Herding and Contrarian Behavior in Financial Markets[*]

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Abstract

Rational herd behavior and informationally efficient security prices have long been considered to be mutually exclusive but for exceptional cases. In this paper we describe conditions on the underlying information structure that are necessary and sufficient for informational herding. Employing a standard sequential security trading model, we argue that people may be subject to herding if and only if, loosely, their information is sufficiently noisy so that they consider extreme outcomes to be more likely than moderate ones and if there is sufficient amount of noise trading. We then show that herding has a significant effect on prices: prices become more volatile than if there were no herding. Liquidity, measured by the bid-ask-spread, is reduced. Furthermore, herding can be persistent and can affect the process of learning. We also outline conditions for contrarian and for no-herding/no-contrarian behavior and thus provide a complete characterization of trading behavior. Our analysis suggests that herding (and contrarian behavior) may be more pervasive than was originally thought. Hence, the paper provides a new perspective on the relevance of herding in financial markets with efficient prices.

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1 Introduction

Informational herding describes a situation in which people rationally disregard their information and follow the crowd. It has long been suspected that such herding behavior may play a role in financial crises or market crashes or booms, but a theoretical proof for the possibility of meaningful financial market herding proved illusive. Indeed, an early result by Avery and Zemsky (1998) showed that in a standard, simple financial market trading setting herding is impossible[1] because the market price would always separated people with good and bad information. Subsequent work that identified some form of herding relied on market frictions that mitigated the separating power of the market price.[2]

In this paper we describe the necessary and sufficient conditions on private information that allow herding and contrarian behavior (moving against one's information and against the crowd), and we also show under which conditions herding is impossible. We do this in a model that imposes no assumptions to mitigate the separating power of the market price.

To illustrate the difficulty of showing herd behavior with a separating price, suppose that people receive only either favorable or unfavorable information. Now imagine that a crowd of people buys frantically. An investor with unfavorable private information will deduce from this behavior that these buyers had favorable information. So would he disregard his unfavorable information and buy, too? The answer is no: while the buys increase the investor's private expectation, they also increase the price, and in this simple model with two kinds of signals (favorable/unfavorable) someone with unfavorable would continue to perceive the security as overvalued.

When allowing more states and more signals the above may no longer applies if the distribution of signals has some specific features and provided there is a sufficient level of uninformed liquidity trading.[3] The simplest possible model that would allow herding is one where there are at least three states and three signals; for this case we provide a complete characterization of trading behavior. To wit, if and only if signals cause their recipients to redistribute probability weight towards extreme values then herding is possible. On the other hand, if and only if a signal causes the recipient to distribute probability weight towards the center, then contrarian behavior is possible.

We follow the microstructure literature and employ a stylized specialist sequential security trading model à la Glosten and Milgrom (1985), and we follow Avery and Zemsky

[1] Two recent experimental studies, Drehmann, Oechssler and Roider (2005) and Cipriani and Guarino (2005), confirm this theoretical result. We will comment more on Avery and Zemsky (1998) below.

[2] E.g. Lee (1998), Cipriani and Guarino (2003), Chari and Kehoe (2004), or Dasgupta and Prat (2005).

[3] The existence of liquidity trading is assumed in most informed-based trading models to avoid no-trade outcomes; for our result we need a slight strengthening of this requirement in that there must be enough noise so that all investors who receive information are indeed willing to trade.

(1998) in defining herding as any history-induced switch of opinion (behavior) by a type of agent in the direction of the crowd. This is in line with the semantic implication of 'herding' as 'going with the crowd'. Moreover, we define a history-induced switch *against* the crowd as "contrarian" behavior. To the best of our knowledge, we are the first to study both herding and contrarian behavior in this manner in a financial market model.

Herding therefore does not signify that everyone acts alike — but in a financial market setting such behavior would actually not be very useful for explaining crises or booms. The reason is that if all traders were to act alike, then actions would be uninformative, so that prices would not react to actions and stay 'constant' for the duration of herding.[4] Moreover, to theoretically ensure that financial markets function, i.e. that for every seller there is a buyer, one usually assumes the presence of a dedicated market maker. Yet this is a modeling trick that is no longer innocuous if all traders were to act alike.[5] In summary, the most one should expect from herding is a substantial shift in market activity that impacts price volatility and liquidity in the short run. This is precisely what we identify here.

To elaborate on the signals that lead to herding and contrarianism, the former is possible if there is a signal with a conditional distribution that is 'U-shaped' in values, the latter if the signal distribution is 'hill-shaped'. Such a U-shaped signal arises if both extreme values generate this signal with large probability. When forming his posterior, the recipient of such a signal will shift weight away from the center to the extremes, thus thickening the posterior distribution's tails. A hill-shaped signal arises, if it is most likely to occur under the middle state; the recipient of such a signal puts most weight on the middle value.

Consequently, the recipient of a U-shaped signal discounts the possibility of the intermediate value and updates the probabilities of *either* extreme value. Moreover, when observing actions that favor either extreme value (buys for the high value, sales for the low value), the recipient of a U-shaped signal also updates faster than an agent who receives only the (noisy) public information. And the intuition is quite simple: the recipient of the

[4]For instance, models with informational cascades such Cipriani and Guarino (2003) and Dasgupta and Prat (2005) have the feature that prices no longer move once the informational cascade starts.
Our argument focusses on informational effects and information externalities and ignores the effect that such mass-uniform behavior and the large order imbalance that would be involved would have; for instance this may lead to payoff externalities akin to a bank-run situation. That notwithstanding, such effects are only second-order consequences of informational herding and thus require a different modeling approach altogether.

[5]In fact, the importance of the role of the market maker comes out very cleanly in Lee (1998)'s insightful paper where booms and crashes occur because the market maker cannot react fast enough to mass behavior and instead has to absorb all trades at the same price (the price adjustment after the rush constitutes the crash outcome). Chari and Kehoe (2004) use a different trick: herding in their model refers to mass-uniform behavior *outside* of capital markets and refers to a fixed investment decision. In Cipriani and Guarino (2003) mass-behavior has two sides: one group of people herds, the other acts as contrarians; both groups are, however, of similar size.

U-shaped signals puts proportionally more weight on extreme outcomes. So if past trading indicates that low values are rather unlikely, then the large weight on the highest outcomes causes such a person to flip, i.e. to switch from selling to buying.

The recipient of a hill-shaped signal does the opposite: he updates extreme values slower and always puts most weight on the middle value. This causes him to take actions that move prices towards this middle value: if prices rise too much, he sells, if prices fall too much, he buys.

There are many real-world situations where signals can have the structure necessary for herding or contrarian behavior. For instance, any upcoming event announcements with uncertain outcomes yields such a structure (e.g. an upcoming merger, a FDA drug approval/dismissal, the results or filing of lawsuits).[6]

More generally even, U-shaped signals may indicate a state of the economy in which uncertainty is wide-spread (e.g. 'will the economy go into recession or 'is the economy really in good shape and current bad news are but a dink in the economy's otherwise robust performance?'). Herding in these situations can arguably have pronounced effects, in particular if policy-makers react to short-term swings in financial markets. Our analysis thus provides an intellectual framework to understand how and why short-term herding may arise.

Our second main result concerns the impact of herding on price volatility and potential mispricings.[7] To tackle this issue we compare price movements in our set-up in which traders learn from each other with those in a hypothetical economy in which agents rely only on their private information and ignore public information (herding is not possible

[6]For instance, in the takeover example, the takeover can be good or bad for the company, or there may be a third middle state in which the takeover does not occur at all; for simplicity assume all three outcomes are equally likely. Two natural signals are one that causes the recipient to put most weight on the takeover being good, and one for the opposite. Agents who receive one of these 'extreme' signals are settled in their ways and will not be herding or following contrarian behavior.
Signals that allow herding and contrarian behavior have a different story. For instance, a signal may indicate that a takeover is extremely unlikely; in which case the recipient of such a 'middle' signal becomes a contrarian by shifting probability weight to the center (no takeover). Someone who gets such a signal may engage in short-selling if prices move up too much. Alternatively, a signal may indicate that a takeover is extremely likely but that it is not clear whether it is a good or bad one. Such a piece of information corresponds to a U-shaped distribution for this signal. Traders with such a signal may 'go with the flow' and engage in herding behavior. Finally, another possibility that a signal is merely a weaker version of one of the extreme signals (it would be monotonic in values); recipients of such a signal would always remain on one side of the market.

[7]This and the following result on herding-persistence is based on settings in which signals satisfy the *Monotone Likelihood Ratio Property* (MLRP). The MLRP is a very standard but strong requirement on signals in the literature, found for instance in rational expectations models or auctions. For instance, the MLRP implies that posteriors allow for a first first-order stochastic dominance ordering and it is thus a convenient tool to ensure that investors' expectations are ordered. This suggests, that models with MLRP should be 'well-behaved'. Arguably, herding and the excess volatility that comes with it is a very strong and surprising result when signals satisfy the MLRP.

here, we refer to this as an economy with naïve traders). We then show that once herding begins, prices respond more to individual trades relative to the situation in which agents ignore public information. The bid-ask-spread widens as a result, thus reducing the most standard measure of 'liquidity'; moreover the volatility of prices increases.

