality is low.

We first describe the model in section 2 and provide some preliminary results in Section 3. Sections 4 and 5 describe demand with and without product information respectively. The equilibrium information disclosure is characterized in Section 6 and Section 7 describes the extension to quality information disclosure.

2 The model

Consumers are interested in buying one unit of one of two goods, which are sold by separate firms. Each product's specification is summarized by a position, "location" on a circle, one for each product. Consumer valuations for each good are determined by their positions on the relevant circle, and a consumer's position on one circle is independent of the position on the other circle. Consumers are uniformly distributed around each circle, and distance disutility ("transport") costs are linear in distance, understood as the closer arc distance between consumer and product. This means that, for a given product positioning, the consumers' gross valuations for the products are uniformly distributed on an interval $[a, b]$, and we suppose for the sequel that a is large enough that all consumers buy one of the two products.[1] We can also normalize $b - a = 1$, and, having made this assumption, we can further set $a = 0$ and so $b = 1$, with the understanding that each consumer will always buy one of the two products (and so we can ignore non-purchase).

There is a known product, sold by Firm 0. This means that this product's specification is known in advance to all consumers. They therefore know their evaluations of this product, in advance of any advertising. There is a visit cost of c_0 for consumers who visit Firm 0 and

[1] To see this, suppose that the gross valuation of a consumer located a distance x away from the product is $b - tx$, where b is interpreted as the reservation price and t is the transportation cost. If the circle has circumference L, valuations are uniformly distributed on $\left[b - \frac{tL}{2}, b\right]$. Hence we set $a = b - \frac{tL}{2}$, and the width of the support of the consumer valuations may be interpreted as the degree of product differentiation.

visiting a firm is required in order to buy its product.

Absent advertising, consumers are unsure of the product specification of product 1 in its circle, and so do not know their valuations. However, they know its location is uniformly distributed, so that any consumer, given her location on the circle, knows that the probability her valuation is below r_1 is $P(r_1) = 1 - r_1$, independent of her (known) valuation r_0. The visit cost for consumers who visit Firm 1 is c_1. We shall in the sequel be able to interpret the visit costs as equal, and different values of c_0 and c_1 will be interpreted as inherent differences in product qualities (so $c_0 > c_1$ will be interpreted as an unknown product of higher "quality" than the competing product).

Firms are able to advertise the prices they charge, and Firm 1 is allowed to advertise its product specification if it so wishes. Doing so will allow consumers to know their realizations of r_1 without visiting Firm 1. If Firm 0 is allowed to advertise Firm 1's product specification and does so, again consumers know their realizations of r_1. We refer to the situation of one firm advertising its rival's product specification as comparative advertising. Even though there is nothing untruthful in it, it may be information that Firm 1 would choose not to reveal on its own. Now, if there is no information on Firm 1's product specification (and hence the value of r_1), consumers must form (rational) expectations of their expected benefits from visiting Firm 1 (at cost c_1) given that they anticipate the option of later visiting and buying from Firm 0.

The search model and demands are given in section below, but we first pause to establish some results that are useful for the later analysis.

3 Some preliminary results

We first give some results for duopoly pricing that are important to the analysis that follows. Since they are used quite extensively, we give them first. The demand curves considered below will satisfy the properties used in these results.

Suppose that there are two products. They are priced at p_0 and p_1. Demand for product 1 depends in an increasing manner on a variable called $\tilde{\Delta}$, which represents the full-price advantage of product 1. It is useful for what follows to write the full price of product i as $p_i^f = p_i + c_i$, and so $\tilde{\Delta} = p_0^f - p_1^f$.

Let then the demand for Firm 1's product be $D_1(\tilde{\Delta})$ and so this is an increasing function. Let the demand for Firm 0's product be given by $D_0 = 1 - D_1$, representing the idea that there is a unit mass of consumers, and each consumer buys one unit of the good from one firm or the other.

We now further assume that $D_1(0) = \frac{1}{2}$ so that firms have an equal share of the market when full prices are equal.

Then we have the following result.

