Non-parametric Bounds in the Presence of Item Non-response, Unfolding Brackets, and Anchoring

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Abstract

Household surveys are often plagued by item non-response on economic variables such as income, savings or wealth. Recent work by Manski shows how bounds on conditional quantiles of the variable of interest can be derived, allowing for any type of non-random response behavior. Including follow up categorical questions in the form of unfolding brackets for initial item non-respondents, is an effective way to reduce complete item non-response. Recent evidence, however, suggests that such design is vulnerable to a psychometric bias known as the anchoring effect. In this paper, we extend the approach by Manski to take account of the information provided by the bracket respondents. We derive bounds under various non-parametric assumptions on anchoring effects. These bounds are applied to earnings in the 1996 wave of the Health and Retirement Survey (HRS). The results show that the categorical questions can be useful to increase precision of the bounds, even if anchoring is allowed for.

Key words: unfolding bracket design, anchoring effect, item non-response, non-parametrics, bounding intervals.
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1 Introduction

Household surveys are often plagued by item nonresponse on economic variables of interest like income, savings or the amount of wealth. For example, in the 1996 wave of the Health and Retirement Survey (HRS), a US panel often used to study socio-economic behavior of the elderly, 12.4% of those who say they have some earnings, do not give their amount of these earnings. Questions on amounts of certain types of wealth often lead to even larger nonresponse rates. In a series of recent papers by Manski it is shown how, in the presence of such non-response, bounds on conditional quantiles of the variable of interest can be derived, allowing for any type of nonrandom response behavior. See, for example, Manski (1989, 1994, 1995). Manski's framework is intuitively appealing, easy to apply, and very flexible, but has the drawback that it often leads to bounds that are too wide to draw meaningful economic conclusions. In this framework, the precision with which features of the distribution of the variable of interest (such as quantiles of the income distribution) can be determined, i.e., the width between the bounds, depends on the probability of non-response. If non-response is substantial, the approach does not lead to accurate estimates of the parameters of interest.

Including follow-up questions in the form of unfolding brackets for initial item non-respondents is an effective way to reduce complete item non-response. In the HRS example given above, 73% of the initial non-respondents answer the question whether or not their earnings exceed $25,000, and most of these also answer a second question on either $50,000 (if the first answer was 'yes') or $5,000 (if the first answer was 'no'). Recent evidence given by Hurd et al. (1998), however, suggests that the follow-up design that is used here leads to an "anchoring effect," a phenomenon well documented in the psychological literature: the distribution of the categorical answers is affected by the amounts in the questions (the "bids" become "anchors"). Experimental studies have shown that even if the anchor is arbitrary and uninformative with respect to the variable of interest, it still produces large effects on the responses (see, for example, Jacowitz and Kahneman, 1995). Using a special survey with randomized initial bids (instead of $25,000 for everybody), Hurd et al. (1998) show that the distribution is biased towards the categories close to the initial bid. They estimate a parametric model to capture this anchoring phenomenon. Their results confirm that the anchoring effect can lead to biased conclusions on the parameters of interest if not properly accounted for. Alternative parametric models for anchoring have been designed by Cameron and Quiggin (1994) and Herriges and Shogren (1996).

This chapter extends the approach by Manski to take account of the information provided by the bracket respondents. Bounds are derived that do and do not allow for an anchoring effect. The latter are based on the assumption that the bracket information is always correct. The former relax this assumption. Instead, they are based on nonparametric assumptions generalizing the existing models of anchoring. In particular, three different assumptions on the nature of anchoring are considered, corresponding to different studies of anchoring in the existing literature.

These bounds are applied to earnings in the 1996 wave of the Health and Retirement Survey. The results show that categorical questions can be useful to increase precision of the bounds, even if anchoring is allowed for. This also helps, for example, to improve the power of statistical tests for equality of earnings quantiles in sub-populations. To prove this point, bounds for respondents with low education are compared to bounds for respondents with high education, showing how the bounds which take account of bracket information can detect differences which are not discovered by bounds based upon full respondents' information only.

The remainder of this paper is organized as follows. Section 2 elaborates on the problems associated with item non-response in economic surveys, and compares different ways to deal with such problems. Building on Manski's approach, Section 3 derives bounding intervals using the unfolding bracket questions information, accounting and not accounting for anchoring effects. Section 4 describes the HRS data used in the empirical work. Section 5 presents the empirical results. Section 6 concludes.

2 Item Non-response in Household Surveys

Item nonresponse in household surveys occurs when individuals do not provide answers to specific questions. The problem is often associated with questions on exact amounts of variables such as income, expenditure, or net worth of some type of assets. Item nonresponse may well be nonrandom, implying that the sample of (item) respondents is not representative for the population of interest. This can affect the estimates of features of the distribution of the variable that suffers from item nonresponse, such as its conditional mean or conditional quantiles given some covariates. This has long been recognized in the economics literature and is known as the selection problem.

There are several ways to handle this problem. The first is to use as many covariates (X) as possible, and to assume that, conditional on X, the response process is independent of the variable of interest. This makes it possible to use parametric or non-parametric regression techniques to impute values for non-respondents, leading to, for example, the hot-deck imputation approach. The key assumption of this approach is that item non-respondents are not systematically different from respondents with the same values of X. See Rao (1996), for an overview of hot-deck imputation, and Juster et al. (1997), for an application and the use of bracket response information in this context.

Since the seminal work by Heckman (see, for example, Heckman, 1979), the common view in many economic examples is that the assumption of random item nonresponse conditional on observed X is often unrealistic and may lead to serious selection bias. Heckman proposed to use a selectivity model instead. This is a joint model of response behavior and the variable of interest, conditional on covariates. See, for example, the survey of Vella (1998). Parametric and semi-parametric selectivity models avoid the assumption that item nonresponse is random conditional on *X*, but require alternative assumptions such as a single index assumption or independence between covariates and error terms.

A new approach to deal with nonrandom item nonresponse was introduced by Manski (1989, 1990). This approach does not make any assumptions on the response process. It uses the concept of identification up to a bounding interval. Manski (1989) shows that in the presence of item nonresponse, the sampling process alone does not fully identify most features of the conditional distribution of a variable *Y* given a vector of covariates *X*. In many cases, however, lower and upper bounds for the feature of interest (such as a value of the distribution function of *Y* given *X*) can be derived. Manski calls these bounds "worst case bounds." Manski (1994, 1995) shows how these bounds can be tightened by adding nonparametric assumptions on monotonicity of the relation between *Y* and nonresponse, or exclusion restrictions on the conditional distribution of *Y*. Manski (1990), Manski et al. (1992), and Lechner (1999) apply the bounds to analyze treatment effects.

The problem of item non-response can be reduced at the data collection level by, for example, carefully designed surveys, careful coding of responses by the interviewer, reducing question ambiguity, giving guarantees for privacy protection, giving respondents the opportunity to consult tax files, etc. A more direct method to reduce item nonresponse is to include categorical questions to obtain partial information from initial non-respondents. Using categorical questions is often motivated by the claim that certain cognitive factors, such as confidentiality or the belief that the interviewer requires an answer that reflects perfect knowledge of the amount in question, can make people more reluctant to disclose information when initially faced with an open-ended question (see, for example, Juster et al., 1997).

Two types of categorical questions are typically used. In some surveys, initial non-respondents are routed to a range card type of categorical question, where they are asked to choose the category which contains the amount (*Y*) from a given set of categories. An alternative set up for categorical questions is that of unfolding brackets. This is used in well-known US longitudinal studies such as the Panel Study of Income Dynamics (PSID), the Health and Retirement Survey (HRS), and the Asset and Health Dynamics Among the Oldest Old (AHEAD). In this type of design, those who answer 'don't know' or 'refuse' to a question on the specific amount, are asked a question such as *'is the amount $B or more?'*, with possible answers 'yes', 'no', 'don't know', and 'refuse'. They typically get two or three such questions consecutively, with changing bids *$B*: a 'yes' is followed by a larger bid and a 'no' by a smaller bid. Those who answer 'don't know' or 'refuse' on the first bid, are full non-respondents. The others are called bracket respondents. They are referred to as complete or incomplete bracket respondents, depending on whether they answer all the bracket questions presented to them by 'yes' or 'no', or end with a ‘don’t know’ or ‘refuse’ answer. The advantage of an unfolding bracket design relative to a range card type of question, is that unfolding brackets can elicit partial information on the variable of interest even if the respondent does not complete the sequence, whereas a range card question might lead to a simple ‘don’t know’ or ‘refuse’.

A problem with unfolding brackets questions is that they may suffer from anchoring effects (see Jacowitz and Kahneman, 1995, and Rabin, 1996, for non-economic examples). A psychological explanation for anchoring effects is that the bid creates a fictitious believe in the individual’s mind: faced with a question related to an unknown quantity, an individual treats the question as a problem solving situation, and the given bid is used as a cue to solve the problem. This can result in responses that are influenced by the design of the unfolding sequence. Hurd et al. (1998) formulate a parametric model which can explain the anchoring patterns observed in the data. We will discuss this model in more detail in Section 3.3. Hurd et al. (1998) estimate their model using an experimental module of the AHEAD data, in which respondents are randomly assigned to different starting bids of an unfolding sequence. They find support for their model and strong evidence of anchoring effects. Alternative parametric models for anchoring effects have been developed by Cameron and Quiggin (1994) and Herriges and Shogren (1996).

The results of Hurd et al. (1998) and others imply that answers to unfolding bracket questions may often be incorrect. They also imply that unfolding bracket questions may not give the same answers as range card questions. In the next section, Manski’s worst case bounds are extended to account for unfolding bracket questions. Various explanations for the anchoring effects are allowed, using nonparametric versions of the assumptions in existing models of anchoring.

3 Theoretical framework

3.1 Worst case bounds; no bracket respondents

First Manski’s (1989) worst case bounds is reviewed for the conditional distribution function of a variable Y, at a given $y \in \mathbb{R}$, and given $X=x \in \mathbb{R}^P$, assuming that there is neither unit nonresponse, nor item nonresponse on X. It is also assumed that reported (exact) values of Y and X are correct, and thus under- or over-reporting the value of Y is excluded. Let FR indicate that Y is (fully) observed and let NR indicate (full) non-response on Y. $F(y|x)$, the conditional distribution function of Y given $X=x$ in the complete population, can then be written as follows.

$$F(y|x) = F(y|x,FR)P(FR|x) + F(y|x,NR)P(NR|x) \tag{1}$$

The assumptions imply that $F(y|x,FR)$ is identified for all x in the support of X, and can be estimated using, for example, some nonparametric kernel regression estimator. The same holds for the conditional probabilities $P(FR|x)$ and $P(NR|x)$. If the assumption is made that, conditional on *X*, response behavior is independent of *Y*, then all expressions in the right hand side of (1) would be identified, since $F(y|x,FR)=F(y|x,NR)$. This is the assumption of exogenous selection. In general, however, response behavior can be related to *Y*, and $F(y|x,NR)$ is not identified, so that $F(y|x)$ is not identified either. Without additional assumptions, all that is known about $F(y|x,NR)$ is that it is between 0 and 1. Applying this to (1) gives,

$$F(y|x,FR)P(FR|x) \leq F(y|x) \leq F(y|x,FR)P(FR|x) + P(NR|x) \qquad \textbf{(2)}$$

These are Manski's worst case bounds for the distribution function. The difference between the upper and the lower bound is equal to $P(NR|x)$. Thus, a lower nonresponse rate leads to narrower and therefore more informative bounds. Additional assumptions can also lead to narrower bounds. Examples are monotonicity or exclusion restrictions, see Manski (1994, 1995).

