Optimal Voting Mechanisms[1]

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1 Introduction

Consider an economics department which must choose between two job candidates, a theorist and an econometrician. Everyone has a certain amount of information regarding the relative abilities of the two candidates. Everyone in the department wants to hire the better candidate, all else equal. However, the theorists have an extra incentive to hire the theorist and the econometricians an extra incentive to hire the econometrician. The other members of the department have no particular incentive to hire in either field. How should the department make the decision?

In this ongoing research, we seek to characterize optimal voting procedures for such problems. Because it seems most realistic for the setting, we assume that monetary transfers between voters are not possible. We also focus entirely on the case of two options and one round of voting, taking as given that a vote is a binary choice. These severe restrictions on the class of mechanisms makes this an unusually difficult mechanism design problem. Consequently, we make several strong simplifying assumptions.

First, we assume that information is binary. That is, each voter observes a signal which takes on one of only two possible values. We assume that the information structure is sufficiently simple that one possible value unambiguously favors one candidate and the other value favors the other. In this respect, a report of a signal favoring a candidate can be interpreted as a vote for that candidate. Without this assumption, the optimal mechanism would have to specify which signals get translated into which votes as well as how votes are aggregated into a decision. The simplification enables us to focus exclusively on the latter.

Second, we assume that the only private information is information about candidate quality. The story above suggests that who has extra incentives to favor one option over the other should be public information. That is, it should be common knowledge that the theorists derive extra utility from hiring the theorist and likewise for the econometricians. However, we take the additional step and assume that the magnitude of this extra utility is also common knowledge among the voters. Without this assumption, the lack of monetary transfers makes the problem extremely difficult. Also, we could not have voters have information about candidate quality and their own private values while retaining the binary signal structure and independence between these two variables.

Finally, we simplify further by making the environment as symmetric as possible across constituencies.

In the next section, we describe the model. In Section 3, we consider an example both to illustrate some of the key factors at play and because the example is interesting in its own right. In the example, we show that an "electoral college" type system can Pareto dominate majority rule when private values are moderately large.

We begin the analysis of optimal mechanisms in Section 4 where we consider the case where only one side has private information. In terms of the story above, suppose the choice is between hiring a theorist or no one and so only the theorists have information. We show by example that the optimal mechanism can be highly complex and not monotonic. That is, the optimal mechanism may have the property that a "vote" for a candidate sometimes makes it *less* likely that that candidate wins. We then turn to a consideration of optimal mechanisms under a constraint of monotonicity. We show that the optimal mechanism with a monotonicity constraint is a generalized version of a supermajority rule.

Finally, in Section 5, we consider an example where there is information on two sides. Surprisingly, we show that the outcome is better if all information is on one side. In terms of the example above, we show that the right candidate is more likely to be selected if the decision is made by three theorists or by three econometricians than if it is made by two members of one group and one from the other.

Related Literature. There is a large literature on the way majority voting aggregates information, following Austen–Smith and Banks [1996] and Fedderson and Pesendorfer [1996, 1997]. Some work in this vein considers voting rules which differ from straight majority, such as Fedderson and Pesendorfer [1998] or Persico [1999], though these seem to only consider varying the majority required.

Two papers which are more directly related are Chwe [1999] and Maug and Yilmaz [2002]. Chwe considers optimal mechanisms in a model similar to ours but where the differences between agents are differences in priors instead of tastes. He characterizes optimal mechanisms for only a very small part of the range of possible parameters however. Maug and Yilmaz consider a model much like ours with two groups of voters with different preferences and binary signals. However, they consider only a limited class of mechanisms. Specifically, they assume that voting is between a status quo and a change. They compare requiring a certain majority among the entire set of agents to effect the change versus requiring a certain fraction of agents in *each* group.

Another related paper is Wolinsky [2002]. He considers a model with one group of voters and gives a partial characterization of the optimal mechanism. His focus is on the contrast between cheap talk versus voting and most of his results concern cheap talk. Also, his assumptions are a little different from ours. As we discuss below, certain of our results can be seen as generalizations of some of his.

2 Model

The two candidates are denoted A and B. There are two states of the world ω_A and ω_B, each with prior probability 1/2. The voters who derive extra utility from electing candidate k ($k = A, B$) are referred to as k *partisans.* There are k_A A partisans, k_B B partisans, and k_I independents. The total number of voters, $k_A + k_B + k_I$, is denoted N and is assumed to be odd. With the exception of an example in the next section, we will take $k_I = 0$.

