

# Competition in Two-Sided Markets\*

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— PRELIMINARY: COMMENTS WELCOME —

## Abstract

There are many examples of markets involving two groups of participants who need to interact via intermediaries. Moreover, these intermediaries usually have to compete for business from both groups. Examples include academic publishing (where journals facilitate the interaction between authors and readers), advertising in media markets (where newspapers or TV channels enable adverts from producers to reach consumers), payment systems (where credit cards can be a convenient method of transaction between consumers and retailers), and telecommunications networks (where networks are used to provide links between callers and those who receive calls). The paper surveys recent theoretical work on these two-sided markets. The main questions are (i) what determines which side of the market is subsidized (if either) in order to attract the other side, and (ii) is the resulting outcome socially efficient?

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# 1 Examples of Two-Sided Markets

There are several examples of markets or institutions involving two groups of participants, say group 1 and group 2, who interact via intermediaries. Surplus is created—or destroyed in the case of negative externalities—when 1 and 2 interact, but this interaction must be mediated in some way. In most interesting cases, “two-sided” network effects are present, and the surplus enjoyed by group 1 depends on the number of group 2 participants with the same intermediary. Consider the following list of examples.

**Example 1** (*CONFERENCES*) *Surplus is usually created when speakers address an audience at a conference. Normally both sides enjoy a direct benefit from the interaction: the audience learns what the speakers have to say, while the speakers enjoy disseminating their work and being the focus of attention. Typically, conferences compete for speakers and audiences. A common way to organize the interaction is for the audience to pay to attend and for the speakers to be paid. Is there a good explanation for this feature? (Note that conferences in which most of the audience also present a paper, and where most of the speakers also listen, do not fit the theme of this paper well since there are not then two distinct groups that need to be attracted, but only the single group of ‘participants’.) Institutions such as music (or other arts) festivals would also fit into this framework.*

**Example 2** (*ACADEMIC JOURNALS*) *Very similar is the market for academic publishing. Journals compete both for authors and for readers. In the economics field, a common (but not universal) way to organize the interaction is for readers to pay and for (successful) authors to publish for free. But in other scientific disciplines, it is commonplace for (successful) authors to pay a substantial per-page charge for publication.*

**Example 3** (*UNIVERSITY TEACHING*) *Staying with the academic theme, universities have to compete for both students and for professors. As with the previous examples, quality as much as quantity is perhaps the relevant dimension here: students care about the “quality” of professors and professors care about the “quality” of their students. (A separate issue is the fact that professors also care about the quality of their fellow professors.) If a university is successful in attracting top students, it may have to offer a lower salary to a professor of given quality than otherwise. It is sometimes suggested, for example, that PhD programs at business schools often run at a loss, and this is acceptable as it gives a more “academic” atmosphere to the institution. All three of these academic examples share the feature that intermediaries **might** not be maximizing profits.*

**Example 4** (*SOFTWARE*) *Software such as “Acrobat” is used for writers disseminate content and for readers to be able to read the content. It is feasible to sell different software to these two groups. Writers are more likely to buy the “writing” software if there are many people with access to the “reading” software, and readers are more likely to invest in the reading software if they expect to receive many documents in the relevant format. A common*

charging arrangement is for “readers” to receive their software for free and for “writers” to pay a fee for their software.

**Example 5 (CONTENT IN MEDIA MARKETS)** People are more likely to buy a newspaper or watch a TV channel, the greater variety of content it contains. In the newspaper context, this content would be “columnists” and the like. In the TV case, it might be “TV personalities”. Most usually, columnists are contracted on an exclusive basis to the newspaper. Clearly, the more such content the paper attracts, the easier it will be able to attract readers. Also, it is plausible that columnists will, in addition to their salary, get benefit from communicating their opinions to a wide readership.

**Example 6 (ADVERTISING IN MEDIA MARKETS)** Advertisers/retailers wish to gain access to potential consumers to tempt them to buy products. Often advertising is ‘bundled’ with other services, such as newspapers, magazines, radio or TV, who act as intermediaries between advertisers and consumers. Revenues from advertising are often used to subsidise the media product for readers/viewers. (Historically, for the case of broadcasting, technology meant that viewers/listeners could not pay directly for the service, and so broadcasters without public funding more-or-less had to find funding from advertisers. But this is less true now for television.) In some special cases, viewers/readers might not benefit directly from advertising, or might actually dislike intrusive advertising. But, for instance in the case of informative advertising, viewers/readers might well benefit from the presence of advertising, as is the case in the next example.

**Example 7 (YELLOW PAGES)** A form of mediated advertising that is not bundled with other services are ‘yellow pages’ directories and similar products. Typically, consumers receive one or more directories for free, while advertisers pay to be included in the book. In the case of competition between directories, if there are costs involved in consulting more than one directory, consumers are perhaps more likely to use the directory with more adverts, while an advertiser will be prepared to pay more to be included in a directory with a wider readership. On the other hand, keeping readership fixed, an advertiser might prefer to be in a directory with fewer other adverts. This might be for two reasons: (i) its own advert might become “lost” among the others, or (ii) a reader might see an advert for a competing product and call that number instead.

**Example 8 (SHOPPING MALLS)** Continuing with the theme of matching consumers to retailers, another example of this is the shopping mall. Often, there are several malls in the relevant area which therefore compete for consumers and/or retailers. Typically, retailers pay rent to malls while consumers have free entry (and might also have additional features, such as free convenient parking, offered to attract them). Typically, consumers care about the number and quality of retailers when they decide which mall to visit, and obviously retailers care about the number consumers coming through the mall. Like the previous yellow pages example, business stealing effects might mean that, all else equal, a retailer might prefer to be in a mall with fewer other (competing) shops.

**Example 9 (PAYMENT SYSTEMS)** *Still continuing with the theme of facilitating interactions between consumers and retailers, consider the various methods of paying for products, including cash and various kinds of ‘card’ payment. Both consumers and retailers may derive direct benefits (in terms of convenience or security) from using one method over another. (This is the relevant surplus created from the interaction, not the surplus created by the consumer buying the product itself, which will, in many cases, occur regardless of the chosen payment method.) To the extent that a consumer only chooses a limited number of payment instruments, he/she will have regard for the number of retailers who choose to accept a given payment method. Similarly, if there are set-up costs in being able to accept a given payment method, a retailer will have regard for the number of consumers who use that method. A common contractual arrangement is for consumers to be able to use a payment card with little or no charge, and for retailers to be asked to cover the costs of the transaction.*

**Example 10 (TELECOMMUNICATIONS WITH CALL EXTERNALITIES)** *Both callers and recipients of calls typically derive some benefit from telephone calls, which are mediated by a telecommunications network. (The recipient’s benefit is termed the “call externality”.) There is often vigorous competition between networks for subscribers. Most subscribers both make and receive calls, and so it is less clear that there are ‘two sides’ in this market. However, if one thinks of “subscription to a network” as one side and “number of calls made to a network” as the other side, then this industry fits into this framework. The more calls a network receives (for instance, because it sets a low charge for delivering calls), the more attractive the network is for subscribers; and naturally, the more subscribers a network has, the more calls it will receive. An inefficient pattern of pricing can sometimes be seen, with networks exploiting their monopoly position over delivering calls to their subscribers and using the proceeds to subsidize connections to the network (perhaps in the form of ‘free mobile handsets’).*

**Example 11 (FISCAL COMPETITION)** *Governments are interested in attracting mobile factors of production (manufacturing plant, football teams, skilled labour) with tax incentives. (Factors are mobile to some extent, and so governments must compete for such factors.) To the extent that factors are complementary in a production process, governments are keen to attract ‘both sides’ of the production process simultaneously. Different factors may have different levels of mobility, however, and this will affect the equilibrium tax incidence.*

**Example 12 (UNREGULATED BANKING)** *Banks act as intermediaries between savers and borrowers. Without effective banking regulation, bank failure is a possibility, and so potential savers will be interested in how secure their funds would be, and hence how well the bank is doing on the lending side of the market. As such, this fits into the topic of this paper. However, effective deposit regulation will largely eliminate this risk, and the consequent externality that otherwise exists between the two sides of the market.*

**Example 13** (*MATCHMAKING SERVICES*) *Similar features are present in the match-making markets, such as those for dating services. Each side of the market cares positively about the number of people on the other side (and perhaps they care negatively about the number of people on their own side). It is sometimes the case that one side of the market is subsidized, and used to ‘attract’ the other side from whom surplus can then be extracted.*

**Example 14** (*SMOKERS AND NON-SMOKERS*) *In a restaurant or aircraft, non-smokers (and even smokers) typically care about the number of people smoking in the space. Unlike many of the previous examples, this is a **negative** inter-group externality. Another difference with the previous examples is that the ‘intermediary’ can affect the extent of the externality, namely, by instituting a non-smoking policy. Such a policy presumably entails a utility loss for smokers, and in a competitive environment, an intermediary must trade off this disutility against the extra attraction it now has for the non-smokers. In the case of airlines (outside of Asia at least), the equilibrium is clearly to have a non-smoking policy, whereas for restaurants the pattern is more mixed.*

This is a highly selective list. It ignores many software applications that exhibit the same features, for instance.<sup>1</sup> And of course there are very many examples of ‘two-sided’ markets with competing intermediaries, and which are more ‘ordinary’ than the examples above. For instance, a ‘firm’ needs to compete for labour (and other factors of production) at the same time as it competes for consumers of its output. However, such examples are different in that participants on one side of the interaction do not really care how well the intermediary does on the other side, but only about the terms on which they deal with the intermediary. In sum, in these more prosaic examples, there are no inter-group network externalities at work.<sup>2</sup>

In addition, there are many examples of important inter-group externalities which are *not* mediated at all. Obvious examples come from economic geography, where one group is more likely to locate in an area where another, complementary group has located (and *vice versa*). This paper does not discuss this well-understood phenomenon further.<sup>3</sup>

This paper is largely about the case of *competitive* intermediation. Thus, to a greater or lesser extent, any profits made on one side of the market by an intermediary are used to attract participants on the other side. Perhaps the main questions are (i) what determines

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<sup>1</sup>See Rochet and Tirole (2001) for several examples of two-sided competition in the software and internet industries.

<sup>2</sup>This discussion assumes that the intermediary is the price setter for the two groups. However, if wages, say, are determined by a bargaining process of some kind (perhaps as in the model of Stole and Zwiebel (1996)), then workers could care about how well the firm does in the final product market since that affects the surplus over which it bargains.

<sup>3</sup>A recent paper that examines this kind of issue is Ellison and Fudenberg (2002). This paper investigates competing (standard) auction markets and, in particular, whether two auction markets can co-exist even if there is no intrinsic product differentiation between them.

which side of the market is subsidized (if either) and (ii) when is the resulting allocation socially efficient?

In broad terms, there are two features that determine the answer to these questions. First, does one side of the market benefit more from the interaction than the other? (If group  $A$  gains little from interacting with group  $B$ , then it is plausible that the former group will need an extra incentive to participate.) Second, are there reasons why one or both sides of the market will use only a *single* intermediary? (Rochet and Tirole (2001) use the term “single-homing” for this phenomenon.) If so, then competition can be expected to be particularly intense on that side. For instance, if people tend to read just a single newspaper, perhaps because of time constraints, then the newspaper has a monopoly position over delivering adverts to their readers and can therefore extract rents from advertisers. These rents are then used to attract readers. By contrast, if people read several newspapers then a given paper has a monopoly in providing access only to its *exclusive* readers, which could be a small group.

## 2 The Literature

*Monopoly Intermediation:* Most of the work on payment systems has been on the monopoly case—see Baxter (1983) for a pioneering contribution, followed more recently by (among others) Rochet and Tirole (2002), Schmalensee (2002) and Wright (2001). See also Baye and Morgan (200) and Baye and Morgan (2001) for an analysis of monopoly intermediation on the internet.

