Costly Contracting in a Long-Term Relationship*

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Abstract

We examine a model of contracting where parties interact repeatedly and can contract at any point in time, but writing enforceable contracts is costly. A contract can describe contingencies and actions at a more or less detailed level, and the cost of writing a contract is proportional to the amount of detail. In each period, parties can save on writing costs by modifying the previous contract rather than drafting a whole new contract. Among other things we find that, if the relationship is relatively durable and uncertainty is relatively high, it is optimal to write a long-term (possibly contingent) contract; otherwise there is ongoing contracting over time. When we allow for informal (self-enforcing) contracts, we find that these tend to be used jointly with formal contracts. We characterize the optimal mix of formal and informal contracts and examine how this changes with the underlying parameters.

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1. Introduction

The costs of writing contracts have interesting implications for the structure of contracts in a long-term relationship, as they generate trade-offs at (at least) two levels: the choice between contingent contracts and spot contracts, and the choice between formal (externally enforced) and informal (self-enforcing) contracts.[1] In this paper we develop a model that sheds light on these trade-offs, and yields interesting predictions on the resulting contractual arrangements.

We consider a multi-task, principal-agent setting with verifiable contingencies and actions, where parties interact repeatedly and can write contracts at any point in time (this includes the possibility of spot contracting, meaning that contracts are written after observing the state of nature and before actions are taken). A contract can describe contingencies and actions at a more or less detailed level, and the cost of writing a contract is proportional (in a sense to be made precise) to the amount of detail. In each period, parties can save on writing costs by modifying the previous contract rather than drafting a whole new contract.

In the first part of the paper (section 2) we focus on the implications of writing costs for the optimal structure of formal contracts, and in particular for the choice between contingent and spot contracts. At the intuitive level, it is not obvious whether the presence of writing costs should favor contingent or spot contracting: on the one hand, spot contracting avoids the cost of describing contingencies; on the other hand, spot contracts must describe the agent's *behavior* repeatedly, and this may push in favor of a contingent contract. Formal analysis can be useful – we believe – to go beyond this initial intuition.

Absent writing costs, the model has little predictive power, as there is a plethora of optimal contracting plans (including a contingent contract, a sequence of spot contracts, and a host of intermediate solutions). But with an arbitrarily small writing cost, the model yields a unique optimum. In particular, the optimum is either (i) a contingent contract, or (ii) a noncontingent contract that is modified every time the need arises. These are two alternative ways to make

[1] We will use interchangeably the expressions "formal" and "externally enforced" contract; likewise for "informal" and "self-enforcing" contract. We refrain from using the terminology "explicit" vs. "implicit" contracts – which is common in the literature – because contracts may be quite explicit even though they cannot be enforced in court.

contractual obligations responsive to the changing environment. We show that a contingent contract tends to be optimal when the relationship is relatively durable and uncertainty is relatively high.

If writing costs are not small, the optimal contracting plan may be incomplete, in the sense that some tasks are regulated by rigid rules or left to the agent's discretion. Regardless of the degree and type of contractual incompleteness, however, the result stated above generalizes, in the following sense: if a *long-term contract* is defined as a contract that is written once and for all, a long-term contract is optimal if the relationship is relatively durable and uncertainty is relatively high. On the other hand, there is likely to be ongoing contracting over time if uncertainty is low or the relationship is not very durable.

The general idea that transaction costs can contribute to explain the use of long-term contracts has already been expressed in the literature at the informal level. For example, Hart and Holmstrom (1987, p. 130) write: "if a relationship is repetitive, it may save on transaction costs to decide in advance what actions each party should take rather than to negotiate a succession of short term contracts."[2] Our model provides a formal examination of this idea for a particular type of transaction costs, namely the costs of writing detailed contracts.[3]

We emphasize that the predictions of our model would be radically different if writing costs were modeled in a more conventional way, and in particular along the lines of Dye's (1985a) well-known model of costly contracting. For example, we show that, with writing costs a' la Dye, if the number of possible states is large enough a contingent contract is dominated by a

[2] Of course there is another theoretical explanation for long-term contracts, which should be viewed as complementary to the transaction-cost explanation: the fact that long-term contracts provide long-term commitment. Long-term commitment may be valuable to induce parties to make relationship-specific (long-term) investments, to facilitate intertemporal smoothing or insurance, or to provide incentives to reveal private information. Papers that highlight these benefits of long-term contracts include Townsend (1982), Lambert (1983), Allen (1985), Rogerson (1985), Harris and Holmstrom (1987), Crawford (1988), Malcomson and Spinnewyn (1988), Rey and Salanie (1990), Fudenberg *et al.* (1990).

[3] Somewhat related to the present paper is Lipman (1997), who analyzes the implications of computation costs for the tradeoff between long-term and short-term contracts. Lipman considers boundedly rational agents who trade repeatedly and can learn the payoff implications of future contingencies only by paying a 'computation cost'. Relatively high computation costs lead to short-term contracts. Low computation costs may lead to long-term contracts. We should also mention that there is a fairly large literature on complexity costs as a cause of contractual incompleteness in a *static* setting. See for example Dye (1985a), Anderlini and Felli (1994, 1999), Krasa and Williams (1999), MacLeod (2000) and Battigalli and Maggi (2000).

sequence of spot contracts.

Two more remarks are in order before we turn to the issue of formal vs. informal contracts. As we mentioned previously, by "writing costs" we mean costs that are proportional to the amount of detail in the contract. Another type of transaction costs that can explain a preference for long-term contracts is fixed per-contract costs.[4] We notice, however, that the implications of fixed contracting costs are very different from those of writing costs. For example, if in our model we replaced our writing costs with a fixed contracting cost, it would always be optimal to write a complete contingent contract (or no contract at all). Fixed contracting costs cannot explain the occurrence of ongoing contracting over time, or of incomplete long-term contracts. Moreover, as will soon become clear, fixed contracting costs cannot explain the simultaneous use of formal and informal contracts.

The second remark concerns Maskin and Tirole's (1999) well-known irrelevance result. They argue, within a static setting, that the presence of unforeseen contingencies (or, by an extension of their argument, the costs of describing contingencies) does not imply inefficiencies in contracting, provided that parties can design an appropriate message-based mechanism to be played after contingencies are observed and before actions are taken. Since in our setting parties have the option of observing the state before contracting, mechanisms *à la* Maskin-Tirole are redundant. As a consequence, the costs of describing contingencies can cause inefficiencies in contracting even if mechanisms *à la* Maskin-Tirole are available. Our analysis thus shows that the Maskin-Tirole critique may lose relevance in a dynamic setting.

In the second part of the paper (section 3) we introduce the possibility of self-enforcing contracts, i.e. contracts that are enforced by reputation mechanisms rather than by external courts. The advantage of a self-enforcing contract is that it can be communicated informally, rather than being written formally, because it need not be enforceable in court, and this saves on writing costs. On the other hand, the absence of an external enforcement mechanism may limit the effectiveness of a self-enforcing contract. Hence there is a trade-off between formal and informal contracting. When we allow for both formal and informal contracting in the model, we find that they tend to be used jointly, with some tasks regulated formally and others regulated

[4] There are a few papers in the macro/labor field where long-term contracts are motivated by the presence of fixed per-contract costs. Examples of this literature are Gray (1976, 1978) and Dye (1985b).

informally. In particular, low-cost tasks are regulated by informal contract, intermediate-cost tasks are regulated by formal contract, and high-cost tasks are left to the agent's discretion. The presence of writing costs can thus contribute to explain the fact that long-term relationships are often managed by a combination of formal and informal contracting.

The relative importance of formal versus informal contracting is captured by the ratio between the number of tasks regulated formally and the number of tasks regulated informally. We find that this ratio need not decrease with the writing cost. Moreover, as the writing cost approaches zero, the optimal contract need not be fully formal, and may even be fully informal. We also find that the relative importance of formal contracting need not decrease with the durability of the relationship, even though a more durable relationship makes informal contracts easier to sustain.

The interaction between formal and informal contracts is analyzed also in Baker *et al.* (1994) and Pearce and Stacchetti (1998).[5] These papers propose a different – and in many respects complementary – explanation for the combined use of the two types of contract. They consider a repeated principal-agent model where parties can write a formal contract based on verifiable signals of the agent's action, and/or an informal contract based on unverifiable signals. Both papers find that it may be optimal to offer a combination of a formal wage and an informal 'bonus'. However, there are important differences between these models and ours, both in the focus of the analysis and in the comparative-statics predictions. We will discuss these differences at length in section 3.

2. A model of formal contracting

We start by modeling the language used to write contracts.

$\Pi^e = \{e_1, e_2, e_{3,}...\}$ is a finite collection of *primitive sentences*, each of which describes an *elementary event* concerning the external environment. For example, e_1: "the passenger has a moustache", e_2: "the passenger's bag is red".

[5] There is also a vast literature on purely self-enforcing contracts. Bull (1987) and Mac Leod and Malcomson (1989) are two prominent examples of this literature.

$\Pi^a = \{a_1, a_2, a_3, ..\}$ is a finite collection of primitive sentences describing *elementary actions* (behavioral events, or *tasks*), for example, a_1 : "check the passenger's passport", a_2 : "search the passenger's bag".

With a slight abuse of terminology, we will use the notation e_k (resp. a_k) to indicate both an elementary event (resp. action) and the primitive sentence that describes it.

We assume that this language is the (only) common-knowledge language for the parties and the courts. This ensures that there are no problems of ambiguous interpretation of the contract.

A *state* is a complete description of the exogenous environment, represented by a valuation function $s : \Pi^e \to \{0, 1\}$, where $s(e_k) = 1$ means that primitive sentence e_k is true at state s and $s(e_k) = 0$ means that primitive sentence e_k is false at state s.[6] In other words, $s(e_k)$ is a dummy variable that takes value one if elementary event e_k occurs and zero otherwise, and a state is a realization of the vector of dummy variables $(s(e_1), s(e_2), ...)$.

Similarly, a *behavior* is a complete description of all elementary actions, represented by a valuation function $b : \Pi^a \to \{0, 1\}$, where $b(a_k) = 1$ means that elementary action a_k is executed, and $b(a_k) = 0$ that a_k is not executed.

To simplify the analysis we assume a very simple payoff structure. There is a one-to-one correspondence between elementary tasks and elementary events. The principal wants task k to be performed if and only if elementary event k occurs. In our airport example, the principal wants the agent to check the passenger's passport if and only if the passenger has a moustache, and to search his bag if and only if the bag is red.

Principal and agent are risk neutral. The principal gets an incremental benefit of one from "matching" e_k with a_k, while he gets zero incremental benefit if there is a "mismatch". Formally, the principal's per-period utility is:

$$\pi(s, b, m) = \sum_{k=1}^{N} [s(e_k)b(a_k) + (1 - s(e_k))(1 - b(a_k))] - m \tag{2.1}$$

where m is the payment to the agent.

