Devaluation without common knowledge

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Abstract

In an economy with a fixed exchange rate regime that suffers a random adverse shock, we study the strategies of imperfectly and sequentially informed speculators that may trigger an endogenous devaluation before it occurs exogenously. The game played by the speculators has a unique symmetric Nash equilibrium which is a strongly rational expectation equilibrium in the set of all strategies with delay. Uncertainty about the extent to which the Central Bank is ready to defend the peg extends the ex ante mean delay between the exogenous shock and the devaluation. We determine endogenously the rate of devaluation.

Keywords: currency crises, fixed exchange rate regime, speculation, endogenous devaluation, imperfect information

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1 Introduction

Can a fixed peg be sustainable when all rational agents know that the current policies will lead to its demise? Is the magnitude of the devaluation related to the length of the peg regime? Can a lack of "transparency" by the Central Bank extend the length of the peg regime? These questions have positive answers in this paper. The essential feature of the framework is that the agents have heterogeneous information: they become gradually informed that the peg is not sustainable, and this information does not pertain to all the agents simultaneously.

When all agents have perfect information on the fundamentals, a peg is sustainable while the Central Bank's reserves are gradually depleted, as shown in the celebrated model of Krugman (1979)[1]. At some point in time, a switch out of the peg takes place with a jump down of the level of reserves[2]. Since agents have perfect information, this jump must take place without devaluation: the "crisis" is defined by a discrete adjustment of the currency holding. The assumption of perfect information is critical for the property of a switch with no devaluation and it implies the perfect anticipation of the timing of the event. There is no capital loss by the agents who hold the domestic currency and no strategic complementarity in running from the domestic currency.

In general, a currency crisis occurs when the Central Bank's reserves are not sufficient to sustain the peg. When there is a large mass of demand for the currency that is speculative, a currency attack with devaluation can be self-fulfilling as a bank run. The coordination problem raises the possibility of multiple equilibria, as highlighted by Obstfeld (1996) for perfectly informed agents in a one-period setting, and by Jeanne and Masson (2000) with multiple periods. But the indeterminacy of the outcome is not satisfying from an analytical point of view and unsuitable for the analysis of policy.

Morris and Shin (1998) have argued that the multiplicity of outcomes of the game between speculators results from the assumption of perfect information and is not realistic. When private informations are only slightly different, the uniqueness of the outcome is restored. In particular, this framework has been used to analyze the impact of "transparency" of the Central Bank's policy on the probability of a successful speculative attack (Bannier and Heinemann, 2005). These models that use the global game technique of Carlsson and Van Damme (1993) have attracted a large amount of attention but they cannot easily be extended to multiple periods, an essential feature of currency crises.

Agents learn over time from the actions of others when the exchange rate floats within a band in Chamley (2003), but the model generates multiple equilibria, and policy has an impact only if it eliminates completely the equilibrium with an attack.

In Guimarães (2004) who much expands on the theoretical analysis of Frankel and Pauzner (2000), there is a unique equilibrium in a dynamic model where the Central Bank abandons the peg if the market rate (the price of the foreign currency just after the peg is abandoned) exceeds some level which is specified as an increasing function

[1] Other "first generation models" include Flood and Garber (1984), and Flood, Garber and Kramer (1996). The latter paper studies the effects of sterilization, by the addition of a bond market. Botman and Jager (2002) build on Krugman (1979) and Flood and Garber (1984) to present a multi-country setting in which coordination and contagion issues can be analyzed.

[2] In the model of Krugman (1979), the inflation rate jumps up after the switch because the government deficit is financed by seignorage. Under perfect foresight, this jump is achieved through a jump in the nominal quantity of money.

of the reserves. Agents have perfect information on the current market rate and the reserves, but frictions restrict their trades.[3] These frictions prevent speculators to unload all their holdings before the devaluation and through the policy function of the Central Bank, they generate a positive relation between the level of reserves and the rate of devaluation.

In this paper, there is no trade friction nor transaction cost. The key feature is the agents' imperfect information about the fundamentals. There is a random occurrence of a permanent adverse shock to the fundamentals that will end with a devaluation, eventually. Agents become gradually informed that such a shock has taken place but they are imperfectly informed about both the timing of the shock and the information of other agents. The peg is abandoned when the reserves of the Central Bank reach some fixed level, possibly zero. An important feature of imperfect information is that agents cannot infer from the level of the Central Bank's reserves the transactions of other speculators, at least during the time of a speculative attack.[4] For simplicity, it is assumed the agents do not observe the level of the Central Bank's reserves.

We show that there is a unique equilibrium, with strong properties of coordination, in which speculators hold currency balances at the instant of the endogenous devaluation due to imperfect information. The excess level of these balances determines endogenously the rate of the devaluation when the fixed exchange rate is abandoned. At each instant before the devaluation, rational speculators balance the risk of a capital loss associated to the devaluation with the benefit of holding the domestic currency. The positive relation between the rate of devaluation and the level of reserves is generated by imperfect information rather than trade frictions[5]. When the agents' information becomes nearly identical, the limit equilibrium is the equilibrium under perfect information and no devaluation of Krugman (1979).

The model is presented in Section 2. In order to focus on the equilibrium properties, we first assume an exogenous rate of devaluation. The sequential arrival of information is modeled after Abreu and Brunnermeier (2003)[6]. A speculator who becomes informed about the occurrence of a shock does not know how many other agents have been informed before him.

The symmetric Nash equilibrium is analyzed in Section 3. Under some parameter

[3]They can adjust their portfolio only at random times determined by an exogenous Poisson process. The total flow of trades at any instant is therefore a function of the reserves and of the fundamental.

[4]During the ERM crisis in 1992-1994 and well after, some information about the reserves were published, but with a lag. Statistics published were incomplete. Moreover, a large fraction of transactions may have other motives than pure speculation.

[5]Broner (2003) also analyzes a model of currency crisis with imperfect information. In his setting, there are two types of agents: the first have perfect information and behave as in Krugman; the second are rational but have no information on the fundamental and act after observing the actions of the agents of the first type. There is a continuum of instants at which the first type of agents can attack the currency and benefit from the excess-reserves of the second type: these excess reserves generate a devaluation and a capital gain for the perfectly informed agents. In our model, agents are identical ex ante and there is a unique equilibrium which is strongly rationalizable.

[6]In the present paper, the assumptions have a nice economic foundation and the analysis is considerably simpler than in Abreu and Brunnermeier (2003): i) in the regime of fixed exchange rate, the asset price is fixed by the Central Bank and trades have indeed no impact on that price; ii) the upper bound on the level of total purchases that triggers a change of regime is naturally imposed by the total amount of reserves of the Central Bank; iii) the jump in the exchange rate which occurs at the end of the first regime can be determined endogenously in a macroeconomic model; iv) there is no transaction cost in this model.

conditions, an agent delays selling the domestic currency after he becomes informed. This equilibrium is unique in a wide set of (non symmetric) strategies where agents do not sell the domestic currency before being informed. It has properties of coordination that are stronger than those of the Nash equilibrium: it survives the iterated elimination of dominated strategies and is therefore a strongly rational expectation equilibrium (Guesnerie (1992)) in the set of all strategies with delay.

