Inefficiencies in Bargaining: Departing from Akerlof and Myerson-Satterthwaite

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Abstract

We consider bargaining problems in which parties have access to outside options. The size of the pie is commonly known and each party privately knows the realization of her outside option. Parties are assumed to have a veto right, which allows them to obtain at least their outside option payoff in any event. Besides, our two agents can receive no subsidy ex post. We show that inefficiencies are inevitable for virtually all distributions of outside options, as long as the size of the surplus generated by the agreement is uncertain and may be arbitrarily small for all realizations of either party's outside option. Our inefficiency result holds true whatever the degree of correlation between the distributions of outside options, and even if it is known for sure that an agreement is beneficial.

1 Introduction

Akerlof (1970) and Myerson-Satterthwaite (1983) are among the most fundamental papers showing the potential inefficiencies caused by asymmetric information.

Akerlof considered a common value setup in which it is common knowledge that the buyer values the object more than the seller and yet there

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is no trade because of the private information held by the seller. Myerson-Satterthwaite considered a bargaining problem between a seller and a buyer who each privately observes the realization of her valuation for the object - it is thus a private value setup. They observed that inefficiencies are inevitable as soon as it is not known for sure who values the good most and the valuations of the seller and the buyer are drawn from independent distributions.

In this paper, we consider a bargaining problem with outside options. The joint value of the agreement is commonly known, and each party i privately observes the realization of her outside option. We further restrict ourselves to bargaining protocols under which each party retains a veto right until the end of the negotiation, and we assume that when a veto right is exercised, each party gets her outside option. These *ex post veto rights* induce constraints on the set of possible equilibrium outcomes. We examine the implications of these constraints using a mechanism design approach.

We show that for virtually all distributions of outside options such that the size of the bargaining surplus is uncertain and may get arbitrarily small for all realizations of either party's outside option, inefficiencies are inevitable. That is, in any equilibrium of any extensive-form game in which the final agreement is subject to the approval of both parties, some inefficiencies must occur.

Observe that we do not require the distributions of outside options to be independent across agents nor do we require that there is some uncertainty as to whether an agreement is beneficial. As a matter of fact, our leading example is such that it is common knowledge that an agreement is beneficial, and yet there are inefficiencies.

Even though we get an inefficiency result, our insight falls outside the scope of Akerlof and Myerson-Satterthwaite' s celebrated results. It falls outside the scope of Akerlof because we consider a private value model as each party knows the realization of her outside option. And it falls outside the scope of Myerson-Satterthwaite because we do not assume that outside options are independently distributed across agents, and inefficiencies may occur, even if it is common knowledge that an agreement is beneficial.

It should be mentioned that whenever it is known that an agreement is desirable our inefficiency result requires that the distributions of outside options of the two parties be correlated, and in particular that a high value of party i's outside option be only compatible with a low value of the outside option of the other party j. [1] With no correlation and when an agreement is known to be desirable, efficiency can be achieved in our setup, as in Myerson-Satterthwaite.

Our finding of bargaining inefficiencies may seem in conflict with the efficiency results obtained in the correlated case by Crémer-McLean (1983) or Johnson et al. (1990). An essential feature of our model is that we require that ex post veto constraints are satisfied. By contrast, the efficiency results previously obtained in the correlated case required that interim participation constraints are satisfied. The extra limits imposed on transfers by ex post veto constraints in turn result in inevitable inefficiencies.

It should be stressed that inefficiencies do not solely arise when the distributions of outside options are nearly independent.[2] Inefficiencies arise for virtually all distributions of outside options whatever their degree of correlation, as long as the size of the surplus generated by the agreement is uncertain and may be arbitrarily small for all realizations of either party's outside option. Our paper can thus be viewed as providing a strong argument as to why private information, even if correlated among agents, is a source of inefficiencies in bargaining.

2 The Impossibility Result

Two parties $i = 1, 2$ bargain over the division of a pie of size V. Each party i has an outside option w_i where $w_i \in [0, V]$. That is, if the parties do not reach an agreement, party i gets w_i. The values of (w_1, w_2) are not commonly known. Party i (but not party j, $j \neq i$) knows the realization of w_i. We let $g(w_1, w_2)$ denote the joint density of (w_1, w_2) on $[0, V]^2$. The

[1] Such negative correlations may be explained by the fact that the two parties are in competition for the same partners when facing the outside option alternative.

[2] The bounds on transfers implied by ex post veto constraints would immediately deliver an impossibility result in the almost independent case, by application of continuity arguments, as in Robert (1991) who considers the case of limited liability and risk-aversion. See also Laffont-Martimort (2000) for a different approach based on collusion among agents.

support of $g(.,.)$ is denoted Γ^g.

One possible interpretation of the model is that parties 1 and 2 may set up a partnership jointly worth V, and that in case they don't, they still have the option of finding out another partner.

Note that we do not require that w_1 and w_2 are independently distributed. We do not either require that there is some uncertainty as to whether an agreement is beneficial. In fact, our most striking result assumes that it is common knowledge that an agreement is beneficial, that is,

$$\Pr(w_1 + w_2 < V) = 1, \tag{1}$$

and that outside options may get arbitrarily close to V. These assumptions entail some form of negative correlation between the outside options, since a large value of w_i (close to V) implies a low value of w_j (close to 0). While slightly restrictive, such a setup applies relatively broadly to bargaining contexts in which the number of alternative partners is limited.[3,4]

Remark: The buyer/seller problem studied by Myerson-Satterthwaite (1983) is formally equivalent to our bargaining problem up to rescaling adjustments. Let v_B and v_S denote the valuations of the buyer and the seller, respectively. The buyer/seller problem studied by Myerson-Satterthwaite can be reformulated as a bargaining problem with outside options in which there is a pie of size 0, the outside option of the seller is her valuation v_S and the outside option of the buyer is $-v_B$.

The bargaining protocols. A bargaining protocol is a process that generates a non-binding proposal, as a function of messages or information

[3] Another possible interpretation of the model is that (i) party 1 and 2 jointly own an asset that they could sell at price V to a third party, (ii) the asset delivers a fixed payment in every period to at most one of the parties, and (iii) parties do not observe the number of payments received by the other party.

[4] Another illustration of the setup is as follows. Think of V as the surplus generated by an invention jointly owned by parties 1 and 2. If parties 1 and 2 do not find themselves a way to divide the surplus they can use a third party who will keep some share of the surplus for himself. While each party i may have a good idea of the share he might get out of this third party he is unsure as to the share that the third party will keep for herself. This clearly leads to a setup like the one just outlined.

transmitted between parties and/or to a third party. Specifically, a non-binding proposal consists of a decision whether or not to share the pie, combined with tentative transfers. We assume that (i) final implementation requires ratification by both parties, and (ii) each party may quit bargaining at any stage, including right before ratification. These bargaining protocols capture bargaining situations in which tentative agreements are generated by agents who do not have the power to commit to make transfers in the course of bargaining. We will also assume that no third party can subsidize the two parties bargaining parties, thus leading to a no subsidy constraint.

We will refer to such situations as *non-binding* bargaining protocols, as the parties are assumed to keep their veto right until a complete agreement is ratified by both parties.

In the mechanism design language to be developed next, the possibility of vetoing the proposal will imply (but will not be equivalent to assuming) that ex post participation constraints must be satisfied. We will further illustrate the differences between ex post participation constraints and ex post veto constraints (see subsection 4.1).

