BARGAINING IN COMMITTEES OF REPRESENTATIVES: THE OPTIMAL VOTING RULE*

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Abstract

Committees are often constituted by representatives of groups of individuals of different sizes, and make decisions by means of a voting rule which specifies what voting configurations can pass a decision. This raises the question of the choice of the optimal voting rule, given the different size of the groups its members represent. In this paper we address this problem assuming that the committee is a bargaining scenario in which negotiations take place 'in the shadow of the voting rule'. That is, a general agreement is looked for, but any winning coalition can enforce an agreement.

Key words: Bargaining in committees, voting, bargaining power.

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1 INTRODUCTION

Committees are often constituted by representatives of groups of individuals of different sizes, and make decisions by means of a voting rule (often a weighted majority rule, but more generally any arbitrary voting rule) which specifies what vote configurations can pass a decision. Examples of committees of representatives of this type are provided by the councils ruling different kinds of organizations, including important political examples as the Council of Ministers in the EU. This raises the question of the 'fair', 'optimal' or more adequate voting rule, given the different sizes of the groups its members represent, if a principle of equal representation is to be implemented. This issue has been approached so far by different authors by modelling the decision-making process as an idealized two-stage process, and assessing the 'decisiveness' (i.e., the probability of being crucial or pivotal) in making a decision that can be imputed to each individual in the different groups assuming that every representative follows the majoritarian opinion in her 'constituency' (see, e.g., Penrose (1946), Owen (1975), Laruelle and Widgrén (1998), Felsenthal and Machover (1998)). In a recent paper Barberà and Jackson (2004), in a different framework, address the issue of the 'efficient' voting rule, for which, under certain assumptions, the expected utility of the voters (or, equivalently, their probability of having the result they want) is maximized.

Either based in the assessment of the likelihood of being 'decisive' or that of being 'satisfied' or 'successful' (Rae (1969), Brams and Lake (1978), Barry (1980), Straffin, Davis, and Brams (1981), see also Laruelle and Valenciano (2004a)), this approach makes sense in the case of a 'take-it-or-leave-it' committee. That is, a committee only entitled to accept or reject proposals submitted to it by some external agency.

On the other hand, the case in which bargaining among groups occurs has been often considered in the economic literature[1]. Usually for two-party negotiations, under different frameworks and with different goals, several noncooperative models have been provided (see, for instance, Jun (1989), Perry and Samuelson (1994), Haller and Holden (1997), and Cai (2000)). In two recent papers this issue has been addressed by Chae and Heidhues (2004) and Chae and Moulin (2004) from an axiomatic point of view in a framework consisting of a bargaining problem plus a partition of the set of players.

In this paper we take a new departure to address the question of the optimal voting rule in a committee of representatives. We assume that the committee is a bargaining scenario in which negotiations take place under or 'in the shadow' of a voting rule. In the cooperative and non cooperative game theoretic literature on bargaining since Nash

[1] See footnote 1 in Chae and Heidhues (2004) for an interesting cuantification based on two leading journals.

(1950) seminal paper, bargaining is supposed 'by definition' to be a process that can be settled only by unanimity. The possible asymmetry of the outcome can only arise from the bargaining environment. In many contexts it is often the case in a committee that uses a (possibly nonsymmetric) voting rule to make decisions that the final vote is merely the formal settlement of a bargaining process in which the issue to be voted upon has been adjusted to gain the acceptance of all members. We base our approach in a previous work (Laruelle and Valenciano, 2004b) in which we address the following question: What agreements can a rational agent expect to arise when faced with the prospect of engaging in such a situation? That is, when a general agreement is looked for, but any winning coalition can enforce an (possibly non unanimous) agreement. Based on this answer, and assuming that a principle of equal representation is to be implemented, here we give one to this question: Which is the optimal voting rule in a bargaining committee of representatives of groups of different sizes?

The rest of the paper is organized as follows. In Section 2 we introduce some basic notation and briefly review the results in Laruelle and Valenciano (2004b) that are required. In Section 3 we address the issue of the optimal voting rule in a bargaining committee of representatives. Section 4 examines some related work. Section 5 concludes with some remarks.