3.2 Partial information from an unfolding bracket sequence

Expressions (1) and (2) do not incorporate information from categorical follow-up questions to initial non-respondents, as discussed in the previous section. In this paper, the bounds in (2) are extended to incorporate information from follow-up questions in the form of an unfolding bracket sequence. The unfolding brackets design was explained in the previous section. Let *B1* be the initial bid. This is assumed to be the same for all initial non-respondents, as is the case in the HRS data. The first bracket question is thus given by

$$\textit{Is the amount } \$B1 \textit{ or more ?} \qquad \textbf{(3)}$$

Individuals can answer 'yes', 'no', or 'don't know'.[1] Those who answer 'don't know' to this initial bid, become full non-respondents. Individuals who answer (3) with 'yes' get the same question with a new bid *B21,* with *B21>B1*. If the answer is 'no', the next bid is *B20,* with *B20<B1*. In the HRS, this second question is usually the final bracket question. In some cases a third question is asked, again with a new bid. This study is limited to the case of two bracket questions, leaving more than two questions as an extension that can be treated along the same

[1] In this study, no distinction is made between the answers 'don't know' and 'refuse'. In the text both are referred to as 'don't know'.

lines. For the sake of the exposition, the case where only one bracket question is asked is considered first.

3.3 Bounds and unfolding bracket response: One bracket question

In this case, three types of respondents can be distinguished: full respondents (*FR*), bracket respondents (*BR*) and (full) non-respondents (*NR*), so that $F(y|x)$ can be written as

$$F(y|x) = F(y|x,FR)P(FR|x) + F(y|x,BR)P(BR|x) + F(y|x,NR)P(NR|x) \quad \textbf{(4)}$$

Full respondents answer the initial question and identify $F(y|x,FR)$, as before. Non-respondents answer 'don't know' to the initial question as well as the bracket question, and, as before, all that is known about $F(y|x,NR)$ is that it is between 0 and 1. Bracket respondents report whether $Y \geq B1$ or not.

Define a variable $Q1$ by $Q1=1$ if the answer to (3) is 'yes', and 0 if the answer is 'no'. Then the bracket respondents identify $P(Q1=1|x,BR)$. For deriving the bounds, it will be useful to write this as

$$\begin{aligned} P(Q1=1|x,BR) = {} & P(Q1=1|Y<B1,x,BR)P(Y<B1|x,BR) + \\ & P(Q1=1|Y \geq B1,x,BR)P(Y \geq B1|x,BR) \end{aligned} \quad \textbf{(5)}$$

Not allowing for an Anchoring effect

If there is no anchoring effect, all bracket respondents answer (3) correctly. This implies that $P(Q1=1|Y<B1,x,BR)=0$ and $P(Q1=1|x,BR)=P(Y \geq B1|x,BR)$, and thus $P(Y<B1|x,BR)$ is identified by the data on bracket respondents. This leads to the following bounds on $F(y|x,BR)$:

$$\begin{aligned} &\textit{for} \quad y < B1 \qquad && 0 \leq F(y|BR,x) \leq P(Q1=0|BR,x) \\ &\textit{for} \quad y \geq B1 \qquad && P(Q1=0|BR,x) \leq F(y|BR,x) \leq 1 \end{aligned} \quad \textbf{(6)}$$

Combining this with the bounds on $F(y|FR,x)$ and $F(y|NR,x)$ yields, for $y < B1$,

$$\begin{aligned} & F(y|FR,x)P(FR|x) \\ & \leq F(y|x) \leq \\ & F(y|FR,x)P(FR|x) + P(Q1=0|x,BR)P(BR|x) + P(NR|x) \end{aligned} \quad \textbf{(7)}$$

For $y \geq B1$, we get

$$F(y|FR,x)P(FR|x) \; + \; P(Q1=0|x,BR)P(BR|x) \\ \leq \; F(y|x) \; \leq \\ F(y|FR,x)P(FR|x) \; + \; P(BR|x) \; + \; P(NR|x) \qquad \textbf{(8)}$$

The bounds in (7) and (8) will both be sharper than the worst case bounds in (2), as long as there are bracket respondents answering ‘yes’ as well as bracket respondents answering ‘no’.

Allowing for the Anchoring Effect

If responses to (3) are vulnerable to the anchoring effect, (6) is no longer valid, since people may give the wrong answer to (3), so that $P(Q1=1|Y<B1,x,BR)$ and $P(Q1=0|Y>B1,x,BR)$ are non-zero. In the Hurd et al. (1998) framework, $Q1$ is based upon comparing Y to $B1+å$, where $å$ is the perception error. Hurd et al. (1998) assume that $å$ is normally distributed with zero mean and is independent of Y and X. In here, the following weaker (non-parametric) assumption is used:

Assumption 1: For all (x,y) in the support of (X,Y),

$$\text{Median}\,[å|X=x,Y=y,BR]=0$$

This assumption implies that the conditional probability that an individual answers question $Q1$ correctly is at least 0.5:

$$P(Q1=1|Y<B1,x,BR) \; = \; P(å \leq Y-B1|B1-Y>0,x,BR) \; \leq \; 0.5 \\ P(Q1=1|Y \geq B1,x,BR) \; = \; P(å \leq Y-B1|B1-Y \leq 0,x,BR) \; \geq \; 0.5 \qquad \textbf{(9)}$$

Applying (9) to (5) gives:

$$P(Q1=1|x,BR) \; \leq \; 0.5P(Y<B1|x,BR) \; + \; P(Y \geq B1|x,BR) \\ P(Q1=1|x,BR) \; \geq \; 0.5P(Y \geq B1|x,BR). \qquad \textbf{(10)}$$

This implies

$$P(Y<B1|x,BR) \; \leq \; 2P(Q1=0|x,BR) \\ P(Y \geq B1|x,BR) \; \leq \; 2P(Q1=1|x,BR). \qquad \textbf{(11)}$$

In other words: the fraction with *Y* smaller than *B1* is at most twice the fraction reporting $Y<B1$; the fraction with *Y* at least *B1* is at most twice the fraction reporting $Y \geq B1$. Compared to the no-anchoring case, the factor 2 reflects the loss of information due to allowing for anchoring.

The bounds on $F(y|x,BR)$ follow immediately:

$$\begin{array}{lll} for \quad y < B1 & & 0 \leq F(y|x,BR) \leq 2P(Q1=0|x,BR) \\ for \quad y \geq B1 & & 1-2P(Q1=1|x,BR) \leq F(y|x,BR) \leq 1 \end{array} \qquad \textbf{(12)}$$

This implies either a non-trivial lower bound or a non-trivial upper bound as long as $P(Q1=1|x,BR)$ is not equal to 0.5. If $P(Q1=1|x,BR)<0.5$, the fraction of bracket respondents with a high value of *Y* is bounded. This leads to a lower bound on $F(y|BR,x)$. If on the other hand, $P(Q1=1|x,BR)>0.5$, not all bracket respondents have a low value of *Y*. This leads to an upper bound on $F(y|BR,x)$. Replacing (6) by (12) and applying this to (4) straightforwardly leads to bounds on $F(y|x)$:

for $y < B1$

$$\begin{array}{c} F(y|FR,x)P(FR|x) \\ \leq F(y|x) \leq \\ F(y|FR,x)P(FR|x) + \min[1,2P(Q1=0|x,BR)]P(BR|x) + P(NR|x) \end{array} \qquad \textbf{(13)}$$

for $y \geq B1$,

$$\begin{array}{c} F(y|FR,x)P(FR|x) + \max[0,1-2P(Q1=1|x,BR)]P(BR|x) \\ \leq F(y|x) \leq \\ F(y|x,FR)P(FR|x) + P(BR|x) + P(NR|x) \end{array} \qquad \textbf{(14)}$$

These bounds are sharper than Manski's worst case bounds in (2) (unless $P(BR|x)=0$ or $P(Q1=1|x,BR)=0.5$). On the other hand, they are wider than the bounds in (7)-(8), which were constructed under the stronger assumption of no anchoring.

Alternative Models for Anchoring

Although the Hurd et al. (1998) model can explain the anchoring phenomena in the data, it may not be the intuitively most appealing way to model anchoring. The model of Herriges and Shogren (1996) allows for anchoring in follow-up questions only, implying the no-anchoring assumption which leads to (6) and (7) for the one bracket question case. The model of Cameron and Quiggin (1994) is specifically designed for two bracket questions. It is straightforward to show that this model is equivalent to the parametric Hurd et al. (1998) model for the case of two bracket questions, although the interpretation of Herriges and Shogren is different.

The motivation of the Hurd et al. (1998) model stems from Green et al. (1998) and Jacowitz and Kahneman (1995). These studies find that, if a high anchor is used, respondents too often report that the amount exceeds the anchor. In terms of our notation this would mean $P(Q1=1)>P(Y\geq B1)$ if $B1$ is large. Jacowitz and Kahneman (1995) report that this finding is not symmetric for their case study, but could well be reversed if the amounts have a natural upper bound instead of a lower bound.

A more specific operational version of an assumption capturing this phenomenon for one bracket question would be

$$\begin{aligned} P(Q1=1|x,BR) \;\geq\; P(Y\geq B1|x,BR) \quad if \quad P(Q1=1|x,BR)\leq 0.5 \\ P(Q1=1|x,BR) \;\leq\; P(Y\geq B1|x,BR) \quad if \quad P(Q1=1|x,BR)\geq 0.5 \end{aligned} \qquad \textbf{(15)}$$

Here '*B1* is large' is specified as 'at most half of the respondents report an amount of at least *B1*.' It is straightforward to show that this assumption is stronger than A1. On the other hand, it is also easy to show that this assumption is satisfied by the parametric specification of Hurd et al. (1998). The underlying intuition is that adding noise to *B1* before comparing it to *Y*, increases the tail probabilities.

Constructing bounds on $P(Y\geq B1|x,BR)$ from (15) is straightforward. If $P(Q1=1|x,BR)\leq 0.5$, (15) leads to an upper bound; if $P(Q1=1|x,BR)\geq 0.5$, (15) leads to a lower bound. A practical problem with estimating these bounds would arise if the estimate of $P(Y\geq B1|x,BR)$ in a given sample would not be significantly different from 0.5.

Finally, a robust finding in the literature is that dichotomous questions usually shift the distribution to the right, compared to open-ended questions. This is particularly so if there is a clear lower bound but no obvious upper bound to the amounts in question. In the WTP (willingness to pay) literature where the amounts are subjective (and reflect how much respondents would be willing to pay for some public good, for example), this phenomenon is known as *yea-saying.* Green et al. (1998) also find this phenomenon for examples of estimates of objective quantities rather than WTP data. Yea-saying would imply the following asymmetric inequality between reported and true fractions.

$$P(Q1=1|x,BR) \;\geq\; P(Y\geq B1|x,BR) \qquad \textbf{(16)}$$

This immediately gives an upper bound on $P(Y \geq B1|x,BR)$. [1]

3.4 More than one unfolding bracket question

With two unfolding bracket questions, those who answer 'yes' to question (3) are given a second question with bid *B21*, where $B21>B1$, and those who answer 'no' get a second question with bid *B20*, where $B20<B1$. Again, they can answer 'yes', 'no' or 'don't know'. In this subsection the assumption is that every bracket respondent answers the second question with 'yes' or 'no', so that all bracket respondents complete the unfolding sequence. This will be generalized in the next subsection.