The payoff to electing j in state ω_j is $1 + z$ for a j partisan and 1 for every other voter. The payoff to electing j in state ω_i, $i \neq j$, is z for a j partisan and 0 for every other voter. In other words, j partisans get a private value of z when their candidate is elected. There is a common value component to the payoffs as well which equals 1 when the "right" candidate is elected and 0 when the "wrong" candidate is elected. Assume $z \in (0, 1)$. We sometimes refer to z as the level of *partisanship* or *bias.*

Each voter i receives a signal σ^i. This signal takes on values either σ_A or σ_B. The signals are conditionally iid with

$$\Pr[\sigma^i = \sigma_A \mid \omega_A] = \Pr[\sigma^i = \sigma_B \mid \omega_B] = q \in \left(\frac{1}{2}, 1\right)$$

Remark 1 The theorist–econometrician story may suggest an alternative information structure where theorists are informed only about the quality of the theory candidate and econometricians are informed only about the quality of the econometrics candidate. That is, we might assume that the information of A partisans is only about the quality of candidate A and analogously for B partisans. On the other hand, one can imagine stories where this would not hold — even where the reverse would be true. For example, suppose the decision is which of two neighborhoods to put a garbage dump in. Then residents of each neighborhood would have private costs associated with their own neighborhood being chosen or, equivalently, private benefits associated with the *other* neighborhood being selected. Hence the partisans for a particular neighborhood would have better information about their less preferred option. Here we consider the simplest, most symmetric information structure possible. We conjecture that our results would be qualitatively similar with other information structures but have not yet verified this.

A mechanism will ask each voter to report his signal. We let $v_i \in \{A, B\}$ denote a report by voter i where $v_i = A$ is a report of signal σ_A and $v_i = B$ a report of signal σ_B. We sometimes refer to a report of σ_A as a vote for A (and likewise for B). We let v denote a typical element of $\{A, B\}^N$, a vote vector. A mechanism will specify a function $m : \{A, B\}^N \to [0, 1]$ where $m(v)$ is the probability candidate A is selected given v. A strategy for voter i is a function $s_i : \{\sigma_A, \sigma_B\} \to \{A, B\}$.

Given a mechanism and a strategy for each voter, we obtain an equilibrium outcome function, say f, which maps vectors of signals to probability distributions on $\{A, B\}$. Let σ denote a typical vector of signals and let $f(\sigma)$ denote the equilibrium probability A is chosen given the mechanism and equilibrium strategies when σ is the vector of signals.

We focus on mechanisms that maximize the probability the right candidate is elected. That is, we seek to maximize

$$\left(\frac{1}{2}\right)\left[\sum_{\sigma \in \{\sigma_A, \sigma_B\}^N} \Pr[\sigma \mid \omega_A] f(\sigma)\right] + \left(\frac{1}{2}\right)\left[\sum_{\sigma \in \{\sigma_A, \sigma_B\}^N} \Pr[\sigma \mid \omega_B](1 - f(\sigma)).\right]$$

When $k_A = k_B$, this is equivalent to maximizing the sum of the payoffs of all voters.

3 An Example

As an illustration of the kinds of considerations that come into play, we begin with an example where $k_A = k_B = k_I = 3$. In this section only, we focus on two particular mechanisms and consider all pure strategy equilibria for those mechanisms. Because there will always be multiple equilibria, for each mechanism, we focus on that pure strategy equilibrium which maximizes the probability the right candidate is elected. Because there are equal number of partisans on each side, this equilibrium also maximizes the sum of the payoffs to all voters. In later sections, we characterize optimal mechanisms.

We say that voter i *votes informatively* if his strategy is $s_i(\sigma_A) = A$ and $s_i(\sigma_B) = B$.

First, consider majority voting. With this mechanism, $m(v) = 1$ iff v contains a majority of A reports and $m(v) = 0$ otherwise. Because the states are equally likely, if everyone votes informatively, the optimal way to use the information is to select A if a majority of the voters receive an A signal. Hence if informative voting is an equilibrium, this mechanism achieves the best possible payoff.

It is not hard to show that the best equilibrium has all voters voting informatively if $z \leq 2q-1$ but has only the independents voting informatively for larger z. For $z > 2q-1$, the A partisans always vote for A, while the B partisans always vote for B.