This is a surprising result because casual intuition suggests that during herding, information is lost or that less information is transmitted. This intuition is, however, incorrect: herding is informationally significant due to the U-shape of the herders' signal distributions. Therefore, short-run price movements are more volatile with herding compared with the case in which social learning does not occur.[8]

Observational learning is a key prerequisite for herding. And since learning is only possible when traders can observe the history of prices and transactions, our excess volatility result also has implications on the effects of 'market transparency'.[9] One can think of naïve traders as acting on the basis of less information than rational traders: for instance, naïve traders could be people who do not observe past or current prices (in a timely fashion); they suffer from a lack of transparency of transactions' data. Yet herding causes the transparent markets to be more volatile, less stable, and less liquid.[10]

We finally show that prices can move substantially during herding and that investors continue to herd as long as trades are 'in the direction of the crowd'. The range of herding-prices can even comprise almost the entire range of feasible prices. Moreover, the longer the herd continues, the more robust it gets. This somewhat contrasts herding in non-financial market settings, such as in the seminal paper by Smith and Sørensen (2000), where the 'overturning principle' applies, i.e. that a single contrarian action can end herd-behavior.

Related Literature. While Avery and Zemsky (1998) is probably best known for their no-herding result, they were also the first to present an example with moving prices and herding. They did, however, attribute their finding to the wrong reasons, asserting that 'multidimensional uncertainty/risk' (investors have a finer information structure than the market) is the cause. Next, in AZ's herding example prices under herding hardly move,[11]

[8]The increase in price-volatility associated with herding is only relative to a hypothetical scenario. Even when herding is possible, in the long-run volatility settles down and prices react less to individual trades. Overall, it is well known that the variance of Martingale price-processes such as ours is bounded by model primitives.

[9]We thank Marco Pagano for suggesting this insight.

[10]Several papers have asserted that liquidity may be lower in transparent markets; see, for instance, Bloomfield and O'Hara (1999,2000) for an experimental analysis or Madhavan, Porter and Weber (2005) for an empirical analysis. For a recent, more complete survey of the literature on market transparency see Madhavan (2000).

[11]To show that extreme price movements (bubbles) with herding are possible they introduce further information asymmetries and thus more risk dimensions; in particular, in addition to the above information structure, they assume that traders have different abilities to interpret the signals and that this is private

and herding is 'self-defeating' because herd-buys themselves eventually stop the herd. Since the underlying 'multidimensional' information structure seemed very specific, and since the implications of herding as identified in their paper seem to be unimportant, it has been concluded that rational herding models are not so relevant to understanding the functioning of efficient financial markets.[12]

We believe that our analysis allows a fresh start to understanding herding in financial markets, for it shows that herding is triggered when signals are sufficiently noisy in the sense of distributing probability weight to both tails of the distribution.[13] Herding then exacerbates price volatility and extreme price movements are possible under not so unlikely situations.

In the next section we outline the basic setup, the assumptions on signal distributions. In Section 3 we define herding and contrarian behavior. Our main results follow in Sections 4, 5, and 6. In Section 4 we discuss which assumptions on signal structures that are necessary and sufficient to ensure that herding occurs with positive probability. In Sections 5 and 6 we then show that herding can persist and explain why herding-prices are more extreme. We discuss our results, their robustness and some further implications[14] in Section 7. All proofs are in the appendix.

2 The Model

Traders arrive in a random sequence and trade a security with an uninformed market maker. The security takes one of three possible liquidation values. Traders can be informed, in which case they receive a conditionally independent signal about the true value of the security, or then can be noise in which case they trade for reasons outside the model. Before meeting a trader, the market maker sets bid and offer prices at which he makes zero-profits. In more detail:

Security: There is a single risky asset with a liquidation value V from a set of three potential values $\mathbb{V} = \{V_1, V_2, V_3\}$ with $V_1 < V_2 < V_3$. The prior distribution over $\mathbb{V}$ is denoted by $\mathsf{Pr}(\cdot)$. To simplify the computation we assume that $\{V_1, V_2, V_3\} = \{0, \mathcal{V}, 2\mathcal{V}\}$,

information. However, even with these further informational asymmetries, the likelihood of large price movements in their set-up during a herd phase is extremely small (of the order of $10^{-6}\times$the probability of a particular sequence of trades, see Chamley 2004).

[12]See, for instance, also Brunnermeier (2001), Bikhchandani and Sunil (2000), Chamley (2004), or Alevy, Haigh and List (2007).

[13]While AZ's signal structure also falls under this category, it is but a knife-edge case in a much more general class.

[14]A more detailed account of these implications is in an earlier working paper version of this paper that is available from the authors upon request.

$\mathcal{V} > 0$ and that the prior distribution is symmetric around V_2; thus $\mathsf{Pr}(V_1) = \mathsf{Pr}(V_3)$.[15]

Traders: There is a pool of traders consisting of two kinds of agents: *Noise Traders* and *Informed Agents.* At each discrete date t one trader arrives at the market in an exogenous and random sequence. Each trader can only trade once at the point in time at which he arrives. We assume that at each date the entering trader is an informed agent with probability $\mu > 0$ and a noise trader with probability $1 - \mu > 0$.

The informed agents are risk neutral and rational. Each receives a private, conditionally i.i.d. signal $S \in \{S_1, S_2, S_3\}$ about V. We assume that the signals are ordered such that $S_1 < S_2 < S_3$.

Noise traders have no information and trade randomly. These traders are not necessarily irrational, but they trade for reasons not included in this model, such as liquidity.[16]

Market Maker: Trade in the market is organised by a market maker who has no private information. He is subject to competition and thus makes zero-expected profits.[17] In every period t, prior to the arrival of a trader, he posts a bid-price bid_t at which he is willing to buy the security and an ask-price p_t^A at which he is willing to sell the security. Consequently he sets prices in the interval $[V_1, V_3]$.[18]

Traders' Actions: Each trader can buy or sell *one* unit of the security at prices posted by the market maker, or he can be inactive. So the set of possible actions for any trader is $\mathbb{A} := \{\text{buy}, \text{hold}, \text{sell}\}$. We denote the action taken in period t by the trader that arrives at that date by $a_t \in \mathbb{A}$.

We assume that noise traders trade with equal probability. Therefore, in any period, a noise-trader buy, hold or sale occurs with probability $\gamma = (1 - \mu)/3$ each.

Information: The structure of the model is common knowledge among all market participants. The identity of a trader and his signal are private information, but everyone can observe past trades and transaction prices. The history (public information) at any date $t > 1$, the sequence of the traders' past actions together with the realised past transaction prices, is denoted by $H_t = ((a_1, \mathsf{p}_1), \ldots, (a_{t-1}, \mathsf{p}_{t-1}))$ for $t > 1$, where a_τ and p_τ are traders' actions and realised transaction prices at any date $\tau < t$ respectively. Also, H_1 refers to the initial history before trade occurs.

The Informed Trader's Optimal Choice: An informed trader enters the market in period t, receives his signal S_t and observes history H_t. We assume, for simplicity, the tie-breaking rule that, in the case of indifference, agents always prefer not to trade. Therefore,

[15] The results of this paper remain valid without these assumptions on symmetry.

[16] As is common in the literature on micro-structure with asymmetric information, we assume that noise traders have a positive weight ($\mu < 1$) to prevent "no-trade" outcomes à la Milgrom-Stokey (1982).

[17] Alternatively, we could also assume a model with many identical market makers setting prices as in Bertrand competition.

[18] The market maker in our model resembles a market 'specialist'.

an informed trader's optimal action is (i) to *buy* if he values the security no less than the ask-price: $\mathsf{E}[V|H_t, S_t] > \mathsf{p}_t^A$, (ii) to *sell* if he thinks the security is worth no more than the bid price: $\mathsf{bid}_t < \mathsf{E}[V|H_t, S_t]$, and (iii) to *hold* in all other cases.

The Market Maker's Price-Setting: To ensure that the market maker receives zero expected profits, bid and ask prices have to be such that at any date t and any publicly available information H_t,

$$\mathsf{ask}_t = \mathsf{E}[V|a_t = \text{buy at } \mathsf{ask}_t, H_t] \text{ and } \mathsf{bid}_t = \mathsf{E}[V|a_t = \text{sell at } \mathsf{bid}_t, H_t]$$

Informed agents are better informed than the market maker. Consequently, if the market maker always sets prices equal to public expectation, $\mathsf{E}[V|H_t]$, he makes an expected loss on trades with informed agents. However, if he sets an ask-price and a bid-price respectively above and below the public expectation, he gains on noise traders, as their trades have no information value. Thus, in equilibrium the market maker makes profit on trades with noise traders to compensate for losses against informed agents. This implies that at any date there is a spread between the bid and ask price; in particular at any date t and for any public information H_t we have $\mathsf{ask}_t > \mathsf{E}[V|H_t] > \mathsf{bid}_t$. Moreover, the spread $\mathsf{ask}_t - \mathsf{bid}_t$ increases in μ, the probability that a trader is informed.

Equilibrium concept: Since the game played by the informed agents is one of incomplete information the appropriate equilibrium concept is the Perfect Bayesian equilibrium.

Long-run behavior of the model: Price formation in our model is standard. Therefore, by standard arguments we have that transaction prices form a Martingale process, and beliefs and prices converge to the truth (see Glosten and Milgrom (1985)). However, as we mentioned in the introduction, here we are interested in short-run behavior and fluctuations.