Lemma 1 *If $c_0 = c_1$, then equilibrium prices satisfy $p_1 = p_0 = \frac{D_0(0)}{-D_0'(0)}$ and full prices are equal. If $c_1 > c_0$, then equilibrium prices satisfy $p_1 < p_0$ and full prices satisfy $p_1^f > p_0^f$, with equilibrium sales satisfying $D_1 < D_0$. The opposite pattern arises for $c_0 > c_1$.*

Proof. The first order conditions are $p_i = \frac{-D_i(\tilde{\Delta})}{D_i'(\tilde{\Delta})}$. Since in any equilibrium, $D_0'\left(\tilde{\Delta}\right) = D_1'\left(\tilde{\Delta}\right)$, then higher prices are associated to higher demands. But this means that, since we have $D_0(0) = 1/2$, the firm with the lower demand has a higher full price. That is, $p_1 < p_0$ holds if and only if $D_1 < D_0$ and if and only if $p_1^f > p_0^f$. Taking the first and last inequalities, this can be only true if $c_1 > c_0$. The other results follow immediately along similar lines. ■

This result means intuitively that a cost (or quality) disadvantage is reflected in a lower mark-up and yet lower demand since the cost disadvantage is only partially "passed on."

The idea motivating the following Lemma is that we later will compare situations where firms can induce different demands through information revelation.

Lemma 2 *Suppose that* $D_1(0) = \widetilde{D}_1(0)$*,* $D_1'(0) = \widetilde{D}_1'(0)$*, and that* $\tilde{\Delta}[D_1(\tilde{\Delta}) - \widetilde{D}_1(\tilde{\Delta})]$ *does not switch sign. Further assume that corresponding revenue functions are quasi-concave. Let* p_0 *be some price charged by Firm* 0*, and let* $\tilde{\Delta}^*$ *be Firm* 1*'s full price advantage at its best response given demand* D_1*.*

- *(i) If* $D_1(\tilde{\Delta}^*) > \widetilde{D}_1(\tilde{\Delta}^*)$*, then the revenue attained under demand* D_1 *is larger than the largest that could be achieved with demand* $\widetilde{D}_1$*.*

  (ii) Conversely if $D_1(\tilde{\Delta}^*) < \widetilde{D}_1(\tilde{\Delta}^*)$ *then Firm 1 could attain a larger revenue with demand* $\widetilde{D}_1$*.*

Proof. The proof of (ii) is obvious. To establish (i), note that marginal revenue is the same with both demands when evaluated at $\tilde{Delta} = 0$. Now suppose, for instance, $\tilde{Delta}^* > 0$, corresponding to a lower price p_1 then, since revenue is quasi-concave, marginal revenue for both demands is negative at $\tilde{Delta} = 0$. Thus, revenue under demand $\widetilde{D}_1$ is also maximized for $\tilde{Delta} > 0$. Now, in accordance with part (i) assume that $D_1(\tilde{\Delta}^*) > \widetilde{D}_1(\tilde{\Delta}^*)$, and thus $D_1(\tilde{\Delta}) > \widetilde{D}_1(\tilde{\Delta})$ for all $\tilde{Delta} > 0$. Thus the maximum revenue that can be achieved with demand $\widetilde{D}_1$ is strictly less than with demand D_1. A symmetric argument proves the result for $\tilde{Delta}^* < 0$ ∎

The last part is straightforward since demand $\widetilde{D}_1$ dominates D_1 when evaluated at the optimal best response for the latter demand curve, it must be strictly more profitable. The other part is more subtle because, while it is true that $\widetilde{D}_1$ is less attractive at equal prices, the option remains of moving onto the part where it dominates D_1. We show this is not profitable.

Figure 1 illustrates the argument for the proof of the first item in the Lemma for $\tilde{\Delta}^* > 0$. Since both demands have the same slope at 0, if the iso-revenue curve going through $(0, 1/2)$ lies below the D_1 demand curve as $\tilde{\Delta}$ becomes positive (which must be the case in order for $\tilde{\Delta}^*$ to be positive) then it must also lie below demand curve $\widetilde{D}_1$; this implies that the best reply to p_0 with demand $\widetilde{D}_1$ is such that $\tilde{\Delta} > 0$ so that the corresponding profit may not be higher than with demand D_1.