[6] To simplify the exposition we describe the basic notation omitting time subscripts. We will introduce time subscripts later in this section, when we describe the timing of the game.

The agent's interests are always in conflict with the principal's, in the sense that his preferred actions are always opposite the principal's preferred actions. Formally, the agent's one-period utility is:

$$U(s, b, m) = m - \sum_{k=1}^{N} \delta_k [s(e_k)b(a_k) + (1 - s(e_k))(1 - b(a_k))], \ (0 < \delta_k < 1) \tag{2.2}$$

The parameter δ_k captures the disutility associated with task k for the agent. The agent's reservation utility is zero. Payoffs are common knowledge to the contracting parties, and the state and the parties' behavior are verifiable in court. Thus, there are no issues of moral hazard or adverse selection. We assume that preferences and realized payoff levels are not verifiable in court, and that the principal cannot "sell the activity" to the agent (i.e., the agent cannot be made the recipient of the gross payoff π).[7]

Next we define a contract and the costs of writing it. A contract is a pair (g, m) where $g = (\gamma_k)_{k=1}^{N}$ is a set of N clauses and m is a transfer from the principal to the agent (wage).[8]

Each clause γ_k regulates a task. Given our simple matching structure between tasks and elementary events, we can restrict our attention to four types of clause: (i) a contingent clause, that constrains the agent to do a_k if and only if e_k occurs, $C_k : [a_k \leftrightarrow e_k]$; (ii) a noncontingent positive clause, constraining the agent to do a_k whatever happens, $R_k : [a_k]$; (iii) a noncontingent negative clause, constraining the agent to do *not* a_k whatever happens, $\bar{R}_k : [\neg a_k]$; (iv) the empty clause, D, that imposes no constraint on the agent (note that since we include the empty clause among the possible clauses, there is no loss of generality in assuming that the number of clauses in the contract is N). For example, if $N = 3$, the set of clauses (R_1, D, C_3) constrains the agent to do a_1 whatever happens and to do a_3 if and only if elementary event e_3 occurs, leaving the agent free with regard to task 2. We denote G the collection of all possible sets of clauses (thus, for any contract (g, m), $g \in G$).

[7] If preferences were verifiable, the first-best outcome could trivially be achieved by a contract of the form "The agent's behavior b must maximize the sum of the parties' utilities." On the other hand, if realized payoff levels were verifiable, the first-best outcome could be achieved by offering the agent a transfer that increases one-for-one with the principal's realized payoff level. And selling the activity to the agent would be equivalent to specifying a contingent transfer as in the previous point.

[8] It can be shown that there is nothing to gain from making payments contingent on the state or on the agent's behavior, due to the assumptions of risk neutrality, verifiable actions and conflict of interests.

Describing a task or an elementary event is costly. To simplify, we assume that the cost of describing a task and the cost of describing an elementary event are both equal to c. It follows that writing a contingent clause C_k costs $2c$, and writing a noncontingent clause (R_k or $\bar{R}_k$) costs c. We also assume that specifying the wage in the contract is costless, thus the cost of writing a set of clauses $g \in G$ is $Cost(g) = 2cN_C^g + cN_R^g$, where N_C^g is the number of contingent clauses and N_R^g is the number of noncontingent clauses. The writing cost is borne entirely by the principal.[9]

Next we describe the timing of the game (and introduce time subscripts in the notation). The parties interact for infinitely many periods and have common discount factor $d \in (0, 1)$. The parameter d can also be interpreted as capturing the stability of the relationship.[10] In each period $t = 1, 2, ...$ the timing is the following: the state of nature $s_t \in S$ is observed, then the principal offers a contract (g_t, m_t) to the agent, where $g_t = (\gamma_{kt})_{k=1}^N$, $\gamma_{kt} \in \{C_k, R_k, \bar{R}_k, D\}$. The principal pays the cost of drafting the contract (g_t, m_t). If the contract is accepted, the principal makes the payment m_t and then the agent acts, both players being constrained by the contract.[11] If the contract is rejected, the agent gets his reservation utility (zero).

In the Markovian equilibrium analyzed in this section, the wage m_t will be set at the minimum level that induces the agent to accept the proposed contract. Since the determination of the wage is a trivial aspect of the analysis, we will focus on the set of clauses. With a slight abuse of our terminology, from now on we will refer to a "contract" simply as its set of clauses.

If at time t the principal wants to offer a different contract than the one at time $t-1$, he can save substantial writing costs by proposing a *modification* of the previous contract, rather than drafting a whole new contract. Contract modifications can take two forms: (i) *amendments*, that is, permanent modifications of the contract; or (ii) *exceptions*, that is, temporary modifications applied only for the current period. We allow the principal to modify the existing contract with

[9] On the "strategic" effects of transaction costs paid by both parties, see Anderlini and Felli (1997).

[10] The parameter d can be interpreted as the composition of two parameters, $d = qd'$, where q is the probability that the game will continue and d' is the 'true' discount factor.

[11] The assumptions that the principal pays before the agent acts, and that the principal cannot pay more than what is specified in the contract, are not essential for the analysis of formal contracting. We will come back to these issues in section 3 when we analyze self-enforcing contracts. Note also that nothing would change if we allowed the principal to choose whether to write the contract before or after the state s_t is observed, as there is no gain from writing the contract before the state is observed.

any set of amendments and exceptions at any point in time.[12]

To capture this idea, we distinguish between the *effective* contract (the contract actually enforced at time t) and the *default* contract. The effective contract at t is given by the default contract at t plus the exceptions at t, and the default contract at t is given by the default contract at $t-1$ plus the amendments at t. The default contract will be the key state variable of our problem, while the amendments and exceptions will be the control variables.

More formally, the *default contract* is a set of clauses

$$\tilde{g}_t : (\tilde{\gamma}_{k,t})_{k \in N},$$

where $\tilde{\gamma}_{k,t} \in \{C_k, R_k, \bar{R}_k, D\}$. The default contract at time t is given by:

$$\tilde{g}_t = \tilde{f}(\tilde{g}_{t-1}, g_t^A) = \left((\tilde{\gamma}_{k,t-1})_{k \in N \setminus K_t^A}, (\alpha_{k,t})_{k \in K_t^A} \right),$$

where $\alpha_{k,t} \in \{C_k, R_k, \bar{R}_k\}$ is the amendment for task k and $g_t^A = (\alpha_{k,t})_{k \in K_t^A}$ is the set of amendments. For each task k, the default clause at $t = 0$ is the empty clause: $\tilde{\gamma}_{k,0} = D$.

The *effective contract* at time t is given by:

$$g_t = f(\tilde{g}_t, g_t^E) = \left((\tilde{\gamma}_{k,t})_{k \in N \setminus K_t^E}, (\varepsilon_{k,t})_{k \in K_t^E} \right),$$

where $\varepsilon_{k,t} \in \{C_k, R_k, \bar{R}_k\}$ is the exception for task k and $g_t^E = (\varepsilon_{k,t})_{k \in K_t^E}$ is the set of exceptions.

The writing cost paid in period t is $Cost(g_t^A) + Cost(g_t^E)$.[13]

[12] Implicit in this formulation is the assumption that the contract for time $t-1$ can be used as default contract for time t, but contracts from earlier dates cannot. For example, we do not allow contract g_t to say "contract g_{t-2} applies with the following modifications...". A more general model would allow for richer 'recalling' possibilities, but we conjecture that, if there is a costs of recalling more remote contracts, the key insights of the analysis would not change.

[13] We assumed that the language described at the outset is the only common-knowledge language. In principle, the parties could construct a new language, for example by attaching a new primitive sentence to each state and to each behavior, and write a contract with the new language. Note that the parties would have to attach a vocabulary that translates the new language into the original one, in order for the courts to be able to interpret the contract. If the relationship is one-shot, the new language cannot be more efficient than the original one, because the cost of writing the vocabulary in the contract is at least as great as its benefits. In a repeated relationship, however, this approach might in principle be efficient. (We thank Leonardo Felli and Luca Anderlini for bringing this point to our attention.) A more general model would allow for this kind of recoding of the language, but we conjecture that the main qualitative results would not be affected.

Note that we are not considering the possibility of multi-period contracts, but this is without loss of generality. We could allow the contract at time t to specify wages or tasks for future dates, but there would be no gains from doing so.[14]

We assume that the stochastic process governing the external environment is a Markov chain;[15] the transition probabilities are denoted by $\mu(s_{t+1}|s_t)$. We will first focus on the special case of an *i.i.d.* process and then show how the results change with persistent shocks. To streamline the exposition, we also assume that in the first period the state is $(1, ..., 1)$, i.e., $s_1(e_k) = 1$ for all k. In the appendix we solve the model dropping this assumption.

In this section we focus on stationary Markov perfect equilibria, that is, subgame perfect equilibria in which current decisions depend only on the state variable, i.e. the current state of the environment and the default contract of the previous period [see Fudenberg and Tirole (1991, Ch. 13)].

The game may have also subgame perfect equilibria that support some cooperation without the aid of formal contracts. These are equilibria where current decisions depend on past behavior ('punishment' strategies). We think of these equilibria as "self-enforcing contracts". The reason we ignore these equilibria in this section is to focus more sharply on the role of formal contracts, and on the issue of long-term versus short-term contracts. We will consider self-enforcing contracts in the next section.

Given the simple structure of the interaction, solving for the stationary Markov perfect equilibria boils down to maximizing the expected discounted value of the surplus net of writing costs. To state the problem formally, we need to introduce the policy function $h : G\times S \to G\times G$, or in more explicit notation, $(g_t^A, g_t^E) = h(\tilde{g}_{t-1}, s_t)$. The policy function induces, at each t, a random value for the surplus net of writing costs, which we denote $\mathbf{NS}_t^h : S^t \to \mathbb{R}$. The problem can then be stated as

$$\max_h \mathrm{E}\left[\sum_{t=1}^{\infty} d^{t-1}\mathbf{NS}_t^h\right] \tag{2.3}$$

[14] This is because (i) there are no gains from long-term commitment, (ii) there are no fixed contracting costs, and (iii) the contract of the previous period can be used as default contract for the current period.

[15] A Markov chain is a Markovian process with one-period memory and stationary transition probabilities (see, e.g., Gallager (1996, p. 103)).

An *optimal contracting plan* is a solution of problem (2.3).

2.1. Independent shocks

As a first step of the analysis, we consider the case in which elementary events are identically and independently distributed across t and k. In the next section we will consider the case of serially correlated events.