2 A BARGAINING COMMITTEE

In this section we summarize the model and results in Laruelle and Valenciano (2004b).

2.1 A two-ingredient model

The first element of our model of a *bargaining committee* is the voting rule. The set $N = \{1, ..., n\}$ labels the *seats* in the committee. As only yes/no voting is considered, a *vote configuration* can be represented by the set of 'yes'-voters. So, any $S \subseteq N$ represents the result of a vote in which only the members of the committee occupying seats in S voted 'yes'. An N-voting rule is specified by a set $W \subseteq 2^N$ of *winning* (i.e., which would lead to passing a decision) vote configurations such that (i) $N \in W$; (ii) $\emptyset \notin W$; (iii) If $S \in W$, then $T \in W$ for any T containing S; and (iv) If $S \in W$ then $N \backslash S \notin W$. $\mathcal{W}$ denotes the set of all such N-voting rules. For voting rule W, $M(W)$ denotes the set of *minimal winning* configurations, i.e., those that do not contain any other winning configuration. For any $S \in M(W)$ $(S \neq N)$, W_S^* denotes the voting rule $W_S^* := W \setminus \{S\}$. For any permutation $\pi : N \to N$, πW denotes the voting rule $\pi W := \{\pi(S) : S \in W\}$. A voting rule W is *symmetric* if $\pi W = W$, for any permutation π.

The n members or *players* of a committee which uses an N-voting rule are labelled by the seats in N that they occupy, and we refer to the subset of players denoted by $S \subseteq N$ as *coalition* S. Thus, depending on the context any $S \subseteq N$ will be referred to either as a vote configuration (of seats) or as a coalition (of players). We will also speak of *winning coalitions* for a given N-rule, with an obvious meaning. A seat/player $i \in N$ is said to be a *null seat/player* in W if, for any S, $S \in W$ if and only if $S \setminus \{i\} \in W$.

We assume that a committee of n (N-labelled) members makes decisions by means of an N-voting rule W in the following sense. They can reach any alternative within a set A, as well as any lottery over them, as long as: (i) a winning coalition supports it, and (ii) no player is imposed upon an agreement worse than the *status quo,* denoted a, where all players will remain if no winning coalition supports any agreement. It is also assumed that every player has expected utility (von Neumann and Morgenstern, 1944) (vNM) preferences over this set of lotteries, so that the relevant information concerning the players' preferences can be encoded à la Nash in utility terms by a feasible set of utility vectors D, together with the particular vector d associated with the disagreement or status quo. Thus, the pair (D, d) is a summary of the situation concerning the players' decision. Accepting this simplification, the whole situation can be summarized by a pair (B, W), where $B = (D, d)$ is a classical n-person bargaining problem that represents the configuration of preferences in the committee, and W is the N-voting rule to enforce agreements. In accordance with this interpretation we assume that $D_d := \{x \in D : x \geq d\}$[2] is *bounded*, and D is a *closed, convex* and *comprehensive* (i.e., $x \leq y \in D \Rightarrow x \in D$) set containing d, such that there exists some $x \in D$ s.t. $x > d$. $\mathcal{B}$ denotes the set of all such bargaining problems. For any permutation $\pi : N \rightarrow N$, $\pi B := (\pi(D), \pi(d))$ denotes the bargaining problem that results from B by π-permutation of its coordinates, so that for any $x \in R^N$, $\pi(x)$ denotes the vector in R^N s.t. $\pi(x)_{\pi(i)} = x_i$. A bargaining problem B is *symmetric* if $\pi B = B$, for any permutation π.

Thus, consistently with the interpretation that accompanied its introduction, any pair $(B, W) \in \mathcal{B} \times \mathcal{W}$ will be referred to as an N-*bargaining committee* (B, W).