Gehrig (1993) provides an interesting early analysis of this issue. (His paper is mainly about the case of a monopoly intermediary.) In his model there is a homogeneous product, and a group of buyers and group of sellers of this product. Buyers differ in their reservation price for a unit of the product, as do sellers. Without intermediation, buyers and sellers must *search* for a suitable trading partner, which entails delay and an uncertain eventual price. Suppose that an intermediary is introduced who can, unlike the other agents, *broadcast* a buying and selling price to all agents. Agents can choose whether to go to the intermediary or to remain in the search market. If the intermediary attracts fewer sellers than buyers, then buyers must be rationed according to some rule, and similarly if there are more sellers than buyers. (If an agent is rationed, she will be returned to the search market.) In this sense there are two-sided network externalities: for a given price, sellers are better off the more buyers there are at the intermediary (and they are worse off with more other sellers), since this reduces the chance that they will be rationed. Gehrig shows that one natural equilibrium involves the intermediary offering a positive bid-ask spread and some agents (those without strong gains from trade) continuing to use the search market. As one would expect, the size of the bid-ask spread increases with the frictions in the search market.

*Early work on two-sided competition with network effects:* Stahl (1988) and Yanelle (1989)

model firms as competing (either sequentially or simultaneously) both for inputs and for outputs. A natural strategy to investigate is for a firm to try to compete hard for inputs (by pricing low in that side of the market), in order to disadvantage its rival in the retail side of the market. When there is a fixed relationship between inputs and outputs (one unit of input is transformed into one unit of output), rationing plays an important role (as with Gehrig (1993)). Because of this, there are inter-group externalities present that affect the competitive outcome. We will discuss a somewhat more tractable model of firms competing both for inputs and for outputs (but without a fixed relationship between the two) in section 4.2 below.<sup>4</sup>

*Call termination in telecommunications:* Section 3.1 of Armstrong (2002b) and Wright (2002) propose a model of competition between mobile telecommunications networks. Here, mobile networks compete to sign up subscribers. They also charge fixed line networks when they deliver calls to their mobile subscribers. The more they charge for this, the fewer calls a mobile subscriber will receive. Since subscribers typically like getting calls, a mobile network must trade off which side of the market to make money on. A common practice is for networks to subsidise their subscribers' charges (via handset subsidies and the like), and to extract profit from the people who call their subscribers.

*Advertising in media markets:* Anderson and Coate (2001) present a model where TV channels compete for viewers (but not using prices), and advertisers wish to gain access to these viewers.<sup>5</sup> Viewers can view adverts as a 'nuisance'. It is assumed that viewers watch only a single channel over the relevant time horizon, and so a channel has a monopoly over providing this access. As a result there is socially too little advertising in equilibrium whenever the 'nuisance factor' of adverts is sufficiently unimportant. (If viewers greatly dislike adverts then there will be too much advertising.)

Rysman (2002) is, to my knowledge, the only structural empirical investigation into markets with two-sided network externalities. This paper estimates the importance of network effects (on both sides of the market) in the market for yellow pages. The theoretical underpinnings of his model are discussed further in section 5.4 below. He estimates that externalities are significant on both sides of the market: users are more likely to use a directory containing more adverts, while an advertiser will pay more to place an advert in a directory that is used by more people.

*Competing Matchmakers:* A class of examples of "competition in two-sided markets" is that of competing matchmakers, such as dating agencies, real estate agents, and internet "business-to-business" websites. The focus of these examples is naturally very much on the

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<sup>4</sup>See Gehrig (1996) and Yanelle (1997) for analyses of the banking industry, where the two sides of the market are "savings" and "loans", that build on these earlier papers.

<sup>5</sup>Page 588 of Armstrong and Vickers (2001) has a short discussion of advertising in media markets, which involves this same kind of model. See also Gabszewicz, Lauussel, and Sonnac (2001) for an analysis of the effect of advertising revenue on the political stance taken by newspapers.



quality of a given match, and heterogeneity of agents plays a crucial role. As a result, there is a rich set of contracting possibilities in these examples: for instance, one might have ‘joining fee’ in combination with a fee in the event of ‘successful match’. For instance, a new entrant might find it useful to charge its clients only in the event of a successful match, since such a strategy means that potential customers are not deterred from joining the new firm by the possibility that they will pay an up-front fee and yet too few other people have joined. These issues are largely ignored in the following analysis. Relevant work in this area is van Raalte and Webers (1998), Caillaud and Jullien (2001) and Caillaud and Jullien (2002). A focus of this work is the possibility of asymmetric outcomes, including those where one intermediary corners the market. Again, I do not investigate this important issue in this paper.

To explore one paper in more detail, consider Caillaud and Jullien (2002). This paper examines a model in which each buyer generates positive surplus with exactly one of the sellers.<sup>6</sup> (Given that there is a continuum of buyers and sellers, there is therefore no scope for ‘bypassing’ the intermediaries by independent search, as in Gehrig (1993), since the chance of meeting your unique match is zero.) If intermediary  $i$  has a fraction  $n_i^l$  of the type  $l$  agents (where  $l$  indexes ‘buyers’ and ‘sellers’, say) then if an agent of the other type joins that intermediary, the chance of that intermediary having your match is  $n_i^l$  and the chance of that intermediary discovering this good match is then  $\lambda n_i^l$ . Thus,  $\lambda < 1$  represents the degree of precision of the intermediary’s matching technology. Therefore, an agent’s benefit from joining a given intermediary is proportional to the fraction of the other type of agent who have also joined the intermediary. Finally, intermediaries are assumed not to be intrinsically differentiated.

Caillaud and Jullien discuss two scenarios. First is the case where, for exogenous reasons, each agent can only use the services of a single intermediary. In this case, since there is no product differentiation, it is socially optimal for there to be a single intermediary. The paper shows that this is also the only equilibrium outcome, and that the ‘successful’ firm makes zero profits. The equilibrium price structure in this case involves charging for successful matches at the maximum possible rate and subsidizing participation. (As mentioned above, a good tactic for an entrant is to generate revenue from transactions fees rather than registration fees. However, if the incumbent does the same then there is no scope for entry.) Second is the case where agents can register with both intermediaries. This might be socially desirable, since there is a better chance of making a match with two intermediaries. (If the parameter  $\lambda$  is less than 1, there is a chance that a single intermediary will fail to make the match, and this chance is reduced with two intermediaries.) Since an intermediary performs a useful function for agents, even if agents also use the rival intermediary, equilibrium will generally involve positive profits. The authors show that the resulting equilibrium is asymmetric: one firm offers a low transactions charge and so “gets the business” whenever that intermediary makes the match, while the other firm only gets the business when it alone has made the

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<sup>6</sup>This feature implies that there are no same-side “congestion” externalities as there are in, say Gehrig (1993), since sellers do not compete against each other for buyers.

match.

*Competing Marketplaces:* Gehrig (1998) presents a model with two marketplaces, in each of which there are located a variety of differentiated firms. Consumers live on a line between these two markets and decide which market to visit on the basis of a comparison of the prices and varieties offered at the two ends. There is a separate tax authority for each market which aims to maximize revenue, which it generates from taxing the transactions within the market. The model assumes is that the fiscal authorities make their choices after firms have made their location decisions, and so there is no scope for using fiscal policy to attract firms to a market. Gehrig investigates competition *within* a market. In section 5.5 I examine a related model of competing “shopping malls”, where the simplifying assumption is made that there is no *intra*-market competition.

*Pricing complementary software products:* Parker and Van Alstyne (2000) present an analysis of two-sided competition in software markets such as Acrobat, and the analysis is done both for monopoly and for competition. They show how a monopolist might well want to distribute a complementary piece of software for free in order to stimulate demand for the fully functioning version. They then use this analysis to suggest why a monopolist in one segment (say, operating systems) might wish to enter a complementary market (say, web browsers) even if it ends up making no money in the competitive market.

A closely related paper, which, like this paper, does not focus on any single industry, is Rochet and Tirole (2001). To be concrete, consider the credit card industry, although their paper discusses many other applications as well. Therefore, one side of the market is the retailers and the other side is the consumers. Retailers are all monopolies, and there is no competition between retailers. Much of their analysis involves a model where each agent obtains a payoff per transaction with the other side of the market, and there are no fixed charges or costs. (This assumption implies the simplifying feature that an agent’s choice of card does not depend on the *number* of agents on the other side who use the same card.) The two credit card companies offer charges for using their payment system: a per-transaction charge to consumers and a per-transaction charge to retailers. Essentially, given these charges, retailers first decide which cards to accept (either one card, both cards, or no cards). Then consumers (who wish to buy a single unit from each monopoly retailer) decide for each retailer whether to use the retailer’s chosen card (if that shop accepts a single card) or, if both cards are accepted, which card to use. Thus, there is an asymmetry in the model: retailers first decide which cards to accept, and then consumers decide which of these acceptable card to use (if any).

A retailer deciding between taking one or both cards faces a trade-off. If it accepts a single card (the card with a high intrinsic benefit or a low charge) then its consumers have a stark choice between paying by this card or not using a card at all. Alternatively, if it accepts both cards then (i) more consumers will choose to pay by some card but (ii) fewer

consumers will choose to use the retailer’s favoured card. If one card reduces its charge to retailers, this will have two effects: firstly, it will cause some retailers who previously did not use either card to join its network, and secondly, it will cause some retailers who previously accepted both cards to accept only the low-charge card. The share of the charges that are borne by the retailers then depends (in part) on how closely consumers view the two cards as being substitutes. If consumers do not switch cards much in response to a price cut, then consumers should pay a large share of the total transaction charge; if consumers view the cards as close substitutes, then the retailers will bear most of the charges in equilibrium.

Rochet and Tirole also consider the case where there are fixed fees as well as per-transaction fees. In this case, whether a consumer decides to use a given card depends upon the number of retailers who accept the card. (This feature was absent from their benchmark model with per-transaction payoffs.) They make the simplifying assumption that consumers choose to use only a single card (if any). I will describe a somewhat similar model (where the chosen application is to shopping malls) in section 5.5 below.

No single, transparent model can hope to encompass all the features of the above examples. Therefore, in the following sections we present three models that cover a variety of plausible situations. First is the benchmark case of a single intermediary (section 3). Second is the case of competing intermediaries, but in the special case where both sides of the market deal with only a single intermediary (section 4). Third is the case where one side of the market deals exclusively with a single intermediary, while the other group deals with all intermediaries in an attempt to gain access to all agents on the other side (section 5).

### 3 Monopoly Intermediation

Here we present an analysis of monopoly intermediation.

This framework does not apply to most of the examples listed in the Introduction. However, perhaps some conferences might fit into this framework, historically “yellow pages” was effectively a monopoly of the incumbent telephone company, and some shopping malls are far enough away from others that the monopoly paradigm might be appropriate.

This section is closely related to section 2 of Rochet and Tirole (2001).<sup>7</sup> There are two

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<sup>7</sup>The two main differences with their analysis are the following. First, in the present model the level of participation is determined by the total surplus  $u_l = \alpha_l n_m - p_l$ , whereas in Rochet and Tirole (2001) participation is determined by the ‘surplus per transaction’, which in our notation means that

$$n_l = \phi_l(\alpha_l - p_l/n_m) .$$

This essentially means that participation *decisions* on one side do not depend on the number of participants on the other side (although the *surplus* does depend on the number on the other side).