The following result summarizes a striking result that will be proven later on:

Proposition 1 *Let* $\Gamma_v = \{(w_1, w_2) \mid w_1 + w_2 < v, w_1 > 0, w_2 > 0\}$. *Suppose that g has a support that contains $\overline{\Gamma_v}$. Suppose further that g is bounded and positive (no smaller than a strictly positive scalar) on its support, and smooth.*[5] *Then for all v close enough to V, inefficiencies must arise in equilibrium in any non-binding bargaining protocol.*

Observe that the inefficiency result of Proposition 1 holds if the support of g coincides with Γ_V in which case it is known for sure that an agreement is beneficial. It also holds for all distributions (correlated or not) with full support on $[0, V]^2$.

At this point, it may be worth stressing a few notable differences with the celebrated impossibility results obtained by Akerlof (1969) and Myerson and Satterthwaite (1983).

[5] By smooth, we mean that g is continuously differentiable with respect to w_1, w_2 on the support of g.

Akerlof (1969) considered a bargaining problem between a buyer and a seller. The seller is privately informed about the quality of the good, and the quality affects the valuations of both the seller and the buyer. Moreover, the buyer is assumed to value the good more than the seller whatever the quality. In a beautiful and simple example, Akerlof shows that no trade can take place in equilibrium. Consider the result of Proposition 1 with a support of g that coincides with Γ_V. As in Akerlof's example, there is no uncertainty as to which alternative is best: an agreement is always beneficial. However, while Akerlof's model and logics crucially depend on the common value character of the payoff specification (i.e., the private information held by the seller affects the buyer's valuation), our model is one of private values, that is, each party's private information is irrelevant to determine the payoff of the other party in the various alternatives.[6] Thus, the logics of our result is radically different from that of Akerlof.

Myerson and Satterthwaite (1983) considered a bargaining problem between a seller and a buyer who are assumed to know their valuation of the good. Hence it is a private value setup like our model. But, Myerson and Satterthwaite (1983)'s impossibility result crucially hinges on the facts that (1) the supports of valuations of the seller and of the buyer overlap - hence it is not common knowledge who values the good most, and (2) the distributions of seller and buyer's valuations are independent. This should be contrasted with our setup in which the distributions of outside options are not independent and there may be no uncertainty as to which alternative is best.[7] Our result can be viewed as providing a considerable generaliza-

[6] In the agreement alternative there is no uncertainty. In the outside option alternative, each party i is assumed to know w_i.

[7] If we assume that the distributions of w_i, $i = 1, 2$ are independent from each other, then we have a result similar to that of Myerson and Satterthwaite. That is, as soon as $\Pr(w_1 + w_2 > V) > 0$ there are inefficiencies, but not otherwise. To see the Myerson-Satterthwaite type of inefficiency, consider the Vickrey-Clarke-Groves mechanism such that the transfers associated with the outside option alternative are set to zero (hence the participation constraints are automatically satisfied). The associated transfer received by party i in the agreement alternative should be set equal to $t_i = V - \hat{w}_j$ where $\hat{w}_j$ denotes the announcement of party j's outside option. It is readily verified that if the efficienct allocation is chosen on the basis of the announced types, it is a dominant strategy to report honestly his true type. The problem is about the budget constraint. Whenever

tion (to the case of correlated distributions) of the fundamental insight that private information is a source of inefficiencies in non-binding bargaining protocols.

It should be mentioned that the veto right that parties can exert at any time is essential for the derivation of our result. If we had allowed parties to surrender their veto rights, then only interim participation constraints would need to be satisfied (as in most mechanism design works using Bayesian Nash implementation). But, ex post veto constraints somehow reduce the transfers that can be made for the various realizations of the outside options. This in turn translates into unavoidable inefficiencies (despite the correlation), as we show. Observe that our impossibility result does not solely arise for distributions of outside options that are nearly independent. It arises for virtually all distributions whatever their degree of correlation. Thus, our result goes far beyond the simple observation that ex post veto constraints impose a continuous transition from the independent distribution case to the correlated distribution case (due to the induced bounds on transfers). It establishes in a strong way that private information even if correlated among agents is an inevitable source of inefficiency in non-binding bargaining protocols.

It should also be mentioned that the requirement of ex post veto constraints is different from the more usual one of ex post participation constraints. For example, when the support of the distribution $g(\cdot,\cdot)$ coincides with $\overline{\Gamma_V}$, ex post participation constraints alone (together with the Bayesian Nash incentive constraints and the ex post no subsidy constraints) need not result in inefficiencies (see subsection 4.1). Contrast this with the result of Proposition 1. The essential reason for this difference is that veto rights can be exerted off the equilibrium path in our setup, which in turn affects the form of the incentive constraints (see below).

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the agreement is optimal, i.e. $w_1 + w_2 > V$, the total transfer recievd by parties 1 and 2 should be $t_1 + t_2 = V + (V - w_1 - w_2) > V$. Hence, the budget constraint cannot be met in this mechanism. By the allocation equivalence principle, it is also immediate to check that no mechanism that induces efficiency can satisfy both the participation constraints of the parties and the budget constraint. (See Williams (1999) or Krishna-Perry (2000) for a related point in the original setup of Myerson-Satterthwaite (1983)).

3 The Mechanism Design Approach

To analyze our bargaining problem it is convenient to develop a mechanism design approach. We posit the problem in the next subsection. Then we proceed to analyze fixed sharing rule mechanisms and the Nash bargaining mechanism. Finally we prove our main impossibility result first for the case of uniform distribution and then for general distributions.

3.1 The Mechanism Design Problem

We start by defining a direct truthful mechanism with veto-rights. Using well-known technics, we will then establish the connection between the mechanism design problem and the original bargaining problem.

A *direct mechanism* is a game in which each party i is asked to report his private information - the report is denoted by $\widehat{w}_i$ - and to each profile of reports $\widehat{w} = (\widehat{w}_1, \widehat{w}_2) \in [0, V]^2$ is associated a probability $q(\widehat{w})$ that agreement is proposed, a distribution of payments $\widetilde{t}_i(\widehat{w})$ in the agreement scenario, and a distribution of payments $\widetilde{t}_i^N(\widehat{w})$ in the no-agreement scenario. A *direct mechanism with veto rights* is a direct mechanism in which each party keeps the right not to ratify the agreement and not to pay the transfers. When this right is exercised, both parties obtain their respective outside option. A direct mechanism with veto rights is *truthful* or incentive compatible if it is an equilibrium strategy for each agent i to report her true private information, i.e. $\widehat{w}_i = w_i$. Without loss of generality, we may restrict attention to direct mechanisms for which in equilibrium, the agreement if proposed is not vetoed.[8]

For any $(w_1, w_2) \in \Gamma^g$, and any $\widehat{w} \in [0, V]^2$, we let $U_i(\widehat{w}_i, \widehat{w}_j; w_i)$ denote the expected utility obtained by party i with outside option w_i when the profile of reports is $(\widehat{w}_i, \widehat{w}_j)$ and party j does not exert her veto right. Since each party keeps his veto right, we have:

$$U_i(\widehat{w}_i, \widehat{w}_j; w_i) \equiv q(\widehat{w})E_{\widetilde{t}_i}(\max(\widetilde{t}_i(\widehat{w}), w_i)) + (1 - q(\widehat{w}))E_{\widetilde{t}_i^N} \max(w_i + \widetilde{t}_i^N(\widehat{w}), w_i)$$

Let $\overline{U}_i(\widehat{w}_i; w_i)$ denote the expected payoff obtained by party i when party

[8] If an agreement is proposed and then vetoed, this can be mimicked by a no-agreement outcome without transfers.

i is of type w_i and reports $\hat{w}_i$ while party j is assumed to report her outside option value truthfully and not to exert her veto right. We have:

$$\overline{U}_i(\hat{w}_i; w_i) = E_{w_j} U_i(\hat{w}_i, w_j; w_i)$$

where the index w_j indicates that the expectation is taken over the possible realizations of w_j, conditional on w_i.