In Section 4, the rate of devaluation is endogenously determined by setting a value of the post-devaluation real quantity of money. This value could be made to depend on an anticipated policy of the Central Bank. As the rate of information increases, the model with endogenous devaluation rate and imperfect information approaches the model of currency crises with perfect information.

The combination of imperfect information and a unique equilibrium enables us to address, in Section 5, the issue of "transparency" of the Central Bank. The policy of the Central Bank is the level of reserves at which it abandons the peg. We show that the agents' uncertainty about this policy extends the *ex ante* mean delay between the exogenous shock and the devaluation. (When the policy of the Central Bank is uncertain, agents have rational expectations about the probabilities over the possible policies). In our framework, the "transparency" of the Central Bank reduces the viability of the peg regime. In the one-period model of Bannier and Heinemann (2005), the Central Bank should provide less information only if the prior of agents about the sustainability of the peg is low, and more information otherwise. Our setting, while different, can perhaps be associated to a low prior since all agents eventually know that the peg is not sustainable. In this setting, the Central Bank should provide less information.

2 The model

Consider an economy with a fixed exchange rate regime compatible with the economy's fundamentals. At some time θ, the economy suffers an adverse shock which changes the fundamentals, and makes the exchange rate incompatible with the fundamentals. The value of θ is determined by an exponential distribution with parameter λ per unit of time. The parameter λ is the probability of the occurrence of an adverse shock per unit of time conditional on no previous shock. The cumulative distribution function is $F(\theta) = 1 - e^{-\lambda\theta}$ and the density is $f(\theta) = \lambda e^{-\lambda\theta}$.

There is a continuum of agents (speculators) of mass normalized to one who hold one unit of wealth each. We assume that each speculator is risk neutral, and can buy a fixed quantity of foreign currency normalized to one at the price of one.[7] The domestic currency yields a return of r per unit of time. The foreign currency yields no return.

Before the adverse shock, the reserves of the Central Bank are exogenously fixed at $\overline{R}$. The adverse shock generates an outflow of reserves. This outflow can be explained by a change in trades or capital movements. It is not observed by the agents. However, the agents know the structure of this outflow. Let the reserves of the Central Bank at time $\theta + s$ be equal to $R(s)$ with $R(0) = \overline{R}$, $R' < 0$ and $R(T) = 0$ for some

[7] We could generalize to the case where the agent can buy a variable and bounded quantity of foreign currency. We will show that delaying more or less than the equilibrium time is strictly iteratively dominated.

$T > 0$ sufficiently large. Once a crisis has begun, the situation of the Central Bank deteriorates. We assume that the reserves of the Central Bank are linear over time:

$$R(s) = \overline{R}(1 - \frac{s}{T}). \tag{1}$$

After some finite interval of time, here time T, the exogenous outflow depletes completely the reserves of the Central Bank assuming no activity by speculators. In this case, a currency crisis occurs exogenously. In general, speculators are active and will sell the currency, thus depleting the reserves at a faster pace. A currency crisis occurs endogenously when the mass of agents who have bought the foreign currency reaches the reserves of the Central Bank. To repeat, a currency crisis is equivalent to the complete depletion of the Central Bank's reserves. It occurs either exogenously or endogenously. When it occurs, the Central Bank devalues the currency by an exogenous value β (the foreign exchange rate appreciates by β), and the game is over. In section 4, the rate of devaluation β will be determined endogenously.

Once a shock has occurred at time θ, speculators become gradually informed about the existence of the shock. Following Abreu and Brunnermeier (2003), we assume that the flow of newly informed speculators is uniform: the agents become informed at a constant rate. The mass of informed agents at time $\theta + s$ is $s\sigma$, for some parameter $\sigma > 0$. A speculator informed at time $t > \theta$ knows only that $\theta < t$: he knows that an exogenous shock has occurred; he does not know when. It follows that he does not know how many other agents are informed.

Let t denote the time when an agent becomes informed about the shock.

A *strategy with delay* corresponds to holding only the domestic currency before time t, and the specification of a measurable function $\mathcal{F}(s)$ for $s \geq 0$ ($\mathcal{F}$ is defined on the positive real numbers equipped with the Lebesgue measure). The function $\mathcal{F}(s)$ specifies the holding of foreign currency of the agent at time $t + s$ contingent on no devaluation before time $t + s$. The agent's portfolio in domestic and foreign currency at time $t + s$ is given by $(1 - \mathcal{F}(s), \mathcal{F}(s))$.

In this model, a *delay strategy* y corresponds to holding only the domestic currency before time $t + y$, and if no devaluation has occurred at time $t + y$, to hold only the foreign currency thereafter.

One should emphasize that the set of strategies with delay is much wider than the set of delay strategies, which could also be called trigger strategies.

2.1 Intuition

The problem with one speculator

Consider a single speculator informed at time t. His problem can be described as follows: If he keeps the domestic currency, he earns r per unit of time. If he switches to the foreign currency, he earns no "dividend" on the currency (per unit of time, it is the probability of a devaluation multiplied by the capital gain of the devaluation). In general, the agent compares these two returns.

When our agent becomes informed, he knows that the depletion of the reserves is ongoing according to equation (1), but he does not know when this process was initiated: he knows only that $\theta < t$. Does he act immediately (assuming there are no other speculator)? In general, no:

He thinks: *I know that the shock has occurred before I got informed, but I think it will take some time before the reserves of the Central Bank are exhausted. In the mean time, I can earn a return r on the domestic currency.*

Our agent should not wait too long: he knows the reserves are exhausted a time T after a shock. The longer he waits, the higher the probability of a sudden devaluation. If he delays, say $T-\epsilon$, and by that time he is lucky that no devaluation has occurred, he knows that the devaluation will occur within ϵ and that this probability is very large.

To summarize: our agent will do a Bayesian analysis, using the model, to compute the instantaneous probability of a devaluation at any time after he becomes informed. Let $\pi(y)$ be this probability if he delays y. We will show later that π is an increasing function. Indeed, the longer the agent delays, the higher the probability that the reserves are about to be completely depleted.

The expected rate of return from holding the domestic currency is r. The expected rate of return from holding the foreign currency is $\beta\pi(y)$. Hence the optimal strategy of the agent is a delay y^* such that $r = \beta\pi(y^*)$. Before y^*, the domestic currency yields a higher return and the agent strictly prefers to hold the domestic currency (conditional on no devaluation). After y^*, the foreign currency yields a higher return and the agent strictly prefers to hold the foreign currency.