To see this, suppose all voters vote informatively and consider the decision by an A partisan. This voter considers only the situations in which he is pivotal. This occurs iff four of the other eight voters vote for A and four for B. Since all voters vote informatively, this means that the information among the other voters consists of four A signals and four B signals. Because of the symmetry of the problem, this is informationally neutral.

Hence conditioning on being pivotal has no information content whatsoever. So the voter votes assuming he is pivotal but taking into account only his own signal.

Clearly, if his signal favors A, he certainly votes for A. Suppose his signal favors B. Since he is assuming he is pivotal, he votes for B iff

$$\Pr[\omega_B \mid \sigma_B] \geq z + \Pr[\omega_A \mid \sigma_B]$$

or, given our assumption that the states are equally likely,

$$q \geq z + 1 - q$$

or $z \leq 2q - 1$. Hence all voters voting informatively is an equilibrium iff $z \leq 2q - 1$.

Intuitively, this is a strong condition. It requires the partisans to be "almost" unbiased in the sense that each partisan's signal by itself is enough to outweigh his bias.

What happens when $z > 2q - 1$? From the above, we see that for a partisan to vote informatively when his signal goes against his candidate, it must be true that being pivotal is bad news about his candidate. But this implies that it is impossible to have some A partisans and some B partisans voting informatively. To see this, suppose to the contrary that at least one A partisan and one B partisan vote informatively. Then, informationally, each sees himself as pivotal in exactly the same situations. Since the A partisan votes informatively, being pivotal must be bad news about A. But then the B partisan would vote for B whenever he is pivotal, a contradiction.

Can we have an equilibrium where some A partisans vote informatively but no B partisans do? It is not hard to show that if a B partisan does not vote informatively, he must be voting for B regardless of his signal. Given this, consider an A partisan who is pivotal. He knows that B gets three votes regardless of information and that A gets no more than two such automatic votes (from the other two A partisans). Hence if the A partisan is pivotal, A must have won among the informative voters by at least a one vote margin. Hence being pivotal is good news about A, not bad. So the A partisan will vote for A regardless of his signal.

Consider as an alternative mechanism a kind of electoral college, where the A partisans, B partisans, and independents are treated like electoral districts. That is, the mechanism selects A if A receives at least two votes from at least two of the three groups of voters. Again, it is not hard to show that informative voting is an equilibrium iff $z \leq 2q-1$. Because majority voting aggregates the information more efficiently than the electoral college, majority voting is preferred in this range.

However, one can show that there is a $\bar{z} \in (2q - 1, 1)$ such that if $z < \bar{z}$, there is an equilibrium in the electoral college where all independents and four partisans vote informatively. More specifically, one A partisan votes for A independently of his signal and one B partisan always votes for B. All other voters follow their signals.

To see how this works, consider an A partisan who is supposed to vote informatively. As before, his vote is optimal conditional on being pivotal. A necessary condition for his vote to be pivotal is that he is pivotal in his "district." That is, it must be true that the other two A partisans split. Because one of them votes for A regardless of his signal, this means that the other A partisan received a B signal. Hence being pivotal in his district is bad news regarding A, exactly the effect we need to induce A to vote informatively.

More simply, an A partisan is pivotal in his district only if half of the other A partisans vote for B. Since the district is predisposed to vote for A, this is bad news about A. The bad news effect helps counteract the voter's partisanship and so encourages him to vote informatively.

Of course, for A to be pivotal, the A district must also be pivotal. That is, it must be true that one of the other two districts is voting for A while the other is voting for B. Because the B district has one voter always voting for B, it is relatively likely to vote for B. Hence this information is slightly good news about A. However, this effect turns out to be smaller, so that the total effect of being pivotal in his district and his district being pivotal is bad news about A.

Note that the A who is supposed to always vote for A is not pivotal in the same circumstances. If he is pivotal, it means the other two A's split, an informationally neutral event. It also means that his district is pivotal, which is mildly positive news about A. Hence for this voter, it is indeed optimal to vote for A given any signal.

Given this, suppose $z \in (2q-1, \bar{z})$. We have seven out of nine voters voting informatively with electoral college, versus only three out of nine voting informatively with under majority voting. While electoral college aggregates less efficiently any given number of informative votes, this problem is more than offset by the advantage of having more informative votes, so the electoral college strictly dominates majority voting.

In fact, the electoral college Pareto dominates majority voting in this situation. To see this, simply note that each candidate is chosen with *ex ante* probability 1/2 in either regime, but the right candidate is chosen with a higher probability under electoral college. Hence even the partisans prefer it.