Not allowing for an anchoring effect

If the assumption is made that all those who answer the bracket questions do this correctly, then, for each bracket respondent, it is known which of the four categories [0, *B20*], [*B20*, *B1*],[*B1*, *B21*] and [*B21*, ∞] contains their value of *Y*. The information is the same as the information provided by a range card question with the same four categories. Bounds on $F(y|x)$ for this case are a straightforward generalization of the bounds in (7) and (8). Denoting the category containing *y* by $[L(y), U(y)]$ (for example, for $B20<y<B1$, $L(y)=B20$ and $U(y)=B1$, etc.), the bounds on $F(y|BR,x)$ are given by

$$F(L(y)|BR,x) \leq F(y|BR,x) \leq F(U(y)|BR,x) \tag{17}$$

Using (4), this leads to bounds on $F(y|x)$ in the same way as for the one bracket question case.

Allowing for the Anchoring Effect

Similar to *Q1* for the first question, define dummy variables *Q20* and *Q21* for those who answer the second bracket question on *B20* and *B21*, i.e., those who answer the first question with 'no' and 'yes', respectively. Thus $Q20=1$ if the respondent reports that the amount is at least *B20*, etc. Apart from the probability $P(Q1=1|x,BR)$ discussed in the one bracket question case, two other

[1] A common test on yea-saying is to compare the estimated distribution for the open-ended respondents with the (upper and lower bound of the) distribution function for the bracket respondents. In absence of selectivity effects, yea-saying would imply that the latter distribution is to the right of the former. In the present framework, however, selectivity effects are not excluded, and the test is not a test on yea-saying only.

probabilities are now identified by the data: $P(Q20=1|Q1=0,x,BR)$ and $P(Q21=1|Q1=1,x,BR)$. In order to derive the bounds, Assumption 1 needs to be generalized. Again, the starting point is the Hurd et al. (1998) framework. This model assumes that the answers *Q1, Q20* and *Q21* to the three bracket questions are based upon comparing *Y* with $B1+å_1$, with $B20+å_{2,0}$, and with $B21+å_{2,1}$, respectively. The errors $å_1$, $å_{2,0}$ and $å_{2,1}$ are assumed to be independent of each other and of *X* and *Y*, and are normally distributed with zero means. The anchoring effects in the data are captured if $å_{2,0}$ and $å_{2,1}$ have smaller variances than $å_1$. The following extension of Assumption 1 is a non-parametric, less restrictive, version of these assumptions.

Assumption 2: For all *(x,y)* in the support of *(X,Y):*

$$\text{Median}[å_1|Y=y,X=x,BR]=0;$$
$$\text{Median}[å_{2,0}|Y=y,X=x,BR,Q1=0]=0;$$
$$\text{Median}[å_{2,1}|Y=y,X=x,BR,Q1=1]=0.$$

A stronger version of this assumption that may look more natural is the assumption that each of the three error terms has median zero, is independent of being a bracket respondent or not, and is independent of *Y,X*, and the other two error terms.

Assumption 2 is weaker than the assumptions of Hurd et al (1998). It implies that each bracket question is answered correctly with probability at least 0.5:

$$\begin{array}{ll} P(Q1=1|Y<B1,x,BR) \leq 0.5; & P(Q1=1|Y\geq B1,x,BR) \geq 0.5 \\ P(Q20=1|Y<B20,Q1=0,x,BR) \leq 0.5; & P(Q20=1|Y\geq B20,Q1=0,x,BR) \geq 0.5 \\ P(Q21=1|Y<B21,Q1=1,x,BR) \leq 0.5; & P(Q21=1|Y\geq B21,Q1=1,x,BR) \geq 0.5 \end{array} \quad \textbf{(18)}$$

In addition to (11), the implication of (18) for those who answer ‘no’ to the first question, is that

$$\begin{array}{l} P(Y<B20|Q1=0,x,BR) \leq 2P(Q20=0|Q1=0,x,BR) \\ P(Y\geq B20|Q1=0,x,BR) \leq 2P(Q20=1|Q1=0,x,BR) \end{array} \quad \textbf{(19)}$$

whereas for those who answer ‘yes’ to the first question, the implication is that

$$P(Y<B21|Q1=1,x,BR) \;\leq\; 2P(Q21=0|Q1=1,x,BR)$$
$$P(Y\geq B21|Q1=1,x,BR) \;\leq\; 2P(Q21=1|Q1=1,x,BR) \qquad \textbf{(20)}$$

The bounds in (11), (19) and (20) can be used to derive bounds on the distribution function for bracket respondents.[2] Since there are many different cases which do not give much insight, the complete overview of the bounds with their derivations is given in the appendix. To illustrate what the results look like, only one example is presented here, the upper bound on $P(Y<B20|x,BR)$:

$$P(Y\leq B20|x,BR) \;\; min \;\; [1,\; 2P(Q20=0|Q1=0,x,BR)]\min[1,\; 2P(Q1=0|x,BR] \qquad \textbf{(21)}$$

If many people say their income exceeds *B1* (that is, if $P(Q1=0|x,BR)$ is low), this limits the number of people whose income can be lower than *B20*. Moreover, if of those who report that their income is lower than *B1*, the majority report it is higher than *B20*, this also limits the number of people with income below *B20*.

Alternative Models for Anchoring

The previous subsection already discussed some alternative assumptions on anchoring for the one bracket question case. The assumptions following the arguments of Jacowitz and Kahneman (1995) basically treat every bracket question separately. In addition to (15), they are:

$$P(Q2k=1|x,BR,Q1=k) \;\geq\; P(Y\geq B1|x,BR,Q1=k) \quad if \quad P(Q2k=1|x,BR,Q1=k) \;\leq\; 0.5$$
$$P(Q2k=1|x,BR,Q1=k) \;\leq\; P(Y\geq B1|x,BR,Q1=k) \quad if \quad P(Q2k=1|x,BR,Q1=k) \;\geq\; 0.5 \qquad \textbf{(22)}$$

for k=0,1. These assumptions can be used to derive bounds on the distribution function for bracket respondents in a similar way as for the Hurd et al. model. The resulting expressions depend on whether the bids are ‘small’ or ‘large’. Appendix A presents the formulas for the case which is relevant for the data used in the empirical section.

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[2] To be precise, a monotonicity assumption (also satisfied by the Hurd et al. model) is used in addition (MON in the appendix).

An intuitively appealing way of allowing for anchoring in the second question is provided by Herriges and Shogren (1996). They formulate a simple model which explicitly allows for an effect of the first bid on the respondent's subjective opinion on the amount *Y*. The essential feature of their model is that although there is no anchoring effect in the first bracket question, the first bid *B1* serves as an anchor for the second bid *B2* (which equals either *B20* or *B21*) so that in the second bracket question the respondent does not compare *B2* to *Y*, but to $Y^* = (1-g) + gB1$. This clearly reflects the intuition behind anchoring: the respondent is uncertain about the true value of *Y*. The bid *B1* is informative about *Y*, and the respondent's new estimate *Y** of *Y* will be drawn towards *B1*. Herriges and Shogren (1996) assume that *g* is a fixed parameter, but also discuss an extension in which *g* can vary with *B1*. They apply their model to willingness to pay data on how much people are prepared to pay for water quality improvement, and find an estimate of *g* of 0.36, with standard error 0.14. In another application, O'Connor et al. (1999) find a similar significantly positive value of *g*.

The Herriges and Shogren (1996) model offers an alternative explanation for the shift in the estimated distribution based upon unfolding bracket questions due to the order of the bids, the main finding in Hurd et al. (1998). On the other hand, the Herriges and Shogren model cannot explain the main finding of Jacowitz and Kahneman (1995), since that finding is related to the first bid, for which there is no anchoring according to the Herriges and Shogren (1996) model.

A natural way to relax the Herriges and Shogren (1996) assumptions is to replace them by the following nonparametric assumptions:

$$\begin{aligned} P(Q1=1|x,BR) &= P(Y \geq B1|x,BR) \\ P(Q20=1|x,BR,Q1=0) &\geq P(Y \geq B20|x,BR,Q1=0) \\ P(Q21=1|x,BR,Q1=1) &\leq P(Y \geq B21|x,BR,Q1=1) \end{aligned} \qquad \textbf{(23)}$$

The first condition states that there is no anchoring in first question, the other two state that anchoring is towards *B1* in the second question. These assumptions can be used to derive bounds on the distribution function for bracket respondents in the same way as in the other models. The results are presented in Appendix A.

3.5 Complete and incomplete bracket respondents

As the previous subsection, the case where at most two bracket questions are asked is considered. Until now it was assumed that all bracket respondents completed the unfolding bracket sequence. In practice, however, some of them answer 'don't know' to the second bracket question. Thus two types of bracket respondents can be distinguished: those who answer both questions with 'yes' or 'no' (*CBR*, complete bracket respondents), and those who only answer one question with 'yes' or 'no' (*IBR*, incomplete bracket respondents). No assumptions are made on the relation between response behavior and the value of *Y*, so the possibility that incomplete bracket respondents are a selective subsample of all bracket respondents is allowed for.

The conditional distribution function for bracket respondents can now be written as follows.

$$F(y|BR,x) = F(y|CBR,x)P(CBR|BR,x) + F(y|IBR,x)P(IBR|BR,x) \quad \textbf{(24)}$$

The probabilities $P(CBR|BR,x)$ and $P(IBR|BR,x)$ are both identified, since it is observed whether bracket respondents are complete or incomplete bracket respondents. Bounds on $F(y|CBR,x)$ can be derived as in Section 3.4, using complete bracket respondents only. Bounds on $F(y|IBR,x)$ can be derived as in Section 3.3, using incomplete bracket respondents only. Combining these and plugging them into (24) leads to bounds on $F(y|BR,x)$. As before, two sets of bounds can be derived, allowing or not allowing for anchoring. The bounds on $F(y|BR)$ can be combined with $F(y|FR,x)$ and bounds on $F(y|NR,x)$ in the same way as in (13)-(15) , and thus yield bounds on $F(y|x)$.

3.6 Bounds on Quantiles

Distributions for variables like income, savings, etc., are often described in terms of (conditional) quantiles. For $\alpha \in [0,1]$, the α-quantile of the conditional distribution of *Y* given $X=x$, is defined as

$$q(\alpha,x) = inf\ \{y:F(y|x) \geq \alpha\} \quad \textbf{(25)}$$

For $\alpha > 1$, we set $q(\alpha,x) = \infty$, and for $\alpha < 0$, $q(\alpha,x) = -\infty$. Following Manski (1994), bounds on these quantiles can be derived by 'inverting' the bounds on the distribution function. All the bounds in Sections 3.1-3.5 can be written as

$$lb(y,x) \leq F(y|x) \leq ub(y,x) \quad \textbf{(26)}$$

for appropriate choices of *lb(y,x)* and *ub(y,x)*, all of them non-decreasing functions of *y*. Inverting this gives the following bounds on the quantiles:

$$inf\ \{y:\ lb(y,x) \geq \acute{a}\} \ \geq\ inf\ \{y:\ F(y|x) \geq \acute{a}\} \ \geq\ inf\ \{y:\ ub(y,x) \geq \acute{a}\} \qquad \textbf{(27)}$$

Plugging in the bounds derived in the previous subsections, yields bounds on the conditional quantiles of *Y*. This is easily illustrated using a graph of the distribution function, with *y* along the horizontal axis and *F(y|x)* along the vertical axis. The bounds on the distribution function squeeze *F(y|x)* in between two curves; the vertical distance between these two curves is the width between the bounds (at each given value of $y \in Y$). Reading the same graph horizontally gives, for a given probability value $\acute{a} \in [0,1]$, a lower and an upper bound on the *á*-quantile.