Classical bargaining problems and simple TU games can be seen as particular cases of this model[3]. The n-person classical bargaining problem corresponds to the case of a committee bargaining *under the unanimity rule.* While when the bargaining element B is the bargaining problem $\Lambda := (\Delta, 0)$, where $\Delta := \left\{x \in R^{\dot{N}} : \sum_{i \in N} x_i \leq 1\right\}$, the pair

[2]We will write for any $x, y \in R^N$, $x \leq y$ ($x < y$) if $x_i \leq y_i$ ($x_i < y_i$) for all $i = 1, .., n$.

[3]As shown in Laruelle and Valenciano (2004b), $\mathcal{B} \times \mathcal{W}$ is isomorphic to a subclass of NTU models, so that these equivalences are exact because this subclass contains all classical bargaining problems and all simple superadditive games.

(Λ, W) is equivalent to the *simple* TU game representing the voting rule, v_W, given by

$$v_W(S) := \begin{cases} 1 & \text{if } S \in W, \\ 0 & \text{if } S \notin W. \end{cases}$$

2.2 Rationality conditions

In (2004b) we impose the following conditions on a map $\Phi : \mathcal{B} \times \mathcal{W} \to R^N$, for vector $\Phi(B, W) \in R^N$ to be considerable as a reasonable expectation of utility levels of a *general* agreement in a bargaining committee (B, W). We impose as prerequisites: $\Phi(B, W) \in D$ (feasibility), and $\Phi(B, W) \geq d$ (individual rationality), if $B = (D, d)$. In addition to this we require:

1. *Efficiency (Eff):* For all $(B, W) \in \mathcal{B} \times \mathcal{W}$, there is no $x \in D$, s.t. $x > \Phi(B, W)$.

2. *Anonymity (An):* For all $(B, W) \in \mathcal{B} \times \mathcal{W}$, and any permutation $\pi{:}N \to N$, and any $i \in N$, $\Phi_{\pi(i)}(\pi(B, W)) = \Phi_i(B, W)$, where $\pi(B, W) := (\pi B, \pi W)$.

3. *Independence of irrelevant alternatives (IIA):* Let $B, B' \in \mathcal{B}$, with $B = (D, d)$ and $B' = (D', d')$, such that $d' = d$, $D' \subseteq D$ and $\Phi(B, W) \in D'$, then $\Phi(B', W) = \Phi(B, W)$, for any $W \in \mathcal{W}$.

4. *Invariance w.r.t. positive affine transformations (IAT):* For all $(B, W) \in \mathcal{B} \times \mathcal{W}$, and all $\alpha \in R^N_{++}$ and $\beta \in R^N$,

$$\Phi(\alpha * B + \beta, W) = \alpha * \Phi(B, W) + \beta,$$

where $\alpha * B + \beta = (\alpha * D + \beta, \alpha * d + \beta)$, denoting $\alpha * x := (\alpha_1 x_1, .., \alpha_n x_n)$, and $\alpha * D + \beta := \{\alpha * x + \beta : x \in D\}$.

5. *Null player* (*NP*): For all $(B, W) \in \mathcal{B} \times \mathcal{W}$, if $i \in N$ is a null player in W, then $\Phi_i(B, W) = d_i$.

The readers can see for themselves the precise correspondence of axioms 1 to 5 with some of Nash's (1950) and Shapley's (1953) axioms. *Eff*, *IIA* and *IAT* (adaptations of Nash's axioms) concern the feasible set, while *An* (adapted from Nash's and Shapley's anonymity) and *NP* (from Shapley's system) concern both.[4].

[4]We omit the arguments in support of each of these conditions, which can be found in Nash's and Shapley's papers. For a careful discussion of Nash's axioms see, e.g., chapter 1 in Binmore (1998).