Let us assume that, for every $k = 1, ..., N$, $t = 2, 3, ...$ and $s_t, s_{t-1} \in S$,

$$\mu(s_t|s_{t-1}) = p^{\sum_k s_t(e_k)}(1-p)^{[N-\sum_k s_t(e_k)]}, \qquad p \in (\frac{1}{2}, 1) \tag{2.4}$$

(recall that $s_1(e_k) = 1$ for all k). The probability that an elementary event occurs is given by p, that is, $p \equiv \Pr\{s_t(e_k) = 1\}$ for all k and $t \geq 2$. We can think of p as capturing the degree of uncertainty in the environment: the higher p, the lower the uncertainty (notice that the variance of dummy variable is decreasing in p).

Given our assumptions, we can derive the optimal contracting plan by looking separately at each task k.[16] It turns out that each task is optimally regulated in one of four ways:

1. A C_k clause written at time $t = 1$, with no subsequent modifications. We will refer to this as a *contingent rule*, denoted by $\mathcal{C}_k$.

2. A default clause R_k followed by an exception every time $s_t(e_k) = 0$ (e_k does not occur). We will refer to this as a *default rule cum exceptions*, denoted $\mathcal{DE}_k$.

3. A default clause R_k with no subsequent modifications. We will refer to this as a *rigid rule*, denoted $\mathcal{R}_k$.

4. No clause at any t (discretion for task k), denoted by $\mathcal{D}_k$.

This is the right juncture to discuss the notion of *contract incompleteness* in this dynamic setting. Contract incompleteness can take two basic forms: (a) *rigidity*, meaning that the

[16] The reader may wonder why we did not simplify the model by assuming that there is a single task. The reason is that, when we allow for self-enforcing contracts, the problem will no longer be separable in the N tasks, as incentive constraints will create interactions between tasks.

contractual obligations do not discriminate sufficiently between states, and (b) *discretion,* in the sense that the contractual obligations do not completely specify the agent's behavior. In this setting, a simple measure of contractual rigidity is the number of tasks regulated by rigid rules, and a measure of discretion is the number of tasks that are left unregulated.

Importantly, the notion of contract incompleteness must be understood in a dynamic perspective. For example, the presence of noncontingent clauses in a contract does not imply that there is contractual rigidity, because the noncontingent clauses may be modified over time. In particular, note that a default rule cum exceptions implements the first best outcome for task k at all times, just as a contingent rule.

It is straightforward to derive the net incremental value of these four rules:

$\mathbf{k}^{th}$ **Rule**	**Incremental Net Value**
$\mathcal{C}_k$	$\frac{1-\delta_k}{1-d} - 2c$
$\mathcal{DE}_k$	$\frac{1-\delta_k}{1-d} - \frac{(1-dp)c}{1-d}$
$\mathcal{R}_k$	$1 - \delta_k + \frac{dp(1-\delta_k)}{1-d} - c$
$\mathcal{D}_k$	0

Table 1

In the next proposition, $N_{\mathcal{C}}$, $N_{\mathcal{DE}}$, $N_{\mathcal{R}}$ and $N_{\mathcal{D}}$ denote respectively the numbers of tasks regulated by rules 1, 2, 3 and 4 above. We refer to a *complete contingent contract* as a contracting plan where each task is handled by a contingent rule, and to a *complete default contract cum exceptions* as a contracting plan where each task is handled by a default rule cum exceptions. (Also, when we use the expression "increasing" or "decreasing" without further specification we mean it in the weak sense.)

Proposition 1. *(i) If c is smaller than some critical level, the optimum is either a complete contingent contract or a complete default contract cum exceptions. The former is preferred if and only if d is higher than the critical level $\overline{d}(p) = \frac{1}{2-p}$.*
(ii) In general, a set of low-δ_k tasks is regulated entirely by $\mathcal{C}_k$ rules or entirely by $\mathcal{DE}_k$ rules; a set of intermediate-δ_k tasks is regulated by $\mathcal{R}_k$ rules; and a set of high-δ_k tasks is left to the agent's discretion (each of these sets may be empty). If $d > \overline{d}(p)$, the contract is written once and for all at $t = 1$.

(iii) $N_{\mathcal{C}}$ is increasing in d; $N_{\mathcal{DE}}$ and $N_{\mathcal{D}}$ are decreasing in d. If d is higher than some critical level, a complete contingent contract is optimal.
(iv) $N_{\mathcal{C}}$ and $N_{\mathcal{D}}$ are decreasing in p; $N_{\mathcal{DE}}$ and $N_{\mathcal{R}}$ are increasing in p.

Absent writing costs, the model has little predictive power, because there is a vast multiplicity of optimal contracting plans. Any contracting plan that implements the first best is optimal. These include a complete long-term contingent contract, a sequence of complete spot contracts, and a whole host of intermediate solutions. However, an arbitrarily small writing cost is sufficient to pin down a unique optimum. Point (i) states that, if the writing cost is small, the optimum is either a complete contingent contract or a complete default contract cum exceptions. These are two ways to implement the first best outcome for all states. The former tends to be optimal when the relationship is more durable (d is higher) and when there is more uncertainty in the environment (p is lower).[17] On the other hand, when uncertainty is low or the relationship is not very durable, the first best can be achieved at lower cost by writing default rules and occasionally negotiating exceptions when low-probability events occur.

If c is higher, the optimal contracting plan may be incomplete. In particular, high-cost tasks are left to the agent's discretion and intermediate-cost tasks are regulated by rigid rules. Low-cost tasks, on the other hand, are regulated by contingent rules or by default rules cum exceptions. Since both of these rules achieve the first best outcome, the more efficient is the one that minimizes (the present expected value of) writing costs, hence their comparison is independent of δ_k. This is why this group of tasks is regulated entirely by contingent rules or entirely by default rules cum exceptions.

If the contract is written once and for all at $t = 1$ (i.e. it is not modified over time), we interpret it as a *long-term contract*. Literally interpreted, this is a one-period contract that is renewed every period. However, an equivalent strategy would be to write a contract with no expiration date at $t = 1$; moreover, a small fixed cost of contracting would make such a contract strictly optimal. Therefore we feel justified in viewing this as a long-term contract. From table 1 it follows immediately that, if $d > \overline{d}(p)$, default-cum-exceptions rules are dominated, hence the

[17] Point (i) implies that a contingent contract is optimal if p is lower than the critical level $\bar{d}^{-1}(d)$, where $\bar{d}^{-1}(\cdot)$ is the inverse of $\bar{d}(\cdot)$.

contract is never modified over time. Our model thus offers a simple (and potentially testable) prediction: a long-term contract tends to be optimal if the relationship is relatively durable or uncertainty is relatively high. Ongoing contracting may be optimal only if uncertainty is low or the relationship is not very stable.[18]

Point (iii) and (iv) look at how the optimal contract is affected by changes in d and p. If the relationship is more stable, the optimal contract tends to be more contingent, and if it is sufficiently stable the model predicts a complete contingent contract. If there is more uncertainty, noncontingent rules (rigid and default-cum-exceptions) are less attractive. This in turn makes the other two options, contingent rules and discretion, more attractive.

2.2. Persistent shocks

Thus far we have assumed that elementary events are serially independent. In this subsection we examine how results change when exogenous shocks are persistent. We will find that, when shocks are persistent, it may be optimal to use *amendments*, rather than exceptions, as a way to adapt the default rules to changing events.

We assume that, for every $t = 2, 3, ...$ and $s_t, s_{t-1} \in S$, the transition probabilities are described by:

$$\mu(s_t|s_{t-1}) = (\frac{1}{2}+\rho)^{\sum_k I_{[s_t(e_k)=s_{t-1}(e_k)]}(s_t)}(\frac{1}{2}-\rho)^{[N-\sum_k I_{[s_t(e_k)=s_{t-1}(e_k)]}(s_t)]},\ \rho \in (0, \frac{1}{2}). \tag{2.5}$$

where $I_{[s(e_k)=s'(e_k)]}(s)$ is an indicator function that takes value one if $s(e_k) = s'(e_k)$ and zero otherwise. (We do not need to assume anything about $\mu(s_1)$.) In words, for each t and k, the probability that $s_t(e_k)$ is equal to $s_{t-1}(e_k)$ is $(\frac{1}{2}+\rho)$, and the probability that $s_t(e_k)$ is different from $s_{t-1}(e_k)$ is $(\frac{1}{2}-\rho)$, and there is "cross-sectional independence". The parameter $\rho \in (0, \frac{1}{2})$ captures the persistence of shocks in the external environment.

Note that, unlike in the previous section, we are assuming that the elementary events e_k and $\neg e_k$ are symmetric. This allows us to focus more sharply on the role of persistence.

[18] The reason we did not state that a long-term contract is optimal *if and only if* $d > \bar{d}(p)$ is the following. It is direct to verify that $d > \bar{d}(p)$ is a necessary and sufficient condition for a long-term contract to be optimal as long as there is at least one task for which the gross surplus exceeds c. If however this is not the case, then a long-term contract may be optimal even if $d < \bar{d}(p)$, because in this case a $\mathcal{DE}_k$ rule is dominated by a $\mathcal{R}_k$ rule.

It turns out that the qualitative results are very similar to those of section 2.1, with the following modification. A default rule cum exceptions can no longer be optimal, and there is a new candidate optimal rule: a *default rule cum amendments*. This is a default clause that is amended every time the realization of $s(e_k)$ changes. The fact that amendments are more efficient than exceptions, as a way to adapt default rules to the changing environment, is an intuitive consequence of persistence.[19] Note that, like contingent rules and default rules cum exceptions, default rules cum amendments implement the first best at all times.

The following proposition highlights the changes in results relative to the previous section.

Proposition 2. *If $\mu(s_t|s_{t-1})$ is described by (2.5), Proposition 1 still holds, provided "exceptions" is replaced with "amendments" and p is replaced with $(\frac{1}{2}+\rho)$.*

This proposition suggests that serial correlation in the states has similar implications as 'intrinsic' asymmetries among states (which we considered in the previous section), with the difference that amendments are now preferred to exceptions. The general insight is that low uncertainty about the future state makes contract modifications (amendments or exceptions) preferable to contingent clauses. When low uncertainty is due to persistence, amendments are preferred. When it is due to intrinsic asymmetries between likely and unlikely states, exceptions are preferred.

Note that a long-term contract is optimal if $d > \bar{d}(\frac{1}{2}+\rho)$, where $\bar{d}(\cdot)$ is an increasing function. Again, we can interpret this result as saying that a long-term contract tends to be optimal when the relationship is relatively durable and uncertainty is relatively high.

A more general model would allow for intrinsically asymmetric states *and* persistent shocks. This would be substantially more complicated to analyze, but we conjecture that the main qualitative insights would not change, except that the optimal contracting plan would probably involve the use of both exceptions and amendments.

[19] Exceptions and amendments are equivalent in the knife-edge case $\rho = 0$.

2.3. Language matters

In this section we argue that the predictions of the model depend heavily on our language-based approach, and could differ radically if writing costs were modeled in a different way. To make this point, we consider a more 'traditional' specification of writing costs, similar in spirit to Dye (1985a).