4 Data

The data used in the empirical section comes from the 1996 wave of the Health and Retirement Survey (HRS). This is a longitudinal study conducted by the University of Michigan on behalf of the US National Institute of Aging. It focuses mainly on aspects of health, retirement and economic status of US citizens born between 1931 and 1941. For this purpose, the study collects individual and household information from a representative sample of this cohort. The data is collected every two years, with the first wave conducted in the Summer of 1992.

Initially the panel consisted of approximately 7,600 households. The respondents are the household representatives that satisfy the age criteria and their partners, regardless of age (second household respondents). This leads to approximately 12,600 individual respondents in the first wave of the panel. Each respondent answers individually to questions on health and retirement issues. Household representatives also answer questions on past and current income and pension plans (including those of their partner), as well as questions at the household level, e.g. on housing conditions, household assets and family structure. If health problems prevent the household representative from answering these questions, someone else (e.g. the spouse) will answer on their behalf. All follow up interviews are conducted over the phone, unless the household has no phone, or health reasons prevent either the household representative or the spouse answering over the phone, in which case the interviewer will visit the household. If respondents die, they are replaced by a remaining household member. This reduces attrition in the panel at the household level.

The 1996 wave has data from 6,739 households, covering 10,887 individuals. In 4,148 of these households, two respondents gave interviews. The remaining 2,591 are single respondent households. To get some insight in the nature of the data, Table 1 shows sample statistics for some background variables. The first column refers to the full sample. The second and third columns refer to the sub-samples of household representatives and second household respondents. The statistics show that 51% of the household representatives are women, and 62% of second household respondents (usually the spouse) are women. There is little difference between educational achievement of household representatives and second household respondents.

Table 1: Means (and standard deviation) and Percentages (with standard errors) for background variables; complete sample

	All Units	Household Representatives	Second Household Respondents
Number of Observations	**10,887**	**6,739**	**4,148**
Age	59.6 (5.62)	60.7 (5.07)	58.6 (6.41)
Percentage males	45 (0.5)	49 (0.6)	38 (0.8)
Education [1]	2.32 (1.02)	2.36 (1.03)	2.25 (0.98)
Percentage home owners	-	79 (0.5)	-
Percentage whites	71 (0.4)	69 (0.6)	76 (0.7)
Percentage hispanics	9 (0.3)	8 (0.3)	11 (0.5)
Percentage black [2]	16 (0.4)	19 (0.5)	9 (0.4)
Percentage other Race	4 (0.2)	4 (0.3)	4 (0.3)
Percentage working	62 (0.5)	62 (0.6)	64 (0.7)
Working for wage/salary	47 (0.5)	46 (0.6)	50 (0.8)
Self-employed	0.09 (0.003)	0.08 (0.003)	0.10 (0.005)
Both working for wage/salary & self-emp.	0.06 (0.002)	0.08 (0.003)	0.04 (0.003)

Notes:

1. Education: educational achievement on a scale of 1 to 4; 1: has completed primary education (up to the 10th grade in the USA education system), 2: has completed high school (up to the 12th grade); 3: some form of college or post-high school education; 4: has completed at least a first degree at university level.

2 Those who describe themselves as black African-American.

The shares of Whites, Blacks and Hispanics reflect the ethnic composition of the cohort represented in the sample. About 62% of the respondents participate in the labor market, most of them are employees. Approximately 80% of the households in the sample are home owners.

The 1996 wave of the HRS panel groups all variables in 11 subsets and a supplement that consists of experimental modules (mostly to check the consistency of answers to previous questions). In the subset named 'Assets and Income', the household representatives provide information about their own incomes, their partner's incomes, household savings, and various other types of net wealth. The bounds derived in Section 3 are applied to the variable 'wages and salaries of the household representative'. This variable shows a significant percentage of initial non-respondents. These are routed to an unfolding bracket sequence where they can disclose partial information on the missing variable.

Wages and salaries of the household representative

All household representatives are asked to provide information on employment status and earned incomes for themselves and their partners. Initially, each household representative is asked if he or she worked for pay during the last calendar year. To this question, 4,145 individuals answered 'yes', 2,097 individuals answered 'no', and the remaining 497 answered 'don't know' or 'refuse'. Each of the 4,145 who answered 'yes' were asked if their earnings during the last calendar year came from self-employment, wages and salaries, or a combination of these two sources: 3,608 individuals declared that all or some of their earnings came from wages and salaries. These individuals are asked the following question.

'About how much wages and salary income did you receive during the last calendar year?'

1 - 'any amount' (in US dollars)
'Don't know'
'Refuse'

A total of 3,160 individuals answered the above question with an exact amount in US dollars, ranging from \$ 0,00 to \$350,000, with a mean of \$29,430 and standard deviation \$26,430. The median was \$25,000. The remaining 448 individuals answered 'don't know' or 'refuse', implying a 12.4% initial nonresponse rate. The latter group was routed to a sequence of unfolding bracket questions as formulated in (3), with starting bid $B1$=\$25,000. The first question thus was:

'Is the amount \$25,000 or more?'

The possible answers were 'yes', 'no', 'don't know,' or 'refuse'. At this initial stage of the unfolding sequence, 119 individuals answered 'don't know' or 'refuse'. Thus the full non-response rate is 3.3%. The remaining 329 individuals form the sample of bracket respondents.

For this variable, the unfolding sequence consists of two questions. Those who answered 'yes' to the initial bid of $25,000 were routed to a second question with bid *B21*=$50,000. Those who answered 'no' were routed to a question with bid *B20*=$5,000. In each case, the question is the same as that given in (3) - only the bid changes. At the second question of the unfolding sequence, individuals can again answer 'don't know' or 'refuse'. They then become incomplete bracket respondents (IBR). For the earnings variable considered in here, 320 individuals completed the sequence of unfolding brackets (CBR), while the other 9 bracket respondents are incomplete bracket respondents.

Table 2 shows some statistics for the sample of individuals with nonzero wages and salaries, partitioned by response behavior. Comparing it with Table 1 shows that the individuals who received wages and salaries are, on average, somewhat younger and less often own their home. The subsample of complete bracket respondents contains a larger percentage of females than the other samples. Likewise, complete bracket respondents have lower educational achievement, are less likely to own their home, and are less often white. The statistics of the incomplete bracket respondents differ substantially from those of the other groups, but this is based upon very few observations.

Table 2: Means (and standard deviations) and Percentages (with standard errors) for some background variables: Sample of respondents who received wages and salaries in the past calendar year

	All employed with wages	Full Respondents (FR)	Full Non-respondents (NR)
Number of Observations	**3602**	**3160**	**113**
Average age	58.6 (4.7)	58.6 (4.7)	59 (4.9)
Percentage Males	50 (0.8)	52 (0.9)	0.45 (4.7)
Education [1]	2.52 (1.01)	2.6 (1.03)	2.6 (0.99)
% Home owners	73(0.7)	74(0.8)	83(3.5)
% White	72 (0.7)	75 (0.8)	72 (4.2)
% Hispanics	8 (0.5)	7 (0.5)	5 (2.1)
% Black [2]	18 (0.6)	16 (0.7)	21 (3.8)
% Other races	2 (0.2)	2 (0.3)	3 (1.6)

Bracket Respondents (BR)

	Complete Bracket Respondents (CBR)	Incomplete Bracket Respondents (IBR)
Number of Observations	**320**	**9**
Average age	58.8 (4.7)	55.7 (3.2)
Percentage Males	38 (2.7)	0.78 (0.14)

Number of Observations	**320**	**9**
Education [1]	2.2 (1.02)	3.1 (1.01)
% Home owners	65(2.7)	89(10.0)
% White	58 (2.8)	78 (14)
% Hispanics	9 (1.6)	0(0)
% Black [2]	32 (2.6)	12 (11)
% Other races	2 (0.8)	10 (10)

Notes: See Table 1

5 Estimates of the Bounds

This section applies the upper and lower bounds derived in Section 3 to earnings of the household representative, as described in Section 4. First, the full sample of respondents is used, not conditioning on covariates. In Section 2, the bounds are estimated separately for high and low educated respondents, and the results are used to determine whether significant differences in the quantiles for the two education levels can be detected. Since conditioning is only with respect to discrete variables, there is no need to use non-parametric smoothing techniques, and the estimates can be computed as functions of (sub-)sample fractions satisfying a given condition.

The width between point estimates of upper and lower bounds reflects the uncertainty due to item non-response. The bounds are illustrated together with estimated confidence bands, to measure uncertainty due to sampling error. For all sets of bounds, these confidence bands are estimated using a bootstrap method, based on 500 (re-)samples drawn with replacement from the original data. The lower and upper bounds are estimated 500 times, and the confidence bands are formed by the 2.5% and 97.5% percentiles in these 500 estimates. This results in two-sided 95% confidence bands for both the upper and lower bound. The figures report the lower confidence band for the lower bound and the upper confidence band for the upper bound. The (vertical) distance between these reflects both the uncertainty due to sampling error and the uncertainty due to item non-response.

5.1 Bounds for all education levels

If item nonresponse is completely random, the full respondents are a representative sample, and the quantiles in the sample of full respondents are consistent estimates of the population quantiles. These estimates are shown in Figure 1a, for all quantiles between 0 and 1.[3] The solid curve connects the point estimates, the dashed curves give point-wise 95% confidence bands for each

[3] The numbering of the figures is designed to simplify comparisons. The codes identify the subpopulation and the underlying anchoring assumptions.

quantile. Bracket respondents and non-respondents are discarded. Table 3 provides basically the same information as Figure 1a. It gives the point estimates and the confidence intervals for some selected quantiles for the full respondents, but now in earnings levels rather than (natural) logs.

Figure 1b shows the estimates of Manski's (1995) worst case bounds, not using the bracket response information. Bracket respondents are treated as non-respondents, and the relevant nonresponse rate in this case is 12.4%. The solid curves are the estimated upper and lower bounds, and the dashed curves are the estimated confidence bands. The horizontal distance between the upper and lower bound equals 0.124, the initial percentage of item nonresponse. To make a comparison possible, the confidence bands for the full respondents quantiles already depicted in Figure 1 are also included. Obviously, these are contained in the worst case bounds, because the worst case bounds allow for the possibility that non-response is completely random. The uncertainty due to non-response appears to be much larger than the uncertainty due to finite sample errors.

Table 4 shows point estimates and confidence intervals for a selection of quantiles corresponding to the worst case bounds in Figure 1b. For example, with at least 95% confidence, the median of wages and salaries is between \$19,500 and \$29,900. Table 4 together with Figure 1a shows that, due to the initial percentage of item nonresponse, the width between upper and lower bound is quite large. This will make it hard to draw meaningful economic conclusions. On the other hand, the width between the curves in Figure 1a is very small, but this comes at the cost of the strong assumption of random nonresponse.

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Table 3: Selected quantiles for the full response sample (n=3,160) (cf. Figure 1a).

Quantiles	Lower estimated confidence interval	Point wise estimated quantile	Upper estimated confidence interval	Standard error for the point estimate
25th Percentile	$11,900	$11,900	$12,800	$352
40th Percentile	$18,500	$19,500	$19,500	$373
50th Percentile	$23,900	$25,000	$25,000	$361
60th Percentile	$28,900	$29,900	$29,900	$346
75th Percentile	$39,000	$39,400	$41,500	$389
90th Percentile	$55,400	$58,000	$59,000	$1,098

Table 4: Estimated bounds and confidence intervals on Respondent's wages and salaries (in US$).Worst case bounds without bracket information (cf. Figure 1b)

Quantiles	Confidence interval (Lower)	Lower bound	Upper bound	Confidence interval (Upper)
25th Percentile	$5,800	$7,700	$13,700	$14,700
40th Percentile	$13,700	$14,700	$22,500	$24,500
50th Percentile	$19,500	$20,800	$27,900	$29,900
60th Percentile	$25,000	$26,000	$34,600	$37,000
75th Percentile	$35,600	$36,900	$50,000	$55,000
90th Percentile	$51,000	$55,000	$350,000	max

The next step is to estimate the extended version of the bounds incorporating the information provided by the bracket respondents. Table 5 summarizes the information provided by these 329 respondents. This table shows that most individuals in this population are complete bracket respondents, only 9 of them are incomplete bracket respondents.

