Competitive Markets with Endogenous Health Risks*

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Abstract

We study a general equilibrium model where agents' preferences, productivity and labor endowments depend on their health status, and occupational choices affect individual health distributions. Efficiency typically requires agents of the same type to obtain different expected utilities if assigned to different occupations. Under mild assumptions, workers with riskier jobs must get higher expected utilities if health affects production capabilities. The same holds if health affects preferences and health enhancing consumption activities are sufficiently effective, so that income and health are substitutes. The converse obtains when health affects preferences, but health enhancing consumption activities are *not* very effective, and hence income and health are complements. Competitive equilibria are first-best if lottery contracts are enforceable, but typically not if only assets with deterministic payoffs are traded. Compensating wage differentials which equalize the utilities of workers in different jobs are incompatible with ex-ante efficiency. Finally, absent asymmetric information, there exist *deterministic* cross-jobs transfers leading to ex-ante efficiency.

Keywords: compensating wage differentials, competitive markets, individual health risks, Pareto efficiency.

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1 Introduction

The paper studies a simple general equilibrium model where the aggregate distribution of health is endogenous, and it is determined jointly with the allocation of labor and consumption goods. The model has the following key features. First, the health distribution of each worker depends on his occupational choice. Second, health affects agents' preferences, productivity and their labor endowments, namely their consumption and production capabilities. Third, occupational choices are indivisible, that is each occupation is defined by an indivisible set of tasks, and each worker can choose at most one occupation together with the associated health distribution.[1] These assumptions capture some of the most significant real-life determinants and effects of individual health status. Indeed, occupational choices generally have both direct and indirect effects on health risks. By influencing the likelihood of work-related injuries and diseases, they directly affect the distribution of future health states. Moreover, they may also change workers' health risks indirectly by determining their location choices, for instance by inducing them to locate in less safe areas (e.g., more crime-ridden or with poorer health facilities). Health status also influences workers' productivity, labor endowment and preferences, as largely documented by the empirical literature (see Rosen, 1986, and Viscusi, 1993, among others). Finally, an important real-world feature of most health risks associated to production activities is that they are diversifiable only to a limited extent. This is due to a non-convexity associated to the specialization of labor, leading most workers to choose a single occupation.

Our analysis encompasses both the direct and the indirect effects of occupational choices on health risks. We study the properties of efficient and equilibrium allocations in a setting where different distributions of health are associated to different occupations. Workers trade in competitive markets, produce several goods, and use financial (insurance) markets to transfer income across individual health states. At a more abstract level, we analyze a competitive set-up where agents (workers) choose among indivisible risky assets (occupations) paying either monetary or non-pecuniary random returns (wages and health, respectively), and where non pecuniary contingent returns are only imperfectly transferable (health status cannot be separated from individuals and can be modified only within certain limits). Other examples of assets with these characteristics include occupations requiring human capital, jobs with unpleasant characteristics, as well as memberships to clubs and organizations. Several of the results of our analysis hold in settings where those assets are traded. For the sake of clarity, however, in this paper we will stick to the health application.

[1] This assumption is imposed for simplicity; in our setting it is sufficient that a worker cannot choose an arbitrarily large number of jobs and offer a small amount of labor in each of them.

The present paper is related to a vast literature on work-related health risks and non pecuniary job attributes, that goes back to Adam Smith (see Evans and Viscusi (1993), Lucas (1972), Rosen (1986) Viscusi (1990, 1993), among many others), and focuses on the determination of the equilibrium wage premia commanded by risky, or otherwise unpleasant, jobs.[2] It characterizes and estimates competitive wage differentials, under the "equilibrium condition" that workers of the same type obtain equal utility levels if assigned to different occupations. Generally, such a condition is derived through a partial equilibrium labor market analysis, or directly imposed as part of the definition of competitive equilibrium. Within this literature, a conventional wisdom has emerged that *utility equalizing* wage differentials lead to market efficiency.[3]

In contrast with this view, we demonstrate that Pareto optimality typically requires workers of the same type to get different (expected) utility levels, if assigned to different occupation. This is a central result of the paper, as it motivates most of our analysis. Moreover, we show that the shadow value of the efficient consumption vectors (calculated at the Pareto-optimal shadow prices) assigned to each particular type of agent typically differs from that of his initial endowment. In other words, efficient allocations are not *budget balancing,* and Pareto optimality requires cross-jobs transfers. As a corollary, markets where wage differentials equalize utilities across occupations typically do not implement first-best allocations.

These findings hinge upon the imperfect transferability of health across agents[4], which makes consumption and production decisions interdependent. Precisely, they rely on a basic optimality argument. Because health risks are specific to occupations, and both preferences and productivity are state-dependent, any pair of ex-ante identical workers with different occupations will generally feature different expected utility functions and budget constraints. For this reason, the equalization of marginal utilities of contingent goods across agents, which is a standard ex-ante efficiency condition, typically prevents either interim efficiency with *equal treatment* (i.e., the utility equalization of agents of the same type assigned to different jobs) or budget balancing.

The inconsistency between ex-ante and interim optimality, and the *need* for Pareto efficient cross-jobs transfers, open a number of important theoretical and policy issues that we investigate in this paper. These issues concern either the efficiency trade-offs between health,

[2]This literature formalizes the Smithian idea that "*the whole of the advantages and disadvantages of the different employments of labour and stock must, in the same neighborhood, be either perfectly equal or continually tending toward equality*".

[3]See, for instance, the textbooks of Ehrenberg-Smith (2003) and Viscusi et al. (2000). In a general equilibrium analysis, however, Cole-Prescott (1997), which studies a moral hazard model, pursues a different view. The present paper has several connections with this article and with the asymmetric information literature.

[4]This imperfect transferability invalidates the separability result between individual consumption and production choices which is standard in welfare analysis (see Mas Colell et al. (1995)).

consumption and production choices; and hence the characterization of first-best allocations, or the implementation of Pareto optima.[5]

The first part of the paper provides a general characterization of Pareto optima. It shows that the properties of ex-ante efficient utility's wedges across occupations and cross-jobs transfers depend both on the riskiness of health distributions and the relative extent to which health affects agents' consumption and production capabilities. Precisely, by ordering the health risk of different occupations according to first-order stochastic dominance we obtain the following results. If health mainly affects production capabilities, efficiency requires workers with riskier jobs to get higher expected utilities under mild conditions. The same holds when health mainly affects preferences, provided that health and consumption (income) are sufficiently good substitutes (which may easily be the case when health enhancing consumption activities are sufficiently effective). In contrast, workers with riskier jobs must obtain lower utilities if health and consumption goods are complements (a condition holding when health enhancing consumption activities are not very effective). Finally, for workers obtaining relatively higher utility levels, the shadow value of consumption is larger than that of their produced and non produced resources.

The second part of the paper develops the competitive analysis. We study two alternative contractual regimes, one where lottery contracts are enforceable and the other where they are unenforceable. In the former, there exist competitive insurance markets to cope with all idiosyncratic risks, as in Malinvaud (1973) and Cass, Chichilnisky and Wu (1996) among others, but only financial contracts with deterministic returns are enforceable. In the latter regime, agents can also "trade" lottery contracts, i.e., assets with random payoffs. The "complete contracts" regime turns out to be the natural benchmark for understanding the welfare properties of competitive markets. The analysis of the case of unenforceability of lotteries, though, is warranted by several reasons. First, all the theoretical and empirical literature on non-pecuniary job attributes and compensating wage differentials, which is a natural reference point for the problem at hand, has only considered contracts with deterministic payoffs. Second, on the empirical side, the use of lottery contracts (or the use of other financial instruments that may replicate allocations obtainable through random contracts) does not appear to be extremely widespread in real markets.[6] Finally, on a theoretical ground, the use of "optimal" random contracts may

[5]Policy issues related to health risks are particularly relevant to the extent that real-life insurance markets for work-related health risks are often heavily regulated, while the rationales for policy interventions are seldom clearly expressed.

[6]Kehoe, Levine and Prescott (2001) show that, if there exists a sufficient number of assets paying units of numeraire in sunspot states of the world, competitive equilibria are first-best efficient. In our setting, however, efficient trades of financial instruments leading to random allocation are typically such that workers must take possibly large *short positions* in the asset markets. This is often impossible in real-life markets also because of

result severely restricted by moral hazard problems, due to the imperfect verifiability of characteristics and outcomes of the random devices needed for their implementation, and especially by limited liability constraints.

In both the contractual regimes that we study a competitive equilibrium exists, "insurance" is traded at fair prices, consumption allocations differ across workers of the same type with different occupations, and equilibrium wage differentials provide a premium for health risks. However, the efficiency properties of competitive equilibria markedly differ in the two environments. If lottery contracts are enforceable both welfare theorems hold. Lotteries ensure ex-ante optimality precisely by allowing agents to make cross-jobs transfers. On the contrary, if lottery contracts are unenforceable, competitive equilibria are interim efficient, but typically not ex-ante efficient. By equalizing the expected utilities of workers of the same type employed in different sectors, competition creates a wedge between their marginal utilities of expected income.

Finally, we show that Pareto optima can be implemented through deterministic cross-transfers' policies. These policies display two key features: they implement cross-subsidies among insurance policies designed for workers choosing different occupations, and impose minimal wages aimed at ensuring a natural non-manipulability requirement of the policy scheme. The transfer received by each worker at the optimum is then determined by the difference between the (shadow) value of his consumption and that of his production and endowment, both calculated at the optimal shadow prices.

Our results are related to the general equilibrium literatures on indivisibilities and on asymmetric information, which stem from the seminal contribution by Prescott-Townsend (1984). In the asymmetric information literature, some versions of the First Welfare Theorem have been proved in the space of lottery contracts[7], while several examples have been developed to show that lotteries can be welfare beneficial, either in the presence of adverse selection or moral hazard, because of their convexifying effects on incentive constraints.[8] Concerning the welfare properties of economies with indivisibilities, Rogerson (1988) provides an example where random contracts implement transfers across workers, which could be reinterpreted as a form of unemployment insurance. Rogerson's results are derived for a very specific class of preferences, and assuming that a completely indivisible labor supply (agents can either work a fixed amount of time or remain unemployed) generates a positive unemployment rate in equilibrium.

incentive problems.

[7]For the analysis of competitive and efficient random allocations see also Allen-Gale (2003), Bennardo-Chiappori (2003), Bisin-Gottardi (2000), Kehoe, Levine and Prescott (2001), Rustichini-Siconolfi (2003), and Bennardo (2005) among others.