In equilibrium, each party i should find it optimal to report the true value of his outside option, as well as accepting the terms of the agreement. This means that

$$\overline{U}_i(w_i; w_i) \geq \overline{U}_i(\hat{w}_i; w_i), \tag{2}$$

and that:[9]

$$\tilde{t}_i(w_1, w_2) \geq w_i, \tag{3}$$

$$\tilde{t}_i^N(w_1, w_2) \geq 0, \tag{4}$$

as otherwise party i would veto the agreement outcome (3) or the no agreement outcome (4).

A direct mechanism with veto right is incentive compatible if and only if conditions (2) and (3) hold for all w_i, $\hat{w}_i$. It is efficient if in addition it implements an agreement if (and only if) $w_1 + w_2 < (\leq) V$ whenever $(w_1, w_2) \in \Gamma^g$. Thus, efficiency requires that for $(w_1, w_2) \in \Gamma^g$:

$$q(w_1, w_2) = \begin{cases} 1 \text{ for } w_1 + w_2 < V \\ 0 \text{ for } w_1 + w_2 > V \end{cases} \tag{5}$$

A further requirement that we will consider is that parties are not subsidized ex post. That is, for all $\hat{w}$ such that $q(\hat{w}) > 0$, and all transfer realizations:

$$\tilde{t}_1(\hat{w}_1, \hat{w}_2) + \tilde{t}_2(\hat{w}_1, \hat{w}_2) \leq V, \tag{6}$$

$$\tilde{t}_1^N(\hat{w}_1, \hat{w}_2) + \tilde{t}_2^N(\hat{w}_1, \hat{w}_2) \leq 0. \tag{7}$$

Condition (7) implies that there is no loss of generality in restricting attention to mechanisms such that $\tilde{t}_i^N(\hat{w}_1, \hat{w}_2) = 0$ (since if $\tilde{t}_i^N(\hat{w}_1, \hat{w}_2) \neq 0$

[9] Note that this last condition is equivalent to the standard ex post participation constraints.

and the announcement profile were $(\hat{w}_1, \hat{w}_2)$ at least one of the parties would prefer vetoing the transfer). From now on, we assume that $\tilde{t}_i^N \equiv 0$.

Our main result will establish that conditions (2),(5) and (3) are not compatible with the no subsidy requirement (6).

The connection with our original bargaining problem is explained below. Consider any non-binding bargaining protocol, an equilibrium of the game associated with this protocol, and assume that it involves no inefficiencies. Denote by $\sigma_i(w_i)$ the strategy used by party i in equilibrium, when his outside option is w_i. To each strategy profile $(\sigma_i(w_i), \sigma_j(w_j))$, we may associate a probability $q^{(k)}(w_i, w_j)$ that an agreement is proposed in stage k, distributions of payments $\tilde{t}_i(\hat{w})$ in the agreement scenario (and possibly distributions of payments in the no-agreement scenario)[10]. Assuming delay is costly, for the equilibrium to involve no efficiency, we should have:

$$q^{(k)}(w_1, w_2) = 1$$

if and only if $k = 1$ and $(w_1, w_2) \in \Gamma^g \cap \Gamma_V$, and since the agreement should not be vetoed in equilibrium, we should have for $(w_1, w_2) \in \Gamma^g \cap \Gamma_V$, and all transfer realizations:

$$\tilde{t}_i(w_1, w_2) \geq w_i.$$

Consider now the strategy that consists in following $\sigma_i(\hat{w}_i)$ during the first stage, and to exercise the outside option if no agreement is proposed by the end of this stage, or if the proposed agreement entails receiving a payment smaller than w_i. The expected payoff associated with that strategy when party i is of type w_i and party j follows $\sigma_j(w_j)$ is denoted $U_i(\hat{w}_i, w_j; w_i)$, and it satisfies:

$$U_i(\hat{w}_i, w_j; w_i) \equiv q^{(1)}(\hat{w})E(\max(\tilde{t}_i(\hat{w}), w_i)) + (1 - q^{(1)}(\hat{w}))w_i$$

Because strategies are in equilibrium, the deviations above must be deterred, which implies that conditions (2) hold for all w_i, $\hat{w}_i$ where again $\overline{U}_i(\hat{w}_i; w_i)$ denotes $E_{w_j} U_i(\hat{w}_i, w_j; w_i)$. It follows that the direct mechanism defined by $(q^{(1)}, \tilde{t}_i, \tilde{t}_i^N \equiv 0)$ is an efficient direct truthful mechanism with veto rights. Since the bargaining games we consider require that the bargaining parties

[10] These would play no role in our veto right paradigm.

receive no subsidy from the outside, if we can prove that any such mechanism must violate condition (6) (i.e., must be subsidized), we will have proved Proposition 1.

Observe that to prove our result, it is enough to prove it for deterministic mechanisms in which for each $(w_1, w_2) \in \Gamma$, $\tilde{t}_i(w_1, w_2)$ takes a unique value.[11] From now on, we refer to this unique realization as $t_i(w_1, w_2)$. With a slight abuse of notation, we will sometimes refer to it as $t_i(w_i, w_j)$.

3.2 Dominant Strategy Implementation.

Fixed sharing rule. Before we elaborate on our main impossibility result it may be worthwhile considering fixed sharing rule mechanisms. A fixed sharing rule is one which says that if an agreement is reached, party i gets t_i and party j gets t_j with $t_i + t_j \leq V$. So the corresponding game can be described as follows: Both parties simultaneously say if they accept the agreement (t_1, t_2). If they both say "yes", the agreement is implemented; otherwise, parties get their outside option.

Obviously, such mechanisms cannot lead the parties to reach an agreement for all feasible (w_i, w_j), unless the support of (w_i, w_j) is included in $[0, t_i] \times [0, t_j]$. To illustrate the claim, suppose that the support Γ of the distribution of (w_i, w_j) contains $\Gamma_v = \{(w_1, w_2) \mid w_1 + w_2 \leq v\}$, with $v > V/2$. Consider any (t_1, t_2) such that $t_1 + t_2 \leq V$ and, say, $t_1 \leq t_2$ (which implies that $t_1 \leq \frac{V}{2}$). When the outside option w_i of party i is above t_i, clearly party i will find it optimal to say "no", and no agreement can be reached. But,

$$\Pr(w_1 > t_1 \text{ or } w_2 > t_2) \geq \Pr(w_1 > t_1) > \Pr(w_1 > V/2),$$

which is positive since $v > V/2$, hence any fixed sharing rule mechanism must result in bargaining inefficiencies.