4 Perfect information

We first use Lemma 1 above to characterize the equilibrium outcome under perfect information that will prevail if information about Firm 1's product is disclosed through advertising. Then a consumer purchases product 1 if and only if

$$r_1 - p_1 - c_1 \geq r_0 - p_0 - c_0$$

or equivalently

$$r_1 \geq r_0 - \tilde{\Delta}.$$

Recalling that $r_0 \in [0, 1]$, it follows that demand for Firm 1 is given by

$$\widetilde{D}_1(\tilde{\Delta}) = \int_{\max\left(\tilde{\Delta},0\right)}^{\min\left(1+\tilde{\Delta},1\right)} [1 - r_0 + \tilde{\Delta}]dr_0 + \max\left\{\tilde{\Delta}, 0\right\},$$

and $\widetilde{D}_0(\tilde{\Delta}) = 1 - \widetilde{D}_1(\tilde{\Delta})$.[2] Notice that $\tilde{\Delta} \in (-1, 1)$ for both firms to have positive demands. In summary:

For $\tilde{\Delta} < -1$, Firm 1's demand is $\widetilde{D}_1(\tilde{\Delta}) = 0$.

In the case $-1 < \tilde{\Delta} < 0$, Firm 1's demand becomes:

$$\widetilde{D}_1(\tilde{\Delta}) = \frac{(1 + \tilde{\Delta})^2}{2} \tag{1}$$

For $0 < \tilde{\Delta} < 1$, Firm 1's demand is

$$\tilde{D}_1(\tilde{\Delta}) = \frac{1 - \tilde{\Delta}^2}{2} + \tilde{\Delta}. \tag{2}$$

[2] Demand can be visualized as the area of the unit square of consumer valuations accorded to each firm. The division line (indifferent consumer type) satisfies $r_1 = r_0 - \tilde{\Delta}$, which is a diagonal line. If $\tilde{\Delta} > 0$, Firm 1 attracts all those consumers for whom $r_0 < \tilde{\Delta}$ irrespective of their valuation of r_1: hence the final term $\max\left\{\tilde{\Delta}, 0\right\}$ in the demand function.

For $\tilde{\Delta} > 1$, Firm 1's demand is $\tilde{D}_1(\tilde{\Delta}) = 1$.

Demand is convex for $\tilde{\Delta} < 0$ (high prices for Firm 1) and concave for $\tilde{\Delta} > 0$ (low prices). The demand derivative is continuous on its support, and so there is no kink.

Since evaluations for the two products are i.i.d. we have $\tilde{D}_1(0) = \frac{1}{2}$, and all assumptions of Lemma 1 are satisfied. Thus the firm for which the visit cost is the lower will charge the lower full price and thus garner a larger share of demand, even if it charges the higher price.[3]

We next find the demands under no advertising. Doing so allows us to directly find conditions for equilibrium advertising behavior without having to find equilibrium prices. However, we need the latter for the later welfare analysis.

5 No advertising

Here we characterize the demand that ensues when neither firm advertises Firm 1's product specification. This describes an equilibrium when either both firms are allowed to advertise Firm 1's product specification (and this is technologically feasible, of course) and neither wishes to do so, or else only Firm 0 can advertise this information, and it does not wish to. We also refer to this case as "price only" information. Note that if prices are not advertised they are correctly anticipated in equilibrium. Since there is no cost of advertising, a firm might as well advertise its price. We must first determine the allocation of consumers to firms in such a situation.

Suppose that the firms set observable prices, p_0 and p_1. Then consider the problem of a consumer who knows her valuation r_0 (as we assume, these are known already) and must decide whether or not to check out Firm 1. She knows that once she goes there and uncovers

[3]Any best reply price for Firm 1 must satisfy $p_1 \in [0, p_0 + c_0 - c_1 - 1]$, where the upper bound is where Firm 1's demand disappears. Hence Firm 1's profit is a continuous function that is defined over a compact set, and so has a maximum. Equilibrium existence follows from Caplin and Nalebuff (1991).

her true valuation, r_1, she will buy the other product zero, if

$$r_1 - p_1 \geq r_0 - p_0 - c_0. \tag{3}$$

Clearly any consumer for whom $r_0 \geq b - p_1 + p_0 + c_0$ will never countenance a visit because nothing can ever be gained (even if the cost, c_1, of searching Firm 1 were zero).