The problem with a continuum of speculators

The problem is essentially the same when there is a continuum of speculators. Our speculator should not delay too long: since other speculators are active, he should delay less than in the above case. Let x be the delay of all the other speculators, and $\pi(y;x)$ the instantaneous probability of a devaluation for our speculator if he delays y. We show later that $\pi(y;x)$ is increasing: the longer our speculator delays after being informed (the larger is y), the larger the instantaneous probability of a devaluation as more and more agents have presumably become informed of the shock and sell after a delay x.

Does he delay at all? Yes.

The "worst" case for him occurs when other speculators act as soon as they become informed. In this case, when our speculator becomes informed, he knows the reserves have been partially depleted at a faster pace than when he was alone. But he is still betting, under suitable parameters of the model, that at the instant he becomes informed, the instantaneous probability of a devaluation is sufficiently low that holding the domestic currency for a while is optimal.

3 Equilibrium

To analyze the workings of the model, we first make the assumption of a symmetric delay strategy. The symmetric equilibrium will be shown in section 3.2 to be the unique strongly rational expectations equilibrium in the much wider set of strategies with delay.

Given that a shock occurred at time θ, let τ denote the time elapsed between θ and the devaluation. Assume first all agents delay for x. Then $\sigma(\tau - x)$ represents the purchases between $\theta + x$ and $\theta + \tau$ and $\overline{R}\tau/T$ represents the exogenous losses of the Central Bank. The equation

$$\sigma(\tau - x) + \overline{R}\frac{\tau}{T} = \overline{R} \tag{2}$$

defines τ as a function of x: the time elapsed between the onset of the deterioration and the devaluation depends on the strategy x of the agents.

Assume now that while all other agents delay for an interval of time x after being informed, an agent who receives a signal about θ at time t deviates and delays for y. In order to compute the agent's rate of return on the foreign currency, we determine his subjective probability of a devaluation.

If no devaluation has occurred after our agent delayed for y, while all other agents delay for x, the reserves of the Central Bank are still positive at time $t + y$. The onset of the deterioration, θ, cannot date back to a very long time (otherwise the reserves would already be depleted). More specifically, the earliest time for the shock is such that the reserves would be depleted immediately after time $t + y$. Given the strategy x, θ satisfies $t - l \leq \theta \leq t$ for some function $l = \phi(y; x)$ of y and x, with

$$\sigma(l + y - x) + \overline{R}\frac{l + y}{T} = \overline{R}, \tag{3}$$

where $\sigma(l+y-x)$ represents the purchases between $\theta+x$ and $\theta+l+y$ and $\overline{R}(l+y)/T$ represents the exogenous losses of the Central Bank.[8]

Equation (3) can be rewritten as follows:

$$l = \phi(y; x) = \frac{x + \frac{\overline{R}}{\sigma} - y(1 + \frac{\overline{R}}{\sigma T})}{1 + \frac{\overline{R}}{\sigma T}}. \tag{4}$$

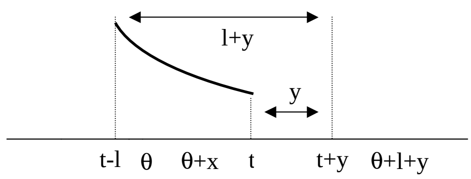

Figure 1

Note that l, the length of the support of θ, is a decreasing function of the delay y: the larger is the delay y, the smaller is l or else the closer is θ to t, the date at which the agent receives some information.

For an agent informed at t which delays for y, θ belongs to the interval $[t-l, t]$; the minimal value taken by θ is $t - l$. If there is a devaluation at $t + y$, then θ is precisely equal to $t - l$.

Once the agent is informed at date t, he revises his belief about θ. After a delay of y, his subjective distribution about θ is the exponential distribution truncated on the support $[t - \phi(y; x), t]$. Its density is $f(\theta; y, x) = Ae^{-\lambda\theta}$ with $1/A = \int_{t-\phi}^{t} e^{-\lambda\theta} d\theta$. Hence,

$$f(\theta; y, x) = \frac{\lambda e^{-\lambda(\theta - t)}}{e^{\lambda\phi(y;x)} - 1}. \tag{5}$$

[8]Whenever $y = x$, we find (2) once we identify τ with $l + y$.

3.1 The reaction function

Given that all other agents delay for an interval of time x, the instantaneous probability of the devaluation for an agent informed at t which delays for y is the exponential distribution of θ truncated on the interval $[t-\phi(y;x),t]$ evaluated at $\theta = t-\phi(y;x)$:

$$\pi(y;x) = \frac{\lambda}{1-e^{-\lambda\phi(y;x)}}. \tag{6}$$

Indeed, a devaluation occurs at $t+y$ if the reserves are depleted at $t+y$, i.e. if θ is at the earliest date of the support of its distribution for the agent, namely $t-\phi(y;x)$. A devaluation occurs according to a Poisson process with an instantaneous probability equal to the density of the distribution of θ at $t-\phi(y;x)$.

For a given strategy x, the function $\phi(y;x)$ is decreasing in y, and hence $\pi(y;x)$ is an increasing function of y: the larger is the delay, the larger is the instantaneous probability of devaluation.

Moreover, an agent knows that the largest delay between the time he gets informed and a devaluation occurs when he is the first to be informed. In this case, the delay is $y_0 = \frac{x+\frac{\overline{R}}{\sigma}}{1+\frac{\overline{R}}{\sigma T}}$. One verifies that $\pi(y;x)$ tends to infinity as y approaches y_0.

We now analyze the optimal delay. Assuming the agent delays for y, let $dy<0$ denote a reduction in the delay and consider the impact of this modification on the gain in case of devaluation, $\beta\pi(y;x)$.

If $\beta\pi(y;x) > r$, then the gain is modified by $(\beta\pi(y;x)-r)dy < 0$ and the agent should delay for an even shorter period of time. Indeed, the gain from the instantaneous probability of a devaluation is greater than the opportunity cost of the interest income on the domestic asset.

Similarly, if $\beta\pi(y;x) < r$, then $(\beta\pi(y;x)-r)dy > 0$ and hence the agent should delay for a longer period of time.

The *arbitrage condition* for buying the foreign currency after a delay of y is

$$\beta\pi(y;x) = r. \tag{7}$$

This condition defines a reaction function of an agent informed at t which delays y, when all other agents delay x.

Using (6), we have

$$\beta\pi(y;x) = \frac{\beta\lambda}{1-e^{-\lambda\phi(y;x)}}. \tag{8}$$

Recall that for a given x, $\beta\pi(y;x)$ is increasing in y. From the expression of ϕ in (4), one can show that the graph of $\beta\pi(\cdot;x)$ is as shown in Figure 2 with $\beta\pi(0;x)>0$. From the figure, it is immediate that there is a unique value $y(x)>0$ such that $\beta\pi(y(x);x)=r$ if and only if $\beta\pi(0;x)<r$. If $\beta\pi(0;x)>r$, then the agent should buy the foreign currency without delay and hence $y(x)=0$ is his optimal strategy.