2.3 Characterizations

Denote by $Nash(B)$ the Nash (1950) (pure) bargaining solution of an n-person bargaining problem $B = (D, d)$, that is,

$$Nash(B) = \arg\max_{x \in D_d} \prod_{i \in N} (x_i - d_i), \tag{1}$$

and by $Nash^w(B)$ the w-weighted asymmetric Nash bargaining solution (Kalai, 1977) of the same problem for a vector of nonnegative weights $w = (w_i)_{i \in N}$, given by[5]

$$Nash_i^w(B) := \begin{cases} \arg_i \max\limits_{x \in D_d} \prod_{j \in J} (x_j - d_j)^{w_j} & \text{if } i \in J, \\ d_i & \text{if } i \in N \setminus J, \end{cases} \tag{2}$$

where $J = \{i \in N : w_i > 0\}$. Nash (1950) characterized (1) as the unique (pure) bargaining solution (i.e., map $\mathcal{B} \to R^N$) satisfying the conditions of 'efficiency', 'anonymity', 'invariance w.r.t. positive affine transformations' and 'independence of irrelevant alternatives'[6]. While Kalai's (1977) solutions emerge by dropping symmetry in Nash system. In this case the 'weights' are usually interpreted as the 'bargaining power' of the players (see, e.g., Binmore (2004)).

The following theorem is a generalization of Nash's characterization.

Theorem 1 *(Laruelle and Valenciano, 2004b) Let $\Phi : \mathcal{B} \times \mathcal{W} \to R^N$ be a solution that satisfies Eff, An, IIA, IAT and NP, then*

$$\Phi(B, W) = Nash^{\Phi(\Lambda, W)}(B), \tag{3}$$

where $\Phi(\Lambda, \cdot) : \mathcal{W} \to R^N$ satisfies efficiency, anonymity and null player.

Therefore, any map $\varphi = \Phi(\Lambda, \cdot) : \mathcal{W} \to R^N$ that satisfies efficiency, anonymity and null player would fit into formula (3) and yield a solution $\Phi(B, W)$ that satisfies the five rationality conditions. Although the main conclusions of this paper are presented in section 3 for any such map $\varphi = \Phi(\Lambda, \cdot)$, we consider worth distinguishing a special case. In Laruelle and Valenciano (2004b) we show that among all such maps[7], substituting the Shapley-Shubik (1954) index in (3) yields the unique solution that is consistent with two different informational scenarios assuming risk on the voting rule. From the axiomatic

[5] In fact, Kalai's solutions do not impose the disagreement payoff on players with weight zero.

[6] Be aware these conditions in Nash's setting, unlike their homonymous in the previous subsection, refer to a map $\mathcal{B} \to R^N$.

[7] The conditions on $\Phi(\Lambda, \cdot)$ bring immediately to mind the Shapley-Shubik (1954) index. But there are other alternatives, for instance, the normalization of any semivalue (Dubey, Neyman and Weber, 1981) meets these conditions. Also some 'power indices,' as the Holler-Packel (1983) index.

point of view, this particular solution is characterized by adding the adaptation to this setting of the condition by means of which Dubey (1975) characterized the Shapley value on the domain of simple games (i.e., the Shapley-Shubik index), which in terms of voting rules can be stated as follows (Laruelle and Valenciano, 2001):

6. *Transfer (T):* For any two rules $W, W' \in \mathcal{W}$, and all $S \in M(W) \cap M(W')$ $(S \neq N)$:

$$\Phi(\Lambda, W) - \Phi(\Lambda, W_S^*) = \Phi(\Lambda, W') - \Phi(\Lambda, W_S'^*). \tag{4}$$

Denote by $Sh(v)$ the Shapley (1953) value of a TU game v, given by

$$Sh_i(v) = \sum_{S:S\subseteq N} \frac{(n-s)!(s-1)!}{n!}(v(S) - v(S\backslash i)),$$

and by $Sh(W)$ the Shapley-Shubik (1954) index of a voting rule W, i.e., the Shapley value of the associated simple game v_W. We have the following result.