Shapes of Signal Distributions. As we mentioned in the introduction, the possibility of herding for any informed agent with signal S depends critically on the shape of the conditional signal distribution that S has; we will henceforth refer to the conditional signal distribution as *csd*. We will also employ the following terminology to describe six different types of csds that any signal S may have:

$$\begin{aligned}
\text{increasing} &\Leftrightarrow \mathsf{Pr}(S|V_1) \leq \mathsf{Pr}(S|V_2) \leq \mathsf{Pr}(S|V_3) \\
\text{decreasing} &\Leftrightarrow \mathsf{Pr}(S|V_1) \geq \mathsf{Pr}(S|V_2) \geq \mathsf{Pr}(S|V_3) \\
\text{U-shaped} &\Leftrightarrow \mathsf{Pr}(S|V_i) > \mathsf{Pr}(S|V_2) \text{ for } i = 1, 3 \\
\text{Hill-shaped} &\Leftrightarrow \mathsf{Pr}(S|V_i) < \mathsf{Pr}(S|V_2) \text{ for } i = 1, 3 \\
\text{Negatively biased} &\Leftrightarrow \mathsf{Pr}(S|V_1) > \mathsf{Pr}(S|V_3) \\
\text{Positively biased} &\Leftrightarrow \mathsf{Pr}(S|V_1) < \mathsf{Pr}(S|V_3) \\
\text{zero biased} &\Leftrightarrow \mathsf{Pr}(S|V_1) = \mathsf{Pr}(S|V_3)
\end{aligned}$$

We shall call a signal *csd-monotonic* if its csd is either increasing or decreasing; we say that is it strictly csd-monotonic if all inequalities are strict. Csd-monotonic thus includes the case of an uninformative signal. Moreover, we refer to negatively/positively bias U-shaped/hill-shaped csds as negative/positive U-shaped/hill-shaped. We will from now on assume that all three signals are different, i.e. for all signals S, S' there exists an i such that $\mathsf{Pr}(S|V_i) \neq \mathsf{Pr}(S'|V_i)$. This assumption ensures that there is always someone who is buying and someone who is selling.

Monotonic Signals. The literature on asymmetric information usually employs signals that satisfy monotonicity in the sense of the *monotone likelihood ratio property* (MLRP) (introduced by Karlin and Rubin (1956) and Milgrom (1981)). This means that for any signals $S_l, S_h \in \mathbb{S}$ and any values $V_l, V_h \in \mathbb{V}$ such that $S_l < S_h$ and $V_l < V_h$ we have

$$\frac{\mathsf{Pr}(S_h|V_h)}{\mathsf{Pr}(S_l|V_h)} > \frac{\mathsf{Pr}(S_h|V_l)}{\mathsf{Pr}(S_l|V_l)}.$$

With the MLRP, there is a natural order of signals because conditional expectations can be ordered, and this order never changes for any trading history. Also the csd for the lowest and the highest signals are csd-monotonic. Formally,

Lemma 1 (Properties of MLRP Signals)
Assume that signals satisfy the MLRP and let $\mathsf{E}[V|S_j] < \mathsf{E}[V|S_i] < \mathsf{E}[V|S_j]$.

(a) *Conditional expectations are monotonic in signals, i.e. for any date t and any history H_t,* $\mathsf{E}[V|S_j, H_t] < \mathsf{E}[V|S_i, H_t] < \mathsf{E}[V|S_j, H_t]$.

(b) *The csd for S_j is strictly decreasing and the csd for S_k is strictly increasing.*

Lemma 1 (a) implies that informed agents' conditional expectations are ordered after any history of trade. Since the MLRP implies First Order Stochastic Dominance, it follows

$\Pr(S\|V)$	V_1	V_2	V_3
S_1	$\frac{5}{9}$	$\frac{1}{3}$	$\frac{2}{9}$
S_2	$\frac{6}{18}$	$\frac{4}{18}$	$\frac{3}{18}$
S_3	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{11}{18}$

decreasing for S_2

$\Pr(S\|V)$	V_1	V_2	V_3
S_1	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{1}{9}$
S_2	$\frac{5}{27}$	$\frac{6}{27}$	$\frac{9}{27}$
S_3	$\frac{7}{27}$	$\frac{1}{3}$	$\frac{5}{9}$

increasing for S_2

$\Pr(S\|V)$	V_1	V_2	V_3
S_1	$\frac{5}{6}$	$\frac{1}{3}$	0
S_2	$\frac{11}{120}$	$\frac{40}{120}$	$\frac{20}{120}$
S_3	$\frac{3}{40}$	$\frac{1}{3}$	$\frac{5}{6}$

hill-shape and positive bias for S_2

$\Pr(S\|V)$	V_1	V_2	V_3
S_1	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{3}{40}$
S_2	$\frac{20}{120}$	$\frac{40}{120}$	$\frac{11}{120}$
S_3	0	$\frac{1}{3}$	$\frac{5}{6}$

hill-shape and negative bias for S_2

$\Pr(S\|V)$	V_1	V_2	V_3
S_1	$\frac{31}{100}$	$\frac{1}{5}$	$\frac{1}{100}$
S_2	$\frac{59}{100}$	$\frac{50}{100}$	$\frac{60}{100}$
S_3	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{39}{100}$

U-shape and positive bias for S_2

$\Pr(S\|V)$	V_1	V_2	V_3
S_1	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{50}$
S_2	$\frac{60}{100}$	$\frac{50}{100}$	$\frac{59}{100}$
S_3	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{39}{100}$

U-shape and negative bias for S_2

Table 1: **Six Examples of MLRP Signal distributions** For very matrix each entry represents the probability of the row-signal given the true liquidation value given by the column. Therefore, for each matrix the sum of the entries in each column add up to 1. In all the above examples, the signal distributions for S_1 and S_3 are csd-monotonic whereas each matrix exhibits a different kind of signal distribution for S_2.

that conditional expectations are ordered ex-ante before any trade. Our result is simply an extension of this observation to expectations after any history.

Lemma 1 (b) says that the conditional probability of the lowest (highest) signal decreases (increases) in the true liquidation value. However, for the remaining signal, no such general rule applies! Csds for S_i can be decreasing, increasing, or they can be hill-shaped or U-shaped with a negative or a positive bias as displayed in Table 1.[19]

3 Herding and Contrarian Behavior: Definitions

We adopt the same definition of herding as in AZ.

Definition 1 (Herding) *A trader with signal S engages in herd-buying in period t after history H_t if and only if* (H1) $\mathsf{E}[V|S] < \mathsf{bid}_1$, (H2) $\mathsf{E}[V|S, H_t] > \mathsf{ask}_t$, (H3) $\mathsf{E}[V|H_t] > \mathsf{E}[V]$. *If at history H_t conditions* (H1)-(H3) *are satisfied for some signal type, then we say that history H_t involves buy-herding. Herd-selling is defined analogously.*

(H1) requires the S type agent to (strictly) prefer to sell ex-ante, before observing other traders' actions; (H2) requires the S type to (strictly) prefer to buy, after observing the

[19]Each information structure is described by a 3×3 matrix; for each such matrix the MLRP is equivalent to all minors of order 2 being positive. This property holds for all matrices.

history H_t; and (H3) requires the public expectation to 'move in the direction' of the herd.

Note that a 'history with buy-herding' only implies that there could be types that buy-herd — it does not mean that the actual trades are by herders.

According to the above definition, agents with a particular signal engage in herding if, as a result of observing the behavior of others, they take a different action from the one that they would take initially. Thus, herding in our set-up (as well as in AZ) represents any history-induced switch of opinion *in the direction of the crowd.*[20] Conditions (H1) and (H2) capture the sense of changing from selling to buying after observing the actions of others; (H3) ensures that the switch occurs following the crowd (see the discussion below).[21]

Definition of Contrarian Behavior. Similar types of agents (with the same signal) may also change their action from selling to buying (or vice versa) as the history unfolds without engaging in herd-behavior. For example, it may be that a type changes, for some trading history, from selling to buying because the market price (public expectations) has fallen. In the literature, such a change of opinion against the crowd is typically referred to as contrarian behavior.[22] Formally, we define contarian-behavior as follows.

Definition 2 (Contrarian) *A trader with signal S is a buy-contrarian in period t after history H_t if and only if* (C1) $\mathsf{E}[V|S] < \mathsf{bid}_1$, (C2) $\mathsf{E}[V|S, H_t] > \mathsf{ask}_t$, (C3) $\mathsf{E}[V|H_t] < \mathsf{E}[V]$. *Contrarian-selling is defined analogously.*

The first two conditions in the definition of contrarian behavior are the same as those for herd behavior: thus for contrarian-buying (C1) requires the S type to (strictly) prefer to sell ex-ante, before observing the action of others and (C2) requires the S type to (strictly) prefer to buy, after observing the history.

[20] In the literature, there are other definitions of herding (and informational cascades). For instance, Smith and Sørensen (2000) and also Cipriani and Guarino (2003) define herding as 'action convergence' — agents of the same 'type' take the same action. They describe an informational cascade as a situation where an agent takes the same decision irrespective of his private signal. Herding in our set-up refers to the actions of a particular signal type, not to all informed agents collectively. In our model the market-maker's zero-profit condition together with the assumption of non-identical signals precludes action-convergence of all informed traders — it is not possible that all informed agents trade on the same side of the market. In Cipriani and Guarino (2003) action convergence refers to specific types with type-characteristics other than just signals — even in their model different types take different actions. In any case, the definition of herding which we (and also AZ) employ is in spirit of herd-mentality: people switch actions to follow the crowd.

[21] In our analysis we look only at the most extreme case when a player switches from selling to buying (or the reverse). One could argue, however, that a switch from holding to buying/selling also constitutes herding. This would require less restrictions but would include further cases of herding information structures that have undesirable properties such as cases where a buy-herder would never sell. To ensure consistency with the literature, we focus on the extreme cases where for some cases a herding candidate would sell and for others buy. We discuss this issue further in Section 7.