Let $\{s^1, ..., s^M\}$ be the set of states and $\{b^1, ..., b^M\}$ the set of behaviors, and assign indices so that it is efficient to do b^j if and only if the state is s^j, for all $j \in M$. Now assume that the cost of describing a state and the cost of describing a behavior are both equal to c. Suppose c is small, so that it is optimal to implement the first best mapping. Keep all other assumptions of our model unchanged.

In this version of the model, if the number of states M is sufficiently large, a sequence of spot contracts is optimal, and in particular it dominates a contingent contract. To see this, note that the cost of a complete contingent contract is $2cM$, while the discounted cost of a complete sequence of spot contracts does not exceed $\frac{c}{1-d}$. The intuition is that spot contracting (i) avoids the costs of describing states, and (ii) requires describing a (weakly) smaller number of behaviors than a contingent contract.

Thus, this alternative specification of writing costs implies that spot contracting is optimal, in stark contrast with our model. This should clarify our statement that the nature of language matters greatly for the predictions of the theory.

2.4. The Maskin-Tirole argument

Maskin and Tirole (1999) have argued that the presence of unforeseen contingencies (or, by a straightforward extension of their argument, the costs of describing contingencies) does not imply inefficiencies in contracting, provided that parties can design an appropriate message-based mechanism to be played after contingencies are observed and before actions are taken. Since in our setting parties are allowed to contract in 'spot' fashion, i.e. after contingencies are observed and before actions are taken, mechanisms *à la* Maskin-Tirole are redundant, hence none of our conclusions changes if such mechanisms are available. In particular, it remains true that the costs of describing contingencies cause inefficiencies in contracting, which is in contrast with Maskin and Tirole's argument. The reason for this apparent divergence in conclusions is that we allow for a repetitive relationship and for costs of describing behavior, while Maskin and Tirole do not.

Recall that in our model, under some parameter values, parties choose to write a contingent contract and incur the corresponding costs of describing contingencies, even if they have the option of writing spot contracts. Suppose the cost of describing an elementary event is distinct from the cost of describing an elementary task, and consider increasing the former, keeping the latter constant. Can this increase inefficiency? The answer is yes. As we have seen, there is a parameter region in which it is optimal to write contingent clauses. This is *a fortiori* true in the extended parameter space where the cost of describing contingencies is distinct from the cost of describing behavior. Therefore, starting from this parameter region, an increase in the costs of describing contingencies decreases the net surplus.

3. Formal and informal contracting

In reality, long-term relationships are often managed by informal (self-enforcing) contracts. It also happens frequently that a relationship is governed by a combination of informal and formal contracts. In this section we examine how the predictions of the model change when parties have the option of using informal as well as formal contracts.

Informal contracts have an advantage over formal contracts, namely that they can be based on informal communication (i.e. communication for the only purpose of reciprocal understanding), as opposed to formal communication (i.e. communication for the purpose of making the contract enforceable in court). Arguably, the cost of the latter is higher than the cost of the former, because for the contract to be enforced by courts it must be written according to the commonly accepted legal standards, which may be quite cumbersome to meet. In particular, it is not sufficient that the language used in the formal contract be common knowledge to the contracting parties; it has to be common knowledge to the parties *and* the courts, and this may require effort and skills (or lawyers).[20]

The shortcoming of informal contracts, on the other hand, is the absence of an external enforcement mechanism. Since an informal contract must satisfy self-enforcement constraints, it may have to be distorted away from the first best. In what follows we will examine more closely this trade-off between formal and informal contracting.

3.1. Efficient equilibria

We assume that there is no cost associated with informal contracts. We could allow for a cost of informal contracts, but this would change the main results in an obvious direction, tilting the balance in favor of formal contracts. Also, what matters most for the trade-off between formal and informal contracts is the *differential* cost of formal versus informal norms, and this is captured in our model by the parameter c.

Consider the game of section 2.1 (where we assume *i.i.d.* shocks). The way we allow for informal contracts is by looking at the set of subgame perfect equilibria, rather than at the Markov perfect equilibrium. In particular, following a common approach in the literature on self-enforcing contracts, we will focus on constrained Pareto-efficient subgame perfect equilibria.

In a subgame perfect equilibrium, players' actions may be regulated by formal or informal norms. The set of formal norms at a given point in time is given by the formal contract that is in effect at that time. Actions that are not regulated by formal norms may be regulated by informal norms, which are enforced by the threat of reverting to a worse equilibrium. Our objective is to understand under what conditions it is efficient to use both formal and informal norms, and if so, which tasks tend to be regulated by one or the other, and how the underlying parameters affect the optimal mix of norms.

In order to simplify the analysis we assume that players are patient enough to make maxmin punishments credible. More specifically, we assume that $d > \overline{d}(p)$, where $\overline{d}(p)$ is the critical level defined in Proposition 1. This restriction ensures the existence of a credible punishment strategy that keeps the principal at his maxmin, as the lemma below states (we already know

[20] See footnote 1 for a discussion of our terminology of formal vs. informal communication.

that the Markov perfect equilibrium keeps the agent at his maxmin). At the end of this section we will discuss how results are likely to change when this condition is not satisfied.

Lemma 1. *Assume $d > \overline{d}(p)$. Then, there is a strategy pair σ^P that keeps the continuation payoff of the principal at his maxmin (zero) in every subgame starting with a move by the principal.*

The punishment strategies that we construct to prove the lemma have roughly the following structure: after a deviation, the informal contract is abandoned and parties revert to the optimal formal contract, and all the surplus from this contract is given to the player that has not deviated. In Appendix we describe this punishment strategy in greater detail.

Having pinned down the off-equilibrium strategies, we can now turn to the equilibrium path of the game. We will restrict our attention to simple equilibrium paths where each task is either regulated according to one of the rules presented in Section 2.1, or is regulated informally with the agent freely choosing the efficient action in every period. This restriction simplifies the analysis and makes it more easily comparable to the analysis of the Markovian equilibrium.[21]

More specifically, for each task k we consider four possibilities: (i) a formal contingent rule, meaning that a C_k clause is written at $t = 1$ and is never modified; (ii) a formal rigid rule, meaning that a R_k clause is written at $t = 1$ and is never modified; (iii) discretion, meaning that the task is not regulated by the formal contract and the agent takes the inefficient action in every period; (iv) an informal contingent rule, meaning that the task is not regulated by the formal contract but the agent takes the efficient action in every period. It can be shown that there is no need to consider informal *rigid* rules, because they would yield lower surplus without relaxing the incentive constraints. Also, given the assumption $d > \overline{d}(p)$, we can ignore formal default-cum-exceptions rules, as they are dominated by formal contingent rules (cf. Proposition 1).

[21] We conjecture that there is no loss of generality in this restriction, in the sense that every equilibrium path is either payoff equivalent or Pareto dominated by a simple equilibrium path. We are able to prove this conjecture "by brute force" in the case of two tasks. But we do not have a complete proof for the general case. The problem is that the incentive constraints yield interactions among tasks, and this makes the general analysis very complex.

Given our restriction on the equilibrium path, we can focus on subgame perfect equilibria of the following form: In each period t, the principal offers a formal contract (g, m_t) – where g is the set of formal (rigid or contingent) clauses and m_t is the transfer[22] – and the agent accepts. The formal clauses are never modified over time, so the principal pays the associated writing costs only in the first period. For a (possibly empty) subset of tasks not regulated by the formal contract, the agent takes the efficient action in every period (informal contingent rules). For the remaining tasks, the agent takes the inefficient action in every period (discretion). As soon as the principal deviates, he is punished according to the strategy pair σ^P of the lemma above. As soon as the agent deviates, he is punished with the Markov perfect equilibrium strategies.

To summarize, the equilibrium path is determined by a wage profile $(m_t)_{t=1}^{\infty}$, where $m_t : S \to \mathbb{R}$,[23] and a partition $\{K_C^I, K_C, K_R, K_D\}$ of the set of tasks, where K_C^I, K_C and K_R denote respectively the sets of tasks regulated by informal contingent, formal contingent and formal rigid rule, and K_D is the set of discretionary tasks.

We can characterize the constrained Pareto-efficient equilibria within this set as the ones that maximize the net present value of the surplus subject to the constraint that players have no incentive to deviate from the equilibrium actions. Formally, let $M_t = \sum_{\tau=t}^{\infty} d^{\tau-t}\mathrm{E}(m_\tau)$ denote the expected present value of wages from period t onward (since states are *i.i.d.*, there is no need to distinguish between conditional and unconditional expectation of the wage in period t). We look for a solution to the following problem (recall that we are assuming $\mu_1(1, ..., 1) = 1$, while μ_t is given by (2.4) for $t \geq 1$):

$$\max_{K_C^I, K_C, K_R, (m_t)_{t=1}^{\infty}} \sum_{k \in K_C^I} (1-\delta_k) + \sum_{k \in K_C} [1-\delta_k - 2c(1-d)] + \sum_{k \in K_R} [(1-d+dp)(1-\delta_k) - c(1-d)] \quad \text{(P)}$$

subject to

$$\sum_{k \in K_C^I} \delta_k \leq dM_{t+1} - \frac{d}{1-d}\left(\sum_{k \in K_C^I \cup K_C} \delta_k + \sum_{k \in K_R} p\delta_k\right), \quad \forall t \geq 1, \quad (\mathrm{IC}_A^1)$$

[22]Given the assumed rules of the game, the principal is not allowed to pay informal "bonuses" (he must pay the exact amount specified in the formal contract). However this is without loss of generality; see the discussion at the end of this section.

[23]Note that we allow the wage in period t to depend on the current state s_t, thus m_t is a random variable (there is no need to consider wage processes with longer memory). It should be clear that, even if the wage is state-dependent, it is not written as a contingent wage in the formal contract, but it is written period by period after observing s, so it involves no writing costs.

$$m_t(s)+dM_{t+1}-\sum_{k\in K_C^I\cup K_C}\delta_k-\sum_{k\in K_R}s(e_k)\delta_k-\frac{d}{1-d}\left(\sum_{k\in K_C^I\cup K_C}\delta_k+\sum_{k\in K_R}p\delta_k\right)\geq 0,\quad \forall (t,s):\mu_t(s)>0, \tag{IC$_A^2$}$$

$$\sum_{k\in K_C^I\cup K_C}1+\sum_{k\in K_R}s(e_k)+\frac{d}{1-d}\left(\sum_{k\in K_C^I\cup K_C}1+\sum_{k\in K_R}p\right)-m_t(s)-dM_{t+1}\geq 0,\quad \forall s\in S, \forall t\geq 2, \tag{IC$_P$}$$

$$\sum_{k\in K_C^I}1+\sum_{k\in K_C}(1-2c)+\sum_{k\in K_R}(1-c)+\frac{d}{1-d}\left(\sum_{k\in K_C^I\cup K_C}1+\sum_{k\in K_R}p\right)-m_1-dM_2\geq 0. \tag{PC$_P$}$$

Inequality (IC$_A^1$) ensures that the agent has no incentive to shirk. Inequality (IC$_P$) says that the principal must have an incentive to offer the equilibrium contract in every state from period $t=2$ on. If all transfers go from the principal to the agent ($m_t(s)\geq 0$ for all t and s), incentive constraints (IC$_A^1$) and (IC$_P$) are sufficient for an equilibrium. But if some transfer goes from the agent to the principal ($m_t(s)<0$ for some t and s), we must ensure that the agent has an incentive to make this payment. Inequality (IC$_A^2$) incorporates such condition. Note that, if $m_t(s)\geq 0$ for all t and s, inequality (IC$_A^2$) is implied by (IC$_A^1$). Finally, (PC$_P$) is the "participation constraint" for the principal.