Second, Rochet and Tirole (2001) assume that costs are incurred on a ‘per-transaction’ basis, i.e. they are proportional to  $n_1 n_2$ , whereas I assume they are incurred on a per-participant basis, i.e., costs are  $n_1 f_1 + n_2 f_2$ . One feature of this modelling approach is that costs are associated directly with participation,

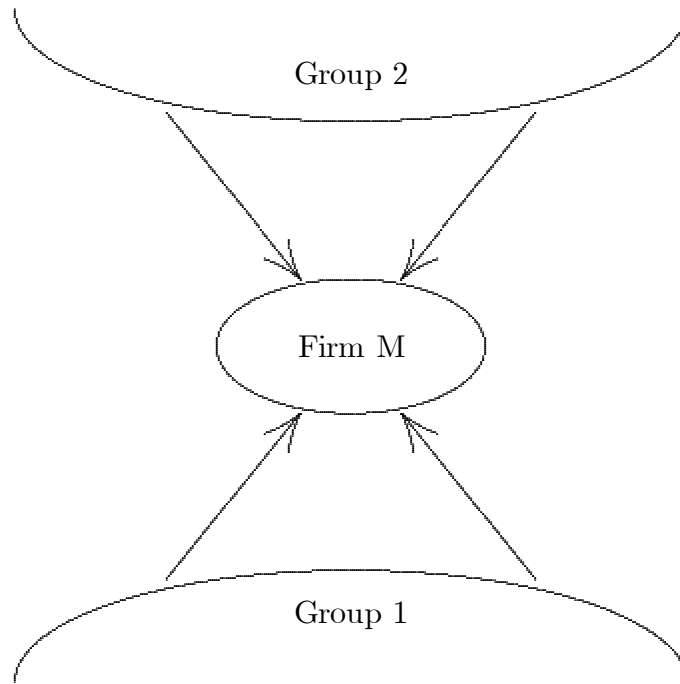


Figure 1: Monopoly Intermediation

groups of agents,  $l = 1, 2$ . If the “utility” (to be defined) offered to group  $l$  is  $u_l$ , suppose that the number of group  $l$  who participate is

$$n_l = \phi_l(u_l)$$

for some increasing function  $\phi_l$ . Here, utility is determined by

$$u_l = \alpha_l n_m - p_l ,$$

so that a member of group  $l$  enjoys additional utility  $\alpha_l$  from each member of the other group  $m$ . Here  $p_i$  is the intermediary’s charge to a group  $i$  participant. The intermediary incurs a fixed cost of serving a group  $l$  participant of  $f_l$ . If we consider the intermediary as offering utilities  $\{u_l\}$  rather than prices  $\{p_l\}$ , then the implicit price for group  $l$  is  $p_l = \alpha_l n_m - u_l = \alpha_l \phi_m(u_m) - u_l$ . Therefore, the intermediary’s profit is

$$\pi = [\alpha_1 \phi_2(u_2) - u_1 - f_1] \phi_1(u_1) + [\alpha_2 \phi_1(u_1) - u_2 - f_2] \phi_2(u_2) .$$

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and so it is possible to discuss which group subsidizes the other, whereas this is not so straightforward in the Rochet-Tirole framework. See also Parker and Van Alstyne (2000) for related analysis of the monopoly model.

Let consumer surplus of group  $l$  be  $v_l(u_l)$ , where  $v'_l(u_l) \equiv \phi_l(u_l)$ . Then total welfare is

$$w = \pi + v_1(u_1) + v_2(u_2) .$$

Notice that both the profit maximizing and welfare maximizing outcomes depend only on the *sum* of externality parameters,  $\alpha = \alpha_1 + \alpha_2$ . It is easily verified that the first-best welfare maximizing outcome involves utilities satisfying:

$$u_l = \alpha \phi_m(u_m) - f_l .$$

Or, since implicit prices are  $p_l = \alpha_l \phi_m(u_m) - u_l$ , we have that socially optimal prices are

$$p_l = f_l - \alpha_m \phi_m(u_m) . \quad (1)$$

As one would expect, prices should ideally be set below cost (if  $\alpha_m > 0$ ) to take account of the externality enjoyed by the other side of the market. The profit-maximizing prices, by contrast, satisfy

$$p_l = f_l - \alpha_m \phi_m(u_m) + \frac{1}{\eta_l} , \quad (2)$$

where

$$\eta_l(u_l) \equiv \frac{\phi'_l(u_l)}{\phi_l(u_l)} > 0$$

measures the responsiveness of participation to increases in utility for group  $l$ . Expression (2) states that the profit-maximizing prices are set above the first-best levels by a factor inversely related to the elasticity of participation. It is possible that the profit-maximizing outcome might involve group  $l$  being offered a subsidised service, i.e.,  $p_l < f_l$ . From (2), this happens if the elasticity of participation  $\eta_l$  is very large and/or if the external benefit  $\alpha_m$  enjoyed by the other group is large.

Finally, consider the Ramsey prices, i.e., the prices that maximize surplus subject to the firm just breaking even. (From (1), the first-best prices are both below cost, and so will cause the firm to run at a loss.) These Ramsey prices are

$$p_l = f_l - \alpha_m \phi_m(u_m) + \frac{\lambda}{\eta_l(u_l)} \quad (3)$$

where  $0 \leq \lambda \leq 1$  is chosen that the firm just breaks even, i.e., so that

$$\phi_1 \left[ \frac{\lambda}{\eta_1} - \alpha_2 \phi_2 \right] + \phi_2 \left[ \frac{\lambda}{\eta_2} - \alpha_1 \phi_1 \right] = 0 ,$$

i.e.,

$$\lambda = \frac{\alpha\phi_1\phi_2}{\phi_1/\eta_1 + \phi_2/\eta_2}.$$

Therefore, from (3) we obtain

$$p_l - f_l = \frac{\alpha}{\eta_l} \frac{\phi_1\phi_2}{\phi_1/\eta_1 + \phi_2/\eta_2} - \alpha_m\phi_m = \phi_m \left[ \alpha \frac{\phi_l/\eta_l}{\phi_1/\eta_1 + \phi_2/\eta_2} - \alpha_m \right]. \quad (4)$$

By construction, (except for knife-edge cases) one side of the market must be subsidised at the expense of the other in the Ramsey problem. From (4), it is group  $l$  rather than group  $m$  that is subsidised if

$$\frac{\phi_l}{\eta_l\alpha_m} < \frac{\phi_m}{\eta_m\alpha_l}.$$

This is the case when (i) group  $l$  is the more ‘elastically supplied’ (all else equal) or (ii) group  $l$  causes the greater external benefit to the other group than *vice versa* (all else equal). For instance, if group 1 essentially enjoys no benefit from interacting with group 2 ( $\alpha_1 \approx 0$ ) then group 1 should certainly be subsidised at the Ramsey optimum. (This would not necessarily be true in the case of profit maximization.)

## 4 Two-Sided Exclusive Intermediation

The second model involves competing intermediaries, but assumes that, for exogenous reasons, each participant (from either group) chooses to join just a single intermediary.

This assumption is too extreme to be strictly applicable in most circumstances. But it is relevant, for instance, for:

- Example 1, if for some reason speakers and listeners only attended a single conference over the relevant period.
- Example 3, since students and teachers belong to a single institution.
- Example 5, when columnists are exclusively employed by a newspaper and when people read a single newspaper.
- Example 8, if consumers only visited a single mall (which is quite plausible) and retailers only located in a single mall (less plausible, especially for chainstores).
- Example 11, if workers and firms located in a single region.
- Example 13, if, instead of dating agencies, we think about ‘dating bars’ and where people do not visit more than one bar in the relevant period.
- Example 14, since diners only go to a single restaurant at a time.

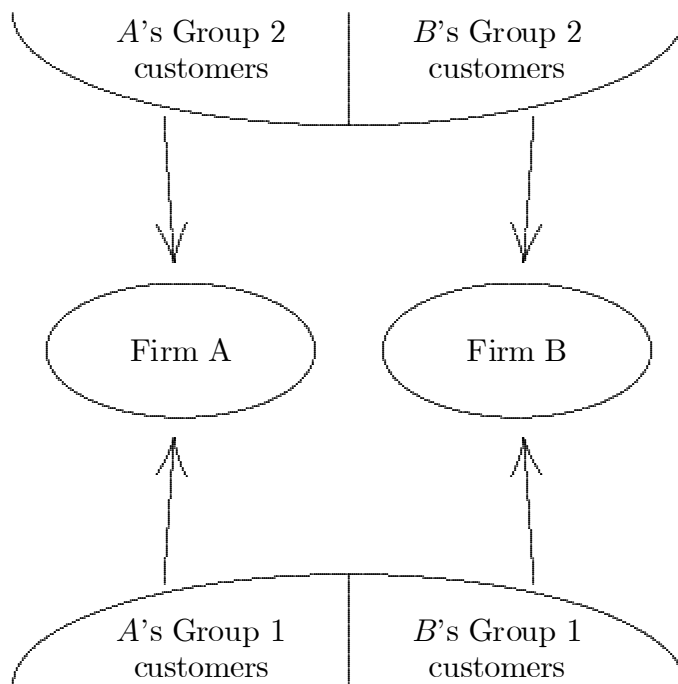


Figure 2: Competitive Exclusive Intermediation

## 4.1 The Model

The model is made up of the following ingredients, which extend the previous monopoly model in the obvious way. There are two groups of participants,  $1$  and  $2$ . (These will be denoted  $l$  or  $m$  as before.) There are two intermediaries,  $A$  and  $B$ , who enable the participants to interact. (These will be denoted  $i$  or  $j$ .) Suppose that intermediary  $i$  offers a group  $m$  participant a utility level  $u_m^i$ . Intermediaries compete for market share within each group. If group  $m$  is offered the choice of utilities  $u_m^A$  and  $u_m^B$  from the two intermediaries, the number who go to intermediary  $i$  is given by the familiar Hotelling specification:

$$n_m^i = \frac{1}{2} + \frac{u_m^i - u_m^j}{2t_m}. \quad (5)$$

Here,  $t_m$  is the differentiation parameter for group  $m$  (which might differ for the two groups).

Next we describe how the utilities  $u_m^i$  are determined. The crucial ingredient is that a member of group  $l$  cares about the number of members of the *other* group  $m$  who go to the same intermediary. (For simplicity, we ignore the possibility that a member of group  $l$  cares also about the number of members of group  $l$  who go to the same intermediary—see Appendix A for a generalized version of this model, which allows for this possibility, as well as the case with more than two groups of participants.) Therefore, utilities are determined in

the following way: if intermediary  $i$  has a number  $n_m^i$  of group  $m$  participants as customers, the utility of a group  $l$  member at this intermediary is

$$u_l^i = \alpha_l n_m^i - p_l^i, \quad (6)$$

where  $p_l^i$  is the price the intermediary charges a group  $l$  member for its services. The parameter  $\alpha_l$  measures the inter-group externality for group  $l$  participants.

We suppose that the network externality parameters  $\alpha_l$  are ‘small’ compared to the differentiation parameters  $t_l$  so that we can focus on symmetric equilibria where each intermediary captures half the market for both groups of participants. (If this were not the case, the network externality effects could outweigh brand preferences, and there would exist equilibria where one intermediary corners both sides of the market.)

Suppose that intermediaries  $A$  and  $B$  offer the four prices  $(p_1^A, p_2^A)$  and  $(p_1^B, p_2^B)$ .<sup>8</sup> Then solving the simultaneous equations (5) and (6) implies (eventually) that

$$n_1^i = \frac{1}{2} + \frac{1}{2} \frac{\alpha_1(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{t_1 t_2 - \alpha_1 \alpha_2}; \quad n_2^i = \frac{1}{2} + \frac{1}{2} \frac{\alpha_2(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{t_1 t_2 - \alpha_1 \alpha_2}. \quad (7)$$

(Assume that brand preferences are more important than network effects in the sense that  $t_1 t_2 - \alpha_1 \alpha_2 > 0$ .) The expressions (7) are perhaps the central insight needed to solve this model. To get some insight into where the expressions come from, the following thought experiment seems useful. Suppose that firm  $i$  decreases its price to group 1 by an amount  $\varepsilon$ , say. Consider this ‘adaptive expectations’ story to infer the total effect on its customer numbers for the two groups:

1. The direct utility of  $A$ ’s group 1 members immediately increases by the price reduction  $\varepsilon$ ;
2. From (5),  $A$  immediately attracts a further  $\varepsilon/2t_1$  group 1 customers;
3. From (6), the *relative* utility to group 2 members from being at  $A$  rather than  $B$  increases by  $\varepsilon\alpha_2/t_1$ ;
4. As a result,  $A$  attracts a further  $\varepsilon\alpha_2/(2t_2t_1)$  group 2 customers;
5. The relative utility to group 1 member from being at  $A$  increases by  $\varepsilon\alpha_1\alpha_2/(t_1t_2)$ ;
6.  $A$  attracts a further  $\varepsilon\alpha_1\alpha_2/(2t_1t_1t_2)$  group 1 customers;
7. The relative utility to group 2 members from being at  $A$  increases by  $\varepsilon\alpha_2\alpha_1\alpha_2/(t_1t_1t_2)$  which causes  $A$  to attract a further  $\varepsilon\alpha_2\alpha_1\alpha_2/(2t_2t_1t_1t_2)$  group 2 customers, and so on *ad infinitum*.