When $z > \bar{z}$, only independents vote informatively in the electoral college. Note that this means that the outcome is determined by majority rule among the independents and hence is the same as that under majority voting. In summary, then, whenever $z > 2q-1$, the electoral college at least weakly Pareto dominates majority voting.

On the other hand, this electoral college mechanism is not the optimal mechanism in general.

4 Optimal Mechanisms with One–Sided Information

We simplify the model to begin the study of optimal mechanisms. As we will see, optimal mechanisms can be very complex even in this simpler setting. In this section, we assume that only the A partisans have information. Effectively, then, we set $k_B = k_I = 0$. However, we continue to consider mechanisms which maximize the probability of electing the right candidate. One can think of this as assuming that only the A partisans are informed, but there are still as many B partisans as A partisans and our objective is to maximize total payoffs.

It is not hard to show that it is without loss of generality to consider a mechanism which depends only on the number of A votes. Hence we can summarize a mechanism by a set of probabilities p_j, $j = 0, \ldots, k_A$, where p_j is the probability of electing A when j A reports are received.

If we do not constrain the mechanism further, the optimal mechanism can be very complex. For example, suppose $k_A = 3$. One can show that the optimal mechanism has $p_0 = 0$ and $p_2 = 1$, but varies p_1 and p_3 in a non–monotonic and discontinuous fashion as z varies between 0 and 1. Specifically, we can divide the range of z's into four ranges. For $z \leq 2q - 1$, majority rule is incentive compatible (i.e., informative voting is an equilibrium of the mechanism) and so it is the optimal mechanism. For the second lowest range of z's, the optimal mechanism is a kind of supermajority mechanism, where A is chosen with probability 1 if there are at least two A votes and is chosen with positive probability if there is only one A vote. In the next lowest range, the optimal mechanism has the probability A is elected non–monotone in the number of A votes. Specifically, A is elected only if there are two A votes and with a certain probability if there are three A votes. Finally, in the highest range of z's, the optimal mechanism has A elected if there are two A votes and with a certain probability if there is only one A vote. In particular, this last mechanism has $p_3 = 0$.

These last two regions are counterintuitive and do not look like mechanisms we appear to see. In addition, the break between the bottom two regions and the top two is discontinuous. More specifically, as we move from the second region into the third, p_1 drops discontinuous from a strictly positive level to 0 and p_3 drops discontinuously from 1 to an interior level.

To understand what is going on in the two high z regions, note that the key issue is to discourage false reporting of A signals. With this in mind, consider a mechanism which elects A only when exactly two voters vote for A. Is such a mechanism incentive compatible? As always, a voter conditions on being pivotal. He is pivotal in two situations. First, he is pivotal if exactly one of the other two voters has an A signal. As before, this is informationally neutral and leaves him preferring to vote for A even if he has a B signal. Second, he is also pivotal if both of the other voters have A signals. However, in this situation, voting for A will cause A to *lose*. Naturally, this tends to discourage false reporting of A signals.

With non–monotonic mechanisms, one can even extract information when $z > 1$. This is surprising since the A partisans then prefer electing A even if they knew the state were ω_B.

In short, when non–monotonic mechanisms are allowed, the analysis is much more complex and can lead to unintuitive predictions. There is one further important concern with non–monotonic mechanisms: they do not appear interpretable as voting. Recall that our interest is in optimal voting mechanisms. We use incentive compatibility to relate reporting an A signal to voting for A. However, we do not believe that any way a mechanism uses a report of an A signal can allow that report to be interpreted as a vote for A. Normally, we think of a vote for A as being an action which makes the group selection of A more likely. Use of reports which violates this surely invalidate the interpretation of the report as a vote.

Given these concerns with non–monotonic mechanisms, we henceforth impose a monotonicity constraint. So we now require the mechanism to satisfy $p_0 \leq p_1 \leq \ldots \leq p_{k_A}$.

One might wonder whether the Revelation Principle still applies. It is clear that any monotonic, incentive compatible outcome can be generated as a truth telling equilibrium of a direct, monotonic mechanism. However, use of the Revelation Principle requires the converse. It is not hard to show that in this environment, any equilibrium outcome of a monotonic mechanism must be monotonic. Hence the set of outcomes generated by some equilibrium of a monotonic mechanism is the set of monotonic, incentive compatible outcomes. Hence we can simply maximize by choosing an outcome subject to incentive compatibility and monotonicity.