[8]See Arnott and Stiglitz (1986), Bennardo-Chiappori (2003), Cole (1990), Garrett (1995), Kehoe, Levine and Prescott (2001), Rogerson (1988), and Bennardo (2005) among others.

In our environment, we prove that random contracts are almost always necessary to achieve efficiency through the market, even if the amount of labor a worker must supply within an occupation is perfectly divisible. More fundamentally, we provide a novel characterization of first-best allocations and cross-jobs transfers for a class of economies with indivisible assets whose returns are monetary as well as non-pecuniary.[9]

2 The Economy

Demography, consumption goods and preferences A continuum of measure 1 of consumers-workers produce C consumption goods. There exists a finite set, $I = \{1, ..., I\}$, of agents' types, and μ_i is the total fraction (measure) of *type* i agents. Agents face health risks that may affect their preferences, endowments and productivity. The set of possible health states, $\Theta = \{\theta_1, ..., \theta_N\}$, is assumed to be finite, and $\theta \in \Theta$ represents a generic health state. In the economy there are $C + 1$ consumption goods, C produced goods and leisure. *Type* i agents have an endowment $e_i \in \Re^C_+$ of produced goods which is the same in all individual states, and an amount L of time which is allocated between work, l, and leisure, x_L. The maximal fraction of time that each agent can devote to work, $L(\theta)$, may depend on his health state; and $L(\theta)$ is weakly increasing in θ.[10] Agents' preferences are assumed to be state (health) dependent and are represented by the utility function $U_i(x, \theta) : \Re^C_+ \times [0, L] \to \Re$ in each health status θ. Moreover, as we want to take into accounts goods such as medical treatments and health-enhancing activities, we consider the possibility that the additional consumption of certain commodities provides utility only in some individual states.[11] Let $\hat{C}(\theta) \subseteq C$ denote the subset of commodities whose consumption provides strictly positive (marginal) utility in the state θ. We shall assume that U_i is n times differentiable, strictly concave and weakly increasing for all θ. In proving existence, we shall also assume that the indifference surfaces of U_i have no intersection with the axes of the Euclidean space $\Re^{\hat{C}(\theta)}_+$ corresponding to the subset of commodities $\hat{C}(\theta)$. Finally, we will, occasionally, impose $D_c U_i(x, \theta) > K$ as $x_c \to 0$, with $K > 0$ sufficiently large,

[9] See however Bennardo (2004) for a similar result on a multicommodity production economy with moral hazard.

[10] This assumption is intended to capture real-life situations where a worker can perform with an appropriate quality standard a labor activity only for a limited amount of time. And the length of this time interval depends on his health state. For instance aircraft pilots, in order to guarantee appropriate safety standards, cannot fly more than a pre-specified number of hours per week. Similarly, a driver, a sportsman or a miner, who typically suffer of overuse syndromes cannot safely perform certain risky activities more than a certain number of hours in a year.

[11] In the same spirit, Makowski-Ostroy (1995), assume that different subsets of existing commodities may enter in the utility function of agents assigned to different occupations.

and $D_cU_i(x,\theta) < k$, as $x_c \to \infty$, with $k > 0$ small, for $c \in \hat{C}(\theta)$.

Technologies and uncertainty Competitive firms produce goods by employing workers, and labor is the only production factor. Firms hire a positive measure of agents, while each worker can supply labor in at most one firm, as specialization prevents workers from performing different jobs. There are $T = C$ production sectors, and only one type of occupation within each sector. The productivity of each single worker is measurable and may depend on his health. Precisely, a *type* i worker who is employed in *sector* t and supplies l_i^t units of labor produces $y_i^t(\theta) = a_i^t(\theta)l_i^t$ units of commodity t in the health state θ, with $a_i^t(.)$ weakly increasing in θ.

The distribution of health of a *type* i agent working in *sector* t is $\langle p_i^t, \Theta\rangle$, with $p_i^t = (p_i^t(\theta_1), ..., p_i^t(\theta_N))$. Finally, health shocks are identically and independently distributed across *type* i workers in the same occupation, and independently distributed across sectors.[12] The endogeneity of the health distribution can be seen as a consequence of the direct effects of labor activities on prospective workers' health; but it can also result from localization choices induced by labor activities.

Timing The economy lasts two periods, $\tau = 0, 1$; at $\tau = 0$, agents trade in financial and labor markets. At $\tau = 1$, health shocks are realized; subsequently agents supply labor, and consumption goods are traded and consumed. The space of enforceable contracts will be defined in Section 5. For notational simplicity, we restrict attention to economies where all agents work in equilibrium, and use the following notation: $x_i^t(\theta)$ is a generic state contingent consumption vector of a *type* i agent employed in *sector* t, with $x_i^t = (x_i^t(\theta))_{\theta\in\Theta}$ and $x = (x_1^t(\theta), ..., x_I^t(\theta))_{\theta\in\Theta}^{t\in T}$; $l_i^t = \{l_i^t(\theta)\}_{\theta\in\Theta}$ is the vector of state contingent labor for a *type* i agent occupied in *sector* t. Finally, $\alpha_i = (\alpha_i^1, ..., \alpha_i^T)$, with $\sum_{t=1}^T \alpha_i^t = 1$, represents an assignment of *type* i workers to production sectors, with $\alpha = (\alpha_i)_{1=1}^I$.

3 Ex-ante and Interim Pareto Optimality

Ex-ante Pareto Optimality Let $u_i^t(x_i^t) = \sum_{\theta\in\Theta} p_i^t(\theta) U_i(x_i^t(\theta), \theta)$ and $\bar{x}_{ic}^t = \sum_{\theta\in\Theta} p_i^t(\theta)x_{ic}^t(\theta)$. By the law of large numbers, a *feasible allocation* of consumption goods and workers, $\langle x, \alpha\rangle$, is defined by the following constraints:

$$(1) \qquad \sum_{i\in I}\mu_i \sum_{t\in T}\alpha_i^t\bar{x}_{ic}^t \le \sum_{i\in I}\mu_i(e_{ic} + \alpha_i^c y_i^c), \quad \forall\, c \in C$$

[12] In this prevention activities are considered. Their effect will be discussed in the last section of the paper, where we also consider economies with aggregate risk.

$$(2) \qquad l_i^t(\theta) + x_{iL}^t(\theta) = L, \ \forall\, \theta \in \Theta, \ t \in T; \qquad \sum_{t\in T} \alpha_i^t = 1, \quad \forall\, i \in I$$

where $y_i^t = \sum_{\theta\in\Theta} p_i^t(\theta)\, a_i^t(\theta) l_i^t(\theta)$ for all t and i. Denote F the set of feasible allocations, and:

$$U = \left\{ \bar{u} = (\bar{u}_2, ..., \bar{u}_I) \in \Re^{I-1} : \exists\, (x,\alpha) \in F, \ s.t., \ \sum_{t\in T} \alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i, \quad \forall\, i = 2, .., I \right\}$$

A (ex-ante) Pareto optimum maximizes $\sum_{t\in T} \alpha_1^t u_1^t(x_1^t)$, subject to $\langle x, \alpha \rangle \in F$, and $\sum_{t\in T} \alpha_i^t u_i^t(x_i^t)$ $\geq \bar{u}_i$ for $i = 2, ..., I$ and $\bar{u} \in U$.

According to this definition, all agents of *type* i face the same probability of being assigned to each occupation; however, they do not necessarily get the same expected utility if assigned to different occupations. Such a condition typically holds in the optima of convex economies, but, in our setting, there is no reason to impose it in the definition of first-best allocations. Finally, the definition above rules out the possibility that an agent obtains a random consumption vector in the optimum conditionally on being assigned to a given occupation. Risk aversion makes this assumption unrestrictive.

Interim Pareto Optimality The following definition of interim Pareto optimality will play a central role in the welfare analysis of equilibria with unenforceable lottery contracts.

An *interim optimal allocation with equal treatment* maximizes $\sum_{t\in T} \alpha_1^t u_1^t(x_1^t)$ subject to : $\langle x, \alpha \rangle \in F$, $\sum_{t\in T} \alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i$ for $i = 2, ..., I$ and $\bar{u} \in U$, and to the additional set of constraints $u_i^t(x_i^t) = u_i^{t'}(x_i^{t'})$ for all pairs (t, t'), with $t \neq t'$, such that $\alpha_i^t > 0, \ \alpha_i^{t'} > 0$.

4 Competitive Equilibria

We shall now define competitive equilibria by assuming that there exist spot markets for all goods, as well as financial markets for insuring *all* risks through assets with *deterministic* payoffs. We study either the case where only deterministic contracts (assets with random payoffs) are *enforceable* or that in which agents can also sign lottery contracts. Considering both cases is useful to fully understand either the beneficial role that random contracts may play in our economy, or the effects of a somewhat natural market friction, that may prevent their use.

4.1 Competitive Equilibrium with *Deterministic Contracts*

Following the approach taken in several contributions of the literature on individual risks (see Malinvaud (1973) and Prescott-Townsend (1984), among others), we assume that competing, risk neutral intermediaries offer securities paying in individual states.[13] Specifically, security payoffs may be contingent on agents' type, occupations and individual health. Let $h^t_{i\theta}$ be a security paying one unit of numeraire in his individual health state θ, and zero otherwise, to a *type* i agent employed in the t-th sector. Denote $z^t_{i\theta}$ and $\hat{z}^t_{i\theta}$ the units of $h^t_{i\theta}$ purchased by *type* i agents employed in *sector* t, and the per capita units of this security offered in the market, respectively. Finally, define $\phi^t_i(\theta)$ the unit price of $h^t_{i\theta}$. Production firms and agents trade at linear prices. Let $w^t_i(\theta)$ denote the state contingent wage of *type* i workers in the t-th occupation, with $w^t_i = (w^t_i(\theta))_{\theta\in\Theta}$.[14] And denote $q = (..., q_c, ...) \in \Re^C_+$ a generic vector of spot prices.[15]

Because of labor supply indivisibilities, it is expositionally convenient[16] to consider the possibility that workers choose their occupation by using mixed strategies. To this end, let $\varphi_i = (\varphi^1_i, .., \varphi^t_i, ..., \varphi^T_i) \in \Delta^T$ a generic probability vector according to which a *type* i worker mixes on occupations. By a standard interpretation of the law of large numbers, φ^t_i is then also the fraction of *type* i agents who "ex-post" are employed in *sector* t.