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<sup>8</sup>Thus we assume that the intermediaries compete in prices, rather than utilities, say. The choice of strategic variable makes a difference to the outcome. See section 2 of Armstrong (2002a) for a comparison of the outcomes when firms compete in prices or compete in utilities in two-sided markets.



Therefore, the total increase in group 1 customers as a result of this price reduction is

$$\frac{\varepsilon}{2t_1} \left\{ 1 + \frac{\alpha_1\alpha_2}{t_1t_2} + \left( \frac{\alpha_1\alpha_2}{t_1t_2} \right)^2 + \dots \right\} = \varepsilon \frac{t_2}{t_1t_2 - \alpha_1\alpha_2} .$$

This gives the own-price elasticity of demand in (7), and a similar infinite series yields the corresponding cross-price elasticity. (This thought experiment also makes clear why we need  $t_1t_2 - \alpha_1\alpha_2 > 0$  for this process to converge. If say, the two firms offer the same initial price pair, and one firm slightly reduced one of its prices, then the above “feedback loop” would mean that that firm captured all members of both groups.)

Turning to the cost side, suppose that an intermediary has a fixed cost  $f_l$  for serving a member of group  $l$ . Therefore, profits for intermediary  $i$  are

$$\pi^i = (p_1^i - f_1)n_1^i + (p_2^i - f_2)n_2^i$$

or

$$\begin{aligned} \pi^i = (p_1^i - f_1) & \left[ \frac{1}{2} + \frac{1}{2} \frac{\alpha_1(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{t_1t_2 - \alpha_1\alpha_2} \right] \\ & + (p_2^i - f_2) \left[ \frac{1}{2} + \frac{1}{2} \frac{\alpha_2(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{t_1t_2 - \alpha_1\alpha_2} \right] . \end{aligned}$$

Further manipulations show that the symmetric equilibrium consists of the following prices

$$p_1 = f_1 + t_1 - \alpha_2 ; p_2 = f_2 + t_2 - \alpha_1 . \quad (8)$$

Therefore, the price-cost margin for group  $l$  is  $t_l - \alpha_m$ , i.e., the equilibrium price-cost margin for a particular group is equal to the differentiation parameter for that group (which represents how ‘competitive’ that side of the market is), minus the externality that joining the intermediary has on the *other* group who have joined the intermediary. Thus, an intermediary will target one group more aggressively than the other if that group is (i) on the more competitive side of the market and/or (ii) causes larger benefits to the other group than *vice versa*.

Each intermediary makes total profit

$$\pi = \underbrace{\frac{1}{2}(t_1 - \alpha_2)}_{\text{profit from group 1}} + \underbrace{\frac{1}{2}(t_2 - \alpha_1)}_{\text{profit from group 2}} .$$

In particular, the cross-group network effects acts to reduce profits compared to the case where  $\alpha_1 = \alpha_2 = 0$ , since firms have an additional reason to compete hard for market share. (This effect is the same as that in the traditional “single-group” network effects model.)

There is no scope for meaningful welfare analysis with this model since prices are just transfers between agents: any (symmetric) pair of prices offered by the two intermediaries will yield the same level of total surplus.

## 4.2 Example: “Columnists and Readers”

This example can be analyzed by direct application of the model in section 4 above.<sup>9</sup> Call readers “group 1” and columnists “group 2”. Suppose that the unit cost of producing a paper is  $f_1$  (where this does not depend on the number of columnists employed). Suppose that there is no cost of employing a columnist (over and above their salary), i.e., that  $f_2 = 0$  in the model. Then expression (8) shows that the equilibrium cover price of a paper is

$$p_1 = f_1 + t_1 - \alpha_2 ,$$

while the salary paid to a columnist is

$$s_2 = \alpha_1 - t_2 .$$

Thus, the fact that a columnist likes a wider readership ( $\alpha_2 > 0$ ) is reflected not in a reduction in his/her salary, but in a reduction of the price of the newspaper. As one would expect, the more ‘competitive’ the market for columnists ( $t_2$ ) the greater the salary that needs to be paid. More interestingly, there is a sense in which columnists are paid ‘too much’: ignoring the component  $t_2$  (which could be small), their salary is  $\alpha_1$  which is the benefit they would cause to the *entire* population of readers, not just the readers of their chosen newspaper. (The benefit to the readers of their newspaper is  $\alpha_1/2$ .) The reason for this apparently excessive rate of remuneration is the knowledge that, if the newspaper fails to employ the columnist, the rival paper will do so, and this will disadvantage it in the battle for readers.<sup>10</sup>

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<sup>9</sup>As mentioned, this model is a variant of the “competition in inputs and outputs” models of Stahl (1988) and Yanelle (1989) where (i) there is product differentiation and (ii) there is no fixed relationship between inputs and outputs. Section 3.2 of Spulber (1999) has a discussion of these models with product differentiation, although he continues to assume a fixed input-output relationship.

<sup>10</sup>The feature that *exclusive* content will command a premium in competitive media markets was also seen in section 3 of Armstrong (1999).

### 4.3 Example: Smokers and Non-Smokers

Consider two restaurants competing for diners. There are two groups of diners: (potential) smokers and non-smokers. Smokers have utility

$$u_s = \begin{cases} v - p & \text{if they can smoke} \\ v - \delta - p & \text{if they can't smoke} \end{cases}$$

(Here  $v$  is the intrinsic utility from the meal, and  $p$  is the price for the meal.  $\delta$  is the disutility from not smoking. Non-smokers have utility

$$u_n = \begin{cases} v - \alpha n_s^i - p & \text{if there are } n_s^i \text{ people actually smoking} \\ v - p & \text{if no one is smoking} \end{cases}$$

Both groups have Hotelling preferences for the two restaurants, and have the same ‘transport cost’ parameter  $t$ . The fraction of smokers in the relevant population is  $\lambda$ , and the likelihood of being a smoker is uncorrelated with brand preference for restaurants. The cost of a meal is  $f$ . Assume that restaurants cannot price discriminate on the basis of smoking, but rather can choose whether or not to ban smoking. In general, the restaurant makes two decisions: the price of a meal  $p$  and whether or not to allow smoking. However, for ease of exposition suppose that prices are exogenously fixed (and symmetric).

If both restaurants follow the same policy—either both allow smoking or both forbid it—then they will share the total market, and each will have a number of customers equal to  $\frac{1}{2}$ . Suppose one restaurant forbids smoking while the other permits it. Then the smoking restaurant attracts a fraction

$$\frac{1}{2} \left( 1 + \frac{\delta}{t} \right) \tag{9}$$

of the smokers, which then number

$$n_s = \frac{\lambda}{2} \left( 1 + \frac{\delta}{t} \right)$$

in all. In this case the utility of non-smokers at the smoking firm is lower than at the non-smoking firm by a margin

$$\frac{\alpha \lambda}{2} \left( 1 + \frac{\delta}{t} \right)$$

and so the smoking firm obtains only a fraction

$$n_n = \frac{1}{2} \left( 1 - \frac{\alpha \lambda}{2} \left( 1 + \frac{\delta}{t} \right) \right) \tag{10}$$

of the non-smokers. (Note that for  $t$  small, so that the market is quite competitive, the asymmetric policy will cause *all* the non-smokers to go to the non-smoking firm, and all the smokers to go to the smoking firm.) In aggregate, then, the smoking firm attracts a total number of customers equal to

$$\frac{\lambda}{2}\left(1 + \frac{\delta}{t}\right) + \frac{1-\lambda}{2}\left(1 - \frac{\frac{\alpha\lambda}{2}\left(1 + \frac{\delta}{t}\right)}{t}\right).$$

This is less than  $\frac{1}{2}$  if

$$(1-\lambda)\alpha\left(1 + \frac{\delta}{t}\right) > 2\delta. \tag{11}$$

In this case both restaurants operating a non-smoking policy is the equilibrium. Otherwise, both restaurants permitting smoking is the equilibrium. (There can be no asymmetric equilibrium in this particular model, since a firm can always guarantee itself a share of diners equal to  $\frac{1}{2}$  by copying the strategy of its rival.)

Consider next social welfare under the various policy combinations. Firms make the same total profit in all cases, so their contribution to welfare can be ignored. If both firms forbid smoking, then total consumer welfare (including transport costs) is

$$W_N = v - p - \frac{t}{4} - \lambda\delta.$$

If both firms permit smoking, then consumer welfare is

$$W_S = v - p - \frac{t}{4} - \frac{(1-\lambda)\lambda\alpha}{2}.$$

Clearly,  $W_N > W_S$  if  $(1-\lambda)\alpha > 2\delta$ .<sup>11</sup> This is a *stronger* condition than that for the equilibrium to be non-smoking in (11). Therefore, in this particular model we see that there is scope for public policy intervention: it might be socially optimal to prohibit smoking in restaurants when the equilibrium is to permit smoking (but never *vice versa*).<sup>12</sup>

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<sup>11</sup>One might also consider welfare under an asymmetric policy, i.e., where exactly one restaurant permits smoking. (This is never an equilibrium in this model.) Compared to the symmetric policies, the asymmetric situation involves better ‘sorting’ between smokers and non-smokers (there are more smokers with smokers and more non-smokers with non-smokers), but worse sorting between customers and restaurants (a customer will sometimes have to travel further to her favored type of restaurant). If transport costs are small, then the former effect certainly dominates, and this is the optimal outcome.

<sup>12</sup>I do not suggest taking this model too seriously. There are at least two problems with the model. First, it is assumed that all non-smokers prefer to eat with smokers rather than not to go out at all. Second, a model with free entry of restaurants is likely to have an asymmetric outcome: a fraction of restaurants will allow smoking (perhaps a fraction similar to  $\lambda$ , the proportion of smokers in the dining population) while the remainder will forbid it.