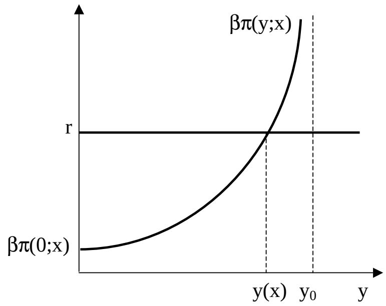

Figure 2

From the previous discussion, the *reaction function* of an agent informed at t which delays for y, given that all the other agents delay for x, is

$$y(x) = \frac{x + \frac{\overline{R}}{\sigma}}{1 + \frac{\overline{R}}{\sigma T}} - \frac{1}{\lambda}\log\left(\frac{r}{r - \beta\lambda}\right). \tag{9}$$

The slope of y as a function of x is smaller than one, and equal to $\frac{1}{1+\frac{\overline{R}}{\sigma T}}$.
We make the following assumption.

Assumption 1: $r > \beta\pi(0;0) = \frac{\beta\lambda}{1-e^{-A}} > \beta\lambda$, where $A = \frac{\lambda\frac{\overline{R}}{\sigma}}{1+\frac{\overline{R}}{\sigma T}}$.

Assumption 1 implies that $y(0) > 0$: the return on the domestic currency is sufficiently high to induce our agent to delay even if other agents do not delay once they become informed. Note that the other agents are not all informed at the same time.

The reaction function is illustrated in Figure 3. From Figure 3, it follows that there is a unique Nash equilibrium; it is the fixed point of the reaction function (9).

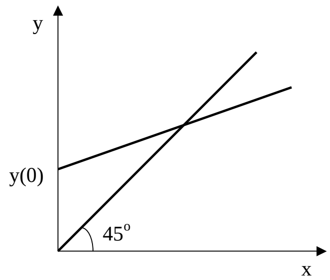

Figure 3

The slope of the reaction function shows that there is strategic complementarity[9] within the set of delay strategies. The properties of the equilibrium strategy y^* are easily computed and are summarized in the next proposition.

[9] A rigorous treatment of strategic complementarity requires the description of an ordering over the space of actions. See Cooper (1999) and Vives (1990). The set of delay strategies has such an ordered structure.

Proposition 3.1 *Under Assumption 1, the unique symmetric Nash equilibrium in the set of delay strategies is given by*

$$y^* = T - (1 + \frac{\sigma T}{\overline{R}})\frac{1}{\lambda}\log(\frac{r}{r - \beta\lambda}).$$

The equilibrium delay strategy y^* is increasing in T, in $\overline{R}$ and in r. It is decreasing in σ, in β and in λ.

The equilibrium delay strategy is increasing in the interest rate and in the initial amount of reserves. The policies of the Central Bank are thus instrumental in the choice of the agent. As expected, the equilibrium delay strategy is negatively related to the gain in case of devaluation β and to the rate of information σ. The smaller the exogenous flow (the larger is T), the larger is the delay. Finally, if the probability λ of the occurrence of an adverse shock per unit of time, conditional on no previous shock, is large, then the delay is small.

Remark From (2), in equilibrium, the time τ between a shock θ and a devaluation is given by

$$\tau = T\frac{\overline{R} + \sigma y^*}{\overline{R} + \sigma T}. \tag{10}$$

As expected, $\tau < T$, and τ is an increasing function of T. Moreover, as T tends to infinity, the ratio τ/T tends to a constant (note that y^* is endogenous to T).

By definition, $1/\sigma$ is the time it takes for all agents to become informed of the shock. When the exogenous flow is small (T is large), all the agents are informed about the shock when a devaluation occurs: $\tau > 1/\sigma$. Using the definition of y^*, this condition is equivalent to

$$1 - \frac{\sigma}{\overline{R}\lambda}\log(\frac{r}{r - \beta\lambda}) > \frac{1}{\sigma T}. \tag{11}$$

Note that the LHS is always positive under Assumption 1.

A devaluation occurs when the exogenous flow is small, and it is not due to the fact that some speculators are not informed about the shock. Indeed, it may be that at the time of devaluation, all agents are informed, and that they all know that all agents are informed (i.e. $\tau > 2/\sigma$)[10]. A discrete devaluation is caused because speculators, who get informed sequentially, do not know their position in the information queue that was initiated by the shock.

3.2 Uniqueness of equilibrium

We now show that the equilibrium y^* in Proposition 3.1 is the unique stable equilibrium in the full set of strategies with delay.[11] For this, we show that y^* is a strongly rational

[10] It takes an interval of time $2/\sigma$ for all agents to know that all agents are informed. Indeed, the last agent to get informed may get informed at time $\theta + 1/\sigma$, in which case, if he thinks he is the first to be informed, he will think that everyone will be informed after another interval of time $1/\sigma$.

[11] Recall that a strategy with delay corresponds to holding only the domestic currency before time t, and the specification of a measurable function $\mathcal{F}(s)$ for $s \geq 0$ specifying the foreign currency holding at time $t + s$, conditional on no devaluation before time $t + s$.

expectation equilibrium (SREE) (Guesnerie (1992)) — i.e. any strategy with delay $y \neq y^*$ is iteratively dominated and y^* is not iteratively dominated.[12] The equilibrium is "stable" in the sense that the sequence of reactions of the agents to any strategy of the others converges to the equilibrium.[13] In showing this result, we strengthen the symmetric Nash equilibrium with strategy y^*. Indeed, a critical issue in this paper is the coordination of agents without common knowledge. We here provide a refinement over Proposition 3.1, as it does not involve strong requirements on the coordination of agents, as compared to the Nash equilibrium.

Any trade is admissible within the fixed constraints. Agents can take any position before the devaluation. The only restriction is that we assume the agents act only after being informed. Note that if an agent takes an action at time t without being informed, his strategy depends only on the time t. An action taken at some time t is successful only if other agents coordinate on the same date. Even if such a coordination could be achieved without some external device, it is not clear that there is a Nash equilibrium strategy where the action depends only on the time t. It is therefore not obvious that there is a Nash equilibrium in which uninformed agents take action. These problems are bypassed to concentrate on the main issues.

Proposition 3.2 *The equilibrium delay strategy y^* in Proposition 3.1 is a strongly rational expectation equilibrium in the set of strategies with delay where agents take action after being informed of the shock.*

The proof can be found in Appendix A. The argument can be paraphrased as follows. Each agent knows that the exogenous flow is sufficient to trigger a devaluation after some finite time. Hence a very long delay is a dominated strategy. Assume there is some x_k such that any delay longer than x_k is dominated. It is shown in the appendix that there is $x_{k+1} < x_k$ such that any delay longer than x_{k+1} is dominated and the sequence $\{x_k\}$ converges to y^*. A similar increasing sequence $\{x'_k\}$ converges to y^*, and it is such that any delay shorter than x'_k is dominated at step k. The equilibrium strategy is shown to be the only one that survives a natural process of elimination of dominated strategies.