[22] Avery and Zemsky use this term too, but their definition of 'contrarian' includes a feature closely related to their definition of monotonicity. To the best of our knowledge we are the first to formalize contrarianism in this way.

The key difference between herding and contrarianism lies in the difference between Conditions (H3) and (C3): the former ensures that the change of action from selling to buying is not due to a decline in the price (public expectation) but instead is *with* the general movement of the crowd. The latter condition, (C3), requires the public expectation to have dropped so that after this history a trader who buys acts *against* the general movement of prices.

In Section 7 we will discuss in more detail what informational herding can and cannot deliver in an environment with efficient prices.

4 Main Results on Herding and Contrarian Behavior

4.1 Necessary Conditions

Signal type S switches from selling to buying buys after observing an increase in prices (herding) only if he updates his expectation faster upwards than the market maker raises the ask-price. Similarly, signal type S buys after a decrease in prices (contrarian behavior) only if he updates his expectation slower downwards than the market maker. The following results spells out the necessary conditions on csds so that such switches are in principle possible.

Proposition 1 (Necessary Conditions for Herding and Contrarian Behavior)
(*a*) *Signal type S buy-/sell-herds $\Rightarrow$ S's csd is negative/positive U-shaped.*
(*b*) *Signal type S acts as buy/sell contrarian $\Rightarrow$ S's csd is negative/positive hill-shaped.*

Notice that this result holds irrespective of whether signals to satisfy the MLRP.

The intuition for necessity of U-shape is somewhat involved. First note that for type S to buy-herd or to become a buy-contrarian, he must have an expectation below the bid-price at the initial history H_1. Since the prior on values is symmetric this implies that signal S must be negatively biased (positive for buying at the initial history),[23] $\mathsf{Pr}(S|V_1) > \mathsf{Pr}(S|V_3)$.

It is straightforward to show that for expectations to rise, the lowest value must be considered less likely than the highest value, i.e. for $\mathsf{E}[V|H_t] > \mathsf{E}[V]$ it must hold that $q_1^t < q_3^t$ and likewise $\mathsf{E}[V|H_t] < \mathsf{E}[V]$ implies $q_1^t > q_3^t$. So suppose now that there is indeed a history with $q_1^t < q_3^t$, so that prices have increased and assume that type S herds.

Consider the case where $\mathsf{Pr}(S|V_2) > \mathsf{Pr}(S|V_1)$. Then type S always shifts less weight towards the tails of his posterior and more towards the posterior's center relative to the

[23]Note that the bias is only required because we assume that priors are symmetric. If, for instance, the prior would favor the highest state, then herding or contrarian behavior can also arise when the signal distributions have no bias.

prior. Moreover, the shift from the tails towards the center is larger for value V_3 than for V_1. This redistribution of probability mass is true for any prior, and thus $q_1^t < q_3^t$, this trader still shifts weight towards the center, and more so for the low state. Thus his expectation is, intuitively, left of the center — for herding it must be to the right.

Now suppose that $\mathsf{Pr}(S|V_2) < \mathsf{Pr}(S|V_1)$. Then type S always shifts *more* weight towards the tails of his posterior and less towards the center. And while the shift into the tails is larger for V_1 than for V_3, when $q_1^t < q_3^t$, state V_3 may be important enough to outweigh states V_2 and V_1 so that type S's expectation moves to the right of the center.

With contrarian behavior the intuition is the reverse.

Corollary (Impossibility of Herding)
Recipients of csd-monotonic signals cannot herd or behave as contrarians.

This corollary is the multi-state extension of AZ's result that herding cannot arise with two states. The intuition is analogous to the one given for no-herding under the hill-shape.

4.2 Sufficient Conditions

Basic Intuition I: Enough Noise Trading. In the argument thus far we alluded to the relation of public and price expectations. To establish the existence of herding or contrarian behavior, however, we must compare bid- and ask-*prices* with private expectations.

The difference is that bid- and ask-prices form a spread around the public expectation. To ensure that this spread has no adverse effect on the possibility of herding, we must ensure that it is 'tight' enough. Tightness of the spread, in turn, depends on the extent of noise trading: the more noise there is, the tighter the spread. The reason is that when the market maker sets the ask price, he accounts for the possibility of noise traders. Since the ask-price is the expectation of the asset's value conditional on the upcoming trade being a buy, the larger is the probability of noise trading, the less information is contained in the buy. And thus the closer are the bid- and ask-prices to the public expectation.

Basic Intuition II: Selling at the Initial History. We have already established that for buy-herding (and buy-contrarianism) we need that the herding candidate's csd is negatively biased, $\mathsf{Pr}(S|V_1) > \mathsf{Pr}(S|V_3)$. This is necessary for $\mathsf{E}[V|S] < \mathsf{E}[V]$. Conditions (H1) and (C1), however, require that type S actually sells at the initial history. So we must require that the spread is tight enough —or that μ be small enough— so that indeed $\mathsf{E}[V|S] < \mathsf{bid}_1$. The critical amount of noise trading that allows (2) to be negative at the initial history is obtained by $\mu^{initial}$. Note that if type S is the only type who has an expectation below the public expectation at the initial history, then $\mu^{initial} = 1$.

Basic Intuition III: Buys after the Right History. Now suppose we can find a history so that $\mathsf{E}[V|S,H_t] > \mathsf{E}[V|H_t]$. To have herding, we need that type S actually buys, which he does if $\mathsf{E}[V|S,H_t] > \mathsf{ask}_t$. Again, to have the private expectation above both the ask-price and the public expectation, the spread must be tight enough, i.e. the level of noise trading must be below a critical level μ^{change}.

So how would such a history look like? We know that herding requires that prices have increased; this occurs if and only if $q_1^t < q_3^t$. We also know that herding is possible only if the signal is U-shaped. Now suppose that the history of trades is such that state V_1 can be ignored relative to states V_2 and V_3. The basic intuition for the sufficiency of the U-shape is then that someone with a U-shaped signal would put more weight on V_3 than V_2, and thus, once state V_1 is insignificant enough he would start herd-buying.

Note that level μ^{change} may depend on the trading history, because it needs to account for the types that are trading when herding starts. For instance, if type S is the only candidate buyer after a certain history, then the critical level μ^{change} is 1.

In other words, the two restrictions on the level of noise trading may not be much of a restriction. It is also intuitively obvious that the critical μ always exists whenever $\mathsf{E}[V|S,H_t] > \mathsf{E}[V|H_t]$. The reason is that $\lim_{\mu\to 0}\mathsf{ask}_t(\mu) = \mathsf{E}[V|H_t]$ so that there must be a critical μ for which $\mathsf{E}[V|H_t] < \mathsf{ask}_t < \mathsf{E}[V|S,H_t]$.

More formally: We first derive the following useful result.

Lemma 2 (Expectation Minus Price)
(i) $\mathsf{E}[V|S,H_t] - \mathsf{p}_t^A$ *has the same sign as*

$$[\beta_2^t \mathsf{Pr}(S|V_3) - \beta_3^t \mathsf{Pr}(S|V_2)] + \frac{q_1^t}{q_3^t}[\beta_1^t \mathsf{Pr}(S|V_2) - \beta_2^t \mathsf{Pr}(S|V_1)] + \frac{2q_1^t}{q_2^t}[\beta_1^t \mathsf{Pr}(S|V_3) - \beta_3^t \mathsf{Pr}(S|V_1)]. \quad (1)$$

(ii) $\mathsf{E}[V|S,H_t] - \mathsf{p}_t^B$ *has the same sign as*

$$\frac{q_3^t}{q_1^t}[\sigma_2^t \mathsf{Pr}(S|V_3) - \sigma_3^t \mathsf{Pr}(S|V_2)] + [\sigma_1^t \mathsf{Pr}(S|V_2) - \sigma_2^t \mathsf{Pr}(S|V_1)] + \frac{2q_3^t}{q_2^t}[\sigma_1^t \mathsf{Pr}(S|V_3) - \sigma_3^t \mathsf{Pr}(S|V_1)]. \quad (2)$$

These expressions can be derived by straightforward manipulations that are outlined in the appendix.

For both buy-herding and for buy-contrarian we need that (2) is negative at the initial history and (1) is positive after some history. In the herding case, the history has to satisfy $\mathsf{E}[V|H_t] > \mathsf{E}[V]$ so that prices have increased. Herders can then be said to chase the trend. In the contrarian case, it must hold that $\mathsf{E}[V|H_t] < \mathsf{E}[V]$ so that prices have decreased. Contrarians thus act against the trend.

Suppose now that for any ϵ, there is some history, $q_1^t/q_l^t < \epsilon\ l = 2, 3$. In particular this implies that one can indeed find a history so that state V_1 can be ignored relative to states

V_2 and V_3. We can thus bound (1) by

$$[\beta_2^t \mathsf{Pr}(S|V_3) - \beta_3^t \mathsf{Pr}(S|V_2)] + \epsilon[\beta_1^t \mathsf{Pr}(S|V_2) - \beta_2^t \mathsf{Pr}(S|V_1)] + 2\epsilon[\beta_1^t \mathsf{Pr}(S|V_3) - \beta_3^t \mathsf{Pr}(S|V_1)]. \quad (3)$$

Reducing ϵ, we can thus make the last two terms arbitrarily small. So if we can find a condition that ensures that the first term in (3) is positive, we would have herding. This condition restricts the level of noise trading as outlined above and it is summarized in the next result.

Lemma 3 (Possibility of Herding and Contrarian Behavior)

(*i*) *Suppose that type S's csd is negative U-shaped. Further assume that for any* ϵ *there exists a history* H_t *such that* $q_1^t/q_l^t < \epsilon$ *for* $l = 2, 3$.
Then there exists a μ_b *so that S buy-herds with positive probability if* $\mu < \mu_b$.