## 5 One-Sided Exclusive Intermediation: “Competitive Bottlenecks”

Next suppose that, while group 1 continues to deal with just a single intermediary, group 2 is happy to deal with both intermediaries.<sup>13</sup>

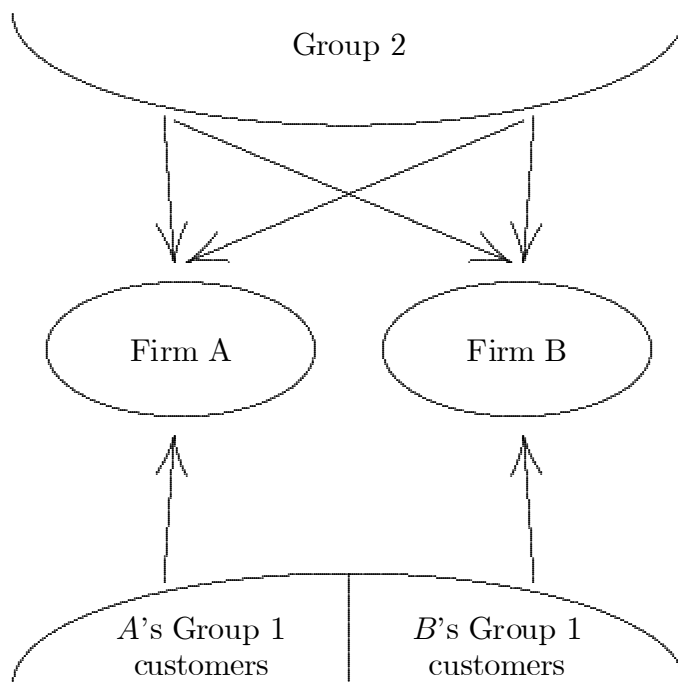


Figure 3: Competitive Bottlenecks

Examples of this kind of market include:

- Example 2, where an author publishes an article to a single journal but where readers typically subscribe to several journals.
- Example 6, if people tend to read a single newspaper (or, less plausibly, watch a single TV channel), and where advertisers are willing to advertise in several outlets in order to make contact with a large pool of potential consumers.
- Example 7, if people tend to consult a single “yellow pages” directory when they need information about a particular service. As in the previous example, advertisers will then tend to advertise in several outlets in order to have a good chance of reaching most potential customers.

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<sup>13</sup>The following model is essentially that of the mobile telephony market analyzed in section 3.1 of Armstrong (2002b), where the term “competitive bottlenecks” was used.

- Example 8, if consumers tend to visit a single mall on a shopping trip (for instance, because of transport costs). Retailers may choose to locate in several malls in order to reach as many potential customers as possible. This example is, in technical terms, different from most of the others listed here since it is natural to suppose that a retailer has a fixed cost per mall, and this acts to complicate the analysis. (By contrast, it seems less essential to model an advertiser as having a fixed cost of preparing an advert *per newspaper*.)
- Example 9, if consumers tend to use a single credit card (perhaps because of transactions costs of dealing with more than one card company). Retailers will most likely wish to have the ability to handle transactions with a variety of cards. However, like the previous mall example, it is plausible that fixed costs of being able to handle a given card by retailers will be important.
- Example 10, if people tend to subscribe to a single, say, mobile network. People on, say, the fixed network will then call people on all mobile networks.
- Example 13, if we think about real estate market where house sellers typically use a single agent to sell the house, and where potential buyers use many agents when looking for a house.

## 5.1 The Model

If intermediary  $i = A, B$  offers utility  $u_1^i$  to a member of group 1, while the rival offers utility  $u_1^j$ , then the former attracts a number

$$n_1^i = \frac{1}{2} + \frac{u_1^i - u_1^j}{2t} \quad (12)$$

of the group 1 members. Here,  $t$  is the relevant product differentiation parameter. Unlike the previous model, this model continues to work even in the competitive limit as  $t$  tends to zero. These utilities are generated in the following way: if a quantity  $X^i$  of the group 2's "output" is available via intermediary  $i$ , and a member of group 1 pays  $p_1^i$  to join, then

$$u_1^i = U(X^i) - p_1^i \quad (13)$$

where  $U$  is some increasing function. (Because group 2 members deal with both intermediaries, it does not make sense to talk about the 'number' of group 2 members on a given intermediary, and so we use the term 'quantity'. To be concrete, think of group 1 as readers of newspapers or yellow pages, and group 2 as advertisers placing a given quantity  $X_i$  of advertising in the outlet  $i$ .) Turning to group 2, suppose that if there are  $n_1^i$  members of group 1 at intermediary  $i$ , then the aggregate utility of group 2 if it supplies quantity  $X$  to the intermediary is

$$n_1^i V(X) - p_2^i X \quad (14)$$

where  $p_2^i$  is the intermediary's price per unit of  $X$  and  $V$  is some (concave) function of its output. Thus group 2 enjoys utility from dealing with intermediary  $i$  that is proportional to  $i$ 's group 1 customer base. Clearly, from (14) the equilibrium quantity choice by group 2 at an intermediary depends only on that intermediary's price per group 1 member,  $p_2^i/n_1^i$ , and not on either (i) the number of group 1 members at the intermediary or (ii) on the terms offered by the other intermediary. Therefore, it makes sense to define an intermediary's "revenue from group 2 per group 1 customer", which we define to be  $R(X)$  if group 2 supplies quantity  $X$ . In fact,

$$R(X) \equiv XV'(X)$$

since  $V'(X)$  is group 2's inverse demand function (per group 1 member), i.e., the price per group 1 member that causes quantity  $X$  to be supplied by group 2.

Suppose that an intermediary incurs a total cost that depends on group 2's quantity  $X$  and the number of its group 1 customers. Suppose for simplicity this cost is proportional to  $n_1^i$ , and so the total cost to intermediary  $i$  is  $n_1^i C(X)$ . (For instance, in the yellow pages example,  $C(X)$  is the cost of producing a directly of size  $X$ .) Therefore, the total profit of intermediary  $i$  is

$$\pi^i = n_1^i(p_1^i + R(X^i) - C(X^i)) . \quad (15)$$

Intermediary  $i$  offers group 1 customers a price  $p_1^i$  and a group 2 quantity  $X^i$ . Equivalently,  $i$  offers group 1 customers a *utility*  $u_1^i$  and a quantity  $X_i$ , in which case expression (13) implies that (15) becomes

$$\pi^i = n_1^i(U(X^i) + R(X^i) - C(X^i) - u_1^i) . \quad (16)$$

Clearly, regardless of the level of utility it chooses to offer its group 1 customers, it will choose the group 2 quantity  $X^*$  satisfying

$$X^* \text{ maximizes } S(X) \equiv U(X) + R(X) - C(X) . \quad (17)$$

The function  $S(X)$  measures the total surplus available to group 1 customers and the intermediary, per group 1 customer, when quantity  $X$  is supplied from group 2. The crucial point to note is that group 2 output is chosen to maximize the total surplus enjoyed by the intermediaries and group 1 participants together. (The interests of group 2 are ignored in the calculation.)

Turning next to competition for group 1 customers, i.e., to how the surplus  $S(X^*)$  is divided between intermediaries and group 1 participants, using (12) and (16) we see that  $i$ 's total profit is

$$\pi^i = \left( \frac{1}{2} + \frac{u_1^i - u_1^j}{2t} \right) (S(X^*) - u_1^i) . \quad (18)$$

It is then straightforward to see that the Nash equilibrium level of group 1 utility,  $u_1$ , is given by

$$u_1 = S(X^*) - t .$$

The equilibrium profit made by an intermediary is the standard Hotelling profit  $\pi = \frac{1}{2}t$ . From (13), the equilibrium price to group 1 members is  $p_1 = t + U(X^*) - S(X^*)$ , or

$$p_1 = C(X^*) + t - R(X^*) . \quad (19)$$

Thus, the equilibrium price for group 1 participants is the standard ‘Hotelling’ price ( $t + C(X^*)$ ) minus the revenue that a group 1 member generates for the intermediary from group 2.

Clearly, it is perfectly possible that group 1 members are subsidised by group 2, in the sense that  $p_1 < C(X^*)$ , and from (19) this inevitably occurs if the market for group 1 customers is highly competitive ( $t \rightarrow 0$ ). Indeed, it is possible that  $p_1$  in (19) is negative. If negative prices are not feasible, then the equilibrium will then involve charging group 1 customers nothing for the service, something which is often seen in some of our examples (yellow pages, shopping malls, and so on).

Turning to overall welfare, social welfare (per group 1 member) is

$$w = \underbrace{V(X) - R(X)}_{\text{surplus of group 2}} + S(X) = V(X) + U(X) - C(X) . \quad (20)$$

Clearly, this involves a *higher* level of group 2 output than that at the competitive equilibrium,  $X^*$ , since now the surplus of group 2 members ( $V(X) - R(X)$ ) is taken into account. (This group 2 surplus is increasing in its output  $X$ .) Even though intermediaries might compete vigorously *for* group 1 members, there is no competition for providing access by group 2 *to* these group 1 members, and this monopoly induces the usual excessive pricing. The market failure is not one of excessive profits—the intermediaries’ overall profits will be small if  $t$  is small—but rather that there is a bad pattern of relative prices for the two groups of participants.

*Free service to group 1:* There are several cases where it is natural to assume that group 1 can consume the service for free. (Examples include yellow pages, which are typically given free to users, and shopping malls, where consumer typically have free access to the mall.) In this case the only that an intermediary can compete for group 1 customers is by offering a high quantity  $X$ . Specifically, if the two group 2 quantities are  $X^i$  and  $X^j$ , firm  $i$  will get

$$n_1^i = \frac{1}{2} + \frac{U(X^i) - U(X^j)}{2t}$$



group 1 customers. In this case profit in (18) is modified to be

$$\pi^i = \left( \frac{1}{2} + \frac{U(X^i) - U(X^j)}{2t} \right) (R(X^i) - C(X^i)). \quad (21)$$

One can then show that the first-order condition for the symmetric equilibrium quantity  $\hat{X}$  is given by

$$U'(\hat{X})[R(\hat{X}) - C(\hat{X})] + t[R'(\hat{X}) - C'(\hat{X})] = 0. \quad (22)$$

Since we must have  $R > C$  if the firm's are to be profitable, this condition shows that  $R' > C'$  at the equilibrium choice of quantity. This is *greater* than the quantity that would be chosen for fixed group 1 shares (where each firm would choose  $X$  to maximize its profit  $R(x) - C(x)$ ). This is to be expected since the firms need to offer a high quantity to attract group 1 customers. However, the comparison of the quantity  $\hat{X}$  with the quantity  $X^*$  in (17), i.e., when group 1 agents pay a fee, does not seem to be straightforward. In competitive markets, where  $t \approx 0$ , (22) shows that the equilibrium will involve firms choosing quantities so that revenue just covers costs:  $R(\hat{X}) = C(\hat{X})$ .

## 5.2 Example: Academic Publishing

This section presents a simple model of the market for academic publishing. The focus is entirely on the *publishing* part of the process, and not on the certification (and other) parts of the process that are also important. It is arguable that new technology has meant that the costs of dissemination have fallen so much that certification has become the central role of academic journals. If so, then the following simple model applies to a previous era.

Suppose that an author has an article to disseminate. If  $n$  people read the article and he pays a dissemination (or 'submission') fee  $P$ , the author's utility is  $bn - P$ . Therefore,  $b$  represents the author's (constant) per-reader benefit when the paper is read. Journals have technology such that the cost of dissemination per article takes the form  $f + cn$ , where  $f$  is a fixed cost per article and  $c$  is the per-reader cost of dissemination.

### 5.2.1 No competition between articles

If  $p$  is the fee for reading the article (or the 'subscription fee'), write  $N(p)$  for the number of readers of the paper and  $V(p)$  for the associated reader surplus function ( $V' \equiv -N$ ). Notice that each article is, for the moment, assumed to be an independent market, and a reader's decision about whether to read one article has no effect on his decision to read another.

A journal's profits from a given article are then

$$\pi = \underbrace{pN(p)}_{\text{revenue from readers}} + \underbrace{P}_{\text{revenue from authors}} - \underbrace{[cN(p) + f]}_{\text{costs}}.$$

Suppose that competition between journals drives their profits to zero.

In this market, journals compete for submissions in terms of the submission fee and the number of readers the article will have. Therefore, the optimal two-sided pricing policy for journals is to maximize author surplus  $bN(p) - P$  subject to the journal break-even constraint  $(p - c)N(p) + P - f \geq 0$ . Clearly, this has the solution in which the subscription fee

$$p \text{ maximizes } (p - c)N(p) + bN(p) .$$

(The submission fee  $P$  is then chosen so that the journal just breaks even.) Thus the journal behaves as a monopoly with marginal cost  $[c - b]$  facing the demand curve  $N(p)$ . In particular, the reading price  $p$  is strictly above this adjusted marginal cost, so that  $p > c - b$ .