4 Endogenous rate of devaluation

First generation models of currency crises are characterized by two exchange rate regimes, separated by the crisis. For simplicity, the demand for money depends only on the domestic inflation rate. In the first regime, the exchange rate is fixed and the government runs a deficit that is financed by the Central Bank. As the exchange rate is fixed, the price level is fixed (by purchasing power parity) and the demand for money, which depends on the inflation rate (0 in this regime) is constant. Hence the assets of the Central Bank remain constant, and the government bonds gradually crowd out the foreign reserves. This process must eventually stop. The exchange rate must eventually be abandoned. Following that event, there is a second regime in which the exchange

[12] Given that all agents have the same set of strategies $J \subset \mathbf{R}$, recall that a strategy y is iteratively dominated if there is a finite sequence of increasing sets $I_0 = \emptyset,...,I_N$, with $y \in I_N$, such that strategies in I_k are strictly dominated when all agents play in the subset of strategies $J \setminus I_{k-1}$.

[13] Bernheim (1984), Pearce (1984).

rate floats and the level of foreign reserves in the Central Bank is constant. In that regime, the deficit which continues to be financed by money creation increases the money supply and the inflation rate is strictly positive and constant (and equal to the rate of appreciation of the foreign exchange). The jump in the inflation rate forces the real demand for money to jump down when the fixed rate regime is abandoned.

Krugman (1979) assumes agents have perfect information. Under this assumption, the exchange rate cannot jump. The jump in the real quantity of money is therefore achieved by a jump in the nominal quantity of money: at the time of the switch, agents run to trade a stock of money equal to the difference in the nominal quantity demanded before and after the switch.

The model of Krugman (1979) remains unsatisfactory because of the perfect foresight assumption. No sudden devaluation takes place in that model. This property does not fit the experiences of currency crises. We now extend the model of the previous sections to address the problem analyzed by Krugman when agents have imperfect information about an exogenous shock that triggers a gradual depletion of the foreign currency reserves of the Central Bank. This model is equivalent to a model where the depletion of reserves is induced by a government deficit. We will show that a devaluation occurs in a currency crisis where agents have incomplete information. When agents have near perfect information, the model will generate the same properties as in Krugman.

Without loss of generality, we assume that the domestic quantity of money is equal to the liabilities of the Central Bank and that it is the sum of the speculators' holdings, K, and a demand for transactions, D.

As in the previous model, speculators hold domestic currency in a regime of fixed exchange rate because of the interest rate premium. After a devaluation, we assume no interest premium and speculators have no demand for the domestic currency. This is a stylized way to think of the model of Krugman where a portfolio equation is defined and in which the inflation rate plays the role of a tax on domestic currency. This "tax" increases in the second regime, leading to a decrease in the demand for domestic currency.

The demand for transactions is set such that $D/P = k$, where P is the exchange rate and k is a parameter. The value of P is equal to 1 under a fixed exchange rate and it is equal to the value determined by the market at the instant after the regime of fixed exchange rate is abandoned. The rate of devaluation β is now endogenous: $\beta = P - 1$.

Let K be the domestic currency holdings of the speculators at the time of devaluation. The quantity of money at that time is therefore $K + D$. Since speculators do not hold domestic currency after the devaluation, we have $(D + K)/P = k$. Hence

$$\beta = \frac{D + K}{k} - 1 = \frac{K}{k}. \tag{12}$$

Let K_0 be the mass of speculators each holding initially one unit of domestic currency. The initial reserves of the Central Bank, $\overline{R}$, are larger than K_0. As in the previous sections, an exogenous shock occurs at some time θ after which there is an exogenous loss of reserves with a flow $\overline{R}/T$ per unit of time, where $\theta + T$ is the time at which an exogenous devaluation will occur.

The structure of information for speculators is the same as in the previous sections with the flow of newly informed agents per unit of time equal to σ.

At the time of devaluation $\theta + \tau$, the holdings of speculators are equal to

$$K = K_0 - \sigma(\tau - y^*), \tag{13}$$

where y^* is the equilibrium delay strategy. Using (2), $\tau = a + \nu y^*$ where $a = \frac{\overline{R}}{\sigma}/[1 + \frac{\overline{R}}{\sigma T}]$ and $\nu = 1/[1 + \frac{\overline{R}}{\sigma T}] < 1$. Hence $K = K_0 - \sigma a + \sigma(1-\nu)y^*$. Substituting this value of K in (12), we get an expression which defines the rate of devaluation, $\tilde{\beta}(y^*)$, as an increasing function of the equilibrium delay, y^*:

$$\tilde{\beta}(y^*) = \frac{1}{k}(K_0 - \frac{\overline{R}}{1 + \frac{\overline{R}}{\sigma T}}) + \frac{y^*}{k}\frac{\frac{\overline{R}}{T}}{1 + \frac{\overline{R}}{\sigma T}}. \tag{14}$$

This property is intuitive: if speculators delay longer, they hold more domestic currency at the time of devaluation and the rate of devaluation must be higher for the money market equilibrium after the devaluation.

Consider now a devaluation rate β. From Proposition 3.1, the equilibrium delay strategy is

$$\tilde{y}^*(\beta) = T - (1 + \frac{\sigma T}{\overline{R}})\frac{1}{\lambda}\log(\frac{r}{r - \beta\lambda}). \tag{15}$$

As already mentioned in that proposition, the delay in the equilibrium strategy is a decreasing function of the rate of devaluation β. When $\beta = 0$, then $\tilde{y}^*(\beta) = T$ and when $\beta = \frac{r}{\lambda}[e^\alpha - 1]/e^\alpha$ with $\alpha = \lambda\overline{R}T/[\overline{R} + \sigma T]$, then $\tilde{y}^*(\beta) = 0$.

The graphs of the functions $\tilde{\beta}(y^*)$ and $\tilde{y}^*(\beta)$ defined in (14) and (15) are represented in Figure 4.

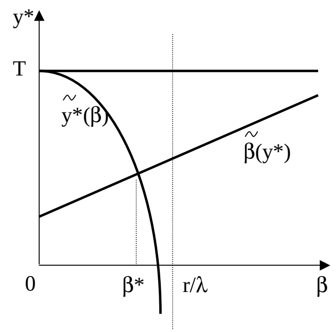

Figure 4

The parameters of the model are such that in (14) for $\tilde{\beta}(y^*) = 0$, $y^* \in (0, T)$. The two schedules have a unique intersection that determines endogenously the rate of devaluation, β^*.