By inspection of the incentive constraints, it is clear that the frontier of attainable SPE payoffs is linear (with slope -45^o) in the relevant range, because we can change the distribution of the surplus by varying the first-period wage without affecting the attainable surplus (as long as each player gets at least his maxmin). This ensures that the constrained Pareto-efficient equilibria are indeed given by the solutions to problem (P).

The following lemma simplifies the analysis of problem (P):

Lemma 2. *(K_C^I, K_C, K_R) is part of some solution of problem (P) if and only if it is a solution of the following auxiliary problem:*

$$\max_{K_C^I,K_C,K_R}\sum_{k\in K_C^I}(1-\delta_k)+\sum_{k\in K_C}[1-\delta_k-2c(1-d)]+\sum_{k\in K_R}[(1-d+dp)(1-\delta_k)-c(1-d)]$$

subject to

$$\sum_{k\in K_C^I}\delta_k\leq\frac{d}{1-d}\left(\sum_{k\in K_C^I\cup K_C}(1-\delta_k)+\sum_{k\in K_R}p(1-\delta_k)\right) \tag{IC}$$

The auxiliary problem stated in the above lemma is derived from the original problem (P) by reducing all the constraints to the single constraint (IC). The left hand side of (IC) is the gain from cheating on the informal rules, while the right hand side of (IC) is the discounted net present value of the surplus. Intuitively, (IC) would be the incentive constraint for the agent in any period t if – in equilibrium – he were to get all the surplus in the following periods. Clearly, this minimizes the agent's incentive to shirk. Therefore, (K_C^I, K_C, K_R) cannot be part of an equilibrium if (IC) is not satisfied. Conversely, if (K_C^I, K_C, K_R) solves the auxiliary problem, it can be implemented by (for example) an equilibrium that gives all the surplus to the agent from period $t = 2$ on and sets the first-period transfer m_1 so that both principal and agent get a nonnegative share of the net present value of the surplus.

The auxiliary problem may admit multiple solutions, for the following reason. For example, suppose that, at a solution of the problem, task 1 is regulated with an informal rule and task 2 with a formal contingent rule. Now modify the contract by regulating task 1 with a formal contingent rule and task 2 with an informal rule. Since informal and formal contingent rules yield the same surplus gross of writing costs, this does not change the value of the objective function and it may well be the case that the (IC) constraint is still satisfied, in which case the modified contract is also optimal. Such indeterminacy prevents clean comparative statics. Therefore we use the following tie-breaking rule: if two choices yield the same present value of net surplus, we assign a preference to the one that implies more slack in the (IC) constraint. We will call (*)-efficient a solution of problem (P) that satisfies our selection criterion.

Let F denote the ratio of tasks regulated formally to the total number of tasks regulated informally and formally. This is a measure of the importance of formal contracting relative to informal contracting. The next proposition characterizes the (*)-efficient equilibria and examines the impact of the key exogenous parameters on F.

Proposition 3. *Assume $d > \overline{d}(p)$. Then, at a (*)-efficient equilibrium:*
(i) If c is smaller than some critical level, a set of low-δ_k tasks is regulated by contingent informal rules and the complementary set of high-δ_k tasks is regulated by contingent formal rules.
(ii) In general, there exist three threshold values, $\delta' < \delta'' < \delta'''$, such that tasks with $\delta_k < \delta'$ are regulated by contingent informal rules, tasks with $\delta' < \delta_k < \delta''$ are regulated by contingent

formal rules, tasks with $\delta'' < \delta_k < \delta'''$ are regulated by rigid formal rules, and tasks with $\delta_k > \delta'''$ are left to the agent's discretion.
(iii) If c is sufficiently high, then no task is regulated formally. However, F need not be decreasing in c. Moreover, as c approaches zero F need not approach one.
(iv) If d is sufficiently close to one, then $F = 0$. However, F need not be decreasing in d.

Part (i) focuses on the case in which the writing cost is small. In this case it is optimal to implement the first best, and in particular, each task is regulated either by a formal contingent rule or by an informal contingent rule. It turns out that low-cost tasks are regulated informally and high-cost tasks are regulated formally. The intuition is as follows. Consider two tasks characterized by different levels of δ_k, and suppose that one task must be regulated formally and the other informally. Which one will be handled informally? The increase in surplus is the same independently of which task is chosen, but the low-δ_k task implies a lower incentive to cheat, hence it is better to regulate this task informally.

If c is higher, it may not be optimal to implement the first best. In particular, as part (ii) states, low-cost tasks are regulated informally, intermediate-cost tasks are regulated formally, and high-cost tasks are left to the agent's discretion. Within the group of tasks regulated formally, lower-δ_k tasks are handled by contingent rules and higher-δ_k tasks by rigid rules.

Not only are formal and informal norms used jointly, but they are complementary, in the sense that they increase each other's value. This is because increasing the number of tasks regulated formally increases the surplus from the relationship, hence it helps relax the incentive constraints of both players, thus making it easier to regulate more tasks informally.[24]

Part (iii) focuses on the impact of changes in c on the relative importance of formal contracting (F). Intuitively, a reduction in c should increase F. However this intuition is not entirely correct: it is possible that a reduction in c results in a lower F. To explain this possibility, consider a small decrease in c. This may have the effect of changing a rigid formal clause into a contingent formal clause. The resulting increase in the available surplus relaxes the incentive constraints, making it possible to regulate informally an additional task, in which case the total

[24] This complementarity is of a similar nature as the one that arises in Baker *et al.* (1994). But see the next subsection for important differences with respect to that paper.

number of formal clauses does not change and the number of informal clauses increases.[25]

The other point to note about the impact of c is that, as this parameter approaches zero, it may *not* be optimal to write a fully formal contract. In fact, as c approaches zero it may even be optimal to regulate all tasks informally. This is certainly the case if d is sufficiently close to one, because in this case the incentive constraints are not binding. We will come back to this point in the next section, where we relate our results to those in Baker *et al* (1994).

Part (iv) looks at the effect of changes in d. If d is sufficiently high, it is possible to support a complete informal contract with a subgame perfect equilibrium (the threshold is $d^* = \frac{1}{N}\sum_{k=1}^{N}\delta_k$). Intuition might suggest that an increase in d should favor informal contracting, since players discount the future less heavily, and this relaxes the incentive constraints. Things however are more subtle, because there is another effect that runs in the opposite direction. We have seen that, in the parameter region under consideration, it is optimal to write the contract once and for all at $t = 1$. This in turn implies that the incidence of writing costs decreases with d, and this pushes in favor of formal contracting. The net effect can go either way.

Next we discuss the role of the parameter restriction $d > \overline{d}(p)$, which ensures that the minimum equilibrium payoff is zero for each player. The condition that d is relatively high is essential for Lemma 1. Consider the extreme case of d equal to zero. Then there is a unique subgame perfect equilibrium in which the principal offers a formal contract and makes a positive profit in every period. At any rate, even though this condition is needed for the lemma, we suspect it is not essential for our qualitative insights. If the condition is not satisfied, it may not be possible to keep the principal at his maxmin payoff in the punishment phase, in which case the principal's incentive constraints will be more stringent, and this is likely to result in fewer tasks being regulated informally. But proposition 3 is still likely to hold, with the only amendment that default-cum-exceptions rules may be preferred to contingent rules.

Before concluding the section, we want to discuss our assumptions about timing, the possibility to pay "bonuses" (i.e. payments in excess of what is specified by the agreed-upon formal contract) and the possibility to sign multi-period contracts.

[25] There is another reason why F may increase with c. It is possible that an increase in c leads to decrease in the number of both formal and informal rules, and that this yields a higher ratio F.

According to the rules of the game, the principal pays the agent before he acts and the payment is exactly the one specified by the offered contract (if accepted). It is clear that each of these assumptions is, by itself, without loss of generality given the other. If bonuses are not allowed, it does not matter exactly when the payment m occurs, because the principal has to pay m independently of whether the agent has shirked on the informally regulated tasks or not. On the other hand, if any payment has to be made before the agent acts, bonuses are redundant, because the best incentive to keep the agent from shirking is still to hold him down to his maxmin from period $t+1$ if he shirks in period t. We now argue that even the *joint* assumption about timing and bonunes is without loss of generality. Suppose that the principal is allowed to pay an informal bonus immediately after the agent acts. In this case, the agent has a stronger incentive not to shirk, because shirking will prevent him from enjoying an immediate reward. On the other hand, with the modified assumption, the principal has an incentive to renege on the informally promised bonus, whereas with our current assumption he can only omit to offer the formal contract specified by the equilibrium. It can be shown that these two effects cancel out.

Similarly, it can be shown that there is no gain from committing to future wages or actions in the current formal contract. The intuitive reason is that this subtracts from the players' ability to punish each other in case of cheating. For example, if the principal commits to a permanent stream of wages in the current contract, he loses the ability to terminate payments in case the agent cheats.

3.2. Relationship to Baker *et al.* (1994) and Pearce and Stacchetti (1998)

Here we discuss briefly the analogies and differences between the model analyzed in the previous section and the two above-mentioned papers. In those papers, as in ours, a combination of formal and informal contracting may be optimal, and the two forms of contracting may be complementary, since the presence of a formal contract may relax the relevant incentive constraints. However, this is where the analogy stops.