(*ii*) *Suppose that type S's csd is negative hill-shaped. Further assume that for any* ϵ *there exists a history* H_t *such that* $q_3^t/q_l^t < \epsilon$ *for* $l = 1, 2$.
Then there exists a μ_b *so that S is a buy-contrarian with positive probability if* $\mu < \mu_b$.

As argued above, the critical amount of noise trading must satisfy two objectives. First, it must allows (2) to be negative at the initial history. Second, after the history so that $q_1^t/q_i^t < \epsilon$, it must ensure that (1) is positive.

Intuitively, a U-shaped csd implies that the second and third terms in expression (1) are both negative. One thus needs that the first term in (1) is positive. The critical value for $\bar{\mu}_b$ does just that solves $\beta_2^t \mathsf{Pr}(S|V_3) - \beta_3^t \mathsf{Pr}(S|V_2) = 0$. Of course, if μ exceeds μ_b, then (1) cannot be positive for all its terms are negative.

For contrarian behavior, we need that the second term in expression (1) is positive (the other two are negative because of the hill-shape). Critical value $\bar{\mu}_b$ thus solves $\beta_1^t \mathsf{Pr}(S|V_2) - \beta_2^t \mathsf{Pr}(S|V_1) = 0$.

The Special Case of Monotone Likelihood Ratio Signals. Lemma 3 leads to the next important question: is there always a history that ensures that q_1^t/q_l^t is small? The answer comes out cleanest and the result is sharpest when we assume that signals satisfy the MLRP.

Theorem 1 (Herding and Contrarian Behavior with MLRP Signals)
Let signals satisfy the MLRP. Then there exist unique critical $\bar{\mu}_b^{herd}, \bar{\mu}_b^{contra}$ *such that*

(*a*) *S_i's csd is negative U-shaped and* $\mu < \bar{\mu}_b^{herd} \Leftrightarrow \exists$ *history* H_t *with buy-herding.*

(*b*) *S_i's csd is negative hill-shaped and* $\mu < \bar{\mu}_b^{contra} \Leftrightarrow \exists$ *history* H_t *with buy-contrarianism.*

First recall that with the MLRP signals, there is always a type S_j that is csd-monotonically decreasing and a type S_k that is csd-monotonically increasing. The required history that induces herding is strikingly simple when signals satisfy the MLRP: it consists simply of sufficiently many more buys than sales.

To see that this is true observe that for no-herding buys the probability of a buy conditional on state l is $\beta_l = \lambda + \mu \mathsf{Pr}(S_k|V_l)$ is monotonic in l because $\mathsf{Pr}(S_k|V_l)$ is monotonic in l. Consequently q_1^t/q_2^t and q_1^t/q_3^t both strictly decrease in the number of buys. Or, put differently, with MLRP signals, when there are many more buys than sales, the conditional probabilities of the states change in a monotonic manner. State can eventually V_1 can be ignored relative to states V_2 and V_3. Since the herding-candidate with a U-shaped signal puts more weight on V_3 than V_2, he would herd once state V_1 is insignificant enough.

A little more formally, we know that an ϵ exists that satisfies relation (3), and since q_1^t/q_2^t and q_1^t/q_3^t decrease in buys, so that for sufficiently many more buys than sales $q_1^t/q_l^t < \epsilon$ $l = 2, 3$ will be satisfied.

Observe also that with MLRP signals there is are unique $\bar{\mu}_b^{\text{herd}}, \bar{\mu}_b^{\text{contra}}$ that ensures the existence of buy-herding/buy-contrarianism. The reason is that expectations of traders are ordered and thus when type S_i buys, type S_k will also always buy; similarly for sales.

The General Case. We are now ready to establish our main result:

Theorem 2 (Existence of Herding and Contrarian Behavior)

(a) *If there is a U-shaped, non-zero biased signal then there is a level of noise trading μ_b so that herding occurs with positive probability.*

(b) *If there is a hill-shaped, non-zero biased signal then there is a level of noise trading μ_b so that contrarianism occurs with positive probability.*

While general, there are two limitations of this result that require a discussion. First, for a given signal structure, there may be more than one noise level μ_b that is compatible with herding. The reason is that with non-MLRP signals the expectations of the signal types may cross. It is thus possible that for different histories, different types are paired to buy or sell. For instance, one can imagine a situation where there are two negative U-shapes and one positive hill-shape. In such a setting, there may be one herding history, where the type with the hill-shaped and one of the types with the negative U-shape buy; and there may be another where only one of the types with U-shaped signal buys while the type with the hill-shaped signal holds (or acts as a sell-contrarian). In each case, a different critical μ applies.

The second limitation is that we cannot claim that whenever there is a negative U-shape, that there will be buy-herding. For instance, there could be a csd with a negative

U-shape, one with a negative hill-shape and one with a positive U-shape. In this situation we can show that the positive U-shape type herds with positive probability, but we cannot show that negative U-shape type also herds (neither can we show that he does not herd).

The following result spells out precisely when the negative U-shape leads to buy-herding.

Proposition 2 (Herding and Contrarian Behavior without MLRP Signals)

(a) *Let signal S_i be negative U-shaped.*

- (i) *If there is no other U-shaped signal then there exists a μ_b so that S_i buy-herds with positive probability if and only if $\mu < \mu_b$.*
- (ii) *If one other signal is U-shaped and the third signal has a weakly positive bias, then there exists a μ_b so that S_i buy-herds with positive probability if $\mu < \mu_b$.*

(b) *Let signal S_i be negative hill-shaped.*

- (i) *If there is no other hill-shaped signal then there exists a μ_b so that S_i is a buy-contrarian with positive probability if and only if $\mu < \mu_b$.*
- (ii) *If another signal is hill-shaped and the third weakly positively biased, then there exists a μ_b so that S_i is a buy-contrarian with positive probability if $\mu < \mu_b$.*

Note that there is always a signal that is not U-shaped because probabilities of signals across each state add to one. It is possible, however, that one other signal is positive U-shaped and that the third is negative hill-shaped. This is the case that the proposition part (a) excludes; likewise for (b). However, turning the statement on its head, in this constellation sell-herding arises with positive probability. The results in parts (ii) lack the 'only if' direction for the same reasons as outlined above: if there is another U-shaped signal (hill-shaped for contrarians), then the critical value μ_b is not necessarily unique.

To show the result (and thus Theorem 2), we must identify a history such that q_1^t/q_2^t and q_1^t/q_3^t (q_3^t/q_2^t and q_3^t/q_1^t for the contrarian case) simultaneously become small.

This is not a simple task when signals do not satisfy the MLRP because it is generally not true that β_i and σ_i are monotonic, i.e. it does not suffice to have a history of sufficiently many more buys than sales to trigger herding. Instead, herding histories usually consist of several parts of the following kind: the first part, say, reduces q_1^t/q_2^t, while not increasing q_1^t/q_3^t by too much. The second part does the reverse by reducing q_1^t/q_3^t, while not increasing q_1^t/q_2^t by too much. The positive bias of the non-U-shaped signal ensure that these histories exist and that separate bounds for q_1^t/q_2^t and q_1^t/q_3^t exist that are not exceeded for specific histories.[24]

[24]Without the positive bias, the necessary bounds may or may not exist; hence the restriction in the statement of the proposition.

It is easiest to grasp the intuition of the underlying procedure for the special case when S_i is U-shaped and S_j is hill-shaped with a zero bias.[25] The first part of the history consists of a large number of holds which decrease the ratio q_1^t/q_2^t while leaving the ratio q_1^t/q_3^t unaffected. The second part of the history consists of buys. While buys increase q_1^t/q_2^t, one can reduce this ratio sufficiently in the first stage so that the subsequent increase still allows the condition for buying, (1), to be positive once q_1^t/q_3^t is small enough.

Observe again that the restriction on noise may be very weak: let S_i be negative U-shaped, S_j monotonic declining and S_k positive hill-shaped. If also $\mathsf{E}[V|S_i] < \mathsf{E}[V|S_j]$ then the initial μ is 1. Moreover, if when S_i buy-herds, S_k holds or acts as a sell-contrarian, then the change-μ is also 1.

5 Price Movements With and Without Social Learning/Herding

In this section we present our main result on the impact of herding and contrarian behavior on liquidity and price volatility. For this section we will assume that signals satisfy the MLRP; as we will argue below, we can derive strong implications on volatility for this case which, taken at face value, is the best-behaved.

To measure and compare liquidity and volatility we need a benchmark; as such we compare price movements when agents can follow the crowd and herd with price movements that may arise when agents ignore public information. In particular, we address the following two questions. First, will buys move prices less with herding/contrarian behavior than when herding/contrarian behavior/social learning is not allowed? And second, will sales move prices more with herding/contrarian behavior than when no herding/contrarian behavior/social learning is allowed? In what follows we focus on price-impacts for buy-herding and buy-contrarian behavior; the effects for sell-herding and sell-contrarianism are analogous.

To answer these questions we compare prices in a buy-herding/contrarian situation, henceforth referred to as the *rational* case, with prices in a hypothetical economy, called *naïve*, that is otherwise identical to the rational world except that

- at each date informed agents with the herding candidate signal signal S_i are naïve and unable (unwilling) to interpret the public information; they therefore buy if their expected value conditional on their private information $\mathsf{E}[V|S_i]$ exceeds the ask price,

[25] AZ's event uncertainty signal structure falls into this category. But as our analysis before illustrates, herding does not arise because there are 'multiple dimensions of uncertainty'. Instead, people herd if they have information that causes them to update probabilities of extreme outcomes faster.

sell if their expected value conditional on their private information is less than the bid price and hold otherwise;

- the market maker sets prices as before taking into account that the strategies of the informed agents with signal S_i are indeed naïve.