Theorem 2 *(Laruelle and Valenciano, 2004b) There exists a unique solution/value* $\Phi : \mathcal{B} \times \mathcal{W} \to R^N$ *that satisfies Eff, An, IIA, IAT, NP and T, and it is given by*

$$\Phi(B, W) = Nash^{Sh(W)}(B). \tag{5}$$

Note that $\Phi(B, W)$ becomes Nash's classical solution of B when W is the unanimity rule, and it becomes the Shapley-Shubik index of v_W when $B = \Lambda$. In other words, Theorem 2 integrates Nash's and Shapley-Dubey's (Shapley (1953), Dubey (1975)) characterizations into one, which extends both and yields a solution to the problem of bargaining under a voting rule given by (5).

Formulae (3) and (5) have a clear interpretation. As Binmore points out, the asymmetric Nash solutions can be justified as reflecting the different 'bargaining power' of the players *"determined by the strategic advantages conferred on players by the circumstances under which they bargain"* (1998, p. 78). In the case of a bargaining committee the voting rule, possibly nonsymmetric, is the source of differences in 'strategic advantages'. Thus, according to formulae (3) and (5), under the assumed conditions in either case, either vector $\Phi(\Lambda, W)$ or $Sh(W)$ gives the 'bargaining power' that the voting rule confers to each member of the committee.

3 A BARGAINING COMMITTEE OF REPRESENTATIVES

Assume that each member i of a committee of n members, labelled by N, represents a group M_i of size m_i. If these groups are disjoint and $M = \cup_{i\in N} M_i$, the cardinal of M is $m = \sum_{i\in N} m_i$. Let us denote by $\mathcal{M}$ the partition $\mathcal{M} = \{M_1, M_2, .., M_n\}$. And assume that it is a bargaining committee in the sense considered in the previous section. It seems clear that, if the different groups are of different sizes, a symmetric voting rule is not adequate (neither 'fair', 'right', or 'optimal'). At least if a principle of equal representation is to be implemented. This raises the issue of the choice of the most adequate voting rule under these conditions.

In general a bargaining committee of representatives will negotiate different issues along time under the same voting rule. In every case, depending on the particular issue, a different configuration of preferences will emerge in the population represented by the members of the committee. Thus it does not make sense to make the 'optimal' voting rule dependent on the preference profile, but it does not make sense either to assume unanimous preferences within every constituency. On the other hand, if there is no relationship at all between the preferences of the members of each group it is not clear on what grounds to found the choice of voting rule for the committee of representatives. In order to found an answer we assume that the configuration of preferences in the represented population is symmetric *within every group*[8] in the following sense.

Assume that $B = (D, d)$ $(d \in D \subseteq R^M)$ is the m-person bargaining problem representing the configuration of preferences of the m individuals in M. We say that a permutation $\pi : M \to M$ respects $\mathcal{M}$ if for all $i \in N$, $\pi(M_i) = M_i$. We say that B is $\mathcal{M}$-*symmetric* if for any permutation $\pi : M \to M$ that respects $\mathcal{M}$, it holds $\pi d = d$, and for all $x \in D$, $\pi x \in D$. In words, B is $\mathcal{M}$-symmetric if for any group (M_i) the disagreement payoff is the same for all its members ($d_k = d_l$, for all $k, l \in M_i$), and fixing in any way the payoffs of the other players in $M \backslash M_i$, the set of feasible payoffs for the players in that group (M_i) is symmetric[9]. Notice that this does not mean at all that all players within every group have the same preferences. But note that if the payoffs of all the players in $M \backslash M_i$ are fixed, the outcome of bargaining within M_i (under unanimity and assuming anonymity) would yield the same payoff to all players in M_i. Thus $\mathcal{M}$-symmetry in B entails the following consequences.

Let M, N, and $\mathcal{M}$, as above, and let $B = (D, d)$ an M-configuration of preferences $\mathcal{M}$-symmetric. Assuming (as a term of reference) the players in M negotiate directly under unanimity, according to Nash bargaining model, the outcome would be $Nash(B)$. On the other hand, as B is $\mathcal{M}$-symmetric, it must be

$$Nash_k(B) = Nash_l(B) \qquad (\forall i \in N, \forall k, l \in M_i).$$

[8]This assumption parallels the one in the two-stage idealization for the assessment of decisiveness in a 'take-it-or-leave-it' commitee of representatives according to which every group decides the vote of its representative by simple majority and all vote configurations are assumed to be equally probable.