The following result generalizes a result in Wolinsky [2002].

Theorem 1 *When it is not constant, the optimal mechanism with $k_B = k_I = 0$ given a monotonicity constraint takes the form*

$$p_j = \begin{cases} 0 & j = 0, \ldots, j_1^* \\ \bar{p} \in (0,1) & j = j_1^* + 1, \ldots, j_2^* - 1 \\ 1 & j = j_2^*, \ldots, N \end{cases}$$

for generic values of z and q. If $z < 2q - 1$, $j_1^* = (N-1)/2$ and $j_2^* = (N+1)/2$, so the mechanism reduces to majority rule.

The result is for generic values of z and q because one can have certain ties in payoffs for specific values of z and q. At such a tie, there may be more than one optimal mechanism. However, these ties are only on a lower dimensional subspace — a tiny change in z leaving q fixed or vice versa would eliminate the tie. Intuitively, the objective function and all constraints are linear in the probabilities. If an indifference curve coincides with a constraint, we may be unable to pin down the optimum. However, a small change will eliminate the problem.

If z is too large, it is impossible to obtain information from the partisans using a monotonic mechanism. Because of the 50–50 prior, any constant mechanism is optimal in such a case.

We conjecture that the interior probability is only at a single value of j but have been unable to show this so far. We can show this for the cases of 3 voters and 5 voters.

To see the intuition, consider the case of 3 voters and assume that $z > 2q - 1$ but is not too large. In this case, the optimal mechanism has $p_2 = p_3 = 1$, $p_0 = 0$, and $p_1 \in (0, 1)$. To see the idea, suppose $p_1 = 1$. Given this, when is a voter pivotal? He is pivotal iff both of the other two voters reports a B signal. This is, of course, the worst possible information regarding A. By hypothesis, z is not too extreme. Specifically, the needed hypothesis is that faced with three B signals, the voter prefers B to win. This implies then that if the voter receives a B signal, he reports it honestly. If he has an A signal, what will he do? In this case, his signal together with the hypothesis that he is pivotal is equivalent to learning that there are two signals favoring B and one favoring A. By hypothesis, $z > 2q - 1$, which exactly says that a one signal advantage for his less preferred candidate is not sufficient to induce him to vote that way. That is, he will report an A signal honestly as well.

In the case where the interior probability is at a single value of j, one obtains an interesting additional result. Specifically,

Theorem 2 *Assume the optimal monotonic mechanism when $k_B = k_I = 0$ has $j_2^* = j_1^* + 2$, so the interior probability is used for only a single value of j. Then the mechanism is the same as that which maximizes the payoff to the A partisans except that the latter would set $p_{j_1^*+1} = 1$.*

In other words, the optimal mechanism from the point of view of the A partisans would be the same as the optimal monotonic mechanism except that the A partisans would prefer to increase the interior probability to 1.

This result has two implications. First, it says that the rest of the group cannot do much better than to simply turn the decision over to the A partisans to make as they like. Second, looking at the same point from a different perspective, it says the the optimal mechanism is relatively robust to collusion. In other words, one might naturally worry that the A partisans might share information among themselves and coordinate their reports to the mechanism. This result implies that the gains to such collusion would be small.

On the other hand, it is important to not overstate this result. For example, with 3 A partisans, the difference between the optimal monotonic mechanism and the mechanism the A partisans would find optimal occurs only when there is only one A vote. However, this happens with probability $q(1-q)$. Even if $q = .8$, this is more than 15%. Of course, for larger number of voters, the probability of any given number of votes for A becomes small. But with large numbers of voters, we expect information aggregation effects to make a wide variety of mechanisms reasonably efficient. Hence the main interest in the problem is with small numbers.

Wolinsky gives results like these for a slightly different model. However, his version of Theorem 1 considers the optimal mechanism without a monotonicity constraint and analyzes it only in the range of parameter values for which the optimal mechanism is monotonic. When monotonicity is a constraint, the possibility that this constraint binds makes the analysis more complicated. In particular, even aside from the differences in the models, Wolinsky's proof does not work for our model. His version of Theorem 2, however, does seem to be driven by the same forces as ours.

5 Two–Sided Information

Given the nature of the solution with one–sided information, it is natural to conjecture that a modified version of an electoral college might be optimal if all the groups have information. We have not computed any examples for the three–sided information case (i.e., where A partisans, B partisans, and independents all have information), but do know that such a conclusion does not follow in general for the two–sided case.