Baker *et al.* (1994) and Pearce and Stacchetti (1998) provide an explanation for the combined use of formal and informal payments (wages and bonuses), whereas our model explains why it may be efficient to regulate some tasks formally and some others informally (note that in our model there is no need for bonuses). Perhaps more importantly, the rationale for mixing formal and informal contracting is very different. In those models, formal and informal contracting are used together because some signals of the agent's action are verifiable and some are not. In our model, the combination of formal and informal contracting is not due to differences in verifiability or writing costs across tasks; rather, it is due to the interaction between writing costs (which are symmetric across tasks) and self-enforcement constraints.[26]

Our model also yields different predictions concerning the interplay between formal and informal contracting. One key result in Baker *et al.* (1994) is that the availability of formal contracts may undermine informal contracts. In particular, if the verifiable signal is sufficiently precise – or in other words, if the imperfections in formal contracting are sufficiently small – an informal contract cannot be sustained. Therefore, a broad prediction of their model is that, if imperfections in formal contracting decrease over time, there comes a point at which informal contracting disappears. In our model, if formal contracting is close to perfect (i.e. if c is close to zero), the optimum typically involves both formal and informal contracting, possibly even a fully informal contract. Thus, our analysis suggests that informal contracting need not disappear as the formal-contracting system becomes more efficient.

The reason for this divergence in results lies in the punishment strategy. Baker *et al.* assume that, if a player cheats, parties revert to the optimal formal contract, with all the surplus from this contract accruing to the principal. This implies that, if formal contracting is close to perfect, it completely fails to deter the principal from cheating. However, in general this is not the most severe credible punishment strategy. Our approach, on the other hand, is to characterize the worst credible punishment strategies, which we are able to do under the parameter restriction $d > \overline{d}(p)$. In this parameter region, each player can be punished with his maxmin payoff, hence changes in c do not affect the severity of the punishment.

Finally, our model yields different predictions on the effect of changes in the discount factor d. In Baker *et al.*, an increase in d always favors informal contracting. In our model, as we remarked earlier, the opposite may happen. This is due to the different nature of the

[26] The discussion in the remainder of this section applies only to Baker *et al.* (1994).

contractual imperfection. In Baker *et al.,* the contractual imperfection (non-verifiability of signals) is exogenous, and is present in each period. In our model, the contractual imperfection (writing costs) is endogenous, and need not be incurred in every period. In fact, in the parameter region $d > \overline{d}(p)$ it is optimal to incur the writing costs only in the first period, hence the dynamic implications of the two kinds of contractual imperfections are very different.

4. Conclusion

Thus far, we have implicitly assumed away an alternative mode of governance that could avoid the costs of writing detailed contracts, namely giving *authority* to the principal. If the principal could instruct the agent on what actions to take in each period, there would be no need to specify contingencies or actions in a contract. In this concluding section we discuss how results would change if we allowed for authority as a governance mode.

As a premise, it is useful to distinguish between *formal* and *informal* authority. We speak of formal authority when the principal's authority is specified in the formal contract, and the agent can be punished by courts for disobeying the principal. We speak of informal authority when the principal's orders are not enforced by courts, but by reputation mechanisms.

Let us focus on formal authority first. It is critical to note that formal authority is enforceable only if two conditions are met: (i) the principal can send verifiable messages to the agent; this requires that messages be written, or at least recorded; and (ii) messages must be expressed in a language understood by the courts. In other words, messages must be *formal.* For this reason, even if a system of formal messages is feasible, it is not clear that its costs would be significantly lower than a system of formal contracts. This might also explain why we rarely observe pure formal-authority relationships in reality.

Informal authority is a more common mode of governance in real organizations. In our model, however, there is no role for such an arrangement, due to the assumption of symmetric information. Given that the principal and the agent have the same information, informal authority cannot improve on an informal contract as we defined it, because in the latter arrangement the agent knows what actions to take under any contingency, hence there is no need for further instructions from the principal. A role for informal authority would probably arise if the principal had private information. An extension of the model in this direction is left for future research.

5. Appendix

Proof of Proposition 1

Here we drop the assumption that $s_1(e_k) = 1$ for all k with certainty. Proposition 1 as stated in the text is still valid, provided we redefine two expressions:

By *default rule cum exceptions*, here we mean the following: The first period in which e_k occurs, say t', clause R_k is introduced in the default contract, and subsequently the exception $\bar{R}_k$ is applied every $\neg e_k$ occurs. If $s_1(e_k) = 0$, clause $\bar{R}_k$ is introduced in the default contract, and replaced by R_k at t'.

By *rigid rule*, here we mean any plan that converges to a *steady state* where there is a default clause R_k or $\bar{R}_k$ with no modifications. We will see that there are several plans that fall in this category and can be optimal under some parameters.

We can assume without loss of generality that the principal chooses at the *beginning* of each period t how he will react to the exogenous shock s_t (his optimally chosen reaction function yields ex post optimal decisions).

We can therefore analyze the resulting dynamic programming problem with restricted state space G, where the principal decides at the beginning of each period t which modifications of the default contract $\widetilde{g}_{t-1}$ he will offer as a function of the external state s_t (yet to be observed). By the additive separability of payoffs we will obtain a value function $v : G \rightarrow \mathbb{R}$ where

$$v\left(\widetilde{g}_{t-1}\right) = \sum_{k=1}^{N} v_k(\widetilde{\gamma}_{k,t-1}).$$

We now derive each component v_k $(k \in N)$ of the value function. For each possible $\widetilde{\gamma}_{k,t-1}$, we can restrict our attention to the following candidate one-period decision rules:[27]

(i) *Cont*(k): include in the default contract a contingent clause independently of the realization s_t.

[27] Note that we can ignore the possibility of simply removing a clause, i.e. replacing it with D. This is due to our assumptions about the payoff structure: in this model having a non-empty clause cannot be worse than having an empty clause.

(ii) *Amend*(k): include in the default contract clause R_k if e_k occurs and $\overline{R}_k$ otherwise.

(iii) *Amend*$(k,+)$: include in the default contract clause R_k if e_k occurs and do nothing otherwise.

(iii') *Amend*$(k,-)$: include in the default contract clause $\overline{R}_k$ if $\neg e_k$ occurs and do nothing otherwise.

(iv) *Except*$(k,-)$: apply the exception $\overline{R}_k$ if $\neg e_k$ occurs and do nothing otherwise.

(iv') *Except*$(k,+)$: apply the exception R_k if e_k occurs and do nothing otherwise.

(v) *Inaction*(k): do not introduce any modification concerning aspect k, independently of the realization s_t.

All other decision rules can be shown to be suboptimal. Furthermore, it can be shown that $v_k(R_k) \geq v_k(\overline{R}_k)$. Intuitively, it is (weakly) better to have a rigid default clause prescribing the right action with probability $p > \frac{1}{2}$ rather than a rigid default clause prescribing the right action with probability $(1-p) < \frac{1}{2}$. This implies that *Amend*$(k,-)$ and *Except*$(k,+)$ cannot be (strictly) optimal and we can safely ignore them. Note also that, since *Inaction*(k) does not change the state variable, *Inaction*(k) is optimal in a given state if and only if it is optimal forever after. The same is true for *Except*$(k,-)$.

Now we consider all the possible values of coordinate k of the state variable $\widetilde{g}_{t-1}$:

- $\widetilde{\gamma}_{k,t-1} = C_k$. Obviously in this case *Inaction*(k) is optimal in the current period and in any future period:

$$v_k(C_k) = 1 - \delta_k + dv_k(C_k).$$

Therefore

$$v_k(C_k) = \frac{1-\delta_k}{1-d}. \tag{5.1}$$

- $\widetilde{\gamma}_{k,t-1} = R_k$. In this case we can restrict our attention to three candidate decision rules:

(i) *Cont*(k), which yields $1 - \delta_k - 2c + dv_k(C_k) = \frac{1-\delta_k}{1-d} - 2c$ (by (5.1)),

(ii) *Except*$(k,-)$, which yields $1 - \delta_k - (1-p)c + dv_k(R_k)$,

(iii) $Inaction(k)$, which yields $p(1-\delta_k) + dv_k(R_k)$.

It follows that

$$v_k(R_k) = \max\{\frac{1-\delta_k}{1-d} - 2c, \frac{1-\delta_k-(1-p)c}{1-d}, \frac{p(1-\delta_k)}{1-d}\}. \tag{5.2}$$

- $\widetilde{\gamma}_{k,t-1} = \overline{R}_k$. In this case the candidate decision rules are:

  (i) $Cont(k)$, which yields $\frac{1-\delta_k}{1-d} - 2c$,

  (ii) $Amend(k,+)$, which yields $1-\delta_k - pc + d\left[pv_k(R_k) + (1-p)v_k\left(\overline{R}_k\right)\right]$

  (iii) $Inaction(k)$, which yields $(1-p)(1-\delta_k) + dv_k\left(\overline{R}_k\right)$.

It follows that

$$v_k\left(\overline{R}_k\right) = \max\{\frac{1-\delta_k}{1-d} - 2c, \frac{1-\delta_k - pc + dpv_k(R_k)}{1-d(1-p)}, \frac{(1-p)(1-\delta_k)}{1-d}\} \tag{5.3}$$

where $v_k(R_k)$ is given by (5.2).

- $\widetilde{\gamma}_{k,t-1} = D$. In this case the candidate decision rules are:

  (i) $Cont(k)$, which yields $\frac{1-\delta_k}{1-d} - 2c$,

  (ii) $Amend(k)$, which yields $1-\delta_k - c + d[pv_k(R_k) + (1-p)v_k\left(\overline{R}_k\right)]$,

  (iii) $Amend(k,+)$, which yields $p(1-\delta_k-c) + d[pv_k(R_k) + (1-p)v_k(D)]$,

  (iv) $Inaction(k)$, which yields $dv_k(D)$.

It follows that

$$v_k(D) =$$
$$\max\{\frac{1-\delta_k}{1-d} - 2c, 1-\delta_k - c + d[pv_k(R_k) + (1-p)v_k\left(\overline{R}_k\right)], \frac{p(1-\delta_k-c) + dpv_k(R_k)}{1-d(1-p)}, 0\} \tag{5.4}$$

where $v_k(R_k)$ and $v_k\left(\overline{R}_k\right)$ are given respectively by (5.2) and (5.3). One can verify that the following intuitive inequalities hold:

$$v_k(D) \leq v_k(\overline{R}_k) \leq v_k(R_k) \leq v_k(C_k). \tag{5.5}$$

Equations (5.1) to (5.4) fully characterize the component v_k ($k \in N$) of the value function for our problem. The optimal decision rule for each $\widetilde{\gamma}_{k,t-1}$ can be derived using the same equations. For example, if $\widetilde{\gamma}_{k,t-1} = R_k$, the optimal decision rule is $Cont(k)$ or $Except(k,-)$ or $Inaction(k)$ depending on whether the maximum element of the set in the right hand side of (5.2) is the first, the second or the third one.