As in section 3.1, we see that when the exogenous flow is small (T is large), all the agents are informed when the devaluation occurs ($\tau > 1/\sigma$). Indeed, assume the exogenous flow is small. Then the vertical intercept of the function $\tilde{\beta}$ gets very large, which implies a very large equilibrium value y^*. We then have $y^* > 1/\sigma$ and thus $\tau > 1/\sigma$ as τ is proportional to y^*.

Recall that σ represents the rate of information or the speed at which the agents get informed. When σ increases, the agents become informed more quickly about the occurrence of the shock. Let us analyze the variations of the rate of devaluation and the equilibrium delay strategy with σ. We have

$$\frac{d\tilde{\beta}}{d\sigma} = -\frac{1}{k}\frac{\overline{R}^2}{(\sigma T)^2}\frac{1}{(1+\frac{\overline{R}}{\sigma T})^2}(T - y^*), \tag{16}$$

which is negative for any $y^* \in (0, T)$, and

$$\frac{d\tilde{y}^*}{d\sigma} = -\frac{1}{\lambda}\frac{T}{\overline{R}}\log(\frac{r}{r-\beta\lambda}) < 0, \tag{17}$$

as found in Proposition 3.1. Hence, an increase in the rate of information decreases the gain in case of devaluation as long as the delay strategy is smaller than T (the time at which the devaluation occurs exogenously), and it decreases the delay.

Proposition 4.1 *The equilibrium value of the endogenous rate of devaluation, β^*, decreases when the rate of information σ increases.*

The limit value of the rate of devaluation as the rate of information tends to infinity is given by

$$lim_{\sigma\to\infty}\tilde{\beta}(y^*) = \frac{1}{k}(K_0 - \overline{R}) + \frac{\overline{R}}{kT}y^*.$$

As σ tends to infinity, the intersection between the functions $\tilde{\beta}(y^*)$ and $\tilde{y}^*(\beta)$, which determines the equilibrium value of the endogenous rate of devaluation, tends to the value of y^* for which $\tilde{\beta}(y^*) = 0$, namely

$$y^* = T(1 - \frac{K_0}{\overline{R}}) > 0. \tag{18}$$

Proposition 4.2 *When the rate of information σ tends to infinity, the equilibrium value of the endogenous rate of devaluation β^* tends to zero. The equilibrium value of the delay tends to y^* defined by $\frac{y^*}{T} = 1 - \frac{K_0}{\overline{R}} > 0$.*

When σ tends to infinity, all agents are informed at nearly the same time. This limit case corresponds to the model of Krugman (1979) of the first generation. Speculators delay y^*, until the level of the Central Bank's reserves is just equal to their own balances, which they trade in at the same time with no capital loss. There is no jump of the exchange rate hence no devaluation when the peg is abandoned.

5 Uncertainty about the policy of the Central Bank

In this last section, we determine whether the Central Bank should reveal or not the extent to which it will defend the peg should a shock occur, when it has a menu of policies.

The initial amount of reserves is $\overline{R}$, but the Central Bank either stops defending the peg if the level of reserves falls below $R_0 - A$ or $R_0 + A$. Without loss of generality, we assume $R_0 = 0$: either the Central Bank borrows from other Central Banks to defend the peg, or it abandons the regime with a positive level of reserves. For simplicity and without loss of generality, we consider only two such reference levels of reserves, and assume that the *ex ante* probability that the Central Bank follows one policy or the other is 1/2. Agents have rational expectations about the policy of the Central Bank and know these probabilities.

The exogenous outflow of reserves is the same as in the model with certainty and its time profile is $R(s) = \bar{R}(1 - s/T)$. Hence, if speculators do not trade, a devaluation will occur exogenously[14] either at date T_1 or date $T_2 > T_1$ with $T_i = A_i T/\overline{R}$, $A_1 = \overline{R} - A$, $A_2 = \overline{R} + A$, and T as in section 2.

We denote by R_i the state in which the reserves are exogenously depleted at T_i. The instantaneous probability of the devaluation of an agent who delays for y given that all other agents delay for an interval of time x, defined in (6) under certainty, is now

$$\pi_u(y;x) = \mu \frac{\lambda}{1 - e^{-\lambda \phi_1(y;x)}} + (1-\mu) \frac{\lambda}{1 - e^{-\lambda \phi_2(y;x)}}, \tag{19}$$

where μ is the probability that the state is R_1 given that the devaluation has not occurred yet and ϕ_i defines the length of the support of θ if the devaluation occurs exogenously at date T_i. A few computations, the details of which can be found in Appendix B, lead to the following proposition.

Proposition 5.1 *Uncertainty about the level of reserves used by the Central Bank to defend the peg leads to an agent's instantaneous probability of a devaluation, π_u, which is smaller than his instantaneous probability of a devaluation under certainty, π.*

Should the Central Bank reveal its policy regarding the reserves or not? In answering this question, we focus on the impact of the policy on the length of the peg regime once a shock has occurred. A thorough analysis would require the specification of an objective function of the Central Bank which goes beyond the scope of the present paper. We here assume that the Central Bank wants to maximize the "sustainability" of the peg, defined as the mean length of the peg once a shock has occurred and the exogenous drain on reserves has started.

Given a distribution of minimal levels of reserves below which the Central Bank would stop to defend the peg, let $\hat{R}$ be the mean of these levels of reserves. The *certainty equivalent policy* is a perfectly anticipated Central Bank's policy in which it stops defending the peg when the level of reserves reaches $\hat{R}$.

Proposition 5.2

- *When there is a distribution of perfectly observed levels of reserves below which the Central Bank would stop defending the peg, the mean delay between the shock and the devaluation is equal to the delay under the certainty equivalent policy.*

[14] A devaluation occurs exogenously when the exogenous outflow generated by the shock depletes completely the reserves of the Central Bank without any intervention of speculators.

- *When there is a distribution of unobserved levels of reserves below which the Central Bank would stop defending the peg, the mean delay between the shock and the devaluation is greater than the delay under the certainty equivalent policy.*

The proposition shows that when agents have perfect information about the policy of the Central Bank, a richer set of policies has no impact on the mean length of the peg after a shock. However, the mean length of the peg is extended when agents have imperfect information about the exact policy followed by the Central Bank within this set. We can infer from Proposition 5.2 that if the objective of the Central Bank consists in maximizing the ex ante expected delay between the shock and the devaluation, it should not reveal any information about the extent to which it will defend the peg.

6 Conclusion

We have considered an economy with a fixed exchange rate regime that has suffered an adverse shock at some random time. Speculators know that a devaluation will occur at some exogenous time in the future. A coordination problem appears as a devaluation may well be triggered by the actions of the speculators before it occurs exogenously. The strategies of the speculators are at the heart of this coordination problem.

In this game, the agents become gradually informed at a constant rate about the occurrence of the shock. A devaluation occurs exogenously or endogenously as soon as the reserves of the Central Bank are completely depleted.