One justification for such naïve behavior (by S_i types) is simply that they do not observe the public history of past actions and prices. Alternatively, one can think of the naïve traders as automata that always buy or sell depending on their signals.[26]

Liquidity. In GM models liquidity is usually measured by the size of the bid ask-spread, for the larger the spread, the higher the adverse selection costs and the lower liquidity. While the spread declines over time, the question we ask is how switches of herding candidates from selling to buying affect the spread.

Casual intuition suggests that buy-herding (or contrarianism) hampers the information transmission so that both the ask-price and the bid-price decrease relative to the rational world. The argument would go, loosely, that (*i*) a 'herd-buy' carries less information than a 'no-herd' buy; and (*ii*) a 'sale' is a stronger negative signal in the rational buy-herd case than in the naïve (S_j and maybe S_i sell) case.

While the intuition for the bid-price is accurate, the intuition for buys is misleading. When buy-herding starts the ask-price *gets larger*. Again, the reason lies in the U-shape of S_i's csd. To see the intuition suppose that buy-herding starts when value V_1 can be ignored relative to both V_2 and V_3. Type S_i puts large weight on signal V_3 an relatively low weight on on V_2. Thus a buy conveys strongly that V_3 occurs that this increases the ask-price.

The result also applies to buy-contrarianism, but the interpretation differs: the hill-shaped S_i type engages in contrarian behavior when prices drop below his favored state, V_2. At this point, value V_3 is considered to be very unlikely and V_2 and V_1 are more likely. When type S_i buys at prices below V_2, his buy is strong signal that the state is not V_1 and thus expectations should move away from the wrong value quickly. Sales during a phase of buy-contrarianism, on the other hand, are a strong indication that the state is neither V_2 nor V_3 and prices move away faster from these values.

Henceforth we use ask_t for the ask-price in the rational world and ask_t^n for the ask-price in the naïve world; similarly for the bid-prices.

[26]The naïve world construction is simply a benchmark to compare the effect of social learning. In the construction of this naïve economy we assume that only S_i types behave naïvely and other informed agents behave as in the rational world (S_j and S_k types always sell and buy, respectively). This assumption is made to ensure that the only difference between the two economies is due to the behavior of the herding types, and to simplify the analysis. Otherwise, we need to allow for S_j and S_k types changing behavior in the hypothetical world (in this case the naïve world may not even have an equilibrium in pure strategies).

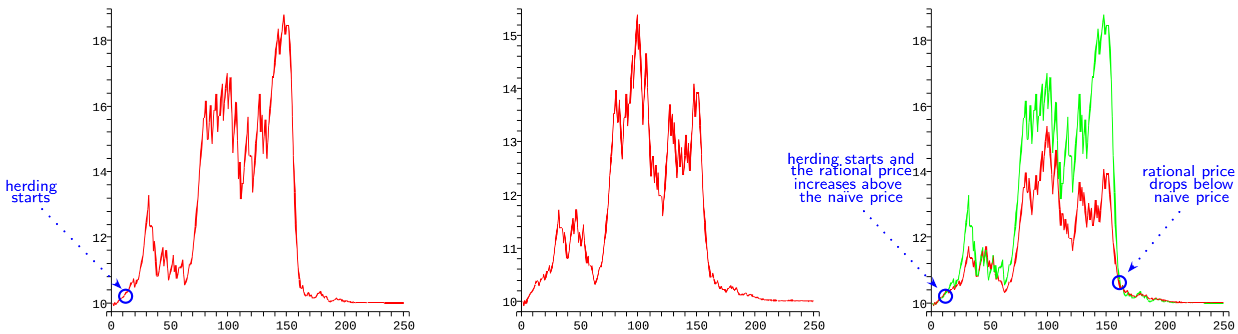

Figure 1: **Simulated Transaction Prices.** The left panel displays a simulation of transaction prices when traders behave rationally (and thus may herd). As can be seen, herding starts for middle prices (i.e. prices close to V_2), and prices during herding can move up substantially ($V_3 = 20$, $V_2 = 10$). The middle panel plots transaction prices for the same sequence of traders, but for "naïve" agents, i.e. the S_2 types merely follow their prior expectation and ignore all information in the trading history. The right panel combines both scenarios. The underlying signal distribution is listed in Appendix D where we use $\mu = \mu_b - 0.001$. The underlying trader sequence is random but for a few (5) buys in the early rounds of trading. The first author will gladly supply the simulation code upon request.

Proposition 3 (The Impact of Herd Behavior on Liquidity)

Consider any history $H_t =$ such that as H_t, the posterior of the market maker is identical for the rational and the naïve case.

(a) Let the rational trader S_i switch from not-buying to buying. Then $\mathsf{ask}_t > \mathsf{ask}_t^n$.

(b) Let the rational trader S_i switch from selling to not-selling. Then $\mathsf{bid}_t < \mathsf{bid}_t^n$.

Volatility. In the context of our sequential trading model we measure volatility by the speed with which prices move.[27] As above with liquidity, one may suspect that herd- or contrarian buys move prices less while sales move them more than in the naïve world. Yet as above, this intuition is accurate for sales, not for buys and for the same reasons that were outlined above.

Before stating the formal result on the comparison between price movements in the two worlds of naïve and rational traders, we need to introduce some further notations and definitions.

First, let $\mathsf{E}_n[V|H_t]$, $\mathsf{p}^A_{t,n}$, $\mathsf{p}^B_{t,n}$, $\beta_{i,n}$ and $\sigma_{i,n}$ be respectively the public (market) expectation, the ask-price, the bid-price, the probability of a buy in state i and the probability of a sale in state i in the naïve world.

[27] Our result does not address the total variance of prices for it is well known (since Glosten and Milgrom (1985), and as was also argued in Avery and Zemsky (1998)), that the absolute variance of price paths is bounded. Moreover, the bound is a function of primitives that have no relation to signals. We are instead concerned with the incremental price variability that herding causes relative to a no-learning benchmark.

Next suppose that $0 < \mu < \mu_b$ (so that buy-herding in the rational world is possible) and consider any history $H_t = (a_1 \ldots, a_{r+b+s})$ of outcomes, with $r > 1, b \geq 0$ and $s \geq 0$, that satisfies the following three conditions:

(N1) for any truncation $H_\tau = (a_1, \ldots, a_\tau)$ a buy-herd/buy-contrarianism in the rational world is possible if and only if $\tau \geq r$,

(N2) the path $(a_{r+1}, \ldots, a_{r+b+s})$ consists of b buys and s sells,

(N3) the posteriors of the market maker at H_r, $\mathsf{Pr}(V_i|H_r)$, are identical for the rational and the naïve case.

For one part of the result we need a minimum level of informed trading μ_{hb}; its exact specification is in Appendix A.

Proposition 4 (The Impact of Herd Behavior on Prices)

Consider any history $H_t = (a_1, \ldots, a_{r+b+s})$ that satisfies (N1)–(N3). Further assume that the S_i-type's csd is negative U-shaped.

(*a*) *Suppose that $b > 0$ and $s = 0$; then $\mathsf{E}[V|H_t] > \mathsf{E}_n[V|H_t]$.*

(*b*) *Suppose that $b = 0$ and $s > 0$. Then there exists $\bar{s} > 0$ such that $\mathsf{E}[V|H_t] < \mathsf{E}_n[V|H_t]$ for any $s \leq \bar{s}$. Moreover, if $\mu_{hb} < \mu < \mu_b$ and $\mathsf{E}[V|H_t, S_2] > \mathsf{E}[V|H_t]$, then $\mathsf{E}[V|H_t] < \mathsf{E}_n[V|H_t]$ for all s.*

(*c*) *For any s there exists $\overline{b}$ such that $\mathsf{E}[V|H_t] > \mathsf{E}_n[V|H_t]$, for all $b > \overline{b}$.*

Part (a) of the above proposition shows that if once herding starts there are only buys then the public expectation at H_t is higher in the rational world than in the naïve economy.

In part (b) if once herding starts there are only sales and the number $\bar{s}$ does not exceed some upper bound then the public expectation (which here coincides with last-period's bid price) will be smaller in the rational world than in the naïve economy; together with (a) this implies that the bid-ask-spread is wider in the rational case and thus prices move more than in the naïve world. Moreover, when the weight of the informed agents is not too small (μ exceeds μ_{hb}),[28] the public expectation in the rational world is and remains below that in the naïve case while people keep *selling*, as long as the rational S_i type remains in the buy-herding mode. There is, of course, a limit to the number of sales because eventually the buys by S_i also increase weight on state V_1 due to the U-shape.

Part (c) shows that if the number of buys b is sufficiently large relative to the number of sales then prices increase more in the rational case than in the naïve case.

[28] Condition $\mu_{hb} < \mu_b$ is generally feasible; for instance for the distribution described in Appendix D, condition (62), $\mu_{hb} \approx .258$, $\mu_b \approx .324$.

Using a series of simulated transaction prices, Figure 1 illustrates the proposition: The left panel displays rational prices, the middle panel displays naïve prices, and the right panel plots both simultaneously. The path generated in the above example is quite common, and is not restricted to a specific sequence of trades.