Specifically, suppose that $k_A = 2$ and $k_B = 1$. It is not hard to show that the optimal mechanism will treat the two A partisans symmetrically. That is, we can consider mechanisms for which the probability A is chosen is p_{ij} when 1 and 2 send a total of i A reports and 3 sends j A reports — so $i = 0, 1, 2$ and $j = 0, 1$. Since we consider only monotonic mechanisms, we require $p_{0j} \leq p_{1j} \leq p_{2j}$ and $p_{i0} \leq p_{i1}$. We do not have a full characterization of the optimal mechanism, but have computed some examples. Typically, the optimal mechanism takes one of two forms. Listing the probabilities in a matrix where p_{ij} is given in the ith row and jth column, one of the forms is

	0	1
2	$1-r$	1
1	r	1
0	0	0

for some $r \in (0,1)$ while the other is

	0	1
2	r_1	1
1	r_1	r_2
0	0	0

for some $0 < r_1 < r_2 < 1$. While there may be some way to interpret this as a kind of electoral college, the interpretation is not obvious.

One interesting observation is that $p_{11} > p_{20}$. That is, holding the level of support (number of A votes) fixed, broader support generates a higher probability of election. One A vote from each group gives A a higher chance of winning than two A votes from the A partisans and none from the B. On the other hand, the opposite is true for the case of two votes favoring B. If there are two votes favoring B, broad–based support would mean that one is from an A partisan and one from the B. Yet the probability of electing B in this case is $1 - p_{10} < 1$. With "narrow" support where both B votes come from the A partisans, the probability of electing B is $1 - p_{01} = 1$. Hence narrow support is stronger for B.

The more surprising result concerns comparing two–sided and one–sided information. For motivation, suppose that the decision must be made not by the group as a whole but by a committee of three voters. Suppose that no independents are informed, so the only choice is how many A partisans and how many B partisans to put on the committee. Because the model is entirely symmetric, the probability of electing the right candidate is the same whether the committee consists of all A partisans or all B partisans and is the same whether it has two A partisans and one B or the reverse. Hence the real choice is between all partisans of one type versus a mix of the two.

A natural intuition would seem to be that it is best to put some representatives of both camps onto the committee. Certainly, this appears to be standard practice, though this may be motivated by fairness considerations rather than information aggregation. To the contrary, we will show that it is *always* weakly better and often strictly better to have the committee made up entirely of partisans from one side.

To show the result, assume voters 1 and 2 are A partisans. We will show that given this, it is better that voter 3 also be an A partisan rather than have him be a B partisan.

As before, since 1 and 2 are both A partisans, we can assume, without loss of generality, that any mechanism treats them symmetrically. That is, we can consider mechanisms for which the probability A is chosen is p_{ij} when 1 and 2 send a total of i A reports and 3 sends j A reports — so $i = 0, 1, 2$ and $j = 0, 1$. Since we consider only monotonic mechanisms, we require $p_{0j} \leq p_{1j} \leq p_{2j}$ and $p_{i0} \leq p_{i1}$.

The payoff to an A partisan who reports A upon receiving a B signal is

$$\begin{array}{c} \Pr[\omega_A \mid \sigma_B][q^2 p_{21} + q(1-q)p_{11} + q(1-q)p_{20} + (1-q)^2 p_{10}](1+z) \\ + \Pr(\omega_B \mid \sigma_B)[(1-q)^2 p_{21} + q(1-q)p_{11} + q(1-q)p_{20} + q^2 p_{10}]z \\ + \Pr(\omega_B \mid \sigma_B)[(1-q)^2(1-p_{21}) + q(1-q)(1-p_{11}) + q(1-q)(1-p_{20}) + q^2(1-p_{10})] \end{array}.$$

The payoff if he reports B honestly instead is the same except that we have one fewer A report — so p_{ij} is replaced by $p_{i-1,j}$ everywhere. Let $x = q(1-q)$. We can write the incentive constraint that the A partisan report a B signal honestly as

$$(\alpha + \gamma)p_{21} + \alpha p_{20} - \alpha p_{01} - \gamma p_{11} - (2\alpha + \beta)p_{10} + (\alpha + \beta)p_{00} \leq 0 \tag{1}$$

where

$$\alpha = x[z - (2q-1)]$$

$$\beta = (2q-1) - z(1-2x)$$

$$\gamma = (2q-1)2x.$$

Note that if $\alpha \leq 0$, then $z \leq 2q - 1$, so majority voting would be incentive compatible regardless of whether we have three A partisans or two A partisans and one B. Hence the interesting case is when $\alpha > 0$. Also, $\gamma > 0$ always. β can be positive or negative.