Note from (3) that there is an extra cost in going to Firm 0 once the consumer is already at Firm 1, which is a source of (hold-up) market power for Firm 1; however, there is a similar power for Firm 0 stemming from the initial cost of going to 1. A priori, it is unclear which way these forces will net out, but we show below that either firm may benefit from this structure.

Hence, the expected benefit to a consumer with valuation $r_0 < b - p_1 + p_0 + c_0$ from going to Firm 1 to check it out is

$$\int_{\max\{0, r_0 - p_0 - c_0 + p_1\}}^{1} (r_1 - (r_0 - p_0 - c_0 + p_1))\, dr_1,$$

where the lower limit is the lowest value of r_1 that the consumer "holding" valuation r_0 will switch for: the consumer will indeed visit if the benefit given above exceeds the search cost c_1. When interior, the indifferent consumer who visits is then characterized by a critical value, $\hat{r}_0$, that satisfies

$$\int_{\max\{0, \hat{r}_0 - p_0 - c_0 + p_1\}}^{1} (r_1 - (\hat{r}_0 - p_0 - c_0 + p_1))\, dr_1 = c_1,$$

and this value therefore determines an extensive margin in the sense that all consumers with $r_0 > \hat{r}_0$ never visit 1.[4] It is convenient to define $\hat{x} = (\hat{r}_0 - p_0 - c_0 + p_1)$ as the critical adjusted valuation: note then that the term $\hat{r}_1 = \max\{0, \hat{x}\}$ corresponds to the lowest feasible value of r_1 that the type holding $\hat{r}_0$ will switch for. It is defined by

$$\int_{\max\{0, \hat{x}\}}^{1} (r_1 - \hat{x})\, dr_1 = c_1.$$

[4] If the resulting $\hat{r}_0$ is less than 0, then no-one visits Firm 1. Conversely, if the $\hat{r}_0$ value exceeds 1, then everyone checks out Firm 1.

which does not depend on prices set by firms and we have $\hat{r}_0 = \hat{x} + p_0 + c_0 - p_1 = \hat{x} + \tilde{\Delta} + c_1$, which is the marginal searcher's identity (when interior to $(0,1)$). It is readily verified that fewer consumers visit Firm 1 if c_1 increases.

This simplifies to

$$\hat{x} = 1 - \sqrt{2c_1},$$

if $c_1 \leq 1/2$ and

$$\hat{x} = \frac{1}{2} - c_1$$

if $c_1 > 1/2$.

Restricting attention to case where demand is strictly between 0 and 1, we have for demand that:

$$D_1(\tilde{\Delta}) = \int_{\max\{0,\tilde{\Delta}+c_1\}}^{\min\{\hat{r}_0,1\}} \left(1 - r_0 + \tilde{\Delta} + c_1\right) dr_0 + \max\{0, \tilde{\Delta} + c_1\}.$$

For $\tilde{\Delta} \geq -c_1$ (which arises if the entrant's price is low relative to the "full price" – i.e., price plus search cost – of going to the incumbent) there are two sub-cases depending on whether $\hat{r}_0 > 1$ or not. The case $\hat{r}_0 > 1$ arises when $\tilde{\Delta} > \min\left\{\frac{1}{2}, \sqrt{2c_1} - c_1\right\}$ (a case that arises only for $c_1 \leq \frac{1}{2}$), and means that $\tilde{\Delta}$ is large enough that all consumers visit Firm 1. Then

$$D_1(\tilde{\Delta}) = \frac{1}{2} - \frac{1}{2}(\tilde{\Delta} + c_1)^2 + \tilde{\Delta} + c_1. \quad \text{Case } \delta$$

Otherwise, $\hat{r}_0 \leq 1$ and

$$D_1(\tilde{\Delta}) = \frac{1}{2} + \tilde{\Delta}. \qquad \text{Case } \gamma$$

For $\tilde{\Delta} < -c_1$, $\hat{r}_0 \leq 1$ and we have

$$\begin{aligned} D_1(\tilde{\Delta}) &= \int_0^{\hat{r}_0} \left(1 - r_0 + \tilde{\Delta} + c_1\right) dr_0 \\ &= \left[-\frac{1}{2}\left(1 - r_0 + \tilde{\Delta} + c_1\right)^2\right]_0^{\hat{r}_0} \\ &= -c_1 + \frac{\left(1 + \tilde{\Delta} + c_1\right)^2}{2}. \quad \text{Case } \beta \end{aligned}$$

Similarly, Firm 0's demand is $1 - D_1$.