Using equations (5.1) to (5.4) and inequalities (5.5), one can derive that the only candidate optimal plans concerning aspect k are the following (a plan describes only the decisions at reachable states):

- $\mathcal{C}_k$: $Cont(k)$ at D and $Inaction(k)$ at C_k. The value of this plan is: $\frac{1-\delta_k}{1-d} - 2c$;
- $\mathcal{DE}_k$: $Amend(k)$ at D, $Amend(k,+)$ at $\overline{R}_k$, $Exception(k,-)$ at R_k. The associated value is $\frac{1-\delta_k}{1-d} - \left(1 + \frac{2-d}{1-d} \cdot \frac{dp(1-p)}{1-d(1-p)}\right) c$;
- $\mathcal{R}_k^+$: $Amend(k)$ at D, $Amend(k,+)$ at $\overline{R}_k$, $Inaction(k)$ at R_k. The associated value is $\frac{\left[1-d(1-p^2)\right](1-\delta_k)}{(1-d)[1-d(1-p)]} - \left(1 + \frac{dp(1-p)}{1-d(1-p)}\right) c$;
- $\mathcal{R}_k^0$: $Amend(k)$ at D, $Inaction(k)$ at R_k and $\overline{R}_k$. The associated value is $\frac{[1-2dp(1-p)](1-\delta_k)}{1-d} - c$;
- $\mathcal{R}_k^*$: $Amend(k,+)$ at D, $Inaction(k)$ at R_k. The associated value is $\frac{p(1-\delta_k)}{1-d} - \frac{pc}{1-d(1-p)}$;
- D_k: $Inaction(k)$ at D. The associated value is 0.

We can now prove parts (i), (ii) and (iii) of the proposition.

(i) If $c \leq \min_k\{1-\delta_k\}$, then $Inaction(k)$ is optimal only if the default clause is contingent (i.e., at C_k) and $Amend(k,+)$ is not optimal when there is no clause (i.e., at D). Therefore, if $c \leq \min_k\{1-\delta_k\}$, $\mathcal{C}_k$ and $\mathcal{DE}_k$ are better than any other plan, for all k. By comparing the respective associated values, one finds that a contingent contract is optimal if and only if

$$(1-d)[1-d(1-p)] < d(2-d)p(1-p)$$

It is direct to verify that this condition is satisfied if and only if $d > \hat{d}(p)$, where $\hat{d}(p)$ is an increasing function satisfying $\hat{d}(\frac{1}{2}) = \frac{2}{3}$ and $\hat{d}(1) = 1$. We note that, under the assumption made in the text that $s_1(e_k) = 1$ for all k, this critical function becomes $\overline{d}(p) \equiv \frac{1}{2-p}$.

(ii) To see how the ranking of different plans for aspect k depends on δ_k we look at the coefficient of δ_k in their values. The ranking between $\mathcal{C}_k$ and $\mathcal{DE}_k$ is clearly independent of δ_k. moreover, since $\frac{1}{2} < p < 1$,

$$\frac{1}{1-d} > \frac{1-d(1-p^2)}{(1-d)\left[1-d(1-p)\right]} > \frac{1-2dp(1-p)}{1-d}, \frac{p}{1-d} > 0.$$

Taking the upper envelope of the plans values as (positive affine) functions of δ_k we obtain a decreasing convex function and two thresholds $\delta_* \leq \delta^*$ (functions of d, p and c); the envelope has slope $-\frac{1}{1-d}$ to the left of δ_* and is flat to the right of δ^*. This means that the $\mathcal{D}_k$ plan is optimal for $\delta_k > \delta^*$, one of the $\mathcal{R}_k$ plans is optimal if $\delta_* < \delta_k < \delta^*$ and the $\mathcal{C}_k$ or $\mathcal{DE}_k$ plan is optimal for $\delta_k < \delta_*$.

For **(iii)**-**(iv)** compare the derivatives of the values of the different plans with respect to d and p. ■

Proof of Proposition 2

Unlike in the proof of Proposition 1, due to the autoregressive nature of the shocks we cannot work with the restricted state space G, but we have to work with the unrestricted state space $G \times S$. The value function is still additively separable in the N dimensions:

$$v\left(\widetilde{g}_{t-1}, s_t\right) = \sum_{k=1}^{N} v_k(\widetilde{\gamma}_{k,t-1}, s_t(e_k)).$$

We now derive each component v_k $(k \in N)$ of the value function using a shortcut. We exploit the symmetries of the model to partition the state space for aspect k in four cells corresponding to the following situations:

D_k (Discretion): there is no clause regulating aspect k in the default contract.

M_k (Match): the default contract contains the rigid k-clause matching the current state of the environment.

NM_k (No Match): the default contract contains the rigid k-clause that does *not* match the current state of the environment.

C_k (Contingent rule): the default contract contains the efficient contingent k-clause.

For each possible situation, one can show that there are only three candidate one-period decision rules:

(i) $Cont[k]$: include in the default contract the contingent clause C_k.

(ii) $Amend[k]$: include in the default the rigid clause matching the current state of the environment (R_k if $s_k = 1$ and $\overline{R}_k$ if $s_k = 0$). Using this rule and independently of the current situation, the system makes a transition to situation M_k with probability $(\frac{1}{2}+\rho)$ and to situation NM_k with probability $(\frac{1}{2} - \rho)$.

(iii) $Inaction[k]$: do not introduce any modification concerning aspect k. Under this rule, if the situation is M_k (or NM_k) the system stays there with probability $(\frac{1}{2} + \rho)$ and makes a transition to NM_k (respectively M_k) with probability $(\frac{1}{2} - \rho)$.

Let us consider the value of each possible situation: with a slight abuse of notation we write $v_k(D_k)$, $v_k(M_k)$, $v_k(NM_k)$ and $v_k(C_k)$.

- C_k (Contingent rule): $Inaction[k]$ is optimal in the current period and in any future period, thus we have
$$v_k(C_k) = \frac{1-\delta_k}{1-d}. \tag{5.6}$$
- M_k (Match): In this case there are two candidate decision rules:

  (i) $Cont[k]$, which yields $\frac{1-\delta_k}{1-d} - 2c$,

  (ii) $Inaction[k]$, which yields $1 - \delta_k + d[(\frac{1}{2} + \rho)v_k(M_k) + (\frac{1}{2} - \rho)v_k(NM_k)]$,

  It follows that
$$v_k(M_k) = \max\left\{\frac{1-\delta_k}{1-d} - 2c, \frac{1-\delta_k + (\frac{1}{2}-\rho)dv_k(NM_k)}{1-d(\frac{1}{2}+\rho)}\right\} \tag{5.7}$$
- NM_k (No Match): In this case there are three candidate decision rules:

(i) $Cont[k]$, which yields $\frac{1-\delta_k}{1-d} - 2c$,

(ii) $Inaction[k]$, which yields $d\left[(\frac{1}{2}-\rho)v_k(M_k) + (\frac{1}{2}+\rho)v_k(NM_k)\right]$,

(iii) $Amend[k]$, which yields $1-\delta_k - c + d\left[(\frac{1}{2}-\rho)v_k(NM_k) + (\frac{1}{2}+\rho)v_k(M_k)\right]$

It follows that

$$v_k(NM_k) =$$
$$\max\left\{\frac{1-\delta_k}{1-d} - 2c, \frac{d(\frac{1}{2}-\rho)v_k(NM_k)}{1-d(\frac{1}{2}+\rho)}, 1-\delta_k - c + d\frac{1-\delta_k - c + d(\frac{1}{2}+\rho)v_k(M_k)}{1-d(\frac{1}{2}-\rho)}\right\}. \tag{5.8}$$

- D_k (Discretion): In this case the candidate decision rules are:

  (i) $Cont[k]$, which yields $\frac{1-\delta_k}{1-d} - 2c$,

  (ii) $Amend[k]$, which yields $1-\delta_k - c + d[(\frac{1}{2}+\rho)v_k(M_k) + (\frac{1}{2}-\rho)v_k(NM_k)]$,

  (iii) $Inaction[k]$, which yields $dv_k(D_k)$.

It follows that

$$v_k(D_k) =$$
$$\max\{\frac{1-\delta_k}{1-d} - 2c, 1-\delta_k - c + d\left[(\frac{1}{2}+\rho)v_k(M_k) + (\frac{1}{2}-\rho)v_k(NM_k)\right], 0\} \tag{5.9}$$

Equations (5.6) to (5.9) fully characterize the component v_k ($k \in N$) of the value function for our problem. One can derive that the only candidate optimal plans concerning aspect k are the following

- $\mathcal{C}_k$: $Cont[k]$ in situation D_k and $Inaction[k]$ in C_k. The value of this plan is: $\frac{1-\delta_k}{1-d} - 2c$.

- $\mathcal{A}_k$: $Amend[k]$ in situations D_k and NM_k, $Inaction[k]$ in M_k. The associated value is $\frac{1-\delta_k}{1-d} - c\left[1 + \frac{d}{1-d}(\frac{1}{2}-\rho)\right]$.

- $\mathcal{R}_k$: $Amend[k]$ in situation D, $Inaction[k]$ in M_k and NM_k. The associated value is $1-\delta_k - c + d(1-\delta_k)\left[\frac{1}{2(1-d)} + \frac{\rho}{1-2d\rho}\right]$.

- $\mathcal{D}_k$: $Inaction[k]$ at all states. The associated value is 0.

Parts (i)-(iv) of the proposition can then be shown with a similar logic as the one used in the proof of proposition 1. ■

Proof of Lemma 1

We exhibit a subgame perfect equilibrium keeping the principal at his maxmin under the parameter restriction $d \geq \overline{d}(p) \equiv \frac{1}{2-p}$. Consider the following strategies: for all $(\widetilde{F}, s)$ (where $\widetilde{F}$ is the default contract) the principal makes amendments and exceptions as in the Markov perfect equilibrium. Wages are determined according to a punishment phase. There are two punishment phases P_P and P_A. The system starts in phase P_P. When the system is in phase P_P the (offered) wage is the net profit generated by the offered contract. When the system is in phase P_A the (offered) wage is the disutility generated by the offered contract. As soon as player i deviates from his strategy the system switches immediately to phase P_i. If the system is in phase P_A the agent accepts the offered contract (and chooses the one-shot best response). Thus, in phase P_A the Markov perfect equilibrium is played. If the system is in phase P_P, the new default set of clauses is $\widetilde{F}'$, the offered contract is (F', m) and the state of nature is s, then the agent accepts if and only if

$$m - \delta(F', s) > dv(\widetilde{F}'), \tag{5.10}$$

where $v(\widetilde{F}')$ is the expected PDV of the flow of net surpluses generated by the Markov perfect equilibrium starting at default $\widetilde{F}'$.

By construction, the agent has no incentive to deviate. In particular, suppose that the state of nature is s, the principal moves the default to $\widetilde{F}'$ and offers (F', m) so that, as a consequence, the system enters (or stays in) phase P_P.

If $m - \delta(F', s) \leq dv(\widetilde{F}')$, the agent is supposed to reject. The expected payoff if the agent conforms is $dv(\widetilde{F}')$. The expected payoff of a one-shot deviation is $m - \delta(F', s)$, because after the deviation the system enters phase P_A where the agent gets his maxmin (zero). Therefore rejection is indeed a best response.