If voter 3 is also an A partisan, the constraint that he honestly report a signal that opposes his preferred candidate takes the form

$$(\alpha + \gamma)p_{21} - (\alpha + \gamma)p_{20} - (\alpha + \beta)p_{01} + 2\alpha p_{11} - 2\alpha p_{10} + (\alpha + \beta)p_{00} \leq 0 \tag{2}$$

while it is

$$-(\alpha + \beta)p_{21} + (\alpha + \beta)p_{20} + (\alpha + \gamma)p_{01} + 2\alpha p_{11} - 2\alpha p_{10} - (\alpha + \gamma)p_{00} \leq 0 \tag{3}$$

if 3 is a B partisan.

First, let us consider the case where voter 3 is a B partisan. In this case, the only interesting range is where $\beta > 0$. To see this, suppose we sum equations (1) and (3) to get

$$\gamma(p_{21} - p_{11}) + \gamma(p_{01} - p_{00}) - \beta(p_{21} - p_{00}) + (2\alpha + \beta)(p_{20} - p_{10}) + 2\alpha(p_{11} - p_{10}) \leq 0.$$

If $\beta \leq 0$, every term on the left–hand side is nonnegative by monotonicity. Hence every term must equal zero. Hence, in particular, $p_{21} = p_{11} = p_{10} = p_{00}$. Hence the only incentive compatible monotonic mechanism is a constant.

Summarizing,

Lemma 1 *If two voters are A partisans and one is a B partisan, then if*

$$z \geq \frac{2q-1}{1-2x},$$

all incentive compatible monotonic mechanisms are constant.

It is also not hard to show that

Lemma 2 *There is always an optimal monotonic mechanism where A is chosen with probability 0 if no voter reports A. If there is an optimal monotonic mechanism which is not constant, then it must have A chosen with probability 1 if every voter reports A.*

In light of these last two results, we focus the discussion on mechanisms with $p_{00} = 0$ and $p_{21} = 1$ and parameters where $\beta > 0$.

Fix any mechanism for the case where voter 3 is a B partisan which is incentive compatible and monotonic. Let $\{p_{ij}\}_{i=0,1,2;j=0,1}$ denote this mechanism. Without loss of generality, we may as well consider only mechanisms with $p_{00} = 0$ and $p_{21} = 1$ from the above. We show that there is a mechanism for the case where 3 is an A partisan which is monotonic, incentive compatible, and yields the same payoff. Let $\{\hat{p}_{ij}\}_{i=0,1,2;j=0,1}$ denote this mechanism. To construct it, set $\hat{p}_{21} = 1$ and $\hat{p}_{00} = 0$. Let $\hat{p}_{20} = \hat{p}_{11} = 1$ and $\hat{p}_{10} = \hat{p}_{01} = 1 - \Delta$ where

$$\Delta = \frac{2}{3}[p_{11} - p_{10}] + \frac{1}{3}[p_{20} - p_{01}].$$

In other words, the $\hat{p}$ mechanism sets the difference in probability of electing A between two votes and one vote equal to the expected difference from the p mechanism. It is not hard to show that the two mechanisms will then yield the same payoff. Note that Δ is a convex combination of differences of probabilities. Since each of these differences must be less than 1, $\Delta \leq 1$. Also,

$$\Delta = \frac{1}{3}[(p_{11} - p_{01}) + (p_{20} - p_{10}) + (p_{11} - p_{10})].$$

Hence monotonicity implies that $\Delta \geq 0$. Because of this, we see that the $\hat{p}$ mechanism is feasible (i.e., all probabilities are well defined) and monotonic. To satisfy incentive compatibility, we require that equations (1) and (2) hold at this point.[1] Given that the preferences of the three voters are the same and that this mechanism treats them symmetrically, it should come as no surprise that these turn out to be the same equation, specifically,

$$(3\alpha + \beta)\Delta \leq \alpha + \beta.$$

To show that this must hold, sum two times equation (1) plus equation (3) to obtain

$$2\gamma + \alpha - \beta + (3\alpha + \beta)p_{20} + (\gamma - \alpha)p_{01} + (\alpha - \gamma)2p_{11} - (3\alpha + \beta)2p_{10} \leq 0$$

or

$$2\gamma + \alpha - \beta + (3\alpha + \beta)(p_{20} - 2p_{10}) + (\alpha - \gamma)(2p_{11} - p_{01}) \leq 0.$$