By contrast, social welfare is maximized when

$$p \text{ maximizes } (p - c)N(p) + bN(p) + V(p)$$

which has the solution

$$p = c - b .$$

Thus this market equilibrium involves too high a reading price (and so too low a submission fee) compared to the social optimum. (Of course, this is the exactly the outcome that would occur if an author could disseminate his article himself, if he had access to the technology with cost-of-dissemination  $f + cn$ .) Notice that the fact that authors like people to read their articles does not mitigate the fundamental problem of monopoly in this industry. An increase in  $b$  causes *both* the privately optimal reading price and the socially optimal reading price to fall. This model is essentially an instance of the general model in section 5 above.

### 5.2.2 Competition between articles

The above stark model can be generalized in the following manner. The population of readers has size normalized to 1. Each reader only has time to read, say,  $m$  articles. Assume for now that all articles are the same quality and that there are more than  $m$  potential articles available. Therefore, readers will choose the  $m$  articles with the lowest subscription fees (and randomize if there are ties). It is then easily verified that the equilibrium in this market is to set the submission fee to extract the authors' surplus, so that  $P = b$ , and to set the subscription fee so that costs are just covered:

$$p = c + f - b .$$

This equilibrium involves journals pricing low to attract readers, and extracting all surplus from authors.

A more general model along these lines is the following. Suppose authors are distinguished by two parameters:  $q$  is the (unambiguous) quality of their article, and  $b$  (as before) is how

much the author wishes to get published (it is exactly how much the author would be willing to pay to get his article published in a journal with wide readership). The parameter  $q$  is measured in money terms, and if a reader reads a quality- $q$  article for the fee  $p$ , his net surplus is  $q - p$ . Notice that  $q + b$  represents the total benefit in publishing the type- $(q, b)$  article.

Suppose that  $\bar{u}$  is the equilibrium net surplus for readers of the marginal article. That is to say, if a journal publishes an article generating net surplus  $q - p > \bar{u}$  it will certainly attract all readers. Then, given  $\bar{u}$ , a journal will be able profitably to publish any article of type  $(q, b)$  such that

$$q + b \geq \bar{u} + [c + k].$$

The utility  $\bar{u}$  will then be determined so that the total number of articles that people want to read ( $m$  in this specific model) equals the number supplied:

$$\bar{u} \text{ satisfies } \# \{ \text{articles such that } q + b \geq \bar{u} + [c + k] \} = m.$$

Authors with type  $(q, b)$  strictly above this line will have strictly positive surplus. (Again, all journals have zero profit.) The predictions of this specific model are

- There will be a menu of ‘journals’ offering differing combinations of article quality  $q$  and subscription fee  $p$ , but all satisfying  $q - p = \bar{u}$ . High quality journals have a high subscription fee.
- Higher quality journals have a lower submission fee, so that  $P = c + k + \bar{u} - q$ . Therefore, an author will send his article to the highest quality journal that is feasible.
- There is no market failure, since the articles with the highest social value (i.e.,  $q + b$ ) are published. The ‘externality’ factor  $b$  is fully internalized by authors’ submission decisions. If an author has a low  $q$ /high  $b$  combination, he will send the article to a low quality journal and pay a high submission fee. He is willing to pay the high fee since he enjoys a high private benefit from getting published. The journal can attract readers since the high submission fee is passed along to readers in the form of a low subscription fee (which compensates for the low quality of its articles).

### 5.3 Example: Advertising in Media Markets

Suppose there are two newspapers,  $A$  and  $B$ , and that the cost of producing and distributing the newspaper is  $f$  per copy (regardless of the number of adverts). Adverts are placed by monopoly producers of new goods. A reader will purchase one unit of a given new good if the price is less than her valuation *and* she sees an advert for the product. Producers are differentiated by the parameter  $\sigma$ : the type- $\sigma$  producer has a good for which all consumers are willing to pay  $\sigma$  (if they see an advert). In particular, due to the unit demand assumption and

that fact that there is no asymmetric information about willingness-to-pay for new products, the equilibrium price for a type- $\sigma$  new product will be  $\sigma$ . Suppose that costs of the new products are normalized to zero. Then, an advert placed in a newspaper is worth  $\sigma$  per reader to the type- $\sigma$  firm.<sup>14</sup> Readers will, in equilibrium, gain no benefit from seeing adverts, since all their surplus is extracted by the monopoly price. Let  $F(\sigma)$  be the cumulative distribution function for  $\sigma$  in the population of producers.

### 5.3.1 Readers buy a single newspaper

This section is based on Anderson and Coate (2001) and concerns the case where newspapers have to compete for readers. The problem can be analyzed using the general model discussed in section 5 above. Suppose that if the newspaper cover prices are  $p_A$  and  $p_B$  respectively, then newspaper  $i$  obtains

$$n_i = \frac{1}{2} + \frac{p_j - p_i}{2t}$$

readers. The crucial assumption here is that readers buy one or other newspaper, but not both. Since readers gain no benefit from advertising in this model, they do not care about the amount of advertising in their paper, and so this does not enter into the market share formula above. (Using the notation of section 5,  $X^i$  is the number of producers who choose to advertise with paper  $i$ , and reader utility of advertising is identically zero, i.e.,  $U(X^i) \equiv 0$ . Also, the per paper unit cost does not depend on advertising volume:  $C(X^i) \equiv f$ .) Suppose that newspaper  $i$  charges  $a_i$  per reader to an advertiser/producer. (We suppose that papers cannot discriminate between producers of different type when they set their advertising rates.) In this case, a producer will agree to advertise if and only if  $\sigma \geq a_i$ . Thus, a newspaper's advertising revenue per reader with the charge  $a_i$  is

$$\hat{R}(a_i) = a_i(1 - F(a_i)) .$$

Since the amount of advertising does not affect the attraction of its newspaper to readers, a newspaper might as well choose its advertising charge to maximize its revenue per reader, i.e. to set the charge  $a^*$  that maximizes  $\hat{R}(\cdot)$ . This is just a special case of the formula (17) above when  $U(X) - C(X)$  is constant. (Note, in this case it is more convenient to write revenue as a function of price  $a$  rather than quantity  $X$ .)

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<sup>14</sup>Thus, this model assumes that an advertiser's payoff is *linear* in the number of people who see the advert. There are at least two reasons why this linearity assumption might be unrealistic. First, Chwe (1998) presents a model where an advertiser's payoff is convex in the number of people who see the advert. (The model is one where there are network effects in consuming the advertised product, and if many people see the advert this might act to change consumer expectations and to move to another equilibrium.) Second, if a seller has limited supplies of the product available for sale (or, more generally, if the cost of production is convex), then the seller only obtains benefit from the advert reaching a certain number of potential consumers.

A newspaper's effective cost per reader is now  $f - \hat{R}(a^*)$ , and so the equilibrium price for readers is given by the usual Hotelling mark-up rule:

$$p_A = p_B = f - \hat{R}(a^*) + t .$$

(Again, this is just a special case of (19).) If  $\hat{R}(a^*) > t$  then  $p < f$  and newspapers are offered for sale below their marginal cost  $f$ . The revenue from advertising is passed onto readers in the form of low newspaper charges. The total profit made by the newspapers is  $t$  (which is exactly the same as if there were no advertising at all). The total profit made by the producers is  $\hat{V}(a)$ , where

$$\hat{V}'(a) = -(1 - F(a)) .$$

(In particular,  $\hat{V}$  is decreasing.) The consumer surplus of readers (ignoring transport costs) is  $-(f - \hat{R}(a) + t)$ . Therefore, total welfare in this market is

$$W(a) = \hat{R}(a) + \hat{V}(a)$$

which is maximized at  $a = 0$ . Market equilibrium, by contrast, involves  $a$  maximizing  $\hat{R}(\cdot)$  which will imply a strictly positive charge for advertising. Therefore, the market equilibrium involves *too little* advertising. Moreover, this distortion is not mitigated by making the market for readers more competitive (i.e., reducing  $t$ ).

### 5.3.2 Overlapping readership

The previous model was somewhat extreme, especially if applied to television (where viewers will watch several different channels over the relevant time frame). Suppose next that the newspapers are independent products rather than substitutes, and so papers do *not* compete directly for readers. Therefore, a fraction of the population will read both newspapers.

The fact that some readers buy both papers means that neither side of the market is characterized by an exclusive choice of intermediary, and so none of the general models in this paper apply to this case. Therefore, to analyze this case in a very partial and preliminary way, consider the following model. Suppose that the fraction of the relevant population who read newspaper  $i$  is  $n(p_i)$ , and that this fraction does not depend on the price for paper  $j$ . Suppose that if the newspapers have readerships  $n_A$  and  $n_B$  then the number who read *both* newspapers is given by some function  $\phi(n_A, n_B)$ . The function  $\phi$  captures the extent of correlation in newspaper buying across the population of readers. If a reader's valuation for paper  $i$  is statistically independent of her valuation for  $j$ , then  $\phi(n_A, n_B) = n_A n_B$ . On the other hand, if valuations are perfectly positively correlated among readers, then  $\phi(n_A, n_B) = \min\{n_A, n_B\}$ . Clearly, newspaper  $i$  has  $n_i - \phi(n_A, n_B)$  exclusive readers.

A producer only needs a consumer to see its advert *once* to make a sale. (This model does not involve 'persuasive' advertising, where repeated sight of an advert might result in

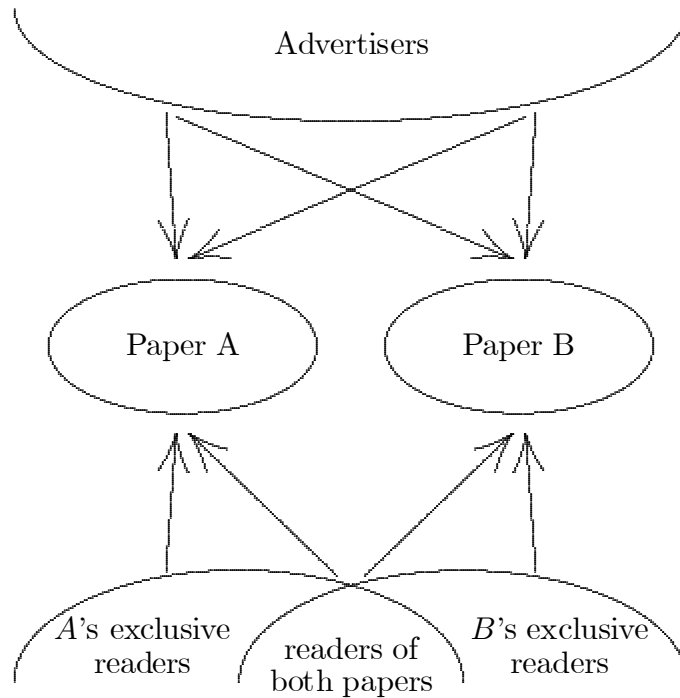


Figure 4: Overlapping Readership

greater likelihood of sale.) Consider first the situation where the two papers have readerships  $n_A$  and  $n_B$ . What are the equilibrium charges to advertisers? Suppose the two lump-sum charges for advertising are  $P_A$  and  $P_B$ . A producer then has four options: not to advertise at all (which yields zero profits), to advertise only in paper  $A$ , to advertise only in paper  $B$ , or to advertise in both outlets. If it advertises only in paper  $i$ , the type- $\sigma$  firm's profit is

$$\sigma n_i - P_i$$

and if it advertises in both outlets its profit is

$$\sigma [n_A + n_B - \phi] - P_A - P_B .$$

Use the subscript 'max' to denote the paper with higher value of  $n_i\sigma - P_i$ , and 'min' for the other paper. Then, if an advertiser chooses to use only a single outlet it will choose paper 'max'. A producer will choose to use both outlets, rather than just outlet 'max', provided that

$$\sigma [n_{\min} - \phi] \geq P_{\min} .$$

In sum, a type- $\sigma$  producer will choose to advertise at least in paper 'max' if

$$\sigma n_{\max} \geq P_{\max} ,$$

and it will choose to advertise in both outlets if the stronger condition

$$\sigma[n_{\min} - \phi] \geq P_{\min}$$

holds.