The contrarian case is analogous:

Corollary (The Impact of Contrarian Behavior on Prices)
Consider any history $H_t = (a_1, \ldots, a_{r+b+s})$ that satisfies (N1)–(N3). *Further assume that the S_i-type's csd is negatively hill-shaped.*

(*a*) *Suppose that $b > 0$ and $s = 0$; then $\mathsf{E}[V|H_t] > \mathsf{E}_n[V|H_t]$.*

(*b*) *Suppose that $b = 0$ and $s > 0$. Then there exists $\bar{s} > 0$ such that $\mathsf{E}[V|H_t] < \mathsf{E}_n[V|H_t]$ for any $s \leq \bar{s}$. Moreover, if $\mu_{hb} < \mu < \mu_b$ and $\mathsf{E}[V|H_t, S_2] > \mathsf{E}[V|H_t]$, then $\mathsf{E}[V|H_t] < \mathsf{E}_n[V|H_t]$ for all s.*

(*c*) *For any s there exists $\overline{b}$ such that $\mathsf{E}[V|H_t] > \mathsf{E}_n[V|H_t]$, for all $b > \overline{b}$.*

6 Large Price Movements during Herding

In the literature, namely, in AZ with the Event Uncertainty information structure, price movements during herding are strictly limited and, more to the point, herding self-defeating: buy-trades end buy-herding by themselves.[29] In our general setting, on the other hand, prices may move significantly during herding. In what follows we shall again restrict attention to the well-behaved case of MLRP signals.

To see the intuition for large price movements, suppose that buy-herding occurs at some date t and consider the effect of more buys after t. Then further herd-buys continue to increase the herding agents' expectations by more than that of the marker maker and thus the herd is not broken. The intuition for this requires some reverse thinking: When buy-herding starts, sales will be attributed to someone puts monotonically less weight on states V_3 than V_2 and on V_2 than V_1.[30] Turned on its head the herd-buys, which stem from the remaining two types taken together put monotonically more weight on state V_3 than V_2 and V_2 than V_1. Suppose now as before that herding starts when state V_1 can be ignored relative to V_2 and V_3. Further buys will thus further reduce the probability of state V_1 relatively to the other two and herding continues.

[29] This is because during herding *all* informed agents that trade take the same action; thus trades in the direction of the herd do not convey information about the high value and hence the expectations of such agents do not move so that a very small price movement stops herding.

[30] In AZ's result, this is not true because buys-herds stem from two types who both have U-shaped signals. Taken together herd-buys increased the probabilities of both states V_1 and V_3.

Of course, if there are many sales, eventually the S_i type's expectation can again drop below the ask-price. Yet the herd is robust — the more herd-buys there are, the more sales it takes to break the herd (because q_1^t/q_i^t, $i = 2, 3$, gets smaller and smaller). This is in contrast to even more advanced herding models (e.g. Smith and Sørensen (2000)) in which herding is very fragile: a *single* action against the herd can collapse the herd.

We further show that in our set-up there exists a set of priors on $\mathbb{V}$ such that herding can start when prices are close to the middle value, V_2. Indeed, the minimum number of same-direction trades necessary to induce herding turns out to be independent of the exact prior. If $\mathsf{Pr}(V_2)$ is sufficiently close to one, then herding can start for a transaction price near V_2.

The left panel in Figure 1 plots simulated transaction prices that illustrate the above points: Herding starts for prices near V_2, and during herding prices rise substantially.

The following result states that prices *can* move during herding. The extent to which herding causes prices to move 'more' (than a benchmark) was discussed in Section 5.

Proposition 5 (Persistence of Herding and the Range of Herd-Prices)
Assume $\mu < \mu_b$ and let signal S_i's csd be U-shaped with a negative bias.

(*a*) Herding towards Extreme Prices. *Suppose that there is there is buy-herding after history $H_{t'}$. Then for any $\epsilon > 0$, there exists history $H_{t'+\tau}$ following $H_{t'}$ so that*
- (*i*) *there is buy-herding at every H_t such that $t' \leq t \leq t' + \tau$ and*
- (*ii*) *the transaction price $\mathsf{p}_{t'+\tau}$ exceeds $V_3 - \epsilon$.*

(*b*) Herding at the Middle Value. *There exists $\mu_* > 0$ such that for every $\epsilon > 0$ there exists a $\delta > 0$ such that if $\mathsf{Pr}(V_2) > 1 - \delta$ and if $0 < \mu < \mu_*$ then herd-buying can start for $\mathsf{p}^* \in (V_2, V_2 + \epsilon)$.*

Contrarian behavior, naturally, has a different interpretation: a contrarian will always chooses an action that pushes prices towards his favorite state, V_2. Suppose prices fell because there were many more buys than sales. Contrarians believe that this price-drop is unwarranted and thus buy. This will increase the probability of state V_2 and V_3 relative to state V_1. When prices with contrarian buying increase 'too much', then the opposite will occur: type S_i will switch to selling. Consequently, contrarian buys will eventually stop contrarianism in the same way as sales triggered it.

7 Extensions, Discussion and Conclusion

What other implications does herding have?[31] As some types of traders may change their trading modes (e.g. during herding), prices become history-dependent. More specifically, as the entry order of traders is permutated, prices with the same population of traders can be strikingly different. Second, herding results in price paths that are very sensitive to changes in some key parameters. In particular, as we noted before, a necessary condition for herding is that the proportion of informed agents is below some critical level. Comparing two situations, one with the proportion of informed agents just below the critical level to trigger herding and one with it just above to prevent herding, prices deviate substantially in the two cases. Finally, herding slows down the convergence to the true value if the herd moves away from that true value, but it accelerates convergence if the herd moves into the right direction. The differences in speeds of convergence speak to the prevalence of herding.

What happens if there are more signals or more values? Our results will intuitively extend to cases with more signals and more values. All that is required is that signals continue to display a U-shape (for herding) or a hill-shape (for contrarian behavior). With 3 states, hill- and U-shape are still well-defined, irrespective of the number of signals, but with more states, this description is no longer the only possible signal structure that can lead to herding. The intuition with many states does, however, remain the same: the herding candidate's signal must distribute probability weight to the tails. We spell out more details in Appendix B.

What is the relation to Avery and Zemsky and what about their result that with monotonic signals, there can't be herding? AZ show that herding cannot occur if all signals satisfy a monotonicity condition. Their condition, however, is non-standard and precludes herding almost by definition. Moreover, AZ's definition of monotonicity does not imply nor is implied by the standard MLRP definition of monotonicity. It is easy to construct an example to show that AZ's definition does not imply the MLRP; to show that the converse does not hold either it suffices to note that the former does not allow herding whereas, as we have shown, the latter does if the middle signal has a U-shaped csd.[32] Finally, AZ's monotonicity definition is not a condition on the primitives of the signal distribution, but it is a requirement on endogenous variables that must hold

[31] An earlier working paper version of our paper contained several numerical simulations and examples that illustrate the claims in this paragraph. Said version also contained illustrations about changes in the number of signals. All these examples are available from the authors upon request; we omit them here to save space.

[32] We have another numerical example of an MLRP csd in which there is no herding and yet AZ's definition of monotonicity is violated. In this example, the csd allows contrarian behavior.

for all trading histories. This makes it difficult to determine ex ante whether or not a given signal distribution is monotonic. It is, of course, straightforward to show that if S is csd-monotonic then S satisfies Avery and Zemsky's definition.[33] All in all AZ's differing conclusion from ours on the possibility of herding with a monotonic information structure is simply to do with the non-standard nature of the definition of monotonicity that they adopt.

A Critical Noise Levels with MLRP Signals

In the main text we mention several critical values for noise. For simplicity of exposition assume that $S_i = S_2$. In what follows we will list the closed form expressions for these values. Define

$$\kappa^{herd} := \frac{\Pr(S_2|V_3) - \Pr(S_2|V_2)}{\rho^{23}_{23}}, \quad \kappa^{contra} := \frac{\Pr(S_2|V_2) - \Pr(S_2|V_1)}{\rho^{12}_{23}},$$
$$\theta := \frac{\Pr(S_2|V_1) - \Pr(S_2|V_3)}{\Pr(V_2)(\rho^{12}_{12} + \rho^{23}_{12}) + (1 - \Pr(V_2))\rho^{13}_{12}},$$

where for any i, j, k, l,

$$\rho^{kl}_{ij} := \Pr(S_i|V_k)\Pr(S_j|V_l) - \Pr(S_j|V_k)\Pr(S_i|V_l).$$

We consider three critical values for μ. First, the bound for μ that insures that signal type S_2 is willing to sell at the initial history (in the case when herding occurs by type S_1, this is trivially satisfied). We denote this by $\mu^{initial}$ and it is

$$\mu^{initial} = \theta_b/(\theta_b + 3).$$

Second, for herding by type $S_i = S_2$, we denote μ^{change}_{herd} as the critical level for μ so that for lower values, herding is possible. For contrarian behavior we define the critical level for μ so that for lower values, contrarian behavior is possible. These two critical levels are thus

$$\mu^{change}_{herd} = \kappa^{herd}/(\kappa^{herd} + 3), \quad \mu^{change}_{contra} = \kappa^{contra}/(\kappa^{contra} + 3).$$

For Proposition 4 we define as follows $\mu_{hb} := \kappa_{hb}/(3 + \kappa_{hb})$ where $\kappa_{hb} = (\Pr(S_2|V_1) - \Pr(S_2|V_3))/\rho^{13}_{12}$.

B Herding with more than three states

With three states, there are only four possible shapes of csds: monotonic increasing, monotonic decreasing, hill-shaped and U-shaped. With more states, the number of possible csd-shapes grows quickly, but we will argue now that some general features of csds suffice to ensure that herding is possible even when there are more states and signals.