If $m - \delta(F', s) > dv(\widetilde{F}')$ the agent is supposed to accept and this is obviously a best response.

Let us check that the principal has no incentive to deviate in phase P_P. Let the overall state be $(\widetilde{F}, s)$, where $\widetilde{F} = (\widetilde{C}, \widetilde{R}, \widetilde{\overline{R}})$, $\widetilde{C}$ is the set of contingent default clauses, $\widetilde{R}$ is the set of rigid default clauses and $\widetilde{\overline{R}}$ is the set of negative, rigid default clauses. Since the principal will be kept at his maxmin from the following period, the only way he can make a profitable one-shot deviation is to make amendments $F^A = (C^A, R^A, \overline{R}^A)$ and exceptions $F^A = (C^E, R^E, \overline{R}^E)$ and offer a wage m such that the new default $\widetilde{F}' = \widetilde{f}(\widetilde{F}, F^A)$ and the resulting new contract $F' = f(\widetilde{F}', F^E)$ satisfy (5.10), making the agent accept, and furthermore $m < \pi(F', s) - Cost(F^A, F^E)$ (where $\pi(F', s)$ is the gross profit generated by F' at s), so that he gets a positive payoff in the current period.

Clearly, a profitable deviation exists if and only if there exists (F^A, F^E) and a state of nature s such that

$$\pi(F', s) - \delta(F', s) - Cost(F^A, F^E) > dv(\widetilde{F}')$$

To show that this is not possible, we will provide a lower bound for the RHS of the above inequality and an upper bound for its LHS, and show that the former is bigger than the latter.

Note that

$$v(\widetilde{F}') = v(\widetilde{C}', \widetilde{R}', \widetilde{\overline{R}}') \geq$$

$$\frac{1}{1-d}\left(\sum_{k\in\widetilde{C}'}(1-\delta_k) + \sum_{k\in\widetilde{R}'\cup\widetilde{\overline{R}}'}(1-p)(1-\delta_k) + \sum_{k\in(C^E\cup R^E\cup\overline{R}^E)\setminus(\widetilde{C}'\cup\widetilde{R}'\cup\widetilde{\overline{R}}')}(1-\delta_k-c)\right),$$

because it is possible to use the default clauses of $\widetilde{F}'$ in all future periods (in each period, each positive rigid clause k yields $1-\delta_k$ with probability $p \geq (1-p)$, and each negative rigid clause k yields $(1-\delta_k)$ with probability $(1-p)$) and to make exceptions in every period with the clauses included in F^E but not included in $\widetilde{F}'$.

On the other hand,

$$\pi(F', s) - \delta(F', s) - Cost(F^A, F^E) \leq \sum_{k\in\widetilde{C}'}(1-\delta_k) + \sum_{k\in\widetilde{R}'\cup\widetilde{\overline{R}}'}(1-\delta_k) + \sum_{k\in(C^E\cup R^E\cup\overline{R}^E)\setminus(\widetilde{C}'\cup\widetilde{R}'\cup\widetilde{\overline{R}}')}(1-\delta_k-c).$$

But $d \geq \frac{1}{2-p}$ implies $1 \leq \frac{d}{1-d}(1-p) \leq \frac{d}{1-d}$. Therefore

$$\sum_{k\in\widetilde{C}'}(1-\delta_k) + \sum_{k\in\widetilde{R}'\cup\widetilde{\overline{R}}'}(1-\delta_k) + \sum_{k\in(C^E\cup R^E\cup\overline{R}^E)\setminus(\widetilde{C}'\cup\widetilde{R}'\cup\widetilde{\overline{R}}')}(1-\delta_k-c) \leq$$

$$\leq \frac{d}{1-d}\left(\sum_{k\in\widetilde{C}'}(1-\delta_k)+\sum_{k\in\widetilde{R}'\cup\widetilde{\widetilde{R}}'}(1-p)(1-\delta_k)+\sum_{k\in(C^E\cup R^E\cup\overline{R}^E)\backslash(\widetilde{C}'\cup\widetilde{R}'\cup\widetilde{\widetilde{R}}')}(1-\delta_k-c)\right).$$

and a profitable deviation is impossible. ■

Proof of Lemma 2

From the constraints of (P) we derive the following set of constraints:

$$dM_2 \geq \sum_{k\in K_C^I}\delta_k + \frac{d}{1-d}\left(\sum_{k\in K_C^I\cup K_C}\delta_k + \sum_{k\in K_R}p\delta_k\right), \tag{5.11}$$

$$dM_2 \leq \frac{d}{1-d}\left(\sum_{k\in K_C^I\cup K_C}1 + \sum_{k\in K_R}p\right), \tag{5.12}$$

$$M_1 \geq \sum_{k\in K_C^I\cup K_C\cup K_R}\delta_k + \frac{d}{1-d}\left(\sum_{k\in K_C^I\cup K_C}\delta_k + \sum_{k\in K_R}p\delta_k\right), \tag{5.13}$$

$$M_1 \leq \sum_{k\in K_C^I}1 + \sum_{k\in K_C}(1-2c) + \sum_{k\in K_R}(1-c) + \frac{d}{1-d}\left(\sum_{k\in K_C^I\cup K_C}1 + \sum_{k\in K_R}p\right). \tag{5.14}$$

Inequality (5.11) is derived from (IC_A^1) evaluated at $t = 1$. Inequality (5.12) is derived from (IC_P) evaluated at $t = 2$ by averaging over states and multiplying both sides of the inequality by d. Inequalities (5.13) and (5.14) are derived, respectively, from (IC_A^2) evaluated at $t = 1$ and from (PC_P), making use of the assumption $s_1(e_k) = 1$ for all k.

Inequalities (5.11) and (5.12) yield

$$\sum_{k\in K_C^I}\delta_k \leq \frac{d}{1-d}\left(\sum_{k\in K_C^I\cup K_C}(1-\delta_k) + \sum_{k\in K_R}p(1-\delta_k)\right), \tag{5.15}$$

while inequalities (5.13) and (5.14) imply that the NPV of the surplus net of writing costs must be non-negative:

$$\sum_{k\in K_C^I}(1-\delta_k) + \sum_{k\in K_C}[1-\delta_k-2c(1-d)] + \sum_{k\in K_R}[(1-d+pd)(1-\delta_k)-c(1-d)] \geq 0. \tag{5.16}$$

We are going to show that a solution to the auxiliary problem can be completed so as to satisfy all the constraints of (P). Since the two problems have the same objective function and the constraint of the auxiliary problem is derived from the constraints of (P), this implies that (K_C^I, K_C, K_R) solves the auxiliary problem if and only if it is part of some solution of the original problem (P).

Let (K_C^I, K_C, K_R) be a solution of the auxiliary problem. A complete solution to problem (P) can be obtained as follows. Focus on wage profiles that are stationary from $t = 2$ on. From period $t = 2$ on, give all the surplus to the agent, so that (IC_P) is satisfied as an equality for each s. This determines the value of M_2 and $m_2(s)$ for each s; it also implies that (IC_A^2) is satisfied for every $t \geq 2$. Substituting this value of M_2 in (IC_A^1) we obtain (5.15), which is satisfied by assumption. Thus (IC_A^1) holds too. Derive m_1 and M_1 by solving (PC_P) as an equality. This way the agent gets all the NPV of the surplus; thus if (5.16) is satisfied the same must hold for (IC_A^2) at $t = 1$. Since "no-contract" is a feasible choice that yields zero surplus, (K_C^I, K_C, K_R) must indeed satisfy (5.16). Therefore, $(K_C^I, K_C, K_R, (m_t)_{t=1}^{\infty})$ satisfies all the constraints of (P). ■

Proof of Proposition 3

We first prove point (ii). We argue in three steps: (1) For any pair of tasks k, k', if $\delta_k > \delta_{k'}$ then it cannot be (*)-efficient to regulate task k informally and task k' by formal contingent clause. Suppose this is the case, and consider swapping the two tasks, so that task k' is now regulated informally and task k by formal contingent clause. The value of the objective does not change, and the constraint gets relaxed; applying our selection criterion, this is preferable to the original contract. (2) For any pair of tasks k, k', if $\delta_k > \delta_{k'}$ then it cannot be optimal to regulate task k by formal contingent clause and task k' by formal rigid clause. Suppose this is the case, and consider swapping the two tasks. This improves the value of the objective without violating the self-enforcement constraint. (3) For any pair of tasks k, k', if $\delta_k > \delta_{k'}$ then it cannot be optimal to regulate task k by formal rigid clause and leave task k' out of the contract. Suppose this is the case, and consider swapping the two tasks. Again, this improves the value of the objective without violating the self-enforcement constraint. The claim follows right away.

Point (i) is an immediate corollary of point (ii), if one notices that with c sufficiently small formal contingent rules dominate formal rigid rules and discretion (it is easy to see that a sufficient condition is $c < \min_k(1 - \delta_k)$: in this case making an exception is always surplus improving, but $d > \overline{d}(p)$ implies that contingent rules dominate default cum exceptions).

The only part of point (iii) that is not straightforward is that F may increase with c. To show this, we display a numerical example. Suppose there are four tasks, with $\delta_1 = 2/3$, $\delta_2 = 3/4$, $\delta_3 = 13/16$ and $\delta_4 = 29/32$, and suppose $p = \frac{1}{2}$, $d = \frac{2}{3}$. In this case, we claim that there exists a critical level c^* such that, if $c < c^*$, the optimum is $\{C_1^I, C_2^I, C_3, C_4\}$, and if c is slightly higher than c^* the optimum is $\{C_1^I, C_2, R_3, D\}$. First note that for these parameter values, contracts $\{C_1^I, C_2^I, C_3, C_4\}$ and $\{C_1^I, C_2, R_3, D\}$ are implementable, i.e. satisfy (5.15), while contracts $\{C_1^I, C_2^I, C_3^I, C_4\}$ and $\{C_1^I, C_2^I, C_3, R_4\}$ are not. Also note that contract $\{C_1^I, C_2, R_3, D\}$ yields higher surplus than $\{C_1^I, R_2, R_3, R_4\}$. These facts imply that the best implementable contract among those that cost $4c$ is $\{C_1^I, C_2^I, C_3, C_4\}$, and the best implementable contract among those that cost $3c$ is $\{C_1^I, C_2, R_3, D\}$. This in turn implies that there is a critical level c^* such that for $c \in (0, c^*)$ the optimum is $\{C_1^I, C_2^I, C_3, C_4\}$ $(F = \frac{1}{2})$, and for c in a right neighborhood of c^* the optimum is $\{C_1^I, C_2, R_3, D\}$ $(F = \frac{2}{3})$. It follows that F may increase if c increases.

We leave it to the reader to construct an example in which a similar result obtains for an increase in d. ■

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