For simplicity, suppose that all producers have the same quality product  $\sigma$ .<sup>15</sup> Then the equilibrium is for each paper to extract advertising revenue for access to its exclusive readers, so that

$$P_i = \sigma[n_i - \phi(n_A, n_B)]. \quad (23)$$

(When both papers set this advertising charge then producers choose to place adverts in both papers. If one paper deviates by setting a higher charge it will lose all its advertisers; if it deviates by setting a lower charge then it will not attract any more advertising and will obtain less revenue from its existing advertisers.) In effect, competition between newspapers for advertising means that each newspaper  $i$  generates revenue only from its exclusive readers. (By contrast, in the previous analysis where readers read a single paper, the papers could charge  $\sigma$  per reader. Here paper  $i$  can charge only  $(1 - \phi/n_i)\sigma$  per reader.)

Having solved the second stage of the game, let us consider the first stage: pricing to attract readers. If paper  $i$  charges the cover price  $p_i$ , obtains readership  $n_i$  while its rival has  $n_j$  readers, its total profit is

$$\begin{aligned} \pi_i &= n_i(p_i - k) + P_i \\ &= \underbrace{n_i(p_i - f)}_{\text{profit from readers}} + \underbrace{\sigma[n_i - \phi(n_A, n_B)]}_{\text{revenue from advertisers}}. \end{aligned} \quad (24)$$

*Perfect correlation:* Consider first the extreme case where  $\phi = \min\{n_A, n_B\}$ . Then the only way that a paper has any exclusive readers, and hence gets any revenue from advertising, is if it sells more copies than its rival. Each reader it attracts in excess of its rival generates advertising revenue  $\sigma$ .

There is no symmetric equilibrium in this example. To see this, let  $p^*$  maximize  $n(p)(p - f)$ . Thus  $p^*$  is the profit-maximizing cover price in the absence of any advertising. Both papers setting the price  $p = p^*$  is not an equilibrium, since it pays one paper to undercut the other in order to obtain some advertising revenue. (There is a second-order loss in profit from readers, but a first-order gain in revenue from advertising.) But neither is both firms setting the same price  $\hat{p} < p^*$  an equilibrium. (Neither paper obtains any advertising revenue, and so it would benefit a paper to increase its cover price to  $p^*$  to maximize its profits from readers.) Let  $p^{**}$  be the price that maximizes

$$n(p)(p - f) + \sigma[n(p) - n(p^*)].$$

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<sup>15</sup>The calculation of the equilibrium advertising charges with heterogeneous producers seems to be complicated, and equilibrium might not exist at all.

(Clearly, this price is strictly lower than  $p^*$ . It is the price that maximizes  $n(p)(p - [f - \sigma])$ , i.e., the monopoly price with a reduced unit cost level.) It is then verified that one paper setting the price  $p = p^*$  and the other setting the price  $p = p^{**}$  is an asymmetric equilibrium. (By construction,  $p^{**}$  is a best response to a rival price  $p^*$ . It is easily verified that the reverse holds as well.) The prediction of this (*ex ante* symmetric) model is therefore an asymmetric outcome: one firm has a lower cover price, a larger readership and obtains some advertising revenue; the other has a high price, low readership and no advertising. The “advertising” paper makes greater profits (since it could choose the price  $p = p^*$  if it wanted).<sup>16</sup>

*General case:* Since  $n_i = n(p_i)$ , from (24) the first-order condition for symmetric Nash equilibrium in this market is

$$n'(p - f) + n + n'[1 - \phi_1]\sigma = 0 \quad (25)$$

where  $\phi_1 = \partial\phi/\partial n_i < 1$ . This again implies a lower cover price than would be the case without advertising (where the cover price would maximize  $n(p)(p - f)$ ), and so the need to compete for advertising makes newspapers also ‘compete’ for readers, even when they operate in entirely separate markets for readers. The reason is that, even though the total number of readers a newspaper attracts does not depend on its rival’s strategy, the number of *exclusive* readers does depend (negatively) on its rival’s readership.

The surplus of producers is their sales revenue, which is  $(2n - \phi)\sigma$ , minus their advertising outlay, which is  $2(n - \phi)\sigma$ , and so this surplus is  $\phi\sigma$ . In effect, where newspaper readership overlaps, which happens for a fraction  $\phi$  of the population, competition implies that a producer obtains access to these readers for free. (It must, however, pay the monopoly rate  $\sigma$  for the exclusive readers.)

*Welfare effects:* Total welfare when both newspapers charge the cover price  $p$  and sell advertising is

$$W = \underbrace{2v(p)}_{\text{consumer surplus}} + \underbrace{2n(p)(p - f) + \sigma [2n(p) - \phi(n(p), n(p))]}_{\text{joint profit of newspapers and producers}}.$$

The first-order condition (25) is actually the first-order condition that maximizes joint profits in the above expression.<sup>17</sup> This is quite intuitive: the profit ‘leakage’ that takes place when papers compete for advertising all flows to advertisers (not readers), and there is no distortion in the overall size of profits. Obviously the cover price  $p$  is too high in social terms, but this is due entirely to the papers’ monopoly position in the market for readers, and the presence of advertising indirectly benefits readers by its effect on intensifying papers’ desire to obtain exclusive readers.

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<sup>16</sup>This asymmetric outcome is reminiscent of the results in Caillaud and Jullien (2002).

<sup>17</sup>Note that  $\phi_1 = \frac{1}{2} \frac{d\phi(n,n)}{dn}$ .



## 5.4 Example: Yellow Pages

Suppose there are two rival producers of ‘yellow pages’,  $A$  and  $B$ .<sup>18</sup> This market can be discussed with a direct application of the model in section 5 above, with readers of directories called “group 1” and the advertisers labelled “group 2”. The amount of advertising in directory  $i$  is  $X^i$ . The utility of a consumer who buys directory  $i$  is

$$u_1^i = U(X^i) - p_1^i$$

where  $U$  is an increasing function and  $p_1^i$  is the price for buying (or using) the directory. (This price could be set equal to zero if directories have to be offered for free to consumers.) Assume that each consumer only uses a single directory. Suppose the number of consumers who use directory  $i$  is given by the familiar Hotelling specification:

$$n_1^i = \frac{1}{2} + \frac{u_1^i - u_1^j}{2t} .$$

Suppose that the benefit to advertisers is proportional to the readership of a directory. Therefore, let  $R(X)$  be the revenue per reader if a directory sets the advertising charge so that quantity  $X$  of advertising is demanded. Suppose that a directory of size  $X$  costs  $C(X)$  per user to produce and distribute. Then expression (17) shows that, if users can be charged for the directory, the equilibrium quantity of advertising will maximize  $U(X) + R(X) - C(X)$  and from (19) the price that users face is  $p_1 = C(X) + t - R(X)$ . Since there is no particular reason to think that directories are intrinsically differentiated,  $t$  is likely to be small. In this case, the optimal price  $p_1$  is negative if the revenue that a user generates,  $R$ , exceeds the cost of the directory,  $C$ . This could be a rationale for why directories are distributed for free.

A complicating factor which is relevant in this market is that directories are typically given to people even if they are not used. Therefore, the cost  $C(X)$  is incurred even if the directory is just thrown away. In this case profit in (21) is modified to be

$$\pi^i = \left( \frac{1}{2} + \frac{U(X^i) - U(X^j)}{2t} \right) R(X^i) - C(X^i) .$$

One can then show that the first-order condition for the symmetric equilibrium advertising quantity  $\hat{X}$  is given by

$$U'(\hat{X})R(\hat{X}) + t[R'(\hat{X}) - 2C'(\hat{X})] = 0 .$$

Clearly, in competitive markets ( $t \approx 0$ ) directories will run at a loss, since they do not recover the cost involved in the duplicate directories.

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<sup>18</sup>This model is adapted from the structural model proposed in section 4 of Rysman (2002). Rysman’s model is more complicated in at least two ways: (i) he allows for the fact that the benefit to advertisers might not be strictly proportional to readership, and (ii) keeping readership constant, an advertiser prefers to be in a directory with *fewer* other adverts. He also assumes that directories compete for advertising by setting quantities rather than prices.

## 5.5 Example: Shopping Malls

The following model ‘almost’ fits into the one-sided exclusive intermediation model of section 5.1 above, except for the fact that shops have *fixed* costs of operation, and their payoff is not directly proportional to the number of consumers. In particular, a shop will only locate in a mall if a certain threshold number of consumers visit the mall, and this feature was not present in the above model. This means that we have to do the analysis from scratch. In the fact, this model is quite close to that presented in section 6 of Rochet and Tirole (2001).

Suppose there are two shopping malls. There are a large number of retailers, and suppose that each has a monopoly position in its product. For modelling simplicity, suppose that these firms differ only in terms of the fixed costs of setting up a shop in a mall. In particular, each visitor to a mall obtains utility  $v$  per shop in the mall, and each shop obtains profit  $\pi$  from each visitor to the mall. The fixed cost of a “type  $\theta$ ” shop is just  $\theta$ , and suppose the number of shops with fixed cost less than  $\theta$  is  $F(\theta)$ . (A shop must incur this cost twice if it locates in both malls.) A mall charges each shop the same rent  $p_s$ , say, and this is a fixed charge (and is not an explicit function, say, of the number of consumers passing through or of the shop’s turnover). If mall  $i$  has  $n_c^i$  consumers and charges  $p_s^i$  to shops, then a type- $\theta$  shop is willing to participate if

$$\theta + p_s^i \leq \pi n_c^i$$

and is willing to do this regardless of whether it chooses to locate also in the rival mall.<sup>19</sup> Therefore, there will then be

$$n_s^i = F(\pi n_c^i - p_s^i) \tag{26}$$

shops locating in mall  $i$ .

Consumers are assumed only to go to a single mall. If their utility is  $u_c^i$  at mall  $i$ , then that mall will get

$$n_c^i = \frac{1}{2} \left( 1 + \frac{u_c^i - u_c^j}{t} \right) \tag{27}$$

consumers. Consumer utility is given by

$$u_c^i = v n_s^i - p_c^i \tag{28}$$

if the mall charges  $p_c^i$  for entry.

The mall has a cost per shop of  $f_s$  and a cost per consumer of  $f_c$ . (Perhaps this latter cost might represent the costs of providing sufficient parking, and so on.) Therefore, the profit of mall  $i$  is

$$n_c^i (p_c^i - f_c) + n_s^i (p_s^i - f_s) . \tag{29}$$

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<sup>19</sup>At least, this is true if shops are “atomistic”. If this were not the case then if a shop already located in mall  $A$  decides to locate also in mall  $B$ , then this will draw consumers away from mall  $A$  and so cause a negative externality on profits from its mall  $A$  branch. Here, though, we ignore this possibility.

First, we can simply derive the equilibrium charge to shops at a symmetric equilibrium. For suppose that each mall has attracted half the population of consumers, and offers equilibrium consumer utility  $u_c$ . Then a mall must be maximizing its profits given this utility  $u_c$ . In other words, consider varying  $p_c^i$  and  $n_s^i$  so that  $u_c = vn_s^i - p_c^i$  is constant. Then from (29) the mall's profit is

$$\frac{1}{2}(vn_s^i - u_c - f_c) + n_s^i(p_s^i - f_s) = \frac{1}{2}(vF(\frac{1}{2}\pi - p_s^i) - u_c - f_c) + F(\frac{1}{2}\pi - p_s^i)(p_s^i - f_s) .$$

Maximizing this with respect to  $p_s^i$  implies that the equilibrium charge to retailers is

$$p_s = f_s + \frac{1}{\eta} - \frac{v}{2} \quad (30)$$

where

$$\eta = \frac{F'(\frac{1}{2}\pi - p_s)}{F(\frac{1}{2}\pi - p_s)} > 0$$

measures the proportional increase in the number of shops as profit opportunities increase. The formula (30) expresses the profit-maximizing charge to shops as a standard “elasticity” rule ( $p = f + 1/\eta$ ) adjusted downwards the external benefit that an additional shop causes the consumers at the mall to enjoy (which is  $\frac{1}{2}v$  when the malls share the consumer market equally). If shops are in very elastic supply (i.e.,  $1/\eta \approx 0$ ) or consumers greatly value each shop ( $v$  is large) then it is optimal to subsidize shops (i.e.,  $p_s < f_s$ ) in order to extract surplus from consumers.