[9]This is equivalent to saying in Chae and Heidhues' (2004) terms that all groups are *homogeneous*.

Namely, in every group all players would receive the same payoff according to Nash bargaining solution. Therefore the optimal solution of the maximization problem (1) that yields $Nash(B)$, coincides with the optimal solution of the same maximization problem when the set of feasible payoff vectors is constrained to yield the same payoff for any two players in a same group. Formally, denote by B^N the N-bargaining problem $B^N = (D^N, d^N)$, where

$$D^N := \left\{ (x_1, ..., x_n) \in R^N : (\underbrace{x_1, .., x_1}_{m_1 - times}, ..., \underbrace{x_n, .., x_n}_{m_n - times}) \in D \right\},$$

and by d^N the vector in R^N whose i-component is, for each $i \in N$, equal to d_k (the same for all $k \in M_i$). Namely, B^N is the bargaining problem that would result by taking one individual from every constituency as representative for bargaining on behalf of it, under the commitment of bargaining symmetrically within that constituency after the level of utility of the other constituencies has been settled. We have that, for all $i \in N$ and all $k \in M_i$,

$$Nash_k(B) = \arg_k \max_{x \in D_d} \prod_{l \in M} (x_l - d_l)$$
$$= \arg_i \max_{x \in D^N_{d^N}} \prod_{j \in N} (x_j - d_j)^{m_j} = Nash^m_i(B^N). \tag{6}$$

That is to say, for the configuration of preferences or M-bargaining problem B, a player k would obtain the same utility level by direct (m-player unanimous) bargaining, as a representative would obtain bargaining on behalf of her/him (and of all the players in the same group) under the configuration of preferences B^N *if each representative were endowed with a bargaining power proportional to the size of the group.* The problem then is how to 'implement' a weighted Nash bargaining solution. In other words and more precisely, *how to implement a bargaining environment that confers to each representative the right bargaining power*[10].

In view of Theorem 1, if a power index (i.e., an efficient, anonymous and ignoring null players map $\varphi : \mathcal{W} \to R^N$) is considered the right assessment of expectations for TU problems, and for some N-voting rule W it holds

$$\frac{\varphi_i(W)}{m_i} = \frac{\varphi_j(W)}{m_j} \qquad (\forall i, j \in N),$$

then *this rule would exactly implement such environment.* In particular, in case the index is the Shapley-Shubik index (Theorem 2), an optimal voting rule would be one for which

$$\frac{Sh_i(W)}{m_i} = \frac{Sh_j(W)}{m_j} \qquad (\forall i, j \in N).$$

[10]Here we use the term 'implementation' in a general sense, and not in the standard technical sense of providing a noncooperative game that yields the desired outcome as an equilibrium.

Then, the previous discussion can be summarized in the following

Theorem 3 *The optimal voting rule in a bargaining committee of representatives is one that gives to each member a bargaining power proportional to the size of the group he/she represents.*

In general no rule will yield exactly the required bargaining weights[11]. Whatever the size of the committee the number of voting rules is finite, while the set of possible combinations of groups' sizes is not. There is the problem then of finding the voting rule closest to optimality[12].

If existence may be a problem, there may also be a problem of multiplicity. For instance, if all the groups are of equal size any symmetric voting rule would be optimal according to Theorem 3. In a case like this there is at least a second point of view to refine the choice: The easiness to make decisions or decisiveness of the rule (Coleman (1971), Felsenthal and Machover (1998))