Pulling all this together, we have that:

Case α: $D_1 = 0$ for $\tilde{\Delta} < \sqrt{2c_1} - c_1 - 1$

Case β: $D_1 = \frac{1}{2}\left(1 + \tilde{\Delta} + c_1\right)^2 - c_1$ for $\sqrt{2c_1} - c_1 - 1 < \tilde{\Delta} < -c_1$

Case γ: $D_1 = \frac{1}{2} + \tilde{\Delta}$ for $-c_1 < \tilde{\Delta} < \min\left\{\sqrt{2c_1} - c_1, 1 - c_1\right\}$

Case δ: $D_1 = \frac{1}{2} - \frac{\left(\tilde{\Delta}+c_1\right)^2}{2} + \tilde{\Delta} + c_1$ for $\sqrt{2c_1} - c_1 < \tilde{\Delta} < 1 - c_1$

Case ε: $D_1 = 1$ for $\tilde{\Delta} > 1 - c_1$.

Note that Case δ does not arise for $c_1 \geq \frac{1}{2}$. Note too that demand is continuous, as is readily derived from calculations at the case switch-points. It also slopes down (as a function of Firm 1's price). It is strictly convex in Case β (D_1" $= 1$), linear in Case γ, and concave (D_1" $= -1$) in Case δ. The first derivative at the switch-point between cases β and γ is -1 and so is continuous; however, there is a kink between cases γ and δ: demand for Firm 0, which is $1 - D_1$ is more inelastic at higher prices than at slightly lower ones. This implies that there can be no equilibrium at the kink: so there could be (in terms of the underlying parameters) either a hole (non-existence) or else an overlap (2 equilibrium types for the same parameter values).

6 Equilibrium information disclosure

Demands have the appropriate properties in order to apply lemmas 1 and 2. It is now straightforward to compare demands under no information and perfect information depending on the sign of $\tilde{\Delta}$ We first show that if $\tilde{\Delta} < 0$, then demand for Firm 1 is larger if information is revealed. We have

$$\begin{aligned} D_1(\tilde{\Delta}) - \widetilde{D}_1(\tilde{\Delta}) &= \frac{1 - 2c_1}{2} + \Delta - \frac{\left(1 + \tilde{\Delta}\right)^2}{2} \\ &= -\frac{\tilde{\Delta}^2}{2} < 0. \end{aligned}$$

This expression holds for $\tilde{\Delta} \geq -c_1$. If $\tilde{\Delta} < -c_1$, then

$$D_1(\tilde{\Delta}) - \widetilde{D}_1(\tilde{\Delta}) = \frac{(\tilde{\Delta} + c_1)^2 - \tilde{\Delta}^2}{2} < 0.$$

Now suppose $\widetilde{\Delta} > 0$. First note that if $\hat{r}_0 > 1$, then all consumers are perfectly informed when they make their purchase decision. However, without advertising, they take this decision after having already incurred c_1 whereas with advertising they need to incur c_1 if and only if she purchases good 1. Thus demand for the unknown product is clearly higher with no information. For $\hat{r}_0 \leq 1$, or equivalently $\tilde{\Delta} < \min\left\{\sqrt{2c_1} - c_1, 1 - c_1\right\}$,

$$\begin{aligned} D_1(\tilde{\Delta}) - \widetilde{D}_1(\tilde{\Delta}) &= \frac{1 - 2c_1}{2} + \Delta - \frac{1 - \tilde{\Delta}^2}{2} - \widetilde{\Delta} \\ &= \frac{\tilde{\Delta}^2}{2} > 0. \end{aligned}$$

To sum up, $D_1 > \widetilde{D}_1$ if and only if $\tilde{\Delta} > 0$ and $D_1 < \widetilde{D}_1$ if and only if $\tilde{\Delta} < 0$. Furthermore, we have $D_1(0) = \widetilde{D}_1(0)$ and $D_1'(0) = \widetilde{D}_1'(0) = 1$. Thus Lemma 2 applies and, as we show below, it may be used to characterize the firms' equilibrium disclosure strategies.