A competitive equilibrium with deterministic contracts is an allocation $(x^{t*}_i, \varphi^{t*}_i)^{t\in T}_{i\in I}$, a collection of vectors $(\widehat{z}^{t*}_i, z^{t*}_i)^{t\in T}_{i\in I}$ and a vector of state contingent prices $(q, \phi^t_i, w^t_i)^{t\in T}_{i\in I}$ satisfying the following conditions.

(I) *Type-i agents maximize utility:*

$$(3) \qquad \left(x^{t*}_i, \varphi^{t*}_i, z^{t*}_i\right)_{t\in T} \in \arg\max_{\varphi_i\in\Delta^T.} \sum_{t\in T} u^t_i(x^t_i)\varphi^t_i$$

$$(4) \qquad s.t., \quad \sum_{c\in C} q_c(x^t_{ic}(\theta) - e_{ic}) = w^t_i(\theta)\,(L - x^t_{iL}(\theta)) + z^t_i(\theta), \quad \forall\, \theta\in\Theta,\ t\in T$$

$$(5) \qquad \sum_{\theta\in\Theta} z^t_i(\theta)\,\phi^t_i(\theta) \le 0, \quad \forall\, t\in T$$

where (4) and (5) are the spot market and initial period budget constraints.

[13] Intermediaries' risk neutrality is, as usual, justified by the assumption of large numbers.

[14] The introduction of individual risks in a competitive settings requires assets' payoffs to be contingent on individual shocks and types; this has been clarified in Malinvaud (1973).

[15] In the absence of aggregate uncertainty, spot market prices are independent from the realizations of individual shocks that wash-out in the aggregate.

[16] It should be clear in the following, however, that for any given equilibrium with mixed strategies, there exists a payoff equivalent equilibrium with pure strategies.

(II) *Production firms and intermediaries set:*

$$(6)\quad l_i^{t*} \in \arg\max \sum_{\theta\in\Theta} p_i^t(\theta)\left(q_t y_i^t(\theta) - w_i^t(\theta)\, l_i^t(\theta)\right) \quad \text{s.t.,} \quad y_i^t(\theta) \le a_i^t(\theta)\, l_i^t(\theta), \quad \forall\, \theta\in\Theta,\ t\in T$$

$$(7)\qquad \widehat{z}_i^{t*} \in \arg\max \sum_{\theta\in\Theta} (\phi_i^t(\theta) - p_i^t(\theta))\widehat{z}_i^t(\theta) \quad \text{s.t.} \quad \sum_{\theta\in\Theta} p_i^t(\theta)\, \widehat{z}_i^t(\theta) \ge 0, \quad \forall\, t\in T,\ i\in I$$

(III) *Consumption, labor and financial markets clear:*

$$(8)\qquad \sum_{i\in I}\mu_i \sum_{t\in T} \varphi_i^{t*} \bar{x}_{ic}^{t*} = \sum_{i\in I}\mu_i (e_{ic} + \varphi_i^{c*} y_i^c), \quad \forall\, c\in C$$

$$(9)\qquad L - x_{iL}^{t*}(\theta) = l_i^{t*}(\theta); \qquad z_i^t(\theta) = \widehat{z}_i^{t*}(\theta), \quad \forall\, \theta\in\Theta,\ t\in T \text{ and } i\in I$$

4.2 Competitive Equilibrium with *Lottery Contracts*

We shall now consider an enlarged set of markets by introducing lottery contracts. In our set-up agents, before making any other market trade, buy lotteries (assets with random payoffs) from financial intermediaries. Following Arnott-Stiglitz (1987), these lotteries will be referred to as *ex-ante random contracts*. Formally, a lottery contract, $\mathcal{C} = ((\gamma, G), \rho(\gamma, G))$, is : *(i)* a finite distribution (γ, G) with probabilities $\gamma = (\gamma^1, ..., \gamma^M) \in \Delta^M$ and payoff support $G = (g^1, ..., g^M) \in \Re^M$, with M finite; and *(ii)* a price $\rho(\gamma, G) \in \Re$. The interpretation is that an agent signing $\mathcal{C}$ pays the price $\rho(\gamma, G)$, and obtains the right to receive the payoff g^m with probability γ^m. A random devise, whose characteristics are publicly verifiable, is then used by the contracting parties. Such a device chooses an *artificial state of the world* by selecting a positive integer $m \in \{1, ..., M\}$ with probability γ^m. Subsequently, the intermediary pays g^m to the agent whenever the integer m is selected. The profit an intermediary earns from $\mathcal{C}$ is $\rho(\gamma, G) - \sum_{m\in M} \gamma^m g^m$.

A general formulation of the competitive equilibrium in the space of random allocations would require all possible lottery contracts (an infinite set) to be priced in equilibrium (see Rustichini-Siconolfi (2003)) and should take into account the possibility that an agent signs several lottery contracts. In order to avoid the technical difficulties arising in working with an infinite dimensional commodity space, as well as a more complex notation, we impose the following unrestrictive assumptions: (i) only the set of fair lottery contracts with payoff support of dimension $M = T$ are offered in the market[17]; (ii) each agent can sign at most one lottery contract; and (iii) will offer labor in *sector* t if and only if he receives the t-th payoff of his lottery contract.

A standard arbitrage argument justifies (i); (ii) is unrestrictive since any finite distribution of net payoffs obtainable by means of N fair lottery contracts can also be achieved through a single fair contract;[18] moreover, by risk aversion, it is always individually optimal to choose a contract with at most $M = T$ payoffs, different from zero. Intuitively, this is because a risk averse agent, conditionally on being assigned to a given production sector, will always prefer a certain payoff, $\hat{g}$, to a non-degenerate lottery, (γ, G), with an expected payoff equal to $\hat{g}$.[19] Finally, (iii) amounts to be a convenient notational convention once (ii) is imposed.

A competitive (Walrasian) equilibrium with lottery contracts is then an allocation $(\tilde{x}_i^t)_{i\in I}^{t\in T}$, a collection of vectors $(\tilde{z}_i^t, \tilde{z}_i^t)_{i\in I}^{t\in T}$, a vector of lottery contracts $(\mathcal{C}_i)_{i\in I}$, and a vector of prices $(\tilde{q}, \tilde{\phi}_i^t, \tilde{w}_i^t)_{i\in I}^{t\in T}$ satisfying the following conditions:

(I) *Type-i agents maximize utility:*

$$(\tilde{x}_i^t, \tilde{z}_i^t, \mathcal{C})_{t\in T} \in \underset{\mathcal{C}_i\in\Gamma}{\arg\max} \sum_{t\in T} \gamma^t u_i^t(x_i^t) \tag{10}$$

$$s.t., \quad \sum_{c\in C} q_c(x_{ic}^t(\theta) - e_{ic}) = w_i^t(\theta)\,(L - x_{iL}^t(\theta)) + z_i^t(\theta) + g^t - \rho(\gamma, G), \quad \forall\, \theta\in\Theta,\ t\in T \tag{11}$$

$$\sum_{\theta\in\Theta} z_i^t(\theta)\,\phi_i^t(\theta) \le 0,\ \forall\, t\in T \tag{12}$$

where (11)-(12) are the first and second period budget constraints, and

$$\Gamma = \left\{((\gamma, G), \rho(\gamma, G)) : \rho(\gamma, G) = \sum_{t\in T}\gamma^t g^t\right\}$$

[17] Notice that, consistently with the definition of lottery contracts, some or even all of its payoffs may be zero.

[18] Such a contract is defined by probabilities and payoffs which are linear combinations of the probabilities and the payoffs of the N fair lottery contracts

[19] More precisely, it is never optimal for a risk averse agent to choose a lottery contract such that: (i) he receives the payoffs g^m and $g^{m'}$, with $g^m \neq g^{m'}$, with positive probabilities γ^m and $\gamma^{m'}$ respectively, and (ii) he chooses to work in *sector* t either when he receives g^m or $g^{m'}$. By convexity, indeed, there exists another fair contract, say $\mathcal{C}'$, which pays $\gamma^m g^m + \gamma^{m'} g^{m'}$ with probability $\gamma^m + \gamma^{m'}$, which, conditionally on working in *sector* t, is strictly preferred to $\mathcal{C}$.

is the set of all fair lottery contracts.

(II) *Production firms and intermediaries solve programs (6) -(7), respectively.*[20]

(III) *Consumption, financial and labor markets clear:*

$$\sum_{i\in I}\mu_i\sum_{t\in T}\widetilde{\gamma}^t\widetilde{\widetilde{x}}^t_{ic}p^t_i(\theta)=\sum_{i\in I}\mu_i(e_{ic}+\widetilde{\gamma}^c\widetilde{y}^c_i),\quad \forall\, c\in C \tag{13}$$

$$L-\tilde{x}^t_{iL}(\theta)=\tilde{l}^t_i(\theta);\qquad \tilde{z}^t_i(\theta)=\tilde{\tilde{z}}^t_i(\theta),\quad \forall\,\theta\in\Theta,\ t\in T\text{ and }i\in I \tag{14}$$

5 Pareto Optimal Allocations

This section characterizes first-best allocations. Let $\lambda=(\lambda_2,...,\lambda_I)$ and $\eta=(\eta_1,...,\eta_C)$ be the vectors of Lagrange multipliers associated, respectively, to the utility constraints, $\sum_{t\in T}\alpha^t_iu^t_i(x^t_i)\geq \bar{u}_i$ for $i=2,...,I$, and the feasibility constraints. Setting $\lambda_1=1$ the first-order conditions with respect to $(x^t_i(\theta),x^t_{iL}(\theta),\alpha^t_i)$ of the (ex-ante) Pareto program are:

$$\lambda_iD_cU_i(x^t_i(\theta),\theta)-\eta_c\mu_i\leq 0,\quad \forall\, c\in C,\ \theta\in\Theta,\ t\in T\text{ and }i\in I \tag{15}$$

$$\lambda_iU_{ix_L}(x^t_i(\theta),\theta)-\eta_ta^t_i(\theta)\mu_i\leq 0,\quad \forall\,\theta\in\Theta,\ t\in T\text{ and }i\in I \tag{16}$$

where (15) and (16) hold with equality whenever $x^t_{ic}(\theta)>0$ and $x^t_{iL}(\theta)>0$.[21]

$$\lambda_i(u^t_i(x^t_i)-u^{t'}_i(x^{t'}_i))-\mu_i(Z^t_i-Z^{t'}_i)=0,\ \forall\ (t,t')\text{ such that }(\alpha^t_i,\alpha^{t'}_i)>0\text{ and }i\in I \tag{17}$$

where:

$$Z^t_i=\sum_{c\in C,\theta\in\Theta}\eta_c\left(p^t_i(\theta)x^t_{ic}(\theta)-e_{ic}\right)-\eta_t\sum_{\theta\in\Theta}p^t_i(\theta)a^t_i(\theta)\left(L-x^t_{iL}(\theta)\right),\quad \forall\, t\in T,\ i\in I$$

[20] This is exactly as in the competitive equilibrium with deterministic contracts.