We have shown that the game played by the speculators has a unique symmetric Nash equilibrium which is a strongly rational expectation equilibrium. We therefore depart from the "second generation models" characterized by multiple equilibria as we present a determinate equilibrium solution, as in Morris and Shin (1998). Moreover, we have determined endogenously the rate of devaluation.

The model could be extended to include additional shocks on the fundamentals. For example, the exogenous outflow that was set here by a random process could be stopped according to a second random process. One can imagine that a policy of the Central Bank that was able to delay a crisis under a permanent shock could avoid entirely a crisis when the shock is transitory.

There are various ways of introducing uncertainty in this model. We have focussed here on an uncertainty regarding the level of reserves used by the Central Bank to defend its currency. We could equally well have considered different rates of information, or else different masses of speculators.

The issue of transparency is crucial in international financial market policies. The results in this paper show that transparency may have adverse effects for the policy maker. However, it is well acknowledged by international organizations that transparency is efficient as it may foster credibility. More research will be devoted to this issue.

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7 Appendix A : Proof of Proposition 3.2

- We prove first that any strategy $y > y^*$ is iteratively dominated.

  Recall that for an agent informed at time t, $\mathcal{F}(s)$ is his foreign currency holding at time $t+s$. His portfolio in domestic and foreign currency at time $t+s$ is defined by $(1-\mathcal{F}(s), \mathcal{F}(s))$.

We will say that an agent does not delay longer than s' if $\mathcal{F}(s) = 1$ for any $s \geq s'$. Note that this definition does not restrict the path of $\mathcal{F}(s)$ for $s < s'$.

The instantaneous expected rate of return of the portfolio specified by the strategy at time $t + s$ is $r + \mathcal{F}(s)[\beta\tilde{\pi}(s) - r]$, where $\tilde{\pi}(s)$ is the instantaneous probability of a devaluation and depends on the strategy of others.

If $\beta\tilde{\pi}(s) > r$, for some interval of time dt, any strategy $\mathcal{F}$ with $\mathcal{F} < 1$ for a set of positive measure in the interval of time dt is strictly inferior to the same strategy where $\mathcal{F}$ is replaced by 1 in that interval of time.

If $\beta\tilde{\pi}(s) < r$, for some interval of time dt, any strategy $\mathcal{F}$ with $\mathcal{F} > 0$ for a set of positive measure in the interval of time dt is strictly inferior to the same strategy where $\mathcal{F}$ is replaced by 0 in that interval of time.

An agent informed at time t knows that a devaluation will occur no later than time $t + T$.

We may assume that for any strategy, $\mathcal{F}(s) = 1$ for $s \geq T$ without any impact on the probability $\tilde{\pi}(s)$ as a devaluation occurs before time $t + T$, where t is the instant an agent is informed.

Assume now that the agents delay no longer than x_k, with $x_1 = T$. The level of reserves at any time after θ is not greater than when all agents delay for the upper-bound x_k. Therefore, the time lag between θ and a devaluation is bounded above by the value found in the symmetric case with all agents delaying x_k. Hence, for an agent informed at t, if no devaluation has occurred by time $t + y$, then $t - \phi(y; x_k)$ is the lower bound of θ for any strategies of the other agents with delay no longer than x_k, and ϕ defined in equation (4). The support of θ is therefore in the interval $[t - \phi(y; x_k), t]$.

Recall that in the symmetric case where agents delay for x_k, a devaluation occurs at $t + y$ when θ is "at" (in the neighborhood of) the lower bound of the support $[t - \phi(y; x_k), t]$.

For any agent, the probability of an instantaneous devaluation depends on his belief about the others' strategies, the only constraint imposed here being that no agent delays more than x_k. We do not assume that the agents follow the same strategy. To each set of possible strategies of the others corresponds a support for θ. Recall that if the belief of our agent about the strategies of others is such that all other agents delay precisely x_k, then the support is $[t-\phi(y; x_k), t]$. Each other set of strategies of the agents is associated to smaller delays, a faster exhaustion of the reserves, and hence a support $[t - a, t]$ with $a < \phi(y; x_k)$.

For any belief about the strategies of the others, the instantaneous probability of a devaluation is the expected value of the instantaneous probability at the lower bounds of all the associated supports. Since each is greater than $\pi(y; x_k)$,[15] we get $\tilde{\pi}(y) \geq \pi(y; x_k)$.

Let x_{k+1} be defined by $\beta\pi(x_{k+1}; x_k) = r$. We know that $\pi(y; x_k)$ is strictly increasing in y. Hence for any $y > x_{k+1}$, and for any strategy of the other agents

[15]Recall that $\pi(y; x_k) = \frac{\lambda}{1-e^{-\lambda\phi(y;x_k)}}$, while for each support of length a, the instantaneous probability at the lower bound is $\frac{\lambda}{1-e^{-\lambda a}}$.

that do not delay longer than x_k, $\tilde{\pi}(y) \geq \pi(y; x_k) > \pi(x_{k+1}; x_k) = \frac{r}{\beta}$. Holding the domestic currency for any $y > x_{k+1}$ (i.e. $\mathcal{F}(y) < 1$ for a set of measure different from zero) is dominated by $\mathcal{F}(y) = 1$.

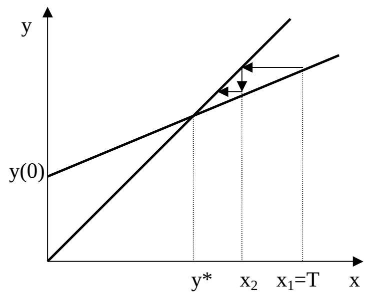

Figure 5

The sequence $\{x_k\}$ is generated by the reaction function y defined in section 3: $x_{k+1} = y(x_k)$. This sequence is monotonically converging to y^*, as illustrated in Figure 5. Therefore, any strategy with a delay beyond y^* is iteratively dominated.

- We now show that any strategy $y < y^*$ is iteratively dominated. An argument analogous to the one used in the previous case proves the result.

  Assume the agents delay at least x_k, with $x_1 = 0$. The time lag between θ and a devaluation is bounded below by the value found in the symmetric case with all agents delaying x_k.

  Let x_{k+1} be defined by $\beta\pi(x_{k+1}; x_k) = r$. Since $\pi(y; x_k)$ is strictly increasing in y, for any $y < x_{k+1}$, then $\tilde{\pi}(y) \leq \pi(y; x_k) < \pi(x_{k+1}; x_k) = \frac{r}{\beta}$.[16]

  Hence holding the foreign currency for any $y < x_{k+1}$ (i.e. $\mathcal{F}(y) > 0$ for a set of measure different from zero) is dominated by $\mathcal{F}(y) = 0$.

  The sequence $\{x_k\}$ is generated by the reaction function y in section 3: $x_{k+1} = y(x_k)$. This sequence is monotonically converging to y^* as illustrated in Figure 6. Therefore, any strategy $y < y^*$ is iteratively dominated.