Dominant strategy implementation. Fixed sharing rule (t_1, t_2) mechanisms are clearly special ones, but they have an interesting property: each

[11] To note this formally, define from a possibly random mechanism a deterministic one with transfers $t_i(w_1, w_2) = E_{\tilde{t}_i}(\tilde{t}_i(w_1, w_2))$. If the conditions (2)-(6) hold for the ransom mechanism, then it is easy to see that they also hold for the deterministic mechanism (because $E_{\tilde{t}_i} \max(w_i, \tilde{t}_i(\cdot)) \geq \max(w_i, E_{\tilde{t}_i} \tilde{t}_i(\cdot))$, and because, since (3) holds, $E_{\tilde{t}_i} \max(w_i, \tilde{t}_i(w_1, w_2)) = E_{\tilde{t}_i} \tilde{t}_i(w_1, w_2)$.

party i has a (weakly) dominant strategy, i.e. say "yes" if $w_i < t_i$ and "no" otherwise. We question below whether there would exist other dominant strategy mechanisms that would improve on these fixed price mechanisms? The next Proposition will show that if implementation in dominant strategy and ex post no subsidy are requested then inefficiency is inevitable whatever the distribution whose support contains Γ_v, when $v > V/2$.

Formally, implementation in dominant strategy implies that the following constraints hold: for all w_i, $\hat{w}_i$, w_j,

$$U_i(w_i, w_j; w_i) \geq U_i(\hat{w}_i, w_j; w_i). \tag{8}$$

We have:

Proposition 2 *Suppose the distribution of* (w_1, w_2) *contains*

$$\Gamma_v = \{(w_1, w_2) \mid w_1 + w_2 \leq v\}.$$

The efficient outcome may only be implemented in dominant strategy while satisfying the (ex post) no subsidy constraint if $v \leq V/2$.

Proof. Suppose $2v > V$ and efficiency can be achieved. This implies that for any $(w_i, w_j) \in \Gamma_v$, $q(w_i, w_j) = 1$ and $t_i(w_i, w_j) \geq w_i$. This implies

$$U_i(w_i, w_j; w_i) = t_i(w_i, w_j)$$

and, for all $\hat{w}_i < v - w_j$,

$$U_i(\hat{w}_i, w_j; w_i) = \max(t_i(\hat{w}_i, w_j), w_i)$$

Constraints 8 thus imply that for all $\hat{w}_i < v - w_j$,

$$t_i(w_i, w_j) \geq \max(t_i(\hat{w}_i, w_j), w_i)$$

hence, since $t_i(\hat{w}_i, w_j) \geq \hat{w}_i$ when $\hat{w}_i < v - w_j$,

$$t_i(w_i, w_j) \geq v - w_j.$$

It thus follows that

$$t_1(0,0) + t_2(0,0) \geq 2v > V,$$

and the no subsidy constraint cannot be satisfied. ■

Note that this result does not follow from Hagerty-Rogerson (1987), who establish a connection between fixed price mechanisms and mechanisms implementable in dominant strategy in the case of ex post budget balanced transfers. Here, we only require that parties receive no subsidy ex post, not that the entire surplus be split between the two parties.[12]

3.3 More general mechanisms: preliminaries

We no longer restrict attention to fixed price mechanisms, nor to mechanisms implementable in dominant strategy. Our purpose in this Subsection is to provide some intuition as to (i) why Bayesian implementation should help obtaining efficient outcomes, and yet (ii) why obtaining full efficiency on the set Γ_V will remain difficult.

To address the first issue, we consider a simple bargaining protocol, inspired from the work of Nash, in which proposals are based on announced outside options. We show that for some distributions over outside options, full efficiency can be obtained with that protocol, while dominant strategy implementation necessarily induce inefficiencies. To address the second issue, we consider a more general version of the Nash bargaining protocol, in which the transfer to party i is only required to be increasing in his announced outside option; and we obtain a first impossibility result in that class.

The Nash Bargaining Protocol.

The Nash Bargaining protocol is described as follows. In the first stage, each party i, $i = 1, 2$ simultaneously announces an outside option $\hat{\omega}_i$. If these announcements are compatible, that is, if the sum $\hat{\omega}_1 + \hat{\omega}_2$ does not exceed V, an agreement is proposed, along with transfers $\tau_1(\hat{\omega}_1, \hat{\omega}_2)$ and $\tau_2(\hat{\omega}_1, \hat{\omega}_2)$ chosen so that each party i obtains, in addition to $\hat{\omega}_1$, half the surplus $V - \hat{\omega}_1 - \hat{\omega}_2$, that is

$$\tau_i(\hat{\omega}_1, \hat{\omega}_2) = \hat{\omega}_i + \frac{V - \hat{\omega}_1 - \hat{\omega}_2}{2} = \frac{V + \hat{\omega}_i - \hat{\omega}_j}{2}$$

[12] In the no subsidy scenario, allocations other than those corresponding to fixed sharing rules can be implemented in dominant strategy.

In case the sum $\hat{w}_1 + \hat{w}_2$ exceeds V, bargaining stops, and each party gets his outside option. In the second stage, parties sequentially report if they accept the deal. If both parties say "yes", the deal is implemented. Otherwise, the outside option alternative is implemented.

Clearly, in the second stage, it is a dominant strategy for party i with outside option w_i to say "yes" (respectively, "no") if $V_i > w_i$ (respectively, $V_i < w_i$). The following proposition characterizes the equilibrium of the outside option announcement stage.

Proposition 3 *Suppose (w_1, w_2) is uniformly distributed on*

$$\Gamma_v = \{(w_1, w_2) \mid w_1 + w_2 \leq v\}.$$

with $3V/4 \leq v \leq V$. The following is an equilibrium of the outside option announcement game: party i with type w_i announces $\hat{w}_i = a(w_i)$ where

$$a(w_i) = \frac{1}{4}V + \frac{2}{3}w_i.$$

There is agreement when $w_1 + w_2 \leq \frac{3V}{4}$. The outside option alternative is implemented when $w_1 + w_2 > \frac{3V}{4}$.

So one Corollary of Proposition 3 is that when $v = 3V/4$, full efficiency can be obtained, which shows that the best (i.e. welfare maximizing) mechanism need not in general be implementable in dominant strategy (see Proposition 2).

Another Corollary of Proposition 3 is that although full efficiency cannot be obtained when $v > 3V/4$, the Nash bargaining protocol performs better than the best fixed price mechanism.[13]

A bargaining protocol with monotone transfers.

The way surplus is shared in the Nash bargaining protocol may seem quite special, and one might hope that for more general transfer functions $\tau_i(\hat{w}_1, \hat{w}_2)$, full efficiency would obtain even as v gets close to V. To get some intuition as to why full efficiency will remain difficult to obtain, we amend

[13] Indeed, in the uniform distribution case considered in Proposition 3, the best (i.e. welfare maximizing) fixed price mechanism corresponds to the symmetric sharing rule $(\frac{V}{2}, \frac{V}{2})$ for which an agreement is reached whenever $w_i < \frac{V}{2}$. It is not difficult to see that the Nash bargaining protocol permits a higher expected welfare in this case.

the Nash bargaining protocol and allow for transfers that are substantially more general than above (and yet not as general as one could imagine). Specifically, we consider any profile of differentiable transfers $\tau_i(\hat{w}_1, \hat{w}_2)$, defined for announcements that are compatible (i.e. $\hat{w}_1 + \hat{w}_2 \leq V$), and satisfying

$$\begin{aligned} \tau_i(\hat{w}_1, \hat{w}_2) &\geq \hat{w}_i \text{ for all } i = 1, 2 \text{ and} \\ \hat{w}_i &\rightarrow \tau_i(\hat{w}_1, \hat{w}_2) \text{ is increasing in } \hat{w}_i \end{aligned}$$