[21] For simplicity we neglect the case where $x^t_{iL}(\theta)=L$ in stating the first-order conditions. Implicitly, we assumed U_{ix_L} sufficiently small at $x^t_{iL}(\theta)=L$.

is the difference between the value of the consumption of a *type* i workers employed in *sector* t and the sum of its endowment and its production, both evaluated at the vector of shadow prices η. Hence Z_i^t is the value of the transfer received in the optimum by a *type* i agent assigned to *sector* t.

As usual, (15) and (16) imply the equality of marginal rates of substitution between state contingent commodities across types. The first-order conditions with respect to α in equation (17) are less standard, but play a crucial role in our analysis. They indicate that the differences in utilities across occupations, $\Delta u_i(t,t') = u_i^t(x_i^t) - u_i^{t'}(x_i^{t'})$, are proportional to $\Delta Z_i(t,t') = Z_i^t - Z_i^{t'}$. Only if $\Delta Z_i(t,t') = 0$, *type* i workers assigned to the occupations t and t', respectively, will get the same utility, and ex-ante and interim optima coincide.

Let $\mathcal{F}(.) = 0$ denote the system of equations (15)-(17). Next proposition shows that $\Delta u_i(t,t')$ typically differs from zero at the solution of $\mathcal{F}(.) = 0$, implying that interim efficiency is generally incompatible with ex-ante efficiency. This is, indeed, one of the distinguishing features of our environment.

In order to prove the result, we need to introduce some notation. Let $t_i = (\langle p_i^t, \Theta \rangle, A_i^t)_{t \in T}$, with $A_i^t = \{a_i^t(\theta_1), ..., a_i^t(\theta_N)\}$, be the *sector* t technology available to *type* i workers. And let $\varepsilon = \langle e, \mathbf{t}, U \rangle$ represent a specific economy defined by an aggregate endowment $e \in \Re_{++}^C$, a vector of production technologies $\mathbf{t} = (t_1, ..., t_I)$ and a profile of utility functions $U = (U_i, ..., U_I)$. The set of possible economies is then defined as $\mathcal{E} = \Re_{++}^C \times \mathcal{T} \times \mathcal{U}$, where $\mathcal{T}$ is the set of all possible technologies, and $\mathcal{U} = \prod_{i=1}^I \mathcal{U}_i$, where $\mathcal{U}_i$ is the set of *type* i admissible utility functions, which will be precisely defined in the appendix.

Proposition 1 *An unique Pareto optimum is associated to each vector of reservation utilities, $\bar{u}$. Moreover, the subset of economies $\mathcal{S} \subset \mathcal{E}$ where ex-ante and interim Pareto optima are different is generic in $\mathcal{E}$, if the number of produced goods is larger than the number of agents' types.*

The proof of this result as well as all the subsequent ones are provided in the appendix.

It uses a transversality argument to prove that the set of solutions of $\mathcal{F}(.)$ typically does not satisfy the interim efficiency constraints.[22]

Proposition 1 has two immediate but very important corollaries. First, ex-ante efficiency typically requires either transfers of resources across workers assigned to different occupations or a random allocation of workers across occupations. Second, compensating wage differentials

[22] Notice that the inconsistency between ex-ante and interim efficiency stated in Proposition 1 holds only under the assumption that health distributions are endogenous.

which equate (expected) utilities of workers assigned to different sectors are typically incompatible with first-best efficiency.

Next proposition shows that the Pareto shadow wages, $\eta_t^P a_i^t(\theta)$, associated to technologies inducing riskier health distributions, in the sense of First-Order Stochastic Dominance (FOSD), are relatively higher in the optimum. Throughout, we shall use the following standard definition of FOSD. For any pair of health distributions, $\langle p_i^t, \Theta\rangle$ and $\langle p_i^{t'}, \Theta\rangle$, $\langle p_i^t, \Theta\rangle$ FOSD $\langle p_i^{t'}, \Theta\rangle$ if $\sum_{\theta\leq\theta_n} p_i^t(\theta) \leq \sum_{\theta\leq\theta_n} p_i^{t'}(\theta), \forall\, \theta_n \in \Theta$, with at least one strict inequality.

Proposition 2 *If $\langle p_i^t, \Theta\rangle$ FOSD $\langle p_i^{t'}, \Theta\rangle$, then $\eta_t^P < \eta_{t'}^P$ for all Pareto optima such that $\alpha_i^{tP} > 0$ and $\alpha_i^{t'P} > 0$ for at least one type $i \in I$.*

Finally, Proposition 3 below, which is a direct corollary of Charateodory Theorem, states that all efficiency gains obtainable by random assignments of workers to occupations can also be achieved through a randomization involving only two occupations for each worker. This finding allows to simplify the efficiency analysis performed in the next section.

Proposition 3 *Given any Pareto optimal allocation $\langle x^P, \alpha^P\rangle$ such that $\sum_{t\in T} \alpha_i^{tP} u_i^t(x_i^{tP}) = u_i$, there exists a pair (t, t') such that $\hat{\alpha}_i^t u_i^t(x_i^t) + (1 - \hat{\alpha}_i^t) u_i^{t'}(x_i^{t'}) = u_i$, with $\hat{\alpha}_i^t = \alpha_i^t/(\alpha_i^t + \alpha_i^{t'})$.*

Propositions 1 and 2 motivate the analysis of the rest of the section, which investigates the determinants of wages' and utilities' differentials across sectors, and of Pareto optimal cross-jobs transfers. Exploiting the result of Proposition 3 we will restrict attention to *two sectors* economies without loss of generality.

5.1 Ex-Ante Efficiency and Optimal Cross-Jobs Transfers

In the rest of the section we study how the effects of health shocks on preferences, endowments, and productivity contribute to determine optimal cross-jobs transfers, as well as the differences between utilities obtained by workers of the same type assigned to different technologies. For this purpose we shall assume that occupations differ for their *health riskiness,* and impose that the health distributions associated to different occupations are ordered by the FOSD criterion.

Merely for expositional purposes, we shall study a simplified setting where two goods are produced by a representative agent. Assume that $\langle p^1, \Theta\rangle$ FOSD $\langle p^2, \Theta\rangle$, and consistently with previous notation, let $x = (x_1, x_2, x_L)$ and $\hat{x} = (x_1, x_2)$.

In order to distinguish the effect of health status on the utility of produced consumption goods from that on the disutility of labor, we shall use the following utility representation:

$U(x,\theta) = f(x,\theta) - \psi(l,\hat{x},\theta)$; where $U(x,\theta)$ satisfies all the assumptions in Section 2, and where $f(x,\theta)$ and $\psi(\hat{x},l,\theta)$ represent the utility of consumption commodities and the disutility of labor, $l = L - x_L$, respectively. Accordingly, both f and ψ may possibly depend on θ; and, by introducing $\hat{x}$ among the arguments of ψ, we also take into account the possibility that consumption activities affect the workers' disutility of labor.[23]

Additionally, in order to derive a sharper characterization of Pareto optima this section we also impose the following assumptions:

A1 All derivatives of U are bounded.

A2 $a^t(\theta) = a(\theta)$ for all t.

A3 $U(.,\theta)$ is supermodular in (x, x_L) for all θ, and $\psi_{l\theta} \leq 0$ for all (x,θ).

A1 is actually unrestrictive since we allow the bounds on the derivatives of U to be arbitrarily large. Assuming that $a^t(\theta)$ is invariant across sectors is also an innocuous normalization whenever $a^t(\theta_1) = 0$ for all t (i.e. whenever workers are unproductive in the worst health state). Supermodularity in (x, x_L) is a simplifying assumption.[24] Finally, as health is typically an input for production, imposing $\psi_{l\theta} \leq 0$ seems a very sensible restriction.

Differently, there exists no *natural* restriction on the sign the vector of cross derivatives $(U_{1\theta}, U_{2\theta})$. Indeed, the sign of $U_{c\theta}$ depends, given the specific nature of health services, on the relative magnitude of two effects that generally go in opposite direction. As the sign of $U_{c\theta}$ turns out to be crucial in the characterization of Pareto optima, in the rest of this section we illustrate these effects; while next section characterizes optima for both the cases where consumption and health are complements or substitutes.

As for production, health is an input also for most consumption activities; as a consequence, the marginal utility of these consumption activities increases with health status. Were this the only channel through which health affects consumption decisions, one should have $U_{c\theta} > 0$ for all c. Health, though, has also a second possibly counterbalancing effect. It arises whenever agents can influence their health by devoting resources to medical treatments or health-enhancing consumption activities. In order to illustrate such an effect, it is convenient to represent agents' *health conditions*, $\hat{\theta}$, by a real valued function $\hat{\theta} = \rho(x,\theta)$ of consumption, x, and initial health, θ, satisfying $\rho_\theta(x,\theta) > 0$ and $\rho_c(x,\theta) \geq 0$. Preferences are then represented by $\hat{U}(x,\rho(x,\theta)) = \hat{f}(x,\rho(x,\theta)) - \hat{\psi}(l,\rho(x,\theta))$.

[23] In real-world, housing facilities, drugs, kindergarten services for parents, etc... reduce the disutility of labor; while goods such as alcohol increase the disutility of labor.

[24] This assumption can be easily relaxed. In order to derive our characterization we only need U_{cx_L} to be *not too negative* for all c.

By differentiating, one has $\hat{U}_{c\theta} = \hat{U}_{c\rho}\rho_\theta + \hat{U}_\rho \rho_{c\theta} + \hat{U}_{\rho\rho}\rho_c\rho_\theta$, hence $U_{c\theta}$ is negative for $\rho_{c\theta} < 0$ and sufficiently small.[25] In fact, assuming $\rho_{c\theta} < 0$ is completely natural in most real-world situations involving health enhancing consumption activities and especially medical treatments. Consider, indeed, a generic treatment, call it c; by its own nature, the treatment is beneficial only in *relatively bad* health states, hence $\rho_{c\theta}$ must be negative for at least a subset of Θ. Moreover, $\rho_{c\theta}$ must also be small if the treatment is marginally very effective in that subset of Θ. Finally, assuming that $\rho_{c\theta}$ does not change sign, seems a sensible assumption in many real-life cases. It becomes even more appropriate whenever, as it is often convenient for both theoretical and practical purposes, c can be interpreted as a total amount of certain types of medical expenses, i.e., a composite good.