However, the important thing to note is that this equilibrium charge to shops maximizes the total surplus available to the consumers and the malls together. (This is precisely analogous to expression (17) in section 5 above, where group 2's contract was chosen to maximize the surplus available to the intermediaries and the group 1 participants together.) The interests of shops are ignored in this calculation.

Turning next to the equilibrium price charged to consumers, notice that

$$\begin{aligned} u_c^i - u_c^j &= (p_c^j - p_c^i) + v(n_s^i - n_s^j) \\ &= (p_c^j - p_c^i) + v [F(\pi n_c^i - p_s) - F(\pi n_c^j - p_s)] \\ &= (p_c^j - p_c^i) + v [F(\pi n_c^i - p_s) - F(\pi(1 - n_c^i) - p_s)] . \end{aligned}$$

Therefore, from (27), given the rival consumer price  $p_c^j$ , in order to attract a number  $n_c^i$  of consumers, mall  $i$  must charge the price

$$p_c^i = p_c^j + t(1 - 2n_c^i) + v [F(\pi n_c^i - p_s) - F(\pi(1 - n_c^i) - p_s)] .$$

From (29) mall  $i$  will therefore choose  $n_c^i$  in order to maximize

$$n_c^i \{ p_c^j + t(1 - 2n_c^i) + v [F(\pi n_c^i - p_s) - F(\pi(1 - n_c^i) - p_s)] - f_c \}$$

$$+F(\pi n_c^i - p_s)(p_s - f_s) .$$

At the symmetric equilibrium, the optimum must involve  $n_c^i = \frac{1}{2}$ , and so the equilibrium charge to consumers is

$$\begin{aligned} p_c &= t + f_c - F'\pi(v + p_s - f_s) \\ &= t + f_c - F'\pi\left(\frac{v}{2} + \frac{1}{\eta}\right) \\ &= t + f_c - \pi F - \frac{1}{2}v\pi F' . \end{aligned} \tag{31}$$

This formula (31) expresses the consumer charge as a standard ‘Hotelling’ price ( $p = t + f$ ) together with *two* downward adjustment terms. The first adjustment term,  $\pi F$ , represents the external benefit that an additional consumer gives to the shops located in the mall. The second adjustment term,  $\frac{1}{2}v\pi F'$ , represents the *indirect* spillover that an additional consumer gives to the existing other consumers at the mall: an additional consumer causes more shops to be located at the mall—an increase of  $\pi F'$  to be precise—and this causes the utility of each existing consumer to increase by  $v$  per additional shop.

*Welfare:* Turning to welfare, given that half consumers visit each mall, social welfare per mall with the charge to shops  $p_s$  is

$$w = \frac{v}{2}F\left(\frac{\pi}{2} - p_s\right) + (p_s - f_s)F\left(\frac{\pi}{2} - p_s\right) + \int^{\frac{\pi}{2} - p_s} \left(\frac{\pi}{2} - \theta - p_s\right) dF(\theta) .$$

Maximizing this gives the socially optimal charge to shops as

$$p_s = f_s - \frac{v}{2} .$$

Thus, shops should be offered mall space at subsidised rates because of the positive externality they exert on consumers. Comparing with (30), we see that equilibrium price for retailing space is too high from a social point of view. Even though the details of the analysis differs from that in section 5, the intuition for the market failure is the same in the two cases: the fact that malls have to compete for consumers but not (directly) for shops means that the equilibrium outcome as given in (30) maximizes the surplus available to malls and shoppers and ignores the profits of shops. Thus the effect is as if the consumers/malls act as a monopoly when setting terms for the shops.

*Competition within malls:* An interesting issue is the equilibrium extent of competition between shops within malls.<sup>20</sup> Thus one could imagine the mall controlling the degree of competition, e.g., the number of shops selling similar products, within the mall itself. In the usual way, more competition will mean less profit per consumer for each shop but each consumer will obtain higher surplus per shop (or per shop type). Thus we would expect that if the mall allowed competition it would make less money from the retailing side of the market but more from the consumer side (if it charged for entry). One hypothesis which could be investigated is: malls would allow competition within the mall if consumers were charged for entry, but if consumers had free entry that malls would restrict competition in order to drive up revenues from shops.

## 5.6 Example: Call Termination in Telecommunications

Consider this stylized model of the market for mobile telephony, which uses the ‘competitive bottlenecks’ framework of section 5. There are two mobile networks,  $A$  and  $B$ , that make calls to, and receive calls from, the fixed telecom network.<sup>21</sup> Here, ‘group 1’ are the subscribers on the mobile networks and ‘group 2’ are people on the fixed network who wish to make calls to these mobile subscribers. Subscribers decide to join one or other mobile network but not both. If utility offered to subscribers on mobile network  $i$  is  $u_1^i$ , then the number of subscribers to  $i$  is

$$n_1^i = \frac{1}{2} + \frac{u_1^i - u_1^j}{2t} .$$

Utility is made up in the following way:

$$u_1^i = u(q^i) + U(X^i) - p^i q^i - P^i .$$

Here  $q^i$  is the number of calls made to the fixed network,  $p^i$  is the price per call to the fixed network,  $X^i$  is the number of calls received by the subscriber from the fixed network, and  $P^i$  is the fixed charge for subscription. Therefore,  $U(\cdot)$  represents the benefit that subscribers obtain when they receive calls (i.e., it is the ‘call externality’).

It is useful to simplify this formula immediately. If  $c$  is the cost of making a call from a mobile to the fixed network, then a mobile network will set  $p^i = c$  in order to maximize its profits for a given level of surplus  $u_i$  in the standard way. Therefore,  $p_i$  is not really an interesting choice variable in the problem, and so set  $p_i = c$  and write  $v = \max_q : u(q) - cq$

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<sup>20</sup>This comments applies equally to the advertising markets discussed in this paper. For instance, a TV channel might charge more for a car advert if it promised not to show a rival’s advert in the same slot. Gehrig (1998) analyzes this issue in a related model.

<sup>21</sup>I assume that mobile subscribers make no calls to other mobile subscribers. If this feature were present then we would enter the territory of network interconnection which is not the topic of this paper.

for a subscriber's resulting surplus from making calls to the fixed network at marginal cost. Then the above expression for utility becomes:

$$u_i = v + U(X_i) - P_i .$$

Suppose that all mobile subscribers are roughly homogenous in terms of how many calls they receive from the fixed network (for a given set of charges). Therefore, suppose that if each mobile subscriber receives  $X_i$  calls from the fixed network, the mobile network obtains revenue  $R(X_i)$  per subscriber. (In this context, this revenue comes in the form of 'call termination' payments, but this makes no difference to the analysis.) Suppose also that a mobile network incurs the cost  $C(X_i)$  when it delivers these  $X_i$  calls to a subscriber.

Applying the previous analysis, we see from (17) that the equilibrium number of calls delivered to mobile subscribers is  $X^*$  per subscriber, where this maximizes  $U+R-C$ . In other words, this quantity is chosen to maximize the surplus available to mobile subscribers and the mobile networks together, and the interests of callers on the fixed network are ignored. From (19) we see that the equilibrium fixed charge for mobile subscription is

$$P = t + f + C(X^*) - R(X^*) .$$

(Here  $f$  is the fixed cost of connecting an additional subscriber to a mobile network, such as the cost of a mobile handset.) Thus, the charge for becoming a mobile subscriber,  $P$ , is equal to the usual Hotelling formula,  $t+f+C(X^*)$ , minus the revenue obtained from callers on the fixed network,  $R(X^*)$ . It is possible that, absent regulation of call termination, this revenue could be large, and that  $P < f$ . In this case, access to the mobile network is subsidized below the associated cost, and this could explain the prevalence of such institutions as 'handset subsidies' in the industry.

As usual in this kind of 'competitive bottleneck' model, total welfare is not maximized since the interest of fixed network callers are not taken into account when the quantity of fixed-to-mobile calls  $X^*$  is chosen. Welfare would be increased if  $X$  were increased, i.e., if the implicit price for calling mobile subscribers from the fixed network were reduced to below the unregulated equilibrium level.

## TECHNICAL APPENDIX

### A Generalized model with two-sided exclusive intermediation

Here we generalize the simple model of section 4 to allow for (i) more than two groups of participants and (ii) members of one group to care about the numbers of the *same* group who have joined their intermediary.

Suppose that each group has Hotelling-style preferences for the two intermediaries, and if the two utilities offered to group  $l$  are  $u_l^A$  and  $u_l^B$ , then firm  $i$  will attract

$$n_l^i = \frac{1}{2} + \frac{u_l^i - u_l^j}{2t_l}$$

customers from this group. Suppose that

$$u_l^i = \sum_{m=1}^M \alpha_{lm} n_m^i - p_l^i.$$

(Hence  $\alpha_{lm}$  is the benefit to a group  $l$  member of an additional member of group  $m$ . There are  $M$  groups of participants in total.) Therefore

$$\begin{aligned} u_l^i - u_l^j &= \sum_{m=1}^M \alpha_{lm} (n_m^i - n_m^j) - (p_l^i - p_l^j) \\ &= \sum_{m=1}^M \alpha_{lm} (2n_m^i - 1) - (p_l^i - p_l^j). \end{aligned}$$

And so

$$n_l^i = \frac{1}{2} + \frac{1}{2t_l} \left\{ \sum_{m=1}^M \alpha_{lm} (2n_m^i - 1) - (p_l^i - p_l^j) \right\}. \quad (32)$$

We look for the symmetric equilibrium when intermediaries set prices. To obtain an explicit formula for how market shares  $n_l^i$  depend on the prices, as in (7) above, would entail a significant amount of linear algebra. However, we can side-step this problem using the following approach. For a *given* set of prices  $\{p_l^j\}$  offered by its rival, there is a one-to-one relationship in (32) between  $i$ 's prices  $\{p_l^i\}$  and the number of customers it attracts  $\{n_l^i\}$ . In fact, if  $i$  sets the customer numbers  $\{n_l^i\}$ , its prices must be given by

$$p_l^i = p_l^j + 2t_l \left( \frac{1}{2} - n_l^i \right) + \sum_{m=1}^M \alpha_{lm} (2n_m^i - 1). \quad (33)$$

Therefore, for given rival prices  $\{p_l^j\}$ ,  $i$ 's profit with  $\{n_l^i\}$  is

$$\pi^i = \sum_{l=1}^M n_l^i \left\{ p_l^j + 2t_l \left( \frac{1}{2} - n_l^i \right) + \sum_{m=1}^M \alpha_{lm} (2n_m^i - 1) - f_l \right\}.$$

Maximizing this expression with respect to  $n_z^i$ , and using the expression (33), yields

$$\frac{\partial \pi^i}{\partial n_z^i} = p_z^i - f_z - 2t_z n_z^i + \sum_{l=1}^M 2n_l^i \alpha_{lz} = 0.$$

Since  $n_l^i \equiv \frac{1}{2}$  at the symmetric equilibrium, we see that the equilibrium prices are

$$p_z = f_z + t_z - \sum_{l=1}^M \alpha_{lz}.$$

This expression generalizes (8) above.

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