Since $\hat{w}_i \rightarrow \tau_i(\hat{w}_1, \hat{w}_2)$ is increasing in $\hat{w}_i$, party i of type w_i has no incentives to understate his outside option. Obtaining full efficiency thus requires that each party i of type w_i has no incentives to overstate his outside option, as otherwise inefficiencies would obtain for the realizations (w_i, w_j) for which w_j is close to $V - w_i$. Thus, for efficiency to obtain, party i with type w_i should prefer reporting he is of type w_i rather than $\hat{w}_i > w_i$. Formally, this requires that for all $\hat{w}_i > w_i$

$$\int_0^{V-w_i} \tau_i(w_i, w_j) g(w_i, w_j) dw_j \geq \int_0^{V-\hat{w}_i} \tau_i(\hat{w}_i, w_j) g(w_i, w_j) dw_j + w_i \int_{V-\hat{w}_i}^{V-w_i} g(w_i, w_j) dw_j$$

or equivalently:

$$\int_0^{V-\hat{w}_i} (\tau_i(\hat{w}_i, w_j) - \tau_i(w_i, w_j)) g(w_i, w_j) dw_j \leq \int_{V-\hat{w}_i}^{V-w_i} (\tau_i(w_i, w_j) - w_i) g(w_i, w_j) dw_j \tag{9}$$

The absence of subsidy ex post implies that

$$\tau_i(w_1, w_2) + \tau_j(w_1, w_2) \leq V.$$

Party j's veto right (together with the observation that parties' announcements must be truthful) implies that

$$w_j \leq \tau_j(w_1, w_2).$$

Thus,

$$\tau_i(w_1, w_2) - w_i \leq V - w_i - w_j.$$

So, if g is bounded, the right hand side of (9) is comparable to $(\hat{w}_i - w_i)^2$. Since our assumptions imply that $\frac{\partial}{\partial w_i} \tau_i$ is bounded from below by a strictly positive number, and since g is assumed to be bounded from below on Γ_V,the left hand side is no smaller than a scalar comparable to $(\hat{w}_i - w_i)$. Therefore inequality (9) cannot hold for $\hat{w}_i$ close enough to w_i.

3.4 The impossibility result.

We now turn to our main impossibility result. We start by considering the simpler case where the distribution over outside options is uniform on Γ_v, because for this distribution the argument is fairly simple, and does not even require assuming no subsidy ex post, but only no subsidy ex ante. We will then move on to the general case.

3.4.1 The case of a uniform distribution.

Assume that outside options are uniformly distributed on Γ_v, and consider a direct truthful mechanism with veto rights that is efficient. We wish to show that efficiency cannot be achieved in this case. To this end, we derive a lower bound on the expected utility $\overline{U}_i(w_i; w_i)$ of agent i whose outside option is equal to w_i. By the use of incentive constraints, we will show that for all $w_i > 0$, we must have

$$\overline{U}_i(w_i; w_i) > v/2 + w_i/2.$$

Taking expectations over the possible realizations of w_i, and since $Ew_i = v/3$ for the uniform distribution, we will then conclude that efficiency is not compatible with no subsidy ex ante as soon as $v/2 + v/6 > V/2$, or equivalently, $v > 3V/4$.

As a matter of fact, computations turn out to be easier when one attempts to obtain a lower bound on $V_i(w_i)$ defined by:

$$V_i(w_i) = (v - w_i)\overline{U}_i(w_i; w_i),$$

To obtain this lower bound, we consider the possibility that party i with outside option w_i overstate his outside option by a small increment $\Delta = \frac{v-w_i}{N}$. This will give us a lower bound on $V_i(w_i)$, which (so we will show) can be written as a function of $V_i(w_i + \Delta)$. We can then consider the possibility that party i with outside option $w_i + \Delta$ overstates his outside option by Δ, which in turn will give a lower bound on $V_i(w_i + \Delta)$. And so on until $V_i(w_i + N\Delta)$, which we know is non negative.

We now start by writing down the incentive compatibility constraint of party i with outside option w_i. Since the mechanism is efficient, we have:

$$V_i(w_i) = \int_{w_j < v - w_i} t_i(w_i, w_j) dw_j.$$

For any $\hat{w}_i \geq w_i$, we also have:

$$\begin{aligned} (v - w_i)\overline{U}_i(\hat{w}_i; w_i) &\geq \int_{w_j < v - \hat{w}_i} t_i(\hat{w}_i, w_j) dw_j + w_i \int_{v - \hat{w}_i}^{v - w_i} dw_j \\ &\geq V_i(\hat{w}_i) + w_i(\hat{w}_i - w_i) \end{aligned}$$

Incentive compatibility thus implies

$$V_i(w_i) \geq V_i(\hat{w}_i) + w_i(\hat{w}_i - w_i) \tag{10}$$

Let $\Delta = \frac{v - w_i}{N}$. We choose $\hat{w}_i = w_i + \Delta$, and obtain the sequence of inequalities:

$$\begin{aligned} V_i(w_i) &\geq V_i(w_i + \Delta) + w_i\Delta \geq V_i(w_i + 2\Delta) + (w_i + \Delta)\Delta + w_i\Delta \\ ... &\geq V_i(w_i + n\Delta) + \Delta \sum_{k=1}^{n-1} (w_i + k\Delta) \\ ... &\geq V_i(w_i + N\Delta) + \Delta(Nw_i + \sum_{k=1}^{N-1} k\Delta) \end{aligned}$$

Since $V_i(w_i + N\Delta) \geq 0$, and since $\sum_{k=1}^{N-1} k\Delta = \frac{v - w_i}{N} N(N-1)/2$, we obtain:

$$V_i(w_i) \geq (v - w_i)[w_i + \frac{N-1}{N}(v - w_i)/2]$$

Since this inequality holds for all N, we finally obtain:

$$\overline{U}_i(w_i; w_i) = \frac{V_i(w_i)}{v - w_i} \geq w_i + (v - w_i)/2 = v/2 + w_i/2,$$

which concludes the proof.

Comments:

1. When (w_1, w_2) is uniformly distributed on Γ_V, another proof of our impossibility result could make use of the impossibility result of Myerson-Satterthwaite obtained in the case of independent distributions. The argument could be as follows. Suppose by contradiction that efficiency could be obtained in the case of a uniform distribution on Γ_V while satisfying the ex

post veto constraints. Then a mechanism stipulating the same transfers and allocations when $(\hat{w}_1, \hat{w}_2) \in \Gamma_V$ and no agreement with no transfer when $(\hat{w}_1, \hat{w}_2) \notin \Gamma_V$ would necessarily result in an efficient and incentive compatible allocation rule when (w_1, w_2) is uniformly distributed on the square $[0, V]^2$. But, this is impossible from Corollary 1 of Myerson-Satterthwaite (1983).