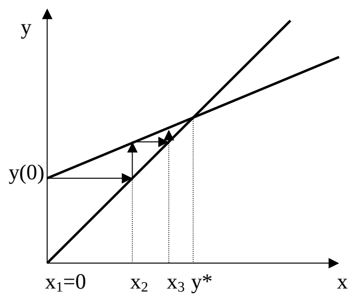

Figure 6

$\square$

[16] The agents delay at least x_k. Each admissible set of strategies of the others with a delay larger than x_k generates a support of θ of length larger than $\phi(y; x_k)$, and leads to a smaller instantaneous probability at the lower bound of the support. Using an argument analogous to the previous case, one gets $\tilde{\pi}(y) \leq \pi(y; x_k)$.

8 Appendix B: Uncertainty about the policy of the Central Bank

Given the notation introduced in section 5, the *instantaneous probability of the devaluation* for an agent that delays for y, given that all other agents delay for an interval of time x, is

$$\pi_u = \mu \frac{\lambda}{1 - e^{-\lambda \phi_1(y;x)}} + (1-\mu) \frac{\lambda}{1 - e^{-\lambda \phi_2(y;x)}}, \tag{20}$$

where μ is the probability that the state is R_1 given that the devaluation has not occurred yet:

$$\mu = P(R_1|S) = \frac{P(S|R_1)P(R_1)}{P(S|R_1)P(R_1) + P(S|R_2)P(R_2)} = \frac{P(S|R_1)}{P(S|R_1) + P(S|R_2)} \tag{21}$$

(as $P(R_1) = P(R_2) = 1/2$), where S denotes the event of being informed about the possible occurrence of a crisis, while the crisis (the devaluation) has not yet occurred, and $l_i = \phi_i(y;x)$ defines the length of the support of θ if the devaluation occurs exogenously at date T_i:

$$l_i = \phi_i(y;x) = \frac{\sigma x + A_i}{\sigma + \frac{\overline{R}}{T}} - y. \tag{22}$$

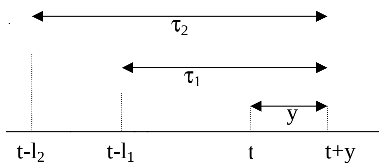

Figure 7

The conditional probability $P(S|R_i)$ is

$$P(S|R_i) = e^{-\lambda(t - \phi_i(y;x))} \int_0^{\phi_i(y;x)} \lambda e^{-\lambda u} du = e^{-\lambda(t-\phi_i(y;x))}(1 - e^{-\lambda \phi_i(y;x)}). \tag{23}$$

Hence

$$\mu = \frac{e^{-\lambda(t-\phi_1(y;x))}(1 - e^{-\lambda\phi_1(y;x)})}{e^{-\lambda(t-\phi_1(y;x))}(1 - e^{-\lambda\phi_1(y;x)}) + e^{-\lambda(t-\phi_2(y;x))}(1 - e^{-\lambda\phi_2(y;x)})} = \frac{e^{\lambda\phi_1} - 1}{e^{\lambda\phi_1} + e^{\lambda\phi_2} - 2} \tag{24}$$

The instantaneous probability of a devaluation is thus given by[17]

$$\pi_u = \lambda \frac{e^{\lambda\phi_1} + e^{\lambda\phi_2}}{e^{\lambda\phi_1} + e^{\lambda\phi_2} - 2}. \tag{25}$$

By definition, $\phi_i(y;x) = \tau_i - y$ (Figure 7), where $\tau_i = a_i + \nu x$, with $a_1 = a - \eta$, $a_2 = a + \eta$, $a = \frac{\overline{R}}{\sigma + \frac{\overline{R}}{T}}$, $\eta = \frac{A}{\sigma + \frac{\overline{R}}{T}}$, and $\nu = \frac{\sigma}{\sigma + \frac{\overline{R}}{T}}$.

[17] Note that for $\phi_1 = \phi_2 = \phi$, then $\pi_u = \pi$: the instantaneous probability of devaluation in case of uncertainty equals the instantaneous probability of devaluation with no uncertainty.

From these observations, the instantaneous probability of a devaluation takes the following form:

$$\pi_u(y;x) = \lambda \frac{\frac{e^{\lambda a_1}+e^{\lambda a_2}}{2}}{\frac{e^{\lambda a_1}+e^{\lambda a_2}}{2} - e^{\lambda(y-\nu x)}}. \tag{26}$$

The function e^x being convex, Proposition 5.1 follows.

Should the Central Bank reveal its policy regarding the reserves or not?

We assume the Central Bank wishes to maximize the *ex ante* expected delay between θ and the devaluation, namely $\tau = \frac{\tau_1+\tau_2}{2}$.

Case 1 The Central Bank *does not* reveal its policy.

In this case, $a_1 = a - \eta$ and $a_2 = a + \eta$, and the *ex ante* expected delay between θ and the devaluation is $\tau = \frac{\tau_1+\tau_2}{2} = a + \nu x$, where x is solution to $\pi_u(x;x) = c$ with $c = r/\beta$ a parameter that depends on the interest rate and the rate of devaluation, and $\pi_u(x;x)$ is the equilibrium value of the instantaneous probability of devaluation in (26):

$$\pi_u(x;x) = \lambda \frac{\zeta}{\zeta - e^{\lambda(1-\nu)x}}, \qquad \text{with} \qquad \zeta = \frac{e^{\lambda(a-\eta)} + e^{\lambda(a+\eta)}}{2}. \tag{27}$$

Case 2 The Central Bank reveals its policy.

In this case, either $a_1 = a_2 = a - \eta$ or $a_1 = a_2 = a + \eta$.

If $a_1 = a_2 = a + \eta$, then the delay between θ and the devaluation is $\tau_+ = a + \eta + \nu x_+$ where x_+ is solution to $\pi(x;x) = c$ with $\pi(x;x)$ the instantaneous probability of the devaluation under certainty (computed in Section 3.1):

$$\pi(x;x) = \lambda \frac{e^{\lambda(a+\eta)}}{e^{\lambda(a+\eta)} - e^{\lambda(1-\nu)x}}. \tag{28}$$

If $a_1 = a_2 = a - \eta$, then the delay between θ and the devaluation is $\tau_- = a - \eta + \nu x_-$ where x_- is solution to $\pi(x;x) = c$ with

$$\pi(x;x) = \lambda \frac{e^{\lambda(a-\eta)}}{e^{\lambda(a-\eta)} - e^{\lambda(1-\nu)x}}. \tag{29}$$

The *ex ante* expected delay between θ and the devaluation is

$$\tau = \frac{\tau_- + \tau_+}{2} = a + \nu(\frac{x_- + x_+}{2}). \tag{30}$$

These observations lead to Proposition 5.2.