Table 5: Information provided by bracket respondents

Group	Bid 1: B1	answer	Bid 2: B21/B20	answer	Resulting bracket bounds	Number
				Yes	**$50,000 — max**	30
		Yes	> $50,000 ?	No	**$25,000 — $50,000**	86
CBR	>$25,000 ?					
		No	> $ 5,000 ?	Yes	**$5,000 — $25,000**	170
				No	**$0 — $5,000**	34
		Yes	> $50,000 ?	DK, RF.	**> $25,000**	9

				Yes	**$50,000 — max**	30
		Yes	> $50,000 ?	No	**$25,000 — $50,000**	86
IBR	>$25,000 ?					
		No	> $ 5,000 ?	DK, RF.	**< $25,000**	0

Bounds accounting for bracket information can allow for anchoring in different ways. To illustrate how the assumptions on anchoring affect the bounds, estimates of the bounds for the population of bracket respondents only are presented first. Figure 2 is based on the assumption of no anchoring (A0 in the figures). Figures 3A1 to 3A3 allow for the three types of anchoring discussed in Section 3, following Hurd et al. (A1 in the figures, (11), (19) and (20) in Section 3), Jacowitz and Kahneman (A2 in the figures, (15) and (22) in Section 3) or Herriges and Shogren (A3 in the figure, (23) in Section 3). In each figure, the confidence bands for the full respondents are also included. The "no anchoring" assumption is obviously stronger than all three anchoring assumptions, and, accordingly, leads to the narrowest bounds for the bracket respondents. Under the no anchoring assumption, the distribution function for the complete bracket respondents is exactly identified at the three bids $5,000, $25,000 and $50,000. Due to the presence of some incomplete bracket respondents, however, the upper and lower point estimates at $50,000 are different.

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Comparing the bounds for bracket respondents in Figure 2 with the full respondents curves suggests that equality of the distribution functions for full respondents and bracket respondents can be rejected at $25,000 but not at $5,000 or $50,000. This is confirmed by a formal (point-wise) tests, see Table 6. The hypothesis that the two distributions are the same, would be valid if there was no anchoring and if the distinction between full respondents and bracket respondents was random. Rejecting this hypothesis at $25,000 thus suggests that at least one of these is violated. The fact that the bracket respondents more often report an income below $25,000 than full respondents, suggests that rejecting the null is not due to "yea-saying" (see Section 3). It could mean, for example, that workers with lower earnings tend not to know their exact income level and therefore answer the bracket questions only.

Table 6: Tests for differences between the distribution functions of full and bracket respondents.

Upper Bounds on Values of the distribution function

	Formulation	**Estimate (s.e)**
FR	P(Y < 5000|FR)	0.101 (0.005)
	P(Y < 25,000|FR)	0.529 (0.0083)
	P(Y < 50,000|FR)	0.853 (0.006)
BR,A0	P(Y < 5000|BR, A0) = P(Q1=0|BR)P(Q20=0|Q1=0)	0.104 (0.017)
	P(Y < 25,000|BR, A0) = P(Q1=0|BR)	0.62 (0.027)
	P(Y < 50,000|BR, A0) = 1 - P(Q1=1|BR)P(Q21=1|Q1=1)	0.901 (0.016)

The Test: This is based on normalizing the difference between FR and A0, A1, A2 and A3 separately. The normalization consists of the square root of the sum of the standard errors of the proportions. The standard error of a proportion is calculated as √[P(1-P)/n], where P is the point estimate of the proportion and n is the number of observations used to estimate it.

HO	Formulation	TEST STATISTIC
FR vs. A0	P(Y < 5000|FR) = P(Q1=0|BR)P(Q20=0|Q1=0)	-0.113
	P(Y < 25,000|FR) = P(Q1=0|BR)	-3.22
	P(Y < 50,000|FR) = 1 - P(Q1=1|BR)P(Q21=1|Q1=1)	-2.81

Allowing for anchoring widens the bounds. Under the Hurd et al. (1998) assumptions, at each value of earnings, the bracket response data provide some additional information. Either the lower bound is more than zero, or the upper bound is less than one, or both. Under the Jacowitz and Kahneman assumptions, the bracket response data do not say anything about F(y|BR) for y between \$5,000 and \$25,000. In general, it is clear that the three different assumptions on anchoring are non-nested: non of them is uniformly more informative than any of the others.

Figures 4 and Figures A1 to Figure A3 combine the bounds for bracket respondents with the information for full respondents, and show the bounds on the quantiles for all respondents in the population.

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As expected, the bounds under no anchoring are narrower than the bounds allowing for anchoring, and all bounds allowing for anchoring are narrower than the worst case bounds in Figure 1b. More precise information on selected quantiles is given in Table 7. For example, under the assumption of random nonresponse, the 95% confidence interval for the 40th earnings percentile has width \$ 1,000 (Table 3), so the median is rather precisely determined. Allowing for nonrandom nonresponse while ignoring the bracket information, the precision deteriorates enormously, and the interval has a width of \$ 10,800 (Table 4). Allowing for nonrandom nonresponse and using the bracket information gives a precision in between these two extremes: \$ 4,900 if no anchoring effects are allowed for; \$ 9,200, \$ 9,400 and \$ 6,000 allowing for the three types of anchoring (Table 7). The "no anchoring" precision is particularly large for the median since the sample median for full-respondents is close to one of the bids (\$ 25,000), where, under A0, the distribution function for bracket respondents is exactly identified.

Table 7: Upper and lower bounds (based on 95% confidence intervals) for quantiles of respondents wages and salaries, not allowing for an anchoring effect (c.f. Figure 4) and allowing for different types of anchoring (c.f. Figures A1 to A3).

Quantiles (Point estimates)	**No anchoring effect (A0)**	**Assumption of Anchoring (A1)**	**Assumption of Anchoring (A2)**	**Assumption of Anchoring (A3)**

25th Percentile				
Lower Bound:	$9,800	$6,800	$6,800	$8,000
Upper Bound:	$13,700	$14,700	$14,700	$13,700
Difference :	$3,900	$7,900	$7,900	$5,700
40th Percentile				
Lower Bound:	$17,900	$14,700	$14,500	$16,900
Upper Bound:	$22,800	$23,900	$23,900	$22,800
Difference :	$4,900	$9,200	$9,400	$6,000
50th Percentile				
Lower Bound:	$23,900	$19,500	$19,500	$21,900
Upper Bound:	$25,000	$27,900	$25,000	$25,000
Difference :	$1,100	$8,400	$5,500	$3,100
60th Percentile				
Lower Bound:	$27,900	$25,000	$25,000	$26,900
Upper Bound:	$31,500	$34,600	$31,500	$31,500
Difference :	$3,600	$9,600	$6,500	$4,600
75th Percentile				
Lower Bound:	$39,400	$35,600	$35,600	$37,900
Upper Bound:	$45,000	$49,700	$46,800	$48,000
Difference :	$5,600	$14,100	$11,200	$8,100
80th Percentile				
Lower Bound:	$44,500	$39,400	$39,400	$43,500
Upper Bound:	$50,000	$52,500	$50,000	$51,000
Difference :	$5,500	$13,100	$10,600	$7,500
90th Percentile				
Lower Bound:	$59,000	$53,900	$53,900	$53,900
Upper Bound:	$69,000	$94,000	$78,000	$84,000
Difference :	$10,000	$40,100	$24,000	$30,100

5.2 Comparing Earnings of the Higher and Lower Educated

Table 8 presents some details on the response behavior of the lower educated (at most high school; levels 1 and 2 in Table 1) and higher educated (more than gigh school; levels 3 and 4 in Table 2) separately. The latter have a somewhat lower initial nonresponse rate than the former, but the difference is small. Mean and median incomes of full respondents are clearly higher for the higher educated than for the lower educated.

Table 8: Sample statistics and response behavior by level of education of household respondent

	All	Low education	High education

Observations in complete sample	6,739	4,110	2,629
Observations with wages and salaries	3,602	1,978	1,624
Number of full respondents	3,160 (88%)	1,713 (86.6%)	1,447 (89.1%)
Mean (std. Deviation)	$29,430 ($26,430)	$22,813 ($18,080)	$38,298 ($31,765)
Median	$25,000	$19,000	$33,000
Number of initial non-respondents	442 (12.3%)	265 (13.4%)	177 (10.9%)
Number of bracket respondents	329 (9.1%)	212 (10.7%)	117 (7.2%)
Number of full non respondents	113 (3.1%)	53 (2.3%)	60 (3.7%)

Figures 6a and 7a show the confidence intervals of all quantiles for the full respondents, for low educated and high educated respondents, respectively. These quantiles are valid estimates for the full populations of low and high educated respondents under the assumption that, conditional on education level, non-response is random, i.e., not related to the earnings level. This assumption, though again quite strong, is different from the random nonresponse assumption underlying Figure 1a, since the estimates are now conditional on education level. Tables 9 and 10 present some more details for selected quantiles for high and low educated respondents, respectively. Comparing related quantiles between these two tables shows that, under the random nonresponse assumption, all considered quantiles (from 0.20 to 0.90) differ significantly between the higher and lower educated. The main issue in the remainder will be whether this conclusion can still be drawn if nonrandom nonresponse is allowed for.

Figure 6b (low educated) and Figure 7b (high educated) present 95% confidence bands for the worst case bounds allowing for nonrandom nonresponse and not using the bracket information.