6.1 Equilibrium pricing and optimal information disclosure.

We start by using Lemma 1 to characterize the firm's pricing behavior depending on whether the visit cost for the unknown firm is above or below that of its competitor. The analysis above shows that both D_1 and $\widetilde{D}_1$ are $\frac{1}{2}$ when evaluated at $\tilde{\Delta} = 0$. We therefore know from Lemma 1 that, independently of the firm's decision on information disclosure, $\tilde{\Delta} > 0$ if and only if $c_0 > c_1$ and $\tilde{\Delta} < 0$ if and only if $c_1 > c_0$.

Consider first a situation where comparative advertising is ruled out so that only the unknown firm may disclose information about its own product. If $c_0 > c_1$, then both the full information and no information price equilibria are such that $\tilde{\Delta} > 0$. As shown in the previous section, this implies that the no information demand for Firm 1 dominates the full information demand and this is true for all $\tilde{\Delta} > 0$. Then Lemma 2(ii) implies that it is profitable for Firm 1 to deviate from the full information equilibrium by not disclosing information while Firm 0 is keeping its price unchanged. Conversely, from Lemma 2(i) it is not optimal for Firm 1 to deviate from the no information equilibrium because there is no price that would yield a higher revenue with the full information demand even if Firm 1 changes its price while Firm 0 keeps its price unchanged.

Thus, if $c_0 > c_1$ then the unknown firm does not advertise product information in equilibrium. Similar arguments may be used to show that if $c_0 < c_1$, then the unknown firm advertises product information in equilibrium.

Now suppose that comparative advertising is allowed. Note that since $D_0 = 1 - D_1$, our results above show that for $\tilde{\Delta} > 0$, demand for Firm 0 is higher under perfect information while for $\tilde{\Delta} < 0$, demand for Firm 0 is higher with no information. Then the arguments above may readily be adapted to establish that if $c_o > c_1$, then Firm 0 would choose to resort to comparative advertising in equilibrium and the unknown firm does not advertise its product while if $c_0 < c_1$ the reverse happens (here we use the tie breaking rule that when a firm is indifferent between disclosing information and not disclosing, it chooses not to: indeed here disclosure by both firms would be an equilibrium since once one firm discloses information, the other is indifferent about information disclosure). Allowing comparative advertising is welfare enhancing to the extent that it improves the match of consumers with products when $c_1 < c_0$. However, if entry costs are too high, it may deter desirable entry.

Intuition for these results is as follows. If the cost of finding out about the unknown product is small relative to the visit cost for the other product, most consumers check out Firm 1 before making their purchase decision; this puts Firm 1 in a strong position because, when they decide, they have sunk their visit cost c_1 whereas they need to incur a visit cost c_0 to purchase the other product. If these consumers were fully informed before starting to search, they would choose between the two products knowing that they would have to incur a visit cost in any case. By contrast, if c_1 is relatively high few consumers would choose to visit Firm 1 if they have no prior information, even though they might like that firm's product a lot. The unknown firm is therefore better off disclosing product information so that those who really like its product know it from the outset.

7 Products with different qualities

The model may readily be reinterpreted as one where the visit cost is identical for both firms but products have different qualities and quality comes in additively in the consumer's surplus. Suppose that the common visit cost is c and qualities are given by q_0 for known product and q_1 for the unknown one. Then, if we define $c_1 = c$ and $c_0 = c + q_1 - q_0$, then the above model may be reinterpreted as one where when $c_1 > c_0$, the unknown product's quality is lower than that of the competing one, whereas if $c_1 < c_0$ the unknown product's quality is higher. Then, assuming that both qualities are known, predictions are that a firm with a high quality product chooses not to disclose match information and would be comparatively advertised by a competitor if such a practice is allowed; by contrast if the product's quality is low the firm advertises match information and the competitor would not wish to comparatively advertise.