Recall that $3\Delta = 2p_{11} - p_{01} + p_{20} - 2p_{10}$, so

$$2\gamma + \alpha - \beta - (\gamma + \beta + 2\alpha)(2p_{11} - p_{01}) + (3\alpha + \beta)3\Delta \leq 0.$$

But $p_{11} \leq 1$ and $p_{01} \geq 0$ implies that $2p_{11} - p_{01} \leq 2$. Hence using $\gamma + \beta + 2\alpha > 0$, we see that this implies

$$2\gamma + \alpha - \beta - 2\gamma - 2\beta - 4\alpha + (3\alpha + \beta)3\Delta \leq 0$$

or

$$(3\alpha + \beta)3\Delta \leq 3(\alpha + \beta)$$

as was to be shown.

To summarize, assume a nonconstant mechanism is optimal in the case where 1 and 2 are A partisans and 3 is a B partisan. Fix any monotonic, incentive compatible mechanism with $p_{21} = 1$ and $p_{00} = 0$. The optimal mechanism must have these properties, but the mechanism we fix can be optimal or not. Then we can find a monotonic incentive compatible mechanism for the case where voter 3 is an A partisan which yields the same payoff. In fact, we can find one which is a supermajority mechanism where A is elected for sure if there are at least two A votes and with some probability if there is only one A vote. Hence the payoff when 3 is an A partisan must be at least as large as the payoff when he is a B partisan.

In fact, it is easy to see the payoff when 3 is an A partisan will often be larger. From the above, we saw that if $z > (2q - 1)/(1 - 2x)$, there is no monotonic incentive compatible mechanism which is not constant if 3 is a B partisan. However, one can show

[1] Of course, incentive compatibility also requires the A partisans to vote for A when they receive an A signal. One can show that this will hold for any choice of p_1.

that it is possible to improve on a constant mechanism is 3 is an A partisan as long as $z \leq (2q-1)(1-x)/(1-3x)$. Note that

$$\frac{1}{1-2x} < \frac{1-x}{1-3x}$$

iff $1-3x < 1-3x+2x^2$ which obviously holds. Hence there is a range of z's for which only constant mechanisms are feasible when 3 is a B partisan, but better mechanisms are feasible when he is an A partisan.

The result presents an intriguing contrast to the results of Krishna–Morgan [2001] for cheap talk games. They consider a model with an interested decision maker who gets information sequentially from one or more informed parties. The decision maker cannot commit to how he will use the information. Analogously to our model, the decision maker knows the preferences of the informed parties. Their main results are the following. First, if faced with a choice between getting information from one party or getting informed from two parties who are biased in the same direction, it is always better to get information from only the less biased of the two. Second, if faced with a choice between getting information from one party or from two parties who are biased in opposite directions, it is always better to get information from the two. By contrast, with only A partisans informed, the probability of the right decision being made is increasing, usually strictly, in the number of A partisans who vote. Furthermore, it is better to have the biases all on the same side rather than on opposite sides as in Krishna–Morgan.

There are at least three potentially important differences between our model and Krishna–Morgan and it is hard to know at this point which differences are key to the difference in results. First, communication is via cheap talk in their model, via voting here. This is clearly important to the fact that more voters, even if biased in the same direction, cannot be bad. If additional voters would have a negative effect, the optimal mechanism would simply ignore them. Because the decision maker in Krishna–Morgan cannot commit himself to ignoring one speaker, this is not possible in their model.

Second, the cheap talk in Krishna–Morgan occurs takes place sequentially, with one speaker observing the message of the other before he speaks. Our voting is simultaneous. The sequentiality is clearly important to their construction in the case where the informed parties have different preferences. See also Lipman–Seppi [1995] for an illustration of the importance of sequential messages.

Third, the speakers have the same information in Krishna–Morgan, while they do not here. Again, this plays an important role in the construction they use.[2]

[2] The difference could also be related to the dimensionality of the information may be relevant. We assume two possible signal realizations for the reasons discussed at the outset; Krishna–Morgan assume a continuum. Relatedly, Battaglini [2002] has shown the importance of the dimension of the space of uncertainty in generating separating equilibria.

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