4 RELATED WORK

The answer provided by Theorem 3 to the optimal voting rule issue is a completely new departure from previous ones, but it is worth noting the formal similarity with some of them. First, with the naive proposal of a weighted majority rule with weights proportional to the size of the groups, which has been longtime criticized but is still used sometimes[13]. Also, in the 'take-it-or-leave-it' scenario, the two-stage idealization yields the 'square root' rule, which solves the normative problem of the fair distribution of 'decisiveness' in such a committee, assuming that every representative follows the majoritarian opinion in her constituency (see, e.g., Penrose (1946), Owen (1975), Laruelle and Widgrén (1998), Felsenthal and Machover (1998)). That is, assuming the group sizes' big enough, the optimal rule is the one for which the Banzhaf (1965) index of each representative is proportional to the square root of the size of the group he/she represents. Curiously enough, in a rather inconsistent way, some US courts accepted the Banzhaf index as a measure of the voting power of the members in a committee, but approved a voting system that made the Banzhaf index of each representative proportional to his district's size (Benoit and Kornhauser, 2002)).

[11] This is the "inverse problem", addressed by Dragan (1991) in the context of TU games.

[12] A similar problem occurs for the 'square root rule' that solves the normative problem of the fair distribution of decisiveness in a 'take-it-or-leave-it' committee commented in the the next section.

[13] See Benoit and Kornhauser (2002, pp. 2252-2259) for an interesting account of the different criterions endorsed by U.S. courts on the issue of "fair and effective representation."

In Harsanyi (1977), where Nash classical bargaining solution is extended to n players, the following 'joint-bargaining paradox' is discussed (cf. 10.7, pp. 203-211). Consider the three-person TU-like bargaining game B in which the set of feasible payoffs is defined by the inequalities $u_1 + u_2 + u_3 \leq 30$, and the disagreement payoff vector is $(0, 0, 0)$. If the three players bargain as different independent agents according to Nash bargaining model, the solution will be $(10, 10, 10)$. *"But suppose players 2 and 3 decide to act as one player and agree that they will split equally the joint payoff that they obtain this way. Then the game will become a two-person game between coalition* $\{2, 3\}$ *and player* 1*. Hence each side will obtain a payoff* $u_1 = u_{23} = 15$*. If players* 2 *and* 3 *later split their joint payoff* u_{23}*, then the final outcome will become* $(15, 7.5, 7.5)$*. Consequently the fact that players 2 and 3 have joined forces has actually decreased their payoffs from 10 to 7.5"* (Harsanyi, 1977, p. 203). He solves the paradox analyzing the situation by means of Zeuthen's (1930) Principle, and offering two explanations. In both cases the explanation shows the *"weakening of the bargaining position"* of the player acting on behalf of the two-player coalition. Moreover, he points out that: *"any possible solution concept will show this behavior if it satisfies the symmetry and the joint-efficiency postulates."* But this is the critical point: symmetry (as anonymity) in the classical setting ignores the possibility of bargaining under asymmetric conditions, or under rules different from unanimity. In fact, the origin of the paradox is admitting for a moment that by bargaining as a single player *"players 2 and 3 have joined forces."* This contradicts common sense views in real-world situations actually, where committees of representatives whose members represent groups of different sizes rarely bargain under unanimity. They often use nonsymmetric voting rules (even if usually chosen on no clear grounds) to bargain under. As Harsanyi put it in somewhat tautological terms: *"If two or more players form a coalition for bargaining purposes, this will tend to strengthen their bargaining position if this organizational change strengthens their determination to obtain better terms and weakens their reluctance to risk a conflict."* This is exactly what an optimal rule would implement, compensating in terms of bargaining power the loss behind Harsanyi's paradox.

It is also interesting to examine some recent work concerning 'group bargaining'. In two recent papers Chae and Heidhues (2004) and Chae and Moulin (2004) address this problem from an axiomatic point of view. Their model consists of a classical bargaining problem plus a partition of the set of players into subsets that represent the bargaining groups. Chae and Heidhues (2004) characterize axiomatically a 'group bargaining solution' for situations where different groups bargain with each other. It is an extension of Nash's solution to the bargaining problem within as well as across groups. They impose a condition of 'Representation of Homogeneous Groups' (RHG), whose interpretation is the following.