Recall the intuition for why the U-shape is needed. First, at the initial history, the buy-herding candidate should sell. Consequently, he must be putting more weight on the lowest than the highest state. Now suppose that after some history, the probability of the lowest state is small enough so that it can be ignored. Then the herding candidate's expectation is only larger than the market maker's if he puts more weight on the highest than the middle state. Together these two imply the sufficiency of the a U-shape.

In a more general setting this intuition carries through: initially, the herding candidate should be selling. For this he must put sufficient weight into the lower part of the distribution relative to the upper part. Now suppose there is a history of trades so that values in some lower part of the distribution can be ignored. Then herding arises if the trader puts relatively more weight into the upper part of this 'truncated' distribution than in its lower part. We will present here an extreme example but it should be clear that more general formulations are possible.

[33] The proof is available from the authors upon request.

For convenience we shall assume signals satisfy the MLRP. A negative bias exists whenever ($\star$) $\mathsf{E}[V|S] < \mathsf{E}[V]$; such a bias can arise with many signal constellations; we provide one in what follows.

Without loss of generality, assume that the number of states $n > 2$ is odd so that $i^* = (n+1)/2$ is the 'middle' state. For simplicity of exposition we will use $p_i = \mathsf{Pr}(S|V_i)$ and we will drop time-subscripts. Moreover, in line with the previous analysis we shall assume that values are on an equal grid, i.e. $\{V_1, V_2, \ldots, V_n\} = \{\mathcal{V}, 2\mathcal{V}, \ldots, n\mathcal{V}\}$, and we shall assume that the prior probability distribution is symmetric, $\mathsf{Pr}(V_{i^*-j}) = \mathsf{Pr}(V_{i^*+j})$. Now define for $j = 1, \ldots, i^*$

$$F_j^-|S = \sum_{i=1}^{j} \mathsf{Pr}(S|V_i) \quad \text{and } F_j^+|S = \sum_{i=1}^{j} \mathsf{Pr}(S|V_{n+1-i}).$$

We then say $F^+|S$ first order stochastically dominates $F^-|S$ is for all i $F_i^+|S > F_i^-|S$.[34] A systematic condition on signals that guarantees a negative bias is as follows:

Lemma 4 (Negative Bias with n Signals)
Let F^+ first order stochastically dominate F^-. Then $\mathsf{E}[V|S] < \mathsf{E}[V]$.

Proof: The middle state, V_{i^*} has no impact on the result. With a symmetric prior and symmetric states, first order stochastic dominance of $\mathsf{Pr}(S|V_i)$ for $i = 1, \ldots, i^*-1$ relative to $\mathsf{Pr}(S|V_{n+1-i})$ for $i = 1, \ldots, i^*-1$ ensures that $|V_{i^*} - \sum_{i=1}^{i^*} q_i \mathsf{Pr}(V_i|S)V_i| > |V_{i^*} - \sum_{i=i^*}^{n} q_i \mathsf{Pr}(V_i|S)V_i|$. Consequently, the expectation for S is below V_{i^*}. □

Next, observe that with MLRP signals the following holds.

Lemma 5 (Action Monotonicity with MLPR Signals)
Assume that signals satisfy the MLRP. Then $\beta_1 < \beta_2 < \ldots < \beta_n$ and $\sigma_1 > \sigma_2 > \ldots > \sigma_n$.

Proof: We will show only $\beta_1 < \beta_2 < \ldots < \beta_n$, the results on sales σ_i follows analogously. To see the result, observe first that with MLRP signals, expectations are ordered in signals: for $i > j$, $\mathsf{E}[V|H_t, S_i] > \mathsf{E}[V|H_t, S_j]$. Thus if signal type S_k buys, so will all $S_l > S_k$. Thus for $i > j$,

$$\begin{aligned} \beta_i - \beta_j \quad &\propto \quad \sum_{l=k}^{n} \mathsf{Pr}(S|V_i) - \sum_{l=k}^{n} \mathsf{Pr}(S|V_j) = 1 - \sum_{l=1}^{n-k-1} \mathsf{Pr}(S|V_i) - \left(1 - \sum_{l=1}^{n-k-1} \mathsf{Pr}(S|V_j)\right) \\ &= \quad \sum_{l=1}^{n-k-1} \mathsf{Pr}(S|V_j) - \sum_{l=1}^{n-k-1} \mathsf{Pr}(S|V_i). \end{aligned}$$

This latter expression is, however, positive since the MLRP implies first order stochastic dominance of the csds. □

We can now show that herding arises if the csd increases at the extreme, i.e. if $\mathsf{Pr}(S|V_n) > \mathsf{Pr}(S|V_{n-1})$.[35]

Proposition 6 (Herding with n States)
Assume that signals satisfy the MLRP. If there exists a signal S with $\mathsf{Pr}(S|V_n) > \mathsf{Pr}(S|V_{n-1})$ and a negative bias then there exists a noise level μ_b so that type S buy-herds with positive probability if $\mu < \mu_b$.

Proof: As is clear from the previous analysis, the condition on noise trading is benign in that it merely ensures a tight bid-ask-spread. In this light, we will present the argument only for the weaker necessary condition for herding $\mathsf{E}[V|S, H_t] > \mathsf{E}[V|H_t]$ and we thus ignore a full description for the necessary noise level μ_b.

[34] Note though that neither expression is a cumulative distribution because $\mathsf{Pr}(S|V_i)$ summed over states does not add to 1.

[35] We conjecture that a more general requirement is that for some left-truncation, the truncated csd is first-order stochastically dominated by a uniform signal (which corresponds to public information).

First, the negative bias guarantees that $\mathsf{E}[V|S] < \mathsf{E}[V]$. Next, we re-write:

$$\mathsf{E}[V|S,H_t] - \mathsf{E}[V|H_t] = \sum_i V \frac{q_i p_i}{\sum_j q_j p_j} - \sum_i V q_i \propto \sum_i \mathcal{V} i \left(\frac{q_i p_i}{\sum_j q_j p_j} - q_i \right)$$

$$\begin{aligned}
\propto \quad & \sum_i i q_i \left(p_i - \sum_j q_j p_j \right) = \sum_i i q_i \left(p_i(1-q_i) - \sum_{j\neq i} q_j p_j \right) \\
= \quad & \sum_i i q_i \left(p_i \sum_{j\neq i} q_j - \sum_{j\neq i} q_j p_j \right) = \sum_i i q_i \left(\sum_{j\neq i} p_j (q_i - q_j) \right).
\end{aligned}$$

Writing out the last expression explicitly yields

$$\begin{aligned}
= \quad & q_1 q_2 (p_1 - p_2) + q_1 q_3 (p_1 - p_3) + \ldots + q_1 q_n (p_1 - p_n) \\
& + 2 q_1 q_2 (p_2 - p_1) + 2 q_2 q_3 (p_2 - p_3) + \ldots + 2 q_2 q_n (p_2 - p_n) \\
& + \ldots \\
& + n q_1 q_n (p_n - p_1) + n q_2 q_n (p_n - p_2) + \ldots + n q_{n-1} q_n (p_n - p_{n-1}) \\
= \quad & q_1 q_2 (p_2 - p_1) + \ldots + q_n q_{n-1} (p_n - p_{n-1}) \\
& + 2 q_1 q_3 (p_3 - p_1) + \ldots + 2 q_n q_{n-2} (p_n - p_{n-2}) \\
& + \ldots \\
& + (n-2) q_2 q_n (p_n - p_2) + (n-1) q_1 q_n (p_n - p_1).
\end{aligned}$$

Concisely expressed this is

$$\sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot q_i q_{i+j} (p_{i+j} - p_i) \propto \sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i q_{i+j}}{q_{n-1} q_n} (p_{i+j} - p_i). \tag{4}$$

As in the 3-state setting the last expression is obtained by dividing the LHS by the probabilities for the two highest states. Since by Lemma 5 the MLRP ensures that $\beta_1 < \beta_2 < \ldots < \beta_n$, for any i, j, expression for a buy at t holds

$$\frac{q_i^{t+1} q_{i+j}^{t+1}}{q_{n-1}^{t+1} q_n^{t+1}} = \frac{\beta_i \beta_{i+j}}{\beta_{n-1} \beta_n} \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t} < \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t},$$

so that $(q_i^t q_{i+j}^t)/(q_{n-1}^t q_n^t) \searrow 0$ for continued buys. Consequently, for sufficiently many more buys than sales,

$$\sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i q_{i+j}}{q_{n-1} q_n} (p_{i+j} - p_i) \rightarrow_{\text{many more buys than sales}} p_n - p_{n-1} > 0. \tag{5}$$

This implies that herding arises with positive probability subject to the usual condition on noise trading.□

In summary: our result straightforwardly extends to settings with many states and many signals.

C Omitted Proofs

Implications of MLRP Signals: Proof of Lemma 1

(a) By standard results on MLRP and stochastic dominance it must be that $\mathsf{E}[V|S_l] < \mathsf{E}[V|S_H]$. By a similar reasoning, at any history H_t, $\mathsf{E}[V|S_l, H_t] < \mathsf{E}[V|S_H, H_t]$ if the following MLRP condition holds at H_t: for any $S_l < S_h$ and any $V_l < V_h$

$$\frac{\Pr(S_h|V_h, H_t)}{\Pr(S_l|V_h, H_t)} > \frac{\Pr(S_h|V_l, H_t)}{\Pr(S_l|V_l, H_t)}. \tag{6}$$