2. When (w_1, w_2) is uniformly distributed on Γ_v, $v < V$, the argument above does not apply, because when $v < \hat{w}_1 + \hat{w}_2 < V$, we only require that agent i with outside option w_i gets at least w_i, not that there is an agreement. However, the following argument would work: One can infer from Myerson-Satterthwaite that in the case of a uniform distribution on $[0, V]^2$, the second-best (requiring only interim participation constraints) leads to having an agreement whenever $w_1 + w_2 < 3V/4$ (see their characterization on pages 276-277). Thus for v, $3V/4 < v < V$, if efficiency could be achieved when (w_1, w_2) is uniformly distributed on Γ_v, one could do strictly better than the second-best found in Myerson-Satterthwaite when (w_1, w_2) is uniformly distributed on $[0, V]^2$, thus contradicting their result.[14]

3. The technique of this subsection however extends to more general distributions of outside options for which the result of Myerson-Satterthwaite could not be used. Specifically, let g_0 denote the uniform distribution on Γ_v. If the distribution g satisfies $0 \geq \frac{\partial g}{\partial w_i}$ and $g_0(1 - \mu) \leq g \leq g_0(1 + \mu)$ with $\mu > 0$, then a simple generalization of our argument gives:[15]

$$\overline{U}_i(w_i; w_i) \geq \frac{v + w_i}{2} \frac{1 - \mu}{1 + \mu},$$

which implies a failure of the ex ante no subsidy constraint when μ is not too large.

[14] Based on this observation, it can also be inferred that the Nash bargaining protocol (which is the analog of the split the difference mechanism first studied by Chatterjee and Samuelson (1983) in the buyer/seller problem) induces the second-best mechanism in the case of a uniform distribution on Γ_v.

[15] Defining $V_i(w_i) = \frac{1}{g_0} \int_{w_j < v - w_i} g(w_i, w_j) t_i(w_i, w_j) dw_j$, it is easy to see that since $\frac{\partial g}{\partial w_i} \leq 0$ and since $g_0(1 - \mu) \leq g$, (10) becomes

$$V_i(w_i) \geq V_i(\hat{w}_i) + w_i(\hat{w}_i - w_i)(1 - \mu)$$

which further implies $V_i(w_i) \geq \frac{v + w_i}{2}(1 - \mu)$. Since $g \leq g_0(1 + \mu)$, we have $\overline{U}_i(w_i; w_i) \geq V_i(w_i)/(1 + \mu)$, which gives us the desired bound.

3.4.2 The general case.

As we have seen the previous argument can be generalized to some distributions (see last comment above), but not to all distributions. To deal with the general case, we propose another argument, in which the ex post no subsidy constraint is now used.

To prove our result, we consider a direct truthful mechanism with veto rights that is efficient and that satisfies the ex post no subsidy constraint, and we establish an upper bound on

$$H_i(w_i) = \int_{w_j < v - w_i} g(w_i, w_j)(V - w_j - t_i(w_i, w_j))dw_j.$$

We will prove that when v is close to V, $H_i(w_i)$ must be close to 0 for all w_i. This in turn will imply that player i's expected utility $\overline{U}_i(w_i; w_i)$ must be close to $E[V - w_j \mid w_i]$, which in turn implies a violation of the ex ante no subsidy constraint since $w_1 + w_2 < V$ for all realizations.

The ex post participation and the no subsidy constraints together imply that for all $(w_i, w_j) \in \Gamma_v$,

$$V - w_j \geq t_i(w_i, w_j) \geq w_i, \tag{11}$$

which implies that $H_i(w_i) \geq 0$. We now use incentive compatibility constraints to derive an upper bound on $H_i(w_i)$. Incentive compatibility requires that for all $\widehat{w}_i > w_i$,

$$\begin{aligned} \int_{w_j < v - w_i} g(w_i, w_j)t_i(w_i, w_j)dw_j \quad &\geq \int_{w_j < v - \widehat{w}_i} g(w_i, w_j)t_i(\widehat{w}_i, w_j)dw_j \\ &\quad + w_i \int_{v - \widehat{w}_i}^{v - w_i} g(w_i, w_j)dw_j, \end{aligned}$$

hence,

$$\begin{aligned} H_i(w_i) \quad &\leq \int_{w_j < v - \widehat{w}_i} g(w_i, w_j)(V - w_j - t_i(\widehat{w}_i, w_j))dw_j \\ &\quad + \int_{v - \widehat{w}_i}^{v - w_i} g(w_i, w_j)(V - w_i - w_j)dw_j \end{aligned}$$

Since there exist $m > 0$ and M such that $m \leq g(w_i, w_j) \leq M$ and $\mid \frac{\partial g}{\partial w_i} \mid \leq M$, $i = 1, 2$, we have

$$\begin{aligned} g(w_i, w_j) \quad &\leq \quad g((\widehat{w}_i, w_j) + \mid g(w_i, w_j) - g((\widehat{w}_i, w_j) \mid \\ &\leq \quad g(\widehat{w}_i, w_j)(1 + M(\widehat{w}_i - w_i)/m), \end{aligned}$$

which can be used to get, defining $\varepsilon = V - v$:

$$H_i(w_i) \leq H_i(\hat{w}_i)(1 + \frac{M}{m}(\hat{w}_i - w_i)) + M[\varepsilon(\hat{w}_i - w_i) + \frac{(\hat{w}_i - w_i)^2}{2}]$$

Let $\Delta = \frac{v - w_i}{N}$, and $\rho = \max(M/m, M)$. We choose $\hat{w}_i = w_i + \Delta$, and obtain the sequence of inequalities:

$$\begin{aligned}
H_i(w_i) &\leq H_i(w_i + \Delta)(1 + \rho\Delta) + \rho\Delta(\varepsilon + \Delta) \\
&\leq H_i(w_i + 2\Delta)(1 + \rho\Delta)^2 + (\varepsilon + \Delta)\rho\Delta(1 + (1 + \rho\Delta)) \\
... &\leq H_i(w_i + n\Delta)(1 + \rho\Delta)^n + (\varepsilon + \Delta)\rho\Delta \sum_{k=0}^{n-1}(1 + \rho\Delta)^k \\
... &\leq (\varepsilon + \Delta)\rho\Delta \sum_{k=0}^{N-1}(1 + \rho\Delta)^k \text{ (since } H_i(w_i + N\Delta) = 0) \\
&\leq (\varepsilon + \Delta)\rho\Delta N(1 + \rho\Delta)^N
\end{aligned}$$

As N gets large, the term $\Delta N(1 + \rho\Delta)^N$ remains bounded (by $Ve^{\rho V}$), and Δ tends to 0. Since the inequalities hold for all N, $H_i(w_i)$ tends to 0 when ε gets to 0, that is, when v tends to V.

4 Discussion

4.1 Ex post participation versus ex post veto constraints.

In this Subsection, we examine the role of ex post veto constraints. To see why these constraints play an important role in our analysis, we now relax them and only impose the standard ex post participation constraints. In a direct truthful mechanism that would be efficient, ex post participation constraints require that

$$t_i(w_1, w_2) \geq w_i \text{ for all } i = 1, 2 \text{ and } (w_1, w_2) \in \Gamma_V.$$

In particular, they do not impose any constraints on transfers when announcement fall outside the support of g, which, we will assume here, coincides with Γ_V.

We exploit this by amending the Nash bargaining protocol described in Section 3.3. We assume that whenever the announcement profile $(\hat{w}_1, \hat{w}_2)$ lies outside Γ_V, both players are severely punished, say by an amount equal to P. Then it is easy to check that when P is large enough, each party has incentives to report his own outside option truthfully, and efficiency results. Ex post participation constraints are satisfied, because for any possible realization of (w_1, w_2), party i of type w_i gets at least w_i. Hence ex post participation constraints alone are not sufficient to undermine efficiency. This is in stark contrast with the Nash bargaining protocol analyzed previously, where ex post veto constraints were imposed.