7.1 Quality revelation

So far, we have assumed that qualities are known to firms and to consumers. We next analyze the situation where consumers do not know qualities unless one or other firm announces this information through its ad. In keeping with the traditional "persuasion game" of Milgrom (1981) and others, we assume that quality cannot be discerned on inspection, and so is not a search characteristic. Instead, quality is only realized after consumption. This means that, if no quality information is provided by firms, consumers buy (and search for horizontal location, when appropriate) only on the basis of expected quality. In this context, we address the question of which firm then announces which information.

To set the stage, we first present the standard "unravelling" result by which the firm of unknown quality always announces its quality in equilibrium. This serves to introduce the generalization of the model (proposed by Shin, 1994) that allows the firm with private information to hide this, and so opens up the possibility that the rival firm may announce when the firm of unknown quality does not do so.

Assume that the quality of the product sold by the unknown firm is $q \in \left[\underline{q}, \bar{q}\right]$, and, as before let q_0 be the actual quality of the known firm. When consumers expect a quality $\tilde{q}$ from the unknown firm, then we write profits as $\pi_i\left(p_0, p_1; q_0, \tilde{q}|NI\right)$, $i = 0, 1$, when no information is revealed about 1's (horizontal) location, and $\pi_i\left(p_0, p_1; q_0, \tilde{q}|LI\right)$, $i = 0, 1$, when information is revealed about 1's location. In both cases, the equilibrium prices are given as in the earlier analysis with $\tilde{q}$ now replacing q_1.[5] Let the corresponding prices be $p_i^n\left(\tilde{q}\right)$ and $p_i^f\left(\tilde{q}\right)$. Clearly, $\pi_1\left(p_0^n\left(\tilde{q}\right), p_1^n\left(\tilde{q}\right); q_0, \tilde{q}|NI\right)$ is increasing in $\tilde{q}$, and $\pi_1\left(p_0^f\left(\tilde{q}\right), p_1^f\left(\tilde{q}\right); q_0, \tilde{q}|FI\right)$ is increasing in $\tilde{q}$. In both cases, Firm 0 does better when taken as higher quality.

Then we get the standard result that, if comparative ads are ruled out, there is no equilibrium at which Firm 1 does not reveal its quality. Suppose there were an equilibrium at which some subset of types did not reveal quality. Then the expected quality over these types would be lower than the highest type purportedly not revealing. It would then profitably wish to reveal its quality, so vitiating the supposed equilibrium. The same argument shows that with comparative advertising, there is no equilibrium at which no firm advertises Firm 1's quality.

The intuition is understood from the standard unravelling argument. First note that if no types reveal quality, then $\tilde{q} < \bar{q}$. But then each type $q > \tilde{q}$ would do better by revealing. In turn, such revelation leads to a reduced $\tilde{q}$ and a further enlarging of the set of types that reveal. In this sense, revelation happens from the "top down." Therefore the unknown quality is revealed by the firm itself: there is no role for the rival to act.

[5] There is no role for quality signaling by prices in the model because quality is not verifiable by consumers.

Following Shin (1994), we will, in future versions, amend the model to allow for the possibility that the quality that is *a priori* unknown to consumers is also unknown to the firms.[6] Specifically, let λ be the probability that q_1 is unknown; otherwise, both firms know it. Firm 1 may now not wish to reveal its quality because it can get away pooling with the possibility it does not know it. More precisely, if consumers do not observe any quality announcement, either it is not known, or else the firm (or firms) have chosen not to reveal it. Understanding which types have incentives to reveal, consumers Bayesian update (actually, they downgrade since they understand that low types will pool with the uninformed type) the quality estimate following no revealed information.[7]

[6] Shin's context is actually quite different from ours, and concerns ...

[7] The ad for quality must have credible information that proves how much the consumer likes it in the vertical dimension. The source of λ is in either not knowing how much consumers like the product, or in not being able to prove it.

Appendix

Full information

We here find the equilibrium prices conditional on consumers knowing product specifications and qualities of both products.