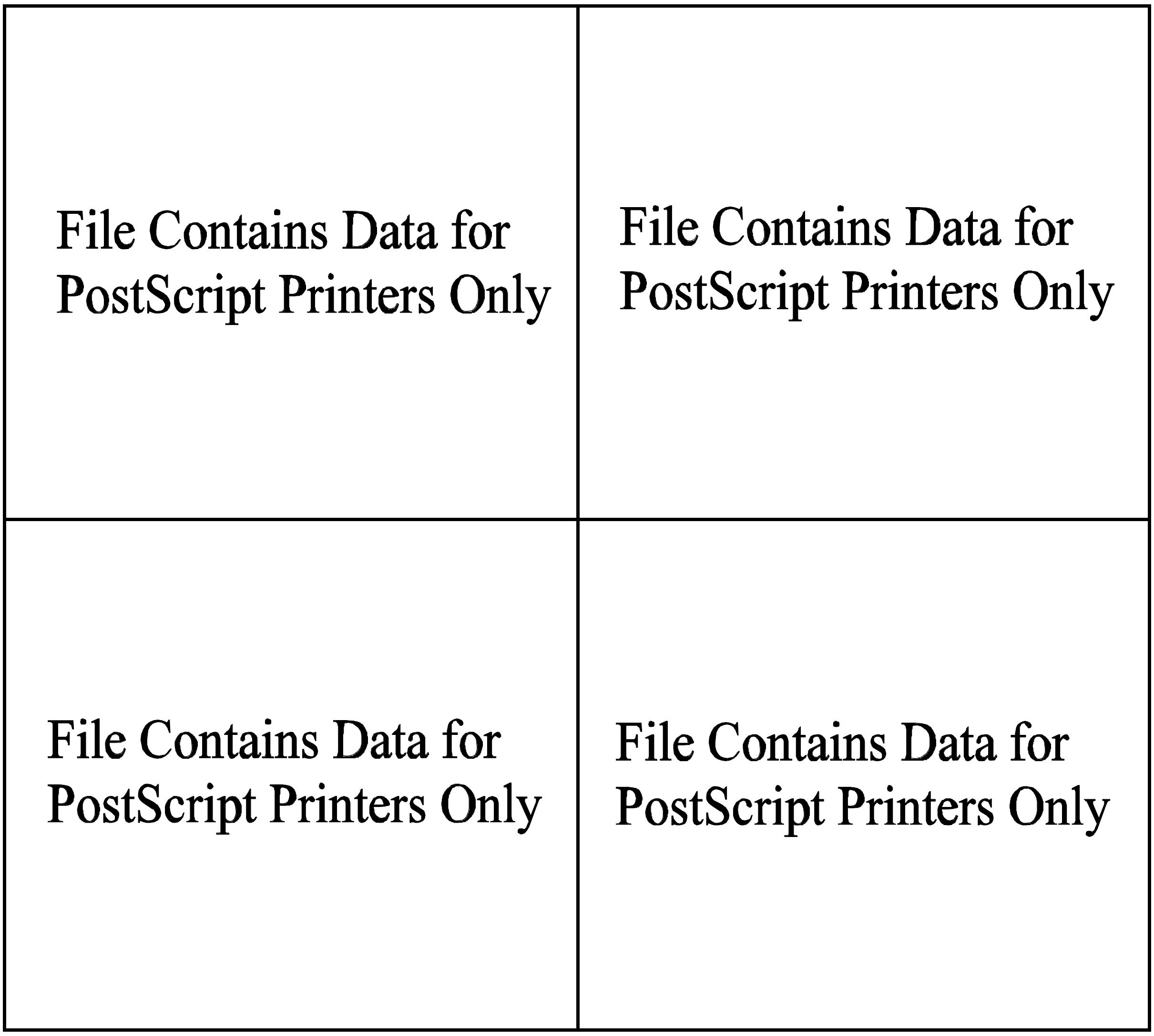


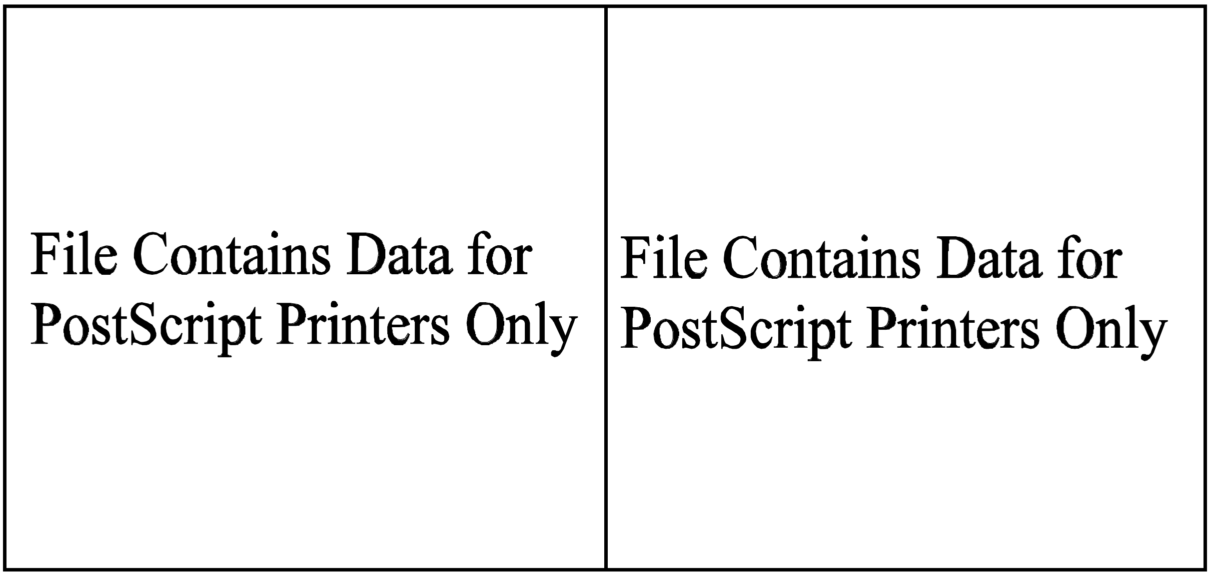
Table 9: Quantiles of the full response sample with low education level (n=1,979) (cf. Figure 6(a)).

Quantiles	Lower confidence band	Point estimate	Upper confidence band	Standard error for the point estimate
25th Percentile	\$9,800	\$9,800	\$10,800	\$439
40th Percentile	\$14,400	\$14,700	\$15,800	\$542
50th Percentile	\$17,900	\$18,700	\$19,500	\$503
60th Percentile	\$21,900	\$23,400	\$24,600	\$620
75th Percentile	\$29,400	\$29,900	\$31,500	\$537
90th Percentile	\$41,500	\$44,500	\$48,000	\$1,279

Table 10: Quantiles using the full response sample with high education level (n=1,623) (cf. Figure 7(a)).

Quantiles	Lower confidence band	Point estimate	Upper confidence band	Standard error for the point estimate
25th Percentile	$15,800	$17,900	$19,500	$790
40th Percentile	$25,900	$27,900	$28,900	$784
50th Percentile	$31,500	$32,500	$34,600	$923
60th Percentile	$37,800	$39,400	$39,700	$601
75th Percentile	$49,500	$49,700	$52,000	$623
90th Percentile	$66,000	$69,000	$74,200	$1,660

Figure 8a compares the 95% confidence bands on the distributions among high and low education level full respondents. Figure 8b compares the worst case bounds for the two populations, again using 95% confidence bands.



Under the strong assumption of random nonresponse, Figure 8a suggests that almost all the earnings quantiles are higher for the high educated than for the low educated. Without imposing this assumption, the worst case bounds in Figure 8b show that the null of equal quantiles is rejected only for the quantiles in the range from about 0.3 to about 0.8.More precisely, Table 11 shows that it is rejected for the 0.40, 0.50, 0.60 and 0.75 quantiles, but not for the 0.20, 0.25, 0.30, 0.80 and 0.90 quantiles.[4] This again illustrates that item nonresponse particularly reduces the information on the quantiles in the tails, where the distribution function is rather flat.

Table 11: Differences between quantiles for the high and low educated using worst case bounds without bracket respondents (cf. Figures 6(b), 7(b) and 8(b)).

	Low Education Standard error	**Low Education Point estimate**	**High Education Point estimate**	**High Education Standard error**	**Test Statistic (one sided 5% critical value: 1.65)**
20th Percentile	\$337	\$9,800	\$6,800	\$761	-3.6
25th Percentile	\$408	\$11,900	\$9,800	\$1,050	-1.87
30th Percentile	\$505	\$13,000	\$14,700	\$960	1.57
40th Percentile	\$390	\$17,900	\$23,900	\$1,138	4.99
50th Percentile	\$572	\$22,500	\$29,900	\$707	8.14
60th Percentile	\$863	\$27,400	\$35,600	\$885	6.64
75th Percentile	\$1,189	\$39,400	\$48,600	\$863	6.26
80th Percentile	\$1,732	\$49,700	\$52,500	\$1,692	1.17
90th Percentile	\$1,970	\$350,000	\$66,000	\$1,465	-115.68

Information on bracket response of high and low educated respondents is included in Tables 12 and 13. The low educated had a higher initial non-response rate, but are quite often willing to answer the bracket questions, so that their full non-response rate is lower than that of the higher educated. Of the bracket respondents, the high educated much more often report that their income exceeds the first bid than the lower educated. This suggests that using the bracket respondents may increase the power of the tests on equality of some of the quantiles. Appendix B shows figures similar to Figures 2 and 3A1-3A3 but separately for the two levels of education: Figures 9 and 10A1-10A3 are estimates based on the low educated sample, and Figures 11 and 12A1-12A3 are based on the high educated sample.

[4] The test statistic is $(Q_H - Q_L)/Ä$), where Q_H is the lower bound point estimate for the high educated and Q_L is the upper bound point estimate for the low educated. $Ä$ is the estimated standard deviation of $(Q_H - Q_L)$ i.e. $Ä = (\hat{ô}^2_H + \hat{ô}^2_L)^{1/2}$, where $\hat{ô}^2_H$ and $\hat{ô}^2_L$ are the bootstrap estimates of the variances of the estimates for Q_H and Q_L. (These are estimated by re-sampling 500 times from the original data, with replacement, and using the sample variance of these 500 estimates.)

Table 12: Information on wages and salaries provided by bracket respondents; low education level (212 observations)

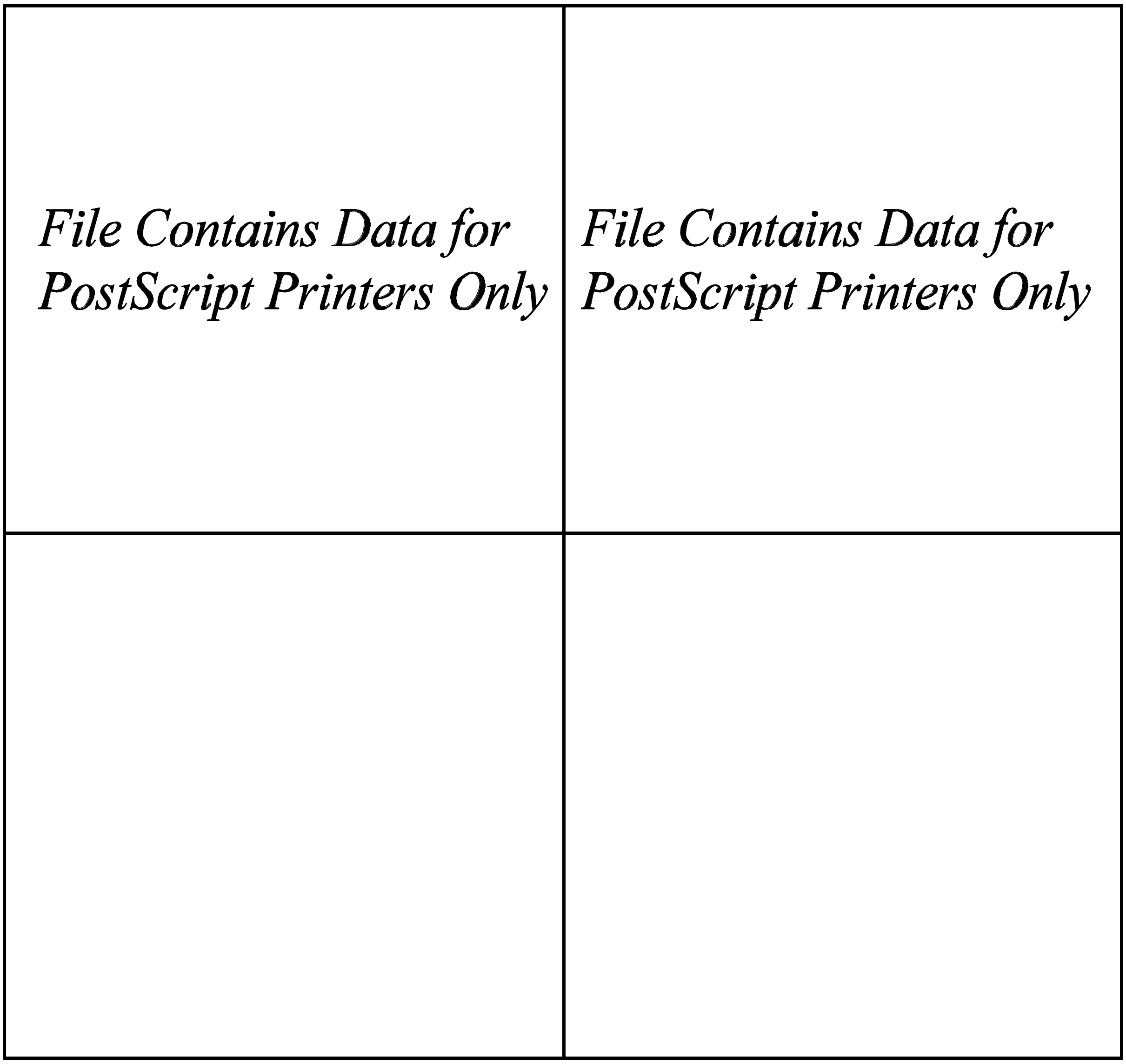
Group	Bid 1: B1	answer	Bid 2: B21/B20	answer	Resulting bracket bounds	Number
				Yes	**$50,000 — max**	6
		Yes	> $50,000 ?	No	**$25,000 — $50,000**	47
CBR	>$25,000 ?					
		No	> $ 5,000 ?	Yes	**$5,000 — $25,000**	133
				No	**$0 — $5,000**	22
		Yes	> $50,000 ?	DK, RF.	**> $25,000**	4
IBR	>$25,000 ?					
		No	> $ 5,000 ?	DK, RF.	**< $25,000**	0