Similar considerations hold with regard to health enhancing consumption activities, ranging from those aimed at satisfying nutritional and housing needs to physical and sports activities. To provide an example, consider individuals who spend a significant fraction of their income for nutritional and housing needs; it is very sensible to assume that: higher levels of consumption improve their health conditions, and that this consumption effect gets stronger for lower health states. This again amounts to impose $\rho_{c\theta} < 0$.[26]

As a conclusion, whether $U_{c\theta}$ is positive or negative it depends on how *effective* treatments and health-enhancing activities are. Whenever they are sufficiently productive, the sign of $\hat{U}_{c\theta}$ is negative under mild assumptions, otherwise it must be positive.[27]

By imposing **A1-A3** in addition to the assumptions stated in Section 2, in the rest of this section we shall study how the direction of cross-transfers among workers facing different health distributions depends upon: (I) *the direct effects of health status on agents well being*, measured by U_θ; (II) *the health effects on consumption choices*, measured by the vector $(U_{1\theta}, U_{2\theta})$; and (III) *the health effects on agents' production choices,* depending on disutility of labor, labor

[25] The first term of this sum represents the effect of θ on the marginal utility of consumption activities, which should be positive as we claimed before; the second term represents the effect of health-enhancing consumption activities on $\hat{\theta}$; while the third addendum captures a second order effect which reinforces that of health-enhancing consumption activities.

[26] As an example, consider the case of an unskilled worker, living in a low-income country which experiences high diffusion rates of contagious diseases (such as malaria or AIDS), and spend a large fraction of his income in buying foods and housing services. Contracting the disease generally impairs his consumption and working aptitudes and reduces his utility. However, the more adequately this worker can satisfy his basic consumption and housing needs (i.e., the larger is his consumption of basic commodities) the smaller should be the effects of the disease on his health conditions. Making this assumption is equivalent to impose $\rho_{c\theta} < 0$.

[27] For simplicity in all the above discussion we only considered the case where $\rho_c \geq 0$. In words, we did not consider the case in which the consumption of some commodities, such as alcohol, smoking, pollution etc. worsens agents' health conditions. These situations can be readily taken into account. For instance, if health reducing consumption activities, c, have a relatively larger impact on the health conditions, ρ, of healthier agents (smoking damages relatively more sportsmen than doormen) then it is easy to verify that $\rho_{c\theta} < 0$.

endowment, and productivity. For the sake of clarity, throughout we consider each of these effects in isolation.

Finally, the distinction between health effects on consumption and production choices just introduced is helpful to understand what transfers across real-world occupations we should observe in efficient, possibly regulated, competitive markets. In the real-world one may distinguish among occupations for which physical or mental health are important prerequisites for productive activities and other jobs requiring only a minimal level of health to be performed satisfactorily. Intuitively, health effects on production should determine the sign of cross transfers for jobs of the former type; while health effects on consumption should be more important otherwise.

5.1.1 Health Effects on Consumption Choices

In the following we study how the properties of Pareto optima are influenced by the effects of health status on agents' preferences for (produced) consumption goods. We illustrate how the effect of health on the (marginal) utility of consumption, measured by $U_{c\theta}(x, \theta)$, and the direct health effect on well being, measured by $U_{\theta}(x, \theta)$, contribute to determine either optimal cross transfers or the sign of $\Delta u^P = \sum_{\theta \in \Theta} p^1(\theta) U(x^{1P}(\theta), \theta) - \sum_{\theta \in \Theta} p^2(\theta) U(x^{2P}(\theta), \theta)$.

To focus on consumption choices, Proposition 4 assumes that workers supply labor inelastically, while either productivity or labor endowments are independent from health status. Arguably, these assumptions describe an agent owing a relatively low amount of human capital whose production choices are only marginally affected by his health. In this case, health should play a minor role in production decisions. Let $\Delta Z^P = Z^{1P} - Z^{2P}$, since the first-order conditions with respect to α^P of the Pareto program imply $\Delta u^P \gtreqless 0$ if and only if $\Delta Z^P \gtreqless 0$. From hereafter in this section, we shall only study the sign of Δu^P.

Proposition 4 *Assume $l^t(\theta) = L$ and $a(\theta) = a$ for all t and θ: (i) if U has increasing differences in (x, θ), then $\Delta u^P > 0$; (ii) if U has decreasing differences in (x, θ), then $\Delta u^P > 0$ whenever $U_{c\theta}/U_{\theta} < k$ for all c, with $k \in \Re$ and sufficiently small; while $\Delta u^P < 0$ whenever $U_{c\theta}/U_{\theta} > K$ for at least one good, c, and $K \in \Re$ sufficiently large; (iii) if $U_{c\theta} > 0$ and $U_{c'\theta} < 0$, for $c \neq c'$, then $\Delta u^P \leq (>) 0$ whenever $U_{c\theta}/|U_{c'\theta}|$ sufficiently small (resp. large).*

The intuition is as follows. Optimality requires risk-averse workers assigned to different occupations to get the same consumption in each individual health state (i.e., $x^1(\theta) = x^2(\theta) = x^P(\theta)$ for all θ). If consumption goods and health are complements optimality also imposes agents' consumption to be larger in better health states; and, for this reason, $U\left(x^P(\theta), \theta\right)$ is increasing in θ. Furthermore, since workers using safer technologies are more likely to experience better health states, they obtain larger utility levels with larger probabilities. Therefore, they get a higher expected utility level, and *(i)* holds. Conversely, if consumption goods and health are substitutes $x^P(\theta)$ is smaller in better health states. If such an effect is sufficiently large to compensate the impact of U_θ, $U\left(x^P(\theta),\theta\right)$ will be decreasing in θ. Thus, workers using riskier technologies obtain a larger utility at the optimum; as stated in *(ii)*. Finally, a careful continuity argument allows to establish part *(iii)*.

Proposition 4, however, does not cover the case where one good is substitute with health, the other is complement, and none of these effects is negligible relatively to the other. In this case, the direction of the optimal transfers depends not only on the magnitude of second cross derivatives but also on the marginal utility of consumption commodities (which, in turn, are affected by aggregate endowments). The main issue then becomes whether there exists a synthetic measure, having empirical correlates, to determine which one of the above effects prevails. Next proposition shows that the cross derivative of the indirect utility with respect to income and health is, indeed, the appropriate measure.

Define $V(q,I(q),\theta) \equiv \max_{x\in\Re_+}\{U(x,\theta)\ s.t.,\ qx \leq I(q)\}$ the state dependent indirect utility associated to the vector of prices q and total wealth $I(q)$. Proposition 5 below shows that the sign of Δu^P is determined by the sign and the magnitude of $V_{I\theta}(q,I(q),\theta)$.

Proposition 5 *Assume $V_{I\theta}$ has constant sign for all q, I, and θ. Then: (i) if $V_{I\theta} > -k$ with k positive and sufficiently small, $\Delta u^P > 0$; (ii) if $V_{I\theta} < -K$ with K positive and sufficiently large, $\Delta u^P < 0$.*

The proof is left to the reader.[28] It simply consists in verifying that $U\left(x^P(\theta),\theta\right)$ is increasing (resp. decreasing) whenever $V_{I\theta}$ is sufficiently large and positive (resp. negative), thereby health and income are sufficiently good complements (resp. substitutes). By FOSD the slope $U\left(x^P(\theta),\theta\right)$ in turn implies the sign of Δu^P.

Finally one can interpret the results stated above in light of the discussion on the sign of $U_{c\theta}$ that we made in the previous section. We can conclude that efficiency requires workers assigned to riskier jobs to get lower expected utilities if health enhancing consumption activities have relatively negligible effects on health, while the converse will often be true otherwise[29].

[28] Note that $U_{c\theta}(x,\theta) > 0$ (resp. < 0) for all c implies $V_{I\theta} > 0$ (resp. > 0), hence Proposition 5 generalizes the result stated in Proposition 4.

[29] Note, however that if one restrict attention to the case where no health henancing activity provides utility directly (as it is the case for medical treatments) Δu^P is negative only if $\rho\left(x^P(\theta),\theta\right)$ is decreasing at least in some subset of Θ. This is not anymore true, tough, as soon as some health enhancing consumption activities increase directly agents utility.

5.1.2 Health Effects on Production Choices

We now focus on the analysis of health effects on production capabilities. We consider first the effects of health status on labor endowment, which are the simplest to analyze, and then the other health effects on production choices. Consistently, in the rest of the section, we assume that health does not effect consumption choices, i.e., $U_{c\theta}(x,\theta) = 0$ for $c = 1, 2$. Once more this extreme assumption is for the sake of clarity. Our goal here is to describe optimal transfers for the case where health effects on consumption are relatively negligible.

Health Effects on Labor Endowment Next proposition demonstrates that agents employed in the riskier sector obtain a larger utility at the optimum whenever health (mainly) affects their labor endowment. To focus on the effects of health risks on labor endowment, we shall assume that health has no direct effect on utility, nor on the marginal utility of labor and productivity. For simplicity, it is also convenient to assume once more that labor supply is completely inelastic. Under this assumption the effects of health shocks on labor supply results magnified, and hence labor supply is completely unaffected by state contingent shadow prices and wages.

Proposition 6 *Assume* $U_\theta\left(x,\theta\right) = U_{x_L\theta}\left(x,\theta\right) = 0,\ a(\theta) = a$ *and* $l^t(\theta) = L\left(\theta\right)$ *for all* θ *and* $t = 1, 2$*, then* $\Delta u^P < 0$ *if* $L(\theta_n) > L(\theta_{n-1})$ *for some* θ_n*.*

Either the proof, which is left to the reader, or the intuition for the result of Proposition 6 are similar to those of Proposition 4. Precisely, in the case at hand, workers get a lower utility in better health states when their labor endowment and labor supply are larger. As a consequence, workers in safer occupations, who work more in average, obtain a lower expected utility.

The rest of the section will look at the more complex case where contingent wages and health status determine labor supply jointly.

Health Effects on the Disutility of Labor We now study the effects of health risks on the disutility of labor. Next proposition shows that workers in the riskier sector get an higher expected utility in the optimum if health effects on the marginal disutility of labor (i.e., on production capabilities) are sufficiently large relatively to the direct impact of health on well being (utility), and if the marginal disutility of labor is "sufficiently increasing"; the converse obtains otherwise.