Consider the case of full information (Perloff-Salop uniform with asymmetric qualities, effectively). Consider the case of $\tilde{\Delta} < 0$; the case of $\tilde{\Delta} > 0$ will then follow by judicious use of symmetry. Note that $\tilde{\Delta} < 0$ corresponds to $c_0 < c_1$ (Lemma?). We first have

$$\tilde{D}_1\left(\tilde{\Delta}\right) = \frac{\left(1+\tilde{\Delta}\right)^2}{2}$$

as in the text. This means that the first order condition yields

$$p_1 = \frac{\left(1+\tilde{\Delta}\right)}{2};$$

we can immediately substitute in for $\tilde{\Delta} = p_0 + c_0 - p_1 - c_1$ to yield a linear reaction function (!)

$$p_1 = \frac{1 + p_0 + c_0 - c_1}{3}.$$

For the other firm, we have the first order condition

$$1 - \frac{\left(1+\tilde{\Delta}\right)^2}{2} - p_0\left(1+\tilde{\Delta}\right) = 0.$$

Substituting in $p_1 = \frac{\left(1+\tilde{\Delta}\right)}{2}$, then

$$1 - 2p_1^2 - 2p_0p_1 = 0.$$

Solving out these equations for prices then gives the solutions as

$$p_0 = \frac{-5\omega + 3\sqrt{8+\omega^2}}{8}$$

and

$$p_1 = \frac{\omega + \sqrt{8+\omega^2}}{8},$$

where $\omega = 1 + c_0 - c_1$. Notice that these price expressions are equal at $\omega = 1$ (symmetry) to 1/2. They also verify $\tilde{\Delta} < 0$, as desired.

The corresponding expressions for $\tilde{\Delta} > 0$ are

$$p_0 = \frac{\hat{\omega} + \sqrt{8+\hat{\omega}^2}}{8},$$

and

$$p_1 = \frac{-5\hat{\omega} + 3\sqrt{8+\hat{\omega}^2}}{8}$$

where $\hat{\omega} = 1 + c_1 - c_0$.

No information (sketch)

Recall the following from the text. First define $\Omega = p_0 + c_0 - p_1$ as the full price difference conditional on having already visited 1.

For demand, we have:

$$D_1(p_1, p_0) = \int_{\max\{0,\Omega\}}^{\hat{r}_0} (1 - r_0 + \Omega)\, dr_0 + \max\{0, \Omega\}$$

For $\Omega \geq 0$ (which arises if the entrant's price is low relative to the "full price" – i.e., price plus search cost – of going to the incumbent) we have

$$D_1(p_1, p_0) = \frac{1}{2} - c_1 + \Omega,$$

Equivalently, $D_1(\tilde{\Delta}) = \frac{1}{2} + \tilde{\Delta}$, which is the version used in the text. Hence $D_1' = -1$, and we have a simple linear demand system (which may appear surprising given the way demand is constructed. Furthermore, note that the "symmetric" version, with consumers knowing their valuations at both firms, does NOT give a linear demand system. This latter system is determined in the text.)

We can immediately determine the equilibrium prices as

$$p_1 = \frac{1}{2} + \tilde{\Delta}$$

and

$$p_0 = \frac{1}{2} - \tilde{\Delta}.$$

These prices depend only on the full price difference! This means we may apply our Lemmas 1 and 2, and also that they apply just as well for qualities.

Now taking the difference of these two equations we can write out and solve for

$$\Omega = \frac{c_0 + 2c_1}{3}.$$

The condition that $\Omega > 0$ implies that $c_0 > -2c_1$; the condition that $\hat{r}_0 < 1$ requires that $\Omega < \min\left\{\sqrt{2c_1}, \frac{1}{2} + c_1\right\}$. Combining these conditions yields the corresponding restrictions on c_0 for this case as $c_0 \in \left(-2c_1, \min\left\{\frac{3}{2}\sqrt{2c_1} - \frac{c_1}{2}, \frac{3}{4} + c_1\right\}\right)$. We also need to check that Firm 0 would not choose to deviate to a price p_0 that would make $\hat{r}_0 = 1$, which is a worry since there is a kink in demand.

Substituting back gives prices as

$$p_1 = \frac{1}{2} + \frac{c_0 - c_1}{3}$$

and

$$p_0 = \frac{1}{2} + \frac{c_1 - c_0}{3}.$$

Note for reference in what follows that the equilibrium profit levels are given by these prices squared. Notice too (also for further reference) that the full price advantage of Firm 1 ($(p_0 + c_0) - (p_1 + c_1)$) is $\frac{c_0 - c_1}{3}$, and so is consistent with Lemma 1.

We do need to show that this is the only possible equilibrium.

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