Table 13: Information on wages and salaries provided by bracket respondents; high education level (117 observations)

Group	Bid 1: B1	answer	Bid 2: B21/B20	answer	Resulting bracket bounds	Number
				Yes	**$50,000 — max**	24
		Yes	> $50,000 ?	No	**$25,000 — $50,000**	39
CBR	>$25,000 ?					
		No	> $ 5,000 ?	Yes	**$5,000 — $25,000**	37
				No	**$0 — $5,000**	12
		Yes	> $50,000 ?	DK, RF.	**> $25,000**	5
IBR	>$25,000 ?					
		No	> $ 5,000 ?	DK, RF.	**< $25,000**	0

Figures 13 and 14 compare the results for the low and high educated including the bracket information, allowing for selective nonresponse and making two different assumptions about anchoring: no anchoring (A0) and anchoring following Hurd et al. (1998) (A1). The results for the other two forms of anchoring lead to similar conclusions.

The figures are organized in the same way as Figure 2 and Figures 3A1-3A3. Formal tests of equality of earnings quantiles of the high and low educated are given in Appendix C. The results of both the informal tests derived from the figures and the formal tests in the tables, are in between the two extreme cases discussed above. Under the no anchoring assumption, the differences between the quantiles considered are all significant. Allowing for anchoring reduces some of the significance levels, and the lowest quantiles are no longer significantly different. But in general, the power of the point-wise tests appears to be substantially larger than if the bracket information is not used at all.



6 Conclusions

Manski's approach consists of estimating a bounding interval around the parameter of interest, such as a (conditional) quantile of the distribution of the variable of interest. The approach allows for selective item nonresponse and avoids the type of assumptions usually associated with parametric and semi-parametric methods. On the other hand, it identifies the unknown parameters up to an upper and a lower bound only. These bounds are extended to take into account that initial non-respondents can provide partial information by answering follow up categorical questions. Nowadays, many household surveys rely on unfolding sequence type of categorical questions to reduce the percentage of item non-response. Several studies have shown that responses to such questions, on variables like income, consumption or savings, can be subject to a psychometric bias known as the anchoring effect: the answer is affected by the wording in the questions and thus can suffer from response errors. Some existing studies model this response error with a parametric set up. In this chapter Manski's worst case bounds are compared both theoretically and empirically to an extended version which do and do not allow for this anchoring effect. In the latter case, the assumptions underlining the nature of the anchoring effect are semi-parametric generalizations of some existing models of anchoring. Using the variable wages and salary of the household representative taken from the 1996 wave of the Household and Retirement Survey, the empirical section shows estimates of Manski's basic worst case bounds that do not use the bracket respondents information and compares these with estimates of the new extended bounds.

For the variable considered, the initial non-response rate is 12.4%. Most of these initial non-respondents answer some unfolding bracket questions, and the percentage of full non-response is only 3.3%. Since the distance between upper and lower bounds is driven by the percentage of item non-response, incorporating information provided by bracket respondents tightens the bounds. Allowing for anchoring effects, however, the gain in information is smaller than under the assumption of no anchoring.

In a final step, the bounds are used to test for equality of quantiles of high and low educated respondents. The findings suggest that according to Manski's basic worst case bounds, only the central quantiles are significantly higher for the higher educated, but the null hypothesis of equality cannot be rejected for the quantiles in the tails. Adding the information provided by bracket respondents improves the power of the tests, and leads to rejecting the null more often. How much the power of the tests increases depends on whether and how anchoring is allowed for.

Manski's bounds are an elegant, intuitively plausible and extremely flexible way to allow for selective non-response. Their flexibility is at the same time their main weakness: the bounds are often so wide that they do not provide enough information for the economic issue under consideration. In chapter has shown that additional information on bracket responses by initial non-respondents can be useful to make the bounds more informative. This is still true if anchoring is allowed for, though to a lesser extent.

The bounds are estimated allowing for different types of anchoring each generalizing a different parametric models in the existing literature. The chapter does not analyze which model of anchoring is most appropriate: this is not a relevant question for this framework. With the current data, however, selective non-response and anchoring interact, and it is hard to say something about anchoring without making strong assumptions about the nature of nonresponse. For an analysis of anchoring itself, therefore, experimental data where all respondents get bids that vary randomly across the sample, such as in the experimental HRS module used by Hurd et al. (1998), is more appropriate. With more knowledge about the nature of the anchoring process, the analysis here could be refined.

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Appendix A: Bounds in Case of Two Bracket Questions with Anchoring

This appendix, shows the derivation of bounds for the case of two bracket questions allowing for anchoring along the lines of Hurd et al. (1998), Jacowitz and Kahneman (1995), and Herriges and Shogren (1996). The bounds are "worst case" in the sense that any type of selective nonresponse or bracket response is allowed for. This implies that the data on full respondents carry no information on the bracket respondents, complete bracket respondents provide no information on incomplete bracket respondents, etc. Bounds on incomplete bracket respondents are straightforward, using the assumptions for the one bracket question case. This appendix therefore focuses on complete bracket respondents. Using (24) and (4), these can be used to obtain bounds for the whole population.

A1: The Hurd et al. (1998) framework

Apart from the zero median assumption (Assumption 2 in the text), a monotonicity assumption is used. Hurd et al. (1998) assume that answers are based upon comparing the amount Y with a perturbed threshold which implies that $P(Q1=1|Y=y,X=x,BR)$ increases with y. It is easy to prove that this implies the following monotonicity condition.

$$P(Q1=1|Y<t,x,BR) \le P(Q1=1|x,BR) \quad for \quad each \quad t \qquad \textbf{(MON)}$$

Together with (11), (19) and (20), this property will be used to determine what the three probabilities $P(Q1=1|x,BR)$, $P(Q20=1|Q1=0,x,BR)$ and $P(Q21=1|Q1=1,x,BR)$ that can be identified from the data, imply for the conditional distribution of Y given $X=x$ among bracket respondents. First, bounds are derived on the distribution function at the bracket values $B20$, $B1$, and $B21$. The bounds on the value of the conditional distribution function at an arbitrary value of y then follow straightforwardly, as in the no anchoring case. For notational convenience, we abbreviate conditioning on $X=x$ and BR by using Pc where $Pc(...)=P(...|x,BR)$ and $Pc(...|...)=P(...|...,x,BR)$.

Upper bound on *Pc(Y<B20)*:

$$\begin{aligned} Pc(Y<B20) &= Pc(Y<B20|Q1=0)Pc(Q1=0) + Pc(Y<B20|Q1=1)Pc(Q1=1) \\ &= Pc(Y<B20|Q1=0)Pc(Q1=0) + Pc(Y<B20)Pc(Q1=1|Y<B20) \qquad \textbf{(A.1)} \\ &\le \min[1,2Pc(Q20=0|Q1=0)]Pc(Q1=0) + Pc(Y<B20)\min[0.5,Pc(Q1=1)] \end{aligned}$$

The inequality in (A.1) is obtained using expression (20) for the first term, whereas for the second term $Pc(Y<B20|Q1=1) \leq Pc(Y<B1|Q1=1)$ is used together with (19) and (MON).Thus,

$$Pc(Y<B20) \leq \min[1,\ 2Pc(Q20=0|Q1=0)]Pc(Q1=0)/(1-\min[0.5,Pc(Q1=1)]) \qquad \textbf{(A.2)}$$

Considering the two cases $Pc(Q1=1)>0.5$ and $Pc(Q1=1) \leq 0.5$ separately, it is easy to see that this can be rewritten as

$$Pc(Y<B20) \leq \min[1,\ 2Pc(Q20=0|Q1=0)]\min[1,2Pc(Q1=0)] \qquad \textbf{(A.3)}$$

Upper bound on *Pc(Y<B1):*

Expression (20) already suggest that $Pc(Y<B1) \leq 2\ Pc(Q1=0)$. The second question gives the following additional information.

$$\begin{aligned} Pc(Y<B1) &= Pc(Y<B1|Q1=0)Pc(Q1=0) + Pc(Y<B1|Q1=1)Pc(Q1=1) \\ &\leq Pc(Q1=0) + Pc(Y<B21|Q1=1)Pc(Q1=1) \\ &\leq Pc(Q1=0) + 2Pc(Q21=0|Q1=1)Pc(Q1=1) \end{aligned} \qquad \textbf{(A.4)}$$

Taken together, the first and second question lead to the bound

$$Pc(Y<B1) \leq \min[2Pc(Q1=0),\ Pc(Q1=0)+2Pc(Q21=0|Q1=1)Pc(Q1=1)] \qquad \textbf{(A.5)}$$

If $Pc(Q1=0) \geq 0.5$ and $Pc(Q21=0|Q1=0) \geq 0.5$, the upper bound in (A.5) is 1. But if at least one of the two probabilities is less than 0.5, the upper bound is smaller than one.

Upper bound on *Pc(Y<B21)*:

$$\begin{aligned} Pc(Y<B21) &= Pc(Y<B21|Q1=0)Pc(Q1=0) + Pc(Y<B21|Q1=1)Pc(Q1=1) \\ &\leq Pc(Q1=0)+\min[1,\ 2Pc(Q21=0|Q1=1)]Pc(Q1=1) \end{aligned} \qquad \textbf{(A.6)}$$

The lower bounds follow by symmetry from (A.3), (A.5) and (A.6).