More generally, consider any profile of differentiable transfers $t_i(w_1, w_2)$ satisfying

$$\begin{aligned} t_i(w_1, w_2) &\geq w_i \text{ for all } i = 1, 2 \text{ and all } (w_1, w_2) \in \Gamma, \text{ and} \\ w_i &\rightarrow t_i(w_1, w_2) \text{ is increasing in } w_i \text{ for all } (w_1, w_2) \text{ in } \Gamma \end{aligned}$$

We are going to show that it is possible to implement the efficient outcome (i.e $q(w) = 1$ iff $w \in \Gamma$). Indeed, choose P large, and set

$$t_i(w_1, w_2) = -P \text{ if } \ (w_1, w_2) \notin \Gamma$$

Since $w_i \rightarrow t_i(w_1, w_2)$ is increasing in w_i, party i of type w_i has no incentives to understate his outside option. He has no incentives to overstate his outside option when the following inequalities hold for all $\hat{w}_i > w_i$

$$\begin{aligned} \int_0^{V-w_i} t_i(w_i, w_j) g(w_i, w_j) dw_j \geq & \int_0^{V-\hat{w}_i} t_i(\hat{w}_i, w_j) g(w_i, w_j) dw_j \\ & + (w_i - P) \int_{V-\hat{w}_i}^{V-w_i} g(w_i, w_j) dw_j \end{aligned}$$

or equivalently:

$$\int_0^{V-\hat{w}_i} (t_i(\hat{w}_i, w_j) - t_i(w_i, w_j)) g(w_i, w_j) dw_j \leq \int_{V-\hat{w}_i}^{V-w_i} (P + t_i(w_i, w_j) - w_i) g(w_i, w_j) dw_j \tag{12}$$

Let m be a lower bound on g and M an upper bound on $\frac{\partial}{\partial w_i} t_i$ and on g, then inequalities (12) are satisfied when the following inequality holds:

$$M^2 V(\hat{w}_i - w_i) \leq Pm(\hat{w}_i - w_i)$$

Thus, picking $P > \frac{M^2 V}{m}$ ensures that efficiency can be obtained if only ex post participation constraints are required.

This result illustrates that ex post participation and ex post veto constraints are quite different. Ex post participation imposes that in equilibrium, players do not regret not having exercised their outside option. In contrast, ex post veto constraints capture the possibility that a party would use his outside option strategically in the bargaining process, pretending to be another type, and yet keeping the option of going out.

4.2 Ex post no subsidy versus Ex ante no subsidy.

As we have seen in Section 3.4, our impossibility result sometimes holds even if we do not require no subsidy ex post, but only no subsidy ex ante. We wish to illustrate here that for some distributions over outside options, the constraint that the agreement is not subsidized ex post is a necessary one. That is, for some distributions of outside options, efficiency can be achieved while satisfying the ex post veto constraints, if only the ex ante no subsidy constraint is required.

We consider a distribution over outside options defined as follows. With probability $p > 0$, outside options are distributed according to a density g_0 with full support on Γ_V. With probability $1 - p$, outside options are distributed uniformly on $F = \{(w_1, V - w_1), w_1 \in [0, V]\}$. We construct below transfers that implement the efficient outcome.

Specifically, we set

$$t_i(w_1, w_2) = w_i \text{ when } w_1 + w_2 < V$$

and

$$t_i(w_1, w_2) = w_i + T(w_i) \text{ when } w_1 + w_2 = V$$

Intuitively, the idea is to subsidize agreement ex post by a substantial amount $T(w_i)$ whenever the announcement falls on the frontier. When party i overstates his outside option, and announces $\hat{w}_i > w_i$, he obtains a transfer equal to $\hat{w}_i$ instead of w_i with probability $p \Pr_{g_0}\{w_j < V - \hat{w}_i \mid w_i\}$. However, with probability $(1 - p)$, he loses the subsidy. So choosing the subsidy $T(w_i)$ so that

$$(1 - p)T(w_i) = p \max(\hat{w}_i - w_i) \Pr_{g_0}\{w_j < V - \hat{w}_i \mid w_i\} \quad (13)$$

ensures that party i has incentives to report his outside option truthfully.

Having defined $T(w_i)$ for all w_i, it remains to check whether ex ante, these subsidies remain smaller than the expected surplus generated by the agreement. To do that, it is sufficient to check that conditional on each w_i, the expected subsidy $(1-p)T(w_i)$ is smaller than half the expected surplus, that is,

$$(1-p)T(w_i) \leq \frac{1}{2} p E_{g_0}(V - w_i - w_j \mid w_i). \tag{14}$$

It is easy to check that (13) and (14) are compatible for some distribution g_0. [16]

4.3 Further comments.

Our main impossibility result assumes that the distribution over outside options has a support that contains Γ_v for v close to V. We wish to illustrate below how departures from this assumption may allow us to implement the efficient outcome.[17]

As Section 3.3 has shown, the assumption that v be close enough to V is important. If $\Gamma^g = \Gamma_v$ with $v \leq V/2$, then efficiency can be obtained with a fixed price mechanism. And, for a uniform distribution, efficiency can still be obtained when $v = 3V/4$. So the fact that the bargaining surplus gets small for some realizations of the outside options is important.

The following example however suggests that it is not sufficient that the surplus gets small for inefficiencies to arise. We assume below that outside

[16] For example, if $g_0(w_1, w_2) = p_0 w_1 w_2$, one obtains

$$\begin{aligned} (1-p)T(w_i) &= p p_0 w_1 \max(\hat{w}_i - w_i) \frac{(V - \hat{w}_i)^2}{2} \\ &= \frac{2}{27} p p_0 w_1 (V - w_i)^3 \end{aligned}$$

and

$$\frac{1}{2} p E_{g_0}(V - w_i - w_j \mid w_i) = \frac{1}{2} p p_0 w_1 \frac{1}{6} (V - w_i)^3.$$

Since $\frac{2}{27} < \frac{1}{12}$, we get the desired inequality.

[17] Note that although full support on Γ_v for v close to V has been assumed, it should be clear that our result also holds if the support of g contains the smaller triangle: $\Gamma_v \cap \{w_1 \geq \underline{w}_1, w_2 \geq \underline{w}_2\}$ where $\underline{w}_1 + \underline{w}_2 < V$. (This would amount to a simple renormalization, with a bargaining surplus now equal to $V - \underline{w}_1 - \underline{w}_2$.)

options are distributed in a band $\Gamma_{a,b}$ defined as

$$\Gamma_{a,b} = \{(w_1, w_2), w_1 \geq 0, w_2 \geq 0, a \leq w_1 + w_2 \leq b\}.$$

We will choose b close to V, meaning that surplus may get small, but we will also choose a close to b, so that surplus cannot be large

Proposition 4 *Suppose (w_1, w_2) is uniformly distributed on $\Gamma^g = \Gamma_{V-2\varepsilon, V-\varepsilon}$, where $\varepsilon > 0$. There exists a dominant strategy mechanism that implements the efficient outcome and that satisfies the ex post no subsidy constraint.*

Proof. Indeed, choose $t_i(w_1, w_2) = V - \varepsilon - w_j$ and $q(w_1, w_2) = 1$ if $(w_1, w_2) \in \Gamma^g$, and no proposal otherwise. It is a dominant strategy to report one's outside option truthfully. Participation constraints are satisfied because $\Gamma^g \subset \Gamma_{V-\varepsilon}$, hence $t_i(w_1, w_2) = V - \varepsilon - w_j \geq w_i$. The ex post no subsidy constraint is satisfied as well because, for any $(w_1, w_2) \in \Gamma^g$

$$t_1(w_1, w_2) + t_2(w_1, w_2) \leq V + [V - 2\varepsilon - w_1 - w_2] \leq V.$$

■

In the example above, the surplus can get as low as ε for some realizations of the outside options. However, it cannot get larger than 2ε, so there is little uncertainty about the size of the surplus.