Consistently, we now assume that health has no effects on marginal utility of consumption, productivity and labor endowment; but in contrast with the analysis performed above we will not anymore assume a completely inelastic labor supply. Let $\sigma_{x_L} = -U_{x_Lx_L}/U_{x_L}$,

Proposition 7 *Assume $f_{c\theta}(x,\theta) = 0$ for all c, $a(\theta) = a$, $L(\theta) = L$ for all θ, and $\sigma_{x_L} > K$, with K sufficiently large for all x_L, then : (i) if U_θ sufficiently large, $\Delta u^P > 0$; (ii) if $|U_{x_L\theta}/\sigma_{x_L}| > \delta$, for some positive δ, $\Delta u^P < 0$ whenever U_θ is sufficiently small.*

The following economic effects determine the results stated above. First, as before, the direct effect of health on utility, captured by U_θ, increases the utility differential in favor of workers in the safer sector, as health losses are less likely in this sector. Moreover, Pareto optimality imposes compensating wage differentials in favor of riskier occupations, as established in Proposition 2. As the individual labor schedule is now increasing in the shadow wage, workers assigned to the riskier occupation supply more labor in each individual health state. For this reason, also the wage effect has a positive impact on Δu^P. On the contrary, though, optimality requires agents to work more in good health states, where the disutility of labor is lower. This effect reduces Δu^P since workers using riskier technologies enjoy *less often* good health status. For U_θ sufficiently small relatively to $U_{x_L\theta}$, the last effect must prevail and Δu^P must be negative, since σ_{x_L} large implies $\Delta l^P(\theta) = l^{1P}(\theta) - l^{2P}(\theta)$ and $\Delta x^P(\theta) = x^{1P}(\theta) - x^{2P}(\theta)$ small. Conversely, Δu^P has a positive sign for U_θ large.

Summarizing, workers in the riskier sector obtain positive transfers and utility differentials whenever health is sufficiently important relatively to wage in determining labor supply, and negative transfers and utility differentials otherwise.

Finally, we will now show that the assumption imposing a lower bound on σ_{x_L} in Proposition 7 can be relaxed at least when preferences are separable with respect to labor and consumption goods. Under separability, that assumption can be replaced by assuming $\psi_{lll} > 0$, which is a common restriction in the applied literature. As it results from the Pareto program, the latter condition is necessary and sufficient for the labor supply schedule to be concave in the (shadow) wage. This restriction, which implies that the individual labor supply is more responsive to wage increases at relatively smaller wages, seems quite realistic in many applications.

Proposition 8 *If $U(x,\theta) = f(\hat{x}) - \psi(l,\theta)$, $\psi_l(0,\theta) \le \varepsilon$ for all θ, with ε sufficiently small, and $\psi_{lll}(l,\theta) > 0$ for all (l,θ), then $\Delta u^P \le 0$ for ψ_θ sufficiently close to 0.*

As $\psi_{lll} > 0$, the difference between labor supply schedules across sectors cannot become "too large" when the shadow wages increase. As a consequence, the effects of the shadow wage on Δu^P cannot overcome the health effect on the marginal disutility of labor.

Health Effects on Productivity We conclude the characterization of Pareto optima by considering the case where health risks affect productivity. To this end, next proposition assumes that workers' productivity is increasing in health, and neglect the effects of health on preferences and labor endowment.

Proposition 9 *Let $\partial a(\theta)/\partial\theta > 0$, $U_\theta(x,\theta) = 0$ and $L(\theta) = L$ for all x and θ, and assume σ_{x_L} has strictly positive upper and lower bounds, if σ_{x_L} and $\partial a(\theta)/\partial\theta$ are sufficiently large for all θ, then $\Delta u^P < 0$.*

The proof of this proposition, as well as the discussion of its economic intuition are omitted, since they are similar to those for Proposition 7. Differently from Proposition 7, however, Proposition 9 does not cover the case of σ_{x_L} small. In fact, it can be showed that signing Δu^P now generally requires specific conditions on health distributions. Nevertheless, we characterize below the Pareto optimum for the particular case, which is often studied in applications, where only one of the two technologies is risky and preferences are separable, i.e., $U(x,\theta) = f(\hat{x}) - \psi(l)$.

Denote $w_\theta = a(\theta)\eta$ and let $l(w_\theta,\theta)$ be the contingent labor supply schedule implicitly defined by optimality conditions; finally, define $\zeta_{l,w} = dl(w_\theta,\theta)/dw_\theta)/(l(w_\theta,\theta)/w_\theta)$.[30] As a preliminary result we need to state the following lemma. Let $h(l) = \psi'(l)l$ and $\sigma_h(l) = h''(l)/h'(l)$,

Lemma 10 *$\partial\zeta_{l,w}/\partial w \gtreqless 0$ for all (l,θ) if and only if $\sigma_\psi(l) \gtreqless \sigma_h(l)$.*

The proof follows from straightforward manipulations of the FOCs of the Pareto program, and is omitted.

Finally, Proposition 11 below shows that efficiency requires agents using riskier technologies to get an higher utility in the optimum whenever $\zeta_{l,w}$ is non decreasing in the shadow wage. This assumption is in line with the empirical findings of the labor supply literature; its interpretation is indeed that agents who are already "working a lot" react less to wage increases.

Proposition 11 *If $p^1(\theta_N) = 1$, then $\Delta u^P \gtreqless 0$ if $\partial\zeta_{l,w}/\partial w \gtreqless 0$.*

6 Characterization of Competitive Equilibria

In this section, we characterize competitive equilibria for economies with enforceable and unenforceable lottery contracts. We begin by proving the existence of a competitive equilibrium. The proof exploits the convexifying effect of large numbers.

Proposition 12 *A competitive equilibrium exists either under enforceable or under unenforceable lottery contracts.*

[30] $\zeta_{l,w}$ is a measure of the sensitivity of the optimal labor schedule with respect to the shadow wage.

Next proposition states the First Welfare Theorem for economies where lottery contracts are enforceable.

Proposition 13 *Under enforceability of lottery contracts, competitive equilibria are first-best.*

The proof is standard and is omitted.

The logic of the First Welfare Theorem is also used to show that competitive equilibria are interim efficient allocations with equal treatment if only deterministic contracts are enforceable.

Proposition 14 *Under unenforceability of lottery contracts, competitive equilibria are interim efficient allocations with equal treatment.*

This result together with Proposition 1 have the following important corollary.

Corollary 15 *Competitive equilibria with deterministic contracts are generically not first-best.*

Next proposition states that agents trade individual securities at fair prices in both contractual regimes, and that state-contingent wages equal the value of state-contingent labor productivity for each type of worker. Furthermore, occupations associated to riskier health distributions command relatively higher contingent wages. Finally, when lotteries are enforceable, the value of consumption for agents of the same type assigned to different occupations typically differs from the sum of the values of their endowment and production. By using lottery contracts, indeed, wealth is optimally transferred across occupations in such a way that agents obtaining the higher (resp. lower) expected utility get a positive (resp. negative) transfer.

Let $\tilde{Z}_i^t = \sum_{c\in C,\theta\in\Theta}(q_c(p_i^t(\theta)x_{ic}^t(\theta) - e_{ic}) - q_t \sum_{\theta\in\Theta} p_i^t(\theta)a_i^t(\theta)\left(L - x_{iL}^t(\theta)\right),$

Proposition 16 *(i) under both contractual regimes* $\phi_i^t(\theta) = g_i^t p_i^t(\theta)$ *for some* $g_i^t \in \Re_+$ *and* $w_i^t(\theta) = q_t a_i^t(\theta)$*, for all* i*,* t *and* θ*; (ii) assume* $a_i^t(\theta) = a_i(\theta)$ *for all* t*,* $w_i^t(\theta) < w_i^{t'}(\theta)$ *if* $\langle p_i^t, \Theta\rangle$ *FOSD* $\left\langle p_i^{t'}, \Theta\right\rangle$ *and strictly positive measures of* type i *agents are assigned to sectors* t *and* t' *under both contractual regimes; (iii) in any equilibrium with lottery contracts such that positive measures of* type i *agents are employed in sectors* t *and* t'*, then* $u_i^t(x_i^t) - u_i^{t'}(x_i^{t'}) \gtreqless 0$ *if and only if* $\tilde{Z}_i^t - \tilde{Z}_i^{t'} \gtreqless 0$.

Part *(i)* of Proposition 16 follows from the linearity of the intermediaries and production firms maximization programs; *(ii)* indicates that *compensating wage differentials* are paid to riskier occupations and it follows from first-order conditions; *(iii)* is a corollary of the optimality analysis performed in Section 5.

7 Second Welfare Theorem and Decentralization

This section studies competitive equilibria with transfers. We shall consider a situation where a policy authority can implement cross-jobs transfers. Will restrict attention to the case where lottery contracts are unenforceable, as unenforceability prevents competitive markets from achieving efficiency.[31]

In the real-world, transfers across workers with different health prospects are implemented through a variety of institutions and policy schemes. Systems of transfers across health insurance policies and occupations on the one hand, and subsidies to health enhancing activities, such as medical treatments, health care etc. on the other hand, are in particular largely diffused. In our set-up these two types of policy instruments play similar roles. For the sake of brevity, we shall only study the effects of cross-subsidies across insurance policies, and discuss only briefly and informally the welfare effects subsidization to health enhancing consumption activities.

We now introduce a class of policy instruments based on deterministic transfers across insurance contracts and on minimal wages. To simplify the definition of transfers policy, in the following we shall assume, without loss of generality, that in equilibrium each agent trades with only one intermediary. Under this assumption an individual vector of assets' trades can be interpreted as an insurance contract.

Let s_i^t the (possibly negative) monetary transfer[32] received by a *type* i agent who signs a health insurance contract designed for *sector* t workers; and denote $f_i^t(\theta)$ the monetary transfer received by a *sector* t firm for each *type* i worker in state θ which it employes. Finally, let $\hat{w}_i^t(\theta)$ the minimal state contingent wage that firms must pay to *type* i workers employed in *sector* t who experiences the health state θ.

A transfers' policy, $\wp = (s, f, \hat{w})$, is a vector $s = (s_i^t)_{i\in I}^{t\in T}$ of subsidies to the workers; a vector $f = (f_i^t(\theta))_{i\in I,\theta\in\Theta}^{t\in T}$ of transfers to production firms and a vector $w = (\hat{w}_i^t(\theta))_{i\in I,\, \theta\in\Theta}^{t\in T}$ of state contingent minimal wages.