Lower bound on *Pc(Y<B20)*:

Pc(Y<B20) = 1- Pc(Y≥B20); a lower bound on *Pc(Y≥B20)* is obtained in the same way as the upper bound on *Pc(Y<B21)* given in (A.6). This gives:

$$Pc(Y \geq B20) \;\leq\; Pc(Q1=1) + \min[1,\; 2Pc(Q20=1|Q1=0)]Pc(Q1=0) \qquad \textbf{(A.7)}$$

And thus

$$\begin{aligned} Pc(Y<B20) \;&\geq\; \{1 - Pc(Q1=1) + \min[1, 2Pc(Q20=1|Q1=0)]Pc(Q1=0)\} \\ &=\; \max[0,\; 1-2Pc(Q20=1|Q1=0)Pc(Q1=0)] \end{aligned} \qquad \textbf{(A.8)}$$

Lower bound on *Pc(Y<B1):*

Pc(Y<B1) = 1 – Pc(Y≥B1); an upper bound on *Pc(Y≥B1)* is obtained in the same way as the upper bound on *Pc(Y<B1)* given in (A.5)

$$Pc(Y \geq B1) \;\leq\; \min[\; 2Pc(Q1=1),\; Pc(Q1=1)+2Pc(Q20=1|Q1=0)Pc(Q1=0)] \qquad \textbf{(A.9)}$$

and (A.9) implies

$$\begin{aligned} Pc(Y<B1) \;&\geq\; \{1-\min[2Pc(Q1=1),\; Pc(Q1=1)+2Pc(Q20=1|Q1=0)Pc(Q1=0)]\} \\ &=\; \max[1-2Pc(Q1=1),\; Pc(Q1=0)(1-2Pc(Q20=1|Q1=0)] \end{aligned} \qquad \textbf{(A.10)}$$

Lower bound on *Pc(Y<B21)*:

Pc(Y<B21) = 1 – Pc(Y≥B21); an upper bound on *Pc(Y≥B21)* is obtained in the same way as an upper bound on *Pc(Y<B20)* as expressed in (A.3),

$$Pc(Y \geq B21) \;\leq\; \min[1,\; 2Pc(Q21=1|Q1=1)]\min[1,\; 2Pc(Q1=1)] \qquad \textbf{(A.11)}$$

and thus a lower bound is obtained as

$$Pc(Y<B21) \;\geq\; \{1-\min[1,2Pc(Q21=1|Q1=1)]\min[1,2Pc(Q1=1)]\} \qquad \textbf{(A.12)}$$

A2: The Jacowitz and Kahneman (1995) Assumption

In this case, expressions (15), (22) and (MON) are the basis for deriving the bounds. The sample analogues of *Pc(Q1=1)* and *Pc(Q21=1|Q1=1)* are smaller than 0.5, while that of *Pc(Q20=1|Q1=0)* is larger than 0.5. Thus *B1* and *B21* are "large" and *B20* is "small." This means that (15) and (22) imply

$$\begin{aligned} Pc(Y\geq B1) &\leq Pc(Q1=1|CBR) \\ Pc(Y\geq B21|Q1=1) &\leq Pc(Q21=1|Q1=1) \\ Pc(Y\geq B20|Q1=0) &\geq Pc(Q20=1|Q1=0) \end{aligned} \qquad \textbf{(JK)}$$

Upper bound on *Pc(Y<B20):*

$$\begin{aligned} Pc(Y<B20) &= Pc(Y<B20|Q1=0)Pc(Q1=0) + Pc(Y<B20|Q1=1)Pc(Q1=1) \\ &\leq Pc(Q20=0|Q1=0)Pc(Q1=0) + Pc(Y<B20)P(Q1=1|Y<B20) \\ &\leq Pc(Q20=0|Q1=0)Pc(Q1=0) + Pc(Y<B20)P(Q1=1) \end{aligned} \qquad \textbf{(A.13)}$$

where (MON) has being used in the last step. Rewriting (A.13) and dividing by P(Q1=0) yields

$$Pc(Y<B20) \;\leq\; Pc(Q20=0|Q1=0) \qquad \textbf{(A.14)}$$

Upper bound on *Pc(Y<B1):*

None of the three assumptions in **(JK)** help to find a non-trivial upper bound, either directly or using the same decomposition used above. Thus all that can be said about this bound is that

$$Pc(Y<B1) \;\leq\; 1 \qquad \textbf{(A.15)}$$

Upper bound on *Pc(Y<B21):*

This immediately follows from (A.15):

$$Pc(Y<B21) \leq 1 \qquad \textbf{(A.16)}$$

Lower bound on *Pc(Y<B20):*

None of the three assumptions in **(JK)** help to find a non-trivial lower bound, so that all can be said about this bound is that

$$Pc(Y<B20) \geq 0 \qquad \textbf{(A.17)}$$

Lower bound on *Pc(Y<B1):*

The first assumption in **(JK)** immediately gives:

$$Pc(Y<B1) \geq Pc(Q1=1) \qquad \textbf{(A.18)}$$

The other two assumptions do not add anything here.

Lower bound on *Pc(Y<B21):*

$$\begin{aligned} Pc(Y<B21) &= Pc(Y<B21|Q1=0)Pc(Q1=0) + Pc(Y<B21|Q1=1)Pc(Q1=1) \\ &\geq Pc(Y<B21)Pc(Q1=0|Y<B21) + Pc(Q21=0|Q1=1)Pc(Q1=1) \\ &\geq Pc(Y<B21)Pc(Q1=0) + Pc(Q21=0|Q1=0)Pc(Q1=1) \end{aligned} \qquad \textbf{(A.19)}$$

where (MON) is used in the last step. Rewriting (A.19) and dividing it by Pc(Q1=1) gives:

$$Pc(Y<B21) \geq Pc(Q21=0|Q1=0) \qquad \textbf{(A.20)}$$

Moreover, the first inequality in (JK) implies directly

$$Pc(Y<B21) \geq Pc(Y<B1) \geq Pc(Q1=1) \qquad \textbf{(A.21)}$$

Combining (A.20) and (A.21) yields the lower bound

$$Pc(Y<B21) \geq \max[Pc(Q1=1),\ Pc(Q21=0|Q1=0)] \qquad \textbf{(A.22)}$$

Thus, with the combining (JK) and (MON) expressions (A.14), (A.15) and (A.18) define non-trivial upper bounds. Likewise, expressions (A.17), (A.18) and (A.22) define non-trivial lower bounds for the complete bracket response population.

A3: The Herriges & Shogren (1996) Model

The assumptions about anchoring in this model can be summarized as

$$\begin{aligned} &Pc(Y<B1)=Pc(Q1=0) \\ &Pc(Y<B21|Q1=1)\leq Pc(Q21=0|Q1=1) \\ &Pc(Y<B20|Q1=0)\geq Pc(Q20=0|Q1=0) \end{aligned} \qquad \textbf{(HS)}$$

The derivations are much easier than in the previous two cases.

Upper bound on *Pc(Y<B20):*

$$Pc(Y<B20) \leq Pc(Y<B1) = Pc(Q1=0) \qquad \textbf{(A.23)}$$

Upper and lower bound on *Pc(Y<B1):*

$$Pc(Y<B1) = Pc(Q1=0) \qquad \textbf{(A.24)}$$

Upper bound on *Pc(Y<B21):*

$$\begin{aligned} Pc(Y{<}B21) &= Pc(Y{<}B21|Q1{=}0)Pc(Q1{=}0) + Pc(Y{<}B21|Q1{=}1)Pc(Q1{=}1) \\ &\leq Pc(Q1{=}0) + Pc(Q21{=}0|Q1{=}1)Pc(Q1{=}1) \end{aligned} \quad \textbf{(A.25)}$$

Lower bound on *Pc(Y<B20):*

$$\begin{aligned} Pc(Y{<}B20) &= Pc(Y{<}B20|Q1{=}0)Pc(Q1{=}0) + Pc(Y{<}B20|Q1{=}1)Pc(Q1{=}1) \\ &\geq Pc(Q20{=}0|Q1{=}0)Pc(Q1{=}0) \end{aligned} \quad \textbf{(A.26)}$$

The lower bounds on B1 and B21 are also given by (A.24). Thus, the set of non-trivial upper bounds are given by (A.23) to (A.25) whereas the non-trivial lower bounds are given by (A.26) and (A.24). In this only (HS) is used, with no need for (MON) in the derivation.

Appendix B

This appendix shows Figures 9 and 10A1-10A3 for the low educated population, and Figures 11 and 12A1-12A3 for the high educated population. In each case the figures compare bounds for bracket respondents (point estimates and 95% confidence bands on these bounds), to the 95% confidence bands for full respondents.

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Appendix C

This appendix presents various (formal) tests for the null of no difference between the high and low educated populations, under different assumptions on anchoring. Tables C1, C2, C3 and C4 should be read together with the informal test carried out with figures 13, 14, 15 and 17 respectively (see Section 5.2)

Table C.1: Testing for differences between earnings quantiles of high and low educated respondents, based on worst case bounds with brackets & no anchoring (cf. Figure 13).

	Low Education standard error	Low education Poin estimate	High Education point estimate	High education standard error	Test Statistic
20th Percentile	$534	$8,900	$12,750	$949	3.53
25th Percentile	$569	$10,800	$17,500	$924	6.17
30th Percentile	$369	$12,750	$20,800	$1,174	6.54

20th Percentile	$534	$8,900	$12,750	$949	3.53
40th Percentile	$554	$17,500	$25,900	$895	7.98
50th Percentile	$598	$21,900	$32,500	$972	9.29
60th Percentile	$6	$25,000	$39,400	$668	21.58
75th Percentile	$1,044	$31,500	$50,000	$934	13.21
80th Percentile	$1,026	$36,900	$56,400	$1,489	10.79
90th Percentile	$125	$50,000	$76,000	$2,675	9.70

Table C.2: Testing for differences between earnings quantiles of high and low educated respondents, based on worst case bounds with brackets & anchoring A1 (cf. Figure 14).

	Low Education standard error	Low education Poin estimate	High Education point estimate	High education standard error	Test Statistic
20th Percentile	$366	$9,800	$7,700	$865	-2.21
25th Percentile	$419	$11,900	$11,900	$1,190	0
30th Percentile	$514	$13,000	$15,800	$1,171	2.19
40th Percentile	$412	$17,900	$25,000	$860	7.42
50th Percentile	$629	$22,500	$29,900	$774	7.42
60th Percentile	$121	$25,000	$35,600	$930	11.31
75th Percentile	$1,017	$34,600	$48,600	$898	10.31
80th Percentile	$1,077	$39,400	$52,500	$1,740	6.42
90th Percentile	$1,054	$50,000	$66,000	$1,375	9.24

Table C.3: Testing for differences between earnings quantiles of high and low educated respondents, based on worst case bounds with brackets & Anchoring A2 (cf. Figure 15).

	Low Education standard error	Low education Poin estimate	High Education point estimate	High education standard error	Test Statistic
20th Percentile	$352	$9,800	$6,800	$763	-3.57
25th Percentile	$418	$11,900	$9,800	$1,064	-1.84
30th Percentile	$489	$13,000	$14,700	$986	1.54
40th Percentile	$391	$17,900	$23,900	$1,147	4.95
50th Percentile	$550	$22,500	$29,900	$741	8.02
60th Percentile	$112	$25,000	$35,600	$951	11.14
75th Percentile	$1,005	$32,500	$48,600	$913	12.60
80th Percentile	$1,007	$36,900	$52,500	$1,703	7.90

20th Percentile	$352	$9,800	$6,800	$763	-3.57
90th Percentile	$887	$50,000	$66,000	$1,514	9.12

Table C.4: Testing for differences between earnings quantiles of high and low educated respondents, based on worst case bounds with brackets & anchoring A3 (cf. Figure 16).

	Low Education standard error	**Low education Poin estimate**	**High Education point estimate**	**High education standard error**	**Test Statistic**
20th Percentile	$550	$8,900	$9,800	$869	0.87
25th Percentile	$570	$10,800	$14,700	$994	3.40
30th Percentile	$364	$12,750	$18,700	$800	6.79
40th Percentile	$580	$17,500	$25,000	$384	10.78
50th Percentile	$666	$21,900	$31,200	$774	9.11
60th Percentile	$8	$25,000	$37,900	$1,052	12.27
75th Percentile	$1,043	$31,500	$49,700	$240	17
80th Percentile	$1,040	$36,900	$52,600	$1,720	7.78
90th Percentile	$1,840	$53,900	$66,000	$1,470	4.85