Obtaining inefficiencies thus requires that (i) the size of the surplus be small for some realizations, and that (ii) there is significant uncertainty about the size of the surplus. But these two conditions alone cannot be responsible for inefficiencies. The two following examples illustrate why it is also important that (iii) the size of the surplus be uncertain and possibly very small for all realizations of either party's outside option.

First, consider the case where outside options are distributed on the square

$$\Gamma^{sq} = \{(w_1, w_2) \mid 0 \leq w_1, w_2 \leq V/2\}.$$

The size of the surplus is small for some realizations, and uncertainty about the size of the surplus is significant. Yet efficiency can be obtained with a simple fixed price mechanism.

Second, consider now the case where outside options are distributed on $\Gamma^{sq} \cup \Gamma_{V-2\varepsilon, V-\varepsilon}$. We have the following Proposition:

Proposition 5 *Assume that with probability $p \in (0,1)$, (w_1, w_2) is uniformly distributed on $\Gamma_{V-2\varepsilon,V-\varepsilon}$, with ε small, and that with probability $1-p$, it is uniformly distributed on Γ^{sq}. Then efficiency can always be achieved.*

Proof. Consider the following direct mechanism. If the announcement vector $(\hat{w}_1, \hat{w}_2)$ lies in $\Gamma^{sq} \cup \Gamma_{V-2\varepsilon,V-\varepsilon}$, an agreement is proposed with transfers equal to

$$t_i(\hat{w}_1, \hat{w}_2) = V - \varepsilon - \hat{w}_j \text{ if } (\hat{w}_1, \hat{w}_2) \in \Gamma_{V-2\varepsilon,V-\varepsilon} - \Gamma^{sq}, \text{ and}$$
$$t_i(\hat{w}_1, \hat{w}_2) = (\frac{V}{2}, \frac{V}{2}) \text{ otherwise.}$$

For any other announcement profile, the outside option is proposed. It is readily verified that reporting the true outside option is a Bayesian Nash equilibrium.[18] ■

In the square example, efficiency obtains, but there is only a degenerate set of realizations of w_i for which the size of the surplus vanishes. In the second example, efficiency can be obtained, despite the fact that there is a non-degenerate set of realizations for which the size of the surplus vanishes: For all realizations w_i below $V/2$, the surplus can either be small (equal to ε) or substantial (equal to $V - w_i$). However there is also a whole range of realizations of w_i (i.e. for w_i above $V/2$), in which the surplus can get small (equal to ε), but it cannot get large (no larger than 2ε). So condition (iii) fails.

5 Conclusion.

We have considered a bargaining problem with outside options, but there are other applications of our setup.

First, note that we would get the same inefficiency result in the buyer/seller setup studied by Myerson and Satterthwaite, as long as the seller can insist on getting a price at least equal to her reservation value and the buyer can

[18] The only potential incentive to deviate is for party 1 to report $\hat{w}_1 > V/2$ when in fact $w_1 \leq V/2$. Yet if he does so, he will get at most V but annoucement will only be compatible with probability $2\varepsilon p$. Whereas by sticking to the truthfull announcement, he would get $V/2$ with probability at least equal to p. Thus when ε is not too large, the latter strategy is clearly preferable.

ensure that he does not pay more than his reservation value for the object for sale (the analog of our ex post veto constraints) and the price received by the seller is no more than the price paid by the buyer (the analog of the no subsidy constraint). More precisely, our inefficiency result in this setup requires that the difference between the seller and buyer's valuations is uncertain and that for all realizations of the seller's reservation value, there is a chance that the reservation value of the buyer is arbitrarily close to that of the seller. But, note that our inefficiency result does not require there is some uncertainty as to whether the seller values more the good than the seller nor does it require that distributions of reservations values be independent between the buyer and the seller.

Another application of our setup is a bargaining setup in which there is no outside option, and there is a potential of a joint venture between parties $i = 1, 2$. If there is a joint venture, each party i has to invest a cost c_i which is not observable nor verifiable and yet known to party i. The benefit of the joint venture is assumed to be commonly known V. The negotiation is about whether to make a joint venture and about how to share the benefit V of it. Of course, the share received by party i should cover the cost c_i in any circumstance as we assume that party i can at any moment decide not to go for the joint venture. This setup is clearly analogous to the one studied above. In particular, when the size of the surplus $V - c_1 - c_2$ is uncertain and can get arbitrarily small for all realizations of c_i, inefficiencies are inevitable when the partnership can receive no subsidy ex post and each partner retains his right not to participate in the joint venture until a complete agreement has been ratified by the two parties.

The analysis of this paper establishes in a strong way that the inefficiencies induced by private information do not solely arise in the case of independent distributions of signals. When parties keep a veto right allowing them to get their outside option value (or reservation value) at any time, private information must generate inefficiencies. It remains to analyze what the second best look like in such situations.[19]

[19] The Nash bargaining protocol was shown to induce the second-best when outisde options are uniformly distributed on Γ_v (see subsection 3.4.1). But, the argument does not carry over to more general distributions with arbitrary forms of correlation.

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6 Appendix

Proof of Proposition 3 The expected gain of party 1 with type w_1 when announcing $\hat{w}_1$ is

$$\begin{aligned} G(w_1, \hat{w}_1) &= \int_{\frac{V-\hat{w}_1+a(w_2)}{2}>w_2} \max(w_1, \frac{V+\hat{w}_1-a(w_2)}{2})\frac{dw_2}{V-w_1} \\ &+w_1 \int_{\frac{V-\hat{w}_1+a(w_2)}{2}<w_2} \frac{dw_2}{V-w_1} \end{aligned}$$

We now check that it is optimal for party 1 to announce $\hat{w}_1 = a(w_1)$. Given the form of $a(\cdot)$ it is readily verified that whenever the announcements are compatible, i.e. $a(w_1) + a(w_2) < V$, we have that $a(w_i) > w_i$ for $i = 1, 2$, hence the Nash bargaining share of each party i is above w_i. This allows us to simplify the expression of $G(w_1, \hat{w}_1)$ when $\hat{w}_1$ lies in a neighborhood of $a(w_1)$ into:

$$\begin{aligned} G(w_1, \hat{w}_1) &= \int_{a(w_2)<V-\hat{w}_1} \frac{V+\hat{w}_1-a(w_2)}{2}\frac{dw_2}{V-w_1} \\ &+w_1 \int_{a(w_2)>V-\hat{w}_1} \frac{dw_2}{V-w_1} \end{aligned}$$

Differentiating $G(w_1, \hat{w}_1)$ with respect to $\hat{w}_1$ yields:

$$\frac{\partial G(w_1, \hat{w}_1)}{\partial \hat{w}_1} = \frac{1}{V-w_1}[(1/2)b(V-\hat{w}_1) - b'(V-\hat{w}_1)(\hat{w}_1 - w_1)]$$

where $b(w) = -\frac{3}{8}V + \frac{3}{2}w$ is the inverse of function $a(\cdot)$. Straightforward computations show that

$$\left.\frac{\partial G(w_1, \hat{w}_1)}{\partial \hat{w}_1}\right|_{\hat{w}_1 = a(w_1)} = 0.$$