Feasible policies must be budget-balancing. Hence,

$$\wp \in \mathcal{P} \equiv \left\{ \wp : \sum_{t\in T, i\in I} \mu_i \varphi_i^t \left(s_i^t + \sum_{\theta\in\Theta} p_i^t(\theta) f_i^t(\theta) \right) = 0 \right\}$$

where φ_i^t represents the measure of *type* i workers who are effectively assigned to *sector* t in an equilibrium with transfers.

[31] The proof that the second welfare theorem holds, under standard assumptions, when random contracts are enforceable may be showed to follow standard arguments.

[32] We will use monetary transfer as a synonimus of "transfer in units of numeraire".

Minimal wages may well induce rationing, and for this reason market clearing rules must now be carefully specified. Throughout we assume that, in any equilibrium with transfers, all commodity as well as asset markets clear at "walrasian" prices without rationing (i.e., exactly as in the absence of transfers), and that firms' labor demand is not rationed as well. Differently, as transfers and minimal wages can generally make some occupations more attractive than others, a rule according to which workers are assigned to each occupation must be specified. We shall assume that whenever *type* i agents receive a larger utility in *sector* t than in *sector* t', for some $t' \neq t$, in equilibrium, the probability that a *type* i agent is assigned to *sector* t is equal to α_i^t, which is precisely the measure of *type* i workers assigned to *sector* t.

The motivation for the clearing rule of consumption and assets markets is the usual one: namely, were firms or agents rationed, they would have an incentive to manipulate prevailing prices.[33]

The same type of argument justifies the assumption that labor demand is never rationed in the equilibrium. Our workers' assignment rule can be seen as the outcome of a *decentralized* job search process where in a first stage workers simultaneously apply for occupations; subsequently, applications are randomly selected whenever the number of workers applying for a job is larger than the number of posted vacancies, and firms offer jobs to the workers; finally, in a third stage, workers, who have possibly received more than one offer, choose their most preferred one. Noteworthy, while this type of assignment mechanism introduces a randomization on agents' labor demand, the transfers policies we consider are completely deterministic, and hence their implementation does not rely on any random device.

Consistently with the above definition of the policy scheme and with the description of market clearing rules, a *rational expectation equilibrium with transfers,* $\{\varphi, x, \alpha, z, p, w, \phi, \wp\}$, is formally defined by the following conditions:

(**I**) *type* i consumers' choose $(x_i^t, \varphi_i^t, z_i^t)_{\theta\in\Theta}^{t\in T}$ by maximizing $\sum_{t\in T} u_i^t(x_i^t)\varphi_i^t$ subject to the budget constraints

$$\sum_{c\in C} q_c(x_{ic}^t(\theta) - e_{ic}) = w_i^t(\theta)\left(L - x_{iL}^t(\theta)\right) + z_i^t(\theta) + s_i^t,\ \forall\ \theta \in \Theta,\ t \in T$$

$$\sum_{\theta\in\Theta} z_i^t(\theta)\,\phi_i^t(\theta) \leq 0\ \forall\ t \in T$$

[33] See, for instance, Mas Colell and others (pp. 315, 1995) for a justification of the walrasian equilibrium notion along these lines.

and to a set of *rationing* constraint of the type

$$\varphi_i^t \leq \alpha_i^t \ \forall\ t \in T$$

indicating that a *type* i agent who offers labor in *sector* t will be assigned to that sector with probability lower or equal to α_i^t, which the measure of *type* i workers effectively assigned to *sector* t in the equilibrium;

(II) production firms' labor demand, l_i^t, and intermediaries assets' supply, $\widehat{z}_i^t$, satisfy the same conditions as in the competitive equilibrium with deterministic contracts (i.e., conditions (6) and (7)) except that, because of the presence of transfers, the *sector* t production firms' objective function is now $\sum_{\theta \in \Theta} p_i^t(\theta)\, l_i^t(\theta)\left(q_t y_i^t(\theta) - w_i^t(\theta) + f_i^t(\theta)\right)$;

(III) the minimal wages' constraints, $w_i^t(\theta) \geq \hat{w}_i^t(\theta)$ for all θ, are satisfied; and

(IV) all feasibility conditions hold.

Next proposition shows that all Pareto optimal allocations can be implemented as equilibria with transfers provided that agents' types are public information. Optimal policy schemes generally hinge on state and sector contingent minimal wages, but do not require transfers across firms. However, in the case of inelastic labor supply, uniform minimal wages suffice to implement Pareto optima, if appropriate transfers across sectors are also implemented.

Proposition 17 *All Pareto optimal allocations can be implemented as equilibria with transfers by policy schemes such that* $f_i^t(\theta) = 0$ *for all* θ*,* i *and* t*. Moreover, if workers' labor supply is inelastic for any positive wage, Pareto optima are implementable through policy schemes such that* $\hat{w}_i^t(\theta) = \hat{w}_i$ *and* $f_i^t(\theta) = \hat{w}_i - \eta_t^P a_i^t(\theta)$ *for all* θ*,* t *and* i*.*

Intuitively, the proposition shows that contingent monetary transfers allow to equalize, at the Pareto optimal shadow prices, the marginal utilities of contingent goods and wealth across occupations. Minimal wages prevent firms from manipulating the transfers' scheme by undercutting wages in the sectors where, at the Pareto shadow prices, workers obtains higher utility levels and labor supply is rationed.

As a remark note also that, by continuity, Pareto improving policy schemes relying only on uniform minimal wages exist whenever the elasticity of labor supply is sufficiently small.[34]

A decentralization result similar to the one stated in the previous proposition can be proved if one considers alternative policy schemes based on (possibly negative) *non-linear* subsidies

[34] Uniform minimal wages and sector dependent minimal wages are both observed in developed countries.

to health-enhancing consumption activities. The logic of the proof remains the same as *non-linear* subsidies to the purchase of health services turn out to be substantially equivalent to cross subsidies.[35] However, it is noteworthy that the non linearity of consumption subsidies is necessary for the implementation of Pareto optima. Indeed, *linear* consumption subsidies would distort individual consumption choices, thereby preventing the equalization of marginal rates of substitution to relative prices.[36] Finally, it is worthwhile mentioning that robust examples can be constructed where simple deterministic cross-transfers policies, that do not discriminate across types, allow to improve upon competitive allocations (see Bennardo and Piccolo 2005). It may be showed that these policies are based on: (i) a uniform, public or regulated insurance scheme implementing cross transfers; and (ii) an opt-out clause allowing agents who prefer to buy insurance at market rates to exit the regulated insurance scheme.

8 Extensions

In the previous sections we made two simplifying assumptions: we assumed away prevention activities and aggregate uncertainty. Both these restrictions may be removed.

Aggregate Uncertainty. Introducing aggregate uncertainty does not involve any analytical complication. All the results of the paper, as well as the analytical arguments extend to the more general case, provided that the number of aggregate states is finite.

Prevention Activities. Introducing prevention requires some carefulness. Prevention activities are naturally described as workers' investments which allow obtaining, at a positive cost, a first-order stochastic shift of the health distributions associated to his occupations. If a pair of health distributions are initially ordered by the FOSD criterion, prevention activities may determine three possible scenarios. In the *first*, prevention technologies are such that the ordering of the two health distributions is preserved after prevention is undertaken. This is the case, for instance, whenever prevention activities are *very* costly, or have a similar impact on the two health distributions. In the *second scenario*, the ordering of the two distributions is reversed after prevention activities are performed. This may happen whenever prevention is relatively much more effective under the riskier health distribution. Finally, there also exists a *third* possible scenario where, once prevention activities are undertaken, health distributions cannot be anymore ordered by the FOSD criterion. As for the *first* case, introducing prevention leaves

[35] The formal proof of this claim are available upon request .

[36] Similarly, policies that do not discriminate across types (either cross subsidies on insurance or subsidies to health services purchases), generally do not allow to equalize, for all possible types, the marginal utility of expected wealth of agents assigned to different occupations.

unaltered the results derived in the paper. In the second case, all our analysis still applies but must be appropriately reinterpreted. Precisely, the ordering of the distributions determining optimal cross transfers and utility differentials is the ex-post one (i.e., the one emerging in equilibrium as a result of prevention activities), and not that holding ex ante. Only in the third case our characterization, which relies on the FOSD criterion, does not anymore apply.[37]

9 Concluding Remarks

The endogeneity of individual health distributions' generates specific "cost-benefit trade-offs" involving agents' occupational choices and their consumption and production capabilities. We studied how these trade-offs shape either the Pareto frontier of the economy or agents' competitive choices. We showed that the relative magnitude of health effects on production and consumption choices determines the sign of Pareto optimal utility differentials across workers who use different technologies as well as that of optimal cross-jobs transfers. Moreover, we proved that competitive equilibria are ex-ante efficient if lottery contracts are enforceable, but not otherwise. As a consequence the unenforceability of lotteries, may justify the introduction of policy schemes implementing cross-transfers across occupations.

All these results have been derived assuming away asymmetric information problems and restricting attention to linear technologies. Differently, several contributions of a recent literature on clubs and firms (Cole-Prescott (1997), Ellickson-Grodal-Scothmer-Zame (1999), Makowski-Ostroy (2003), Zame (2005)) have focused on the complex issue of the pricing of institutions, firms, and occupations in general equilibrium settings, where complementarities between consumption and production activities play a crucial role. Our conjecture, based on the analysis of the present paper, is that the generic inconsistency between ex-ante and interim optimality continues to hold in most of the settings studied in the club and in the asymmetric information literature. A result in this spirit is obtained by Bennardo (2005), which characterizes optimal transfers in a moral hazard set-up where health effects are not considered, but occupations affect agents' consumption choices via incentive constraints.

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[37] All the results mentioned in this section are formally proved in a more extended version of this paper. Their proofs are available on request.