Every member of a homogeneous group obtains what he would obtain if he alone bargained on behalf of the group. They show that by adding this condition to Nash's axioms a unique solution is characterized. Namely, the *asymmetric Nash solution in which the weight of every representative is the reciprocal of the size of the group he belongs to*. That is, the reciprocal of what our Theorem 3 prescribes! The explanation is easy. They *impose* the indifference for any player between bargaining (under unanimity) directly or as a representative. But this can only be achieved by 'penalizing' representatives proportionally to the size of the group they represent. Again this is the effect of taking symmetric bargaining under-unanimity as the only conceivable way of bargaining[14].

In Chae and Moulin (2004) the family of asymmetric Nash solutions where the bargaining power of an agent in a group of size m_i is m_i^α, with $\alpha \geq -1$, is characterized axiomatically. As they point out: *"One benchmark of the family F^α is the group-insensitive solution F^1: this solution is the ordinary symmetric Nash bargaining solution, ignoring the partition altogether"*. In other words, these are exactly the weights for which (6) holds, and this is the solution that an optimal rule would implement[15].

5 CONCLUSIONS

From the point of view of voting power theory and collective decision or 'constitutional' design, this paper contributes to some clarification. Namely, an alternative to the traditional approach to the issue of the optimal voting rule, only adequate for a 'take-it-or-leave-it' committee, has been provided. A new approach consistent with the idea of a bargaining committee is the main contribution of this paper. In real world situations possibly things are most often not that black-and-white: Often the same committee acts sometimes as a 'take-it-or-leave-it' committee, and others as a bargaining committee. In any case the two clear-cut extreme cases are valuable terms of reference for more complex ones.

Let us examine the foundations of the recommendation implicit in Theorem 3. On the one hand, Nash bargaining solution can be interpreted in positive terms as a prediction

[14] They also discuss in this setting Harsanyi's 'joint-bargaining paradox', which occurs with their solution, and is explained as an effect of the reduction of the multiple "rights to talk" of the individuals of a group to a single right and thereby benefitting the outsiders. As a consequence the results do not explain why individuals would bargain in groups if bargaining takes place in the way described by the model, on the contrary the paper gives arguments against this practice. Though, the authors give two resonable explanations: either groups are given exogenously, formed by reasons that lie outside the bargaining situation, or the situation is not a pure-bargaining one. They also provide a family of solutions obtained by using a replication process that may reverse the 'paradoxical' effects of RHG.

[15] They show how this solution as well as those associated with $\alpha > 1$ are free from the 'joint-bargaining paradox'.

of the outcome of negotiations among ideally rational players in a perfectly transparent environment (e.g., Binmore (2004))[16]. While a positive interpretation is plausible for the case of two bargainers, this interpretation is not that credible for a large number of bargainers. In some cases a committee represents thousands or even millions of individuals (consider, e.g., the Council of Ministers of the EU). In such cases 'direct bargaining' is unthinkable in practical terms, but still Nash solution can be used as an ideal term of reference with normative purposes. The same applies to its extensions given by Theorems 1 and 2, which are at the base of Theorem 3.

At the foundations an egalitarian principle of equal representation has also been assumed. This justifies the desideratum of a voting rule for the committee such that all represented people see as indifferent direct bargaining and leaving it in the hands of a committee of representatives (at least under some ideal symmetry conditions). But there are situations in which the members of a committee represent groups qualitatively different. For instance, in the council governing a department or a university there are usually representatives of different categories of faculty, students and administrative staff. In this case, in addition to number, there is a second possible source to justify asymmetry. In this case the recommendation of Theorem 3 should be adjusted[17].

Even assuming that the optimal rule exists there is still the problem of the determination (or indetermination) of the preferences of the representatives for arbitrary (i.e., no necessarily $\mathcal{M}$-symmetric) configurations of preferences. In real world situations this condition will possibly fail to occur in most cases. Only in the ideal case of $\mathcal{M}$-symmetry the principle of equal representation is enough to determine an answer. But this idealization seems a reasonable term of reference if a voting rule is to be chosen.

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