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10 Appendix

Proof of Proposition 1

The proof of the uniqueness part relies on a standard convexity argument. In order to prove the genericity result, we need to formally define the utility space $\mathcal{U}_i$. Following the literature[38]

[38] See Geanakoplos-Polemarchakis (1986) and Citanna, et al. (1994) for a detailed discussion.

assume that, beyond all assumptions stated in Section 2, agents' preferences satisfy the following property: a sequence $U_{ik}(x_i,\theta)$ in $\mathcal{U}_i$ converges to $U_i(x_i,\theta) \in \mathcal{U}_i$ if and only if $U_{ik}(x_i,\theta)$, $DU_{ik}(x_i,\theta)$ and $D^2U_{ik}(x_i,\theta)$ uniformly converge to $U_i(x_i,\theta)$, $DU_i(x_i,\theta)$ and $D^2U_i(x_i,\theta)$, respectively, for all θ, on any compact subset of $\Re_+^C \times [0,L]$.[39]

Let $\xi = (x,\alpha,\eta,\lambda)$ define the vector of variables in the Pareto program. We consider first the case where the solution of the Pareto program is internal and $D_cU_i(x,\theta) > 0$ for all i, c and θ. A Pareto optimum then solves:

$$\mathcal{F}(\xi,\varepsilon,\bar{u}) = \left(\begin{array}{c} \lambda_i D_c U_i(x_i^t,\theta) - \eta_c \mu_i \quad \forall\, c \in C \\ -\lambda_i U_{ix_L}(x_i^t,\theta) + \eta_t a_i^t(\theta)\mu_i \\ \lambda_i(u_i^t(x_i^t) - u_i^T(x_i^T)) - \mu_i(Z_i^t - Z_i^T) \ \ \forall\, t \neq T \\ \sum_{i\in I} \mu_i\left(\overline{x}_i - e_i\right) - \sum_{i\in I}\mu_i y_i \\ \sum_{t\in T} \alpha_i^t u_i^t(x_i^t) - \bar{u}_i \ \ \forall\, i \neq 1 \end{array} \right)_{\theta\in\Theta, t\in T, i\in I} = \mathbf{0}$$

for some vector of Pareto weights, $\bar{u} = (\bar{u}_i)_{i=2}^I$. Assume now, without loss of generality, $\alpha_1^t \in (0,1)$ for $t = 1$ and $t = T$. Now for any arbitrary economy $\varepsilon \in \mathcal{E}$, define the extended system of equations $\mathcal{G}(\xi,\varepsilon,\bar{u}) \equiv (\mathcal{F}(\xi,\varepsilon,\bar{u}),\ (u_1^1(x_1^1) - u_1^T(x_1^T))) = 0$, which is obtained by adding one interim efficiency condition to $\mathcal{F}(.) = 0$. Finally let $\mathcal{S}_{\bar{u}} = \{\varepsilon \in \mathcal{E} : \mathcal{G}(\xi,\varepsilon,\bar{u}) = 0\}$ be the subset of economies where a solution $\xi(\varepsilon,\bar{u})$ of $\mathcal{G}(.)$ exists for a given $\bar{u}$. We will show that ex-ante and interim Pareto optima are generically different, by proving the equivalent statement that the complement of $\mathcal{S}_{\bar{u}}$ is open and dense.

(i) *Density*

The space, $\mathcal{E}$, of economies is infinite dimensional. However, as *density* is a local property, one may restrict attention to a properly defined subset of $\mathcal{E}$. Specifically, we will consider the linear subspace of $\mathcal{U}$ defined as follows. Fix arbitrarily an $\bar{\varepsilon} \in \mathcal{E}$ and a vector $\bar{u}$, and let $x^P(\bar{\varepsilon},\bar{u})$ be the Pareto optimal allocation associated to $\bar{\varepsilon}$, and to a particular vector of Pareto weights, $\bar{u}$. Given an utility profile $\hat{U} \in \mathcal{U}$, consider the perturbed utility functions $U_i(x_i,\theta) = \hat{U}_i(x_i,\theta) + \kappa_i(\theta) + \beta_i(\theta)(x_i - x_i^P(\theta|\bar{\varepsilon},\bar{u}))$ where, for all θ and i, $\kappa_i(\theta)$ is a scalar and $\beta_i(\theta)$ denotes a $(C+1)$ dimensional vector. Assume $|\kappa_i(\theta)|$ and $\|\beta_i(\theta)\|$ sufficiently small for all (θ,i). The set of certainty utility functions, $\widehat{\mathcal{U}}$, defined by all possible perturbations obtainable in this way is a finite dimensional, linear subspace of $\mathcal{U}$. We shall prove density on $\widehat{\mathcal{E}} = \mathcal{E} \times \mathcal{T} \times \widehat{\mathcal{U}}$. Define $\widehat{\mathcal{S}}_{\bar{u}} = \left\{\varepsilon \in \widehat{\mathcal{E}} : \mathcal{G}(\xi,\varepsilon,\bar{u}) = 0\right\}$ and let $(\xi^P(\bar{u}),\varepsilon(\bar{u}))$ a generic point such that $\mathcal{G}(.) = 0$. We will show that the complement of $\widehat{\mathcal{S}}_{\bar{u}}$ is dense by proving that $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P(\bar{u}),\varepsilon(\bar{u}))$, the matrix

[39] In words, we assume that $\mathcal{U}_i$ is endowed with the subspace topology of the C^2 uniform convergence topology on compact sets. Notice also that $\mathcal{U} = \prod_{i=1}^I \mathcal{U}_i$ is endowed with product topology.

associated to the Jacobian of $\mathcal{G}(.)$ evaluated at $(\xi^P(\bar{u}), \varepsilon(\bar{u}))$, has full row rank (i.e., $\mathcal{G}(.)$ is transversal to zero). Let $e_c = \sum_{i\in I} \mu_i e_{ic}$ for all c, and $e = (..., e_c, ...) \in \Re^C$; moreover, define $a_i(\hat{\theta}) = (a_i^1(\hat{\theta}), .., a_i^{T-1}(\hat{\theta})) \in \Re^{T-1}$ and $a(\hat{\theta}) = (a_i(\hat{\theta}))_{i=1}^I \in \Re^{(T-1)\times I}$ for $\hat{\theta} \in \Theta$. Finally, let $\kappa_i^t = \sum_{\theta\in\Theta} p_i^t(\theta)\kappa_i(\theta)$ for all pairs (t, i) with $\kappa^T = (\kappa_i^T)_{i=2}^I \in \Re^{I-1}$, and $\beta_L = (\beta_{1L}(\theta), .., \beta_{IL}(\theta)) \in \Re^I$. Straightforward elementary operations imply that the rank of $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^P(\bar{u}), \varepsilon(\bar{u}))$ is equal to the rank of the matrix[40]:

$$\mathbf{A} = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\left(\begin{array}{cccccccc} & x & e & a(\hat{\theta}) & \kappa^T & \kappa_1^1 & \beta_L \\ \text{FOCs}(x) & \mathbf{H} & \mathbf{0} & \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{E} \\ \text{FEAs} & \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \text{FOCs}(\alpha) & * & \mathbf{0} & \mathbf{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{u}\text{-CONS.} & * & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ u_1^1(x_1^1) - u_1^T(x_1^T) = 0 & * & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} \end{array}\right)$$

where $\mathbf{H}$, the submatrix of the Hessians, has full rank since preferences are strictly convex; $\mathbf{I}$ is a C dimensional identity matrix; $\mathbf{B}$ has all entries equal to zero, except for the ones corresponding to the first-order conditions (FOCs) with respect to $x_{iL}^t(\theta)$ which are equal to $\eta_t a_i^t(\theta)\mu_i$ for all (i, t); $\mathbf{C}$ is a $(T-1) \times I$ dimensional square matrix with all null entries, but the ones of the principal diagonal which are equal to $p_i^t(\theta) l_i^t(\theta)$ for all (i, t); and $\mathbf{E}$ has all null entries except for the elements corresponding to FOCs with respect to $x_{iL}^t(\theta)$, which are equal to 1 for all (i, t).

Indeed, $\mathbf{A}$ is obtained by summing the columns corresponding to e (multiplied by appropriate scalars) to the ones corresponding to a and by summing the rows corresponding to the utility constraints to the ones corresponding to the FOCs with respect to α. Now define,

$$\mathbf{M}_i = \left(\begin{array}{cccc} & x_i & a_i(\hat{\theta}) & \beta_{Li} \\ \text{FOCs}(x_i) & \mathbf{H}_i & \mathbf{B}_i & \mathbf{E}_i \\ \text{FOCs}(\alpha_i) & * & \mathbf{C}_i & \mathbf{0} \end{array}\right), \text{ for all } i = 1, .., I$$

where "$*$" indicates generic submatrices. As $\mathbf{C}$, $\mathbf{I}$ and all the Hessians submatrices $\mathbf{H}_i$ are non-singular, $\mathbf{A}$ has full row rank if $\mathbf{M}_i$ has full row rank for all i. Finally, by summing the columns of $\mathbf{M}_i$ corresponding to β_{Li} (multiplied by appropriate scalars) to the ones corresponding to x_i

[40] This can be easily verified by using the condition $u_1^1(x_1^1) - u_1^T(x_1^T) = 0$ to rewrite the FOC with respect to α_1^1 in $\mathcal{G}$ as $Z_1^1 - Z_1^T = 0$.

and $a_i(\hat{\theta})$, respectively, one obtains:

$$\hat{\mathbf{M}}_i = \begin{array}{c} \\ \text{FOCs}(x_i) \\ \text{FOCs}(\alpha_i) \end{array} \begin{pmatrix} x_i & a_i(\hat{\theta}) & \beta_{Li} \\ \hat{\mathbf{H}}_i & \mathbf{0} & \mathbf{E}_i \\ * & \mathbf{I}_i & \mathbf{0} \end{pmatrix} \quad \text{where,} \quad \hat{\mathbf{H}}_i = \begin{pmatrix} \mathbf{I}.. & ..\mathbf{0}.. & ..\mathbf{0}.. & ..\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ *.. & ..\mathbf{I}.. & ..\mathbf{0}.. & ..\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}.. & ..*.. & ..\mathbf{I}.. & ..\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}.. & ..\mathbf{0}.. & ..*.. & ..\mathbf{I} \end{pmatrix}$$

It is then straightforward to verify that $\hat{\mathbf{M}}_i$ has full rank and so does $\mathbf{M}_i$. This implies that $\mathbf{A}$ has full rank. Thus $\mathcal{G}(.)$ is transversal to zero and $\widehat{\mathcal{S}}_{\bar{u}}$ is dense whenever the solution of the Pareto program is internal. Finally, the proof extends to the case where $\mathcal{F}(.)$ has corner solutions such that $\alpha_i^t \in \{0, 1\}$, say $\tilde{\alpha}_i^t = 0$, for some pairs (i, t) and for some consumption goods, say $\tilde{x}_{ic}^t(\theta) = 0$. Indeed, in this case it suffices to replace $\mathcal{G}(.) = 0$ by a system $\mathcal{G}'(.)$ which differs from $\mathcal{G}(.) = 0$ only because $\tilde{\alpha}_i^t$ and $\tilde{x}_{ic}^t(\theta)$ are fixed to zero in all the equations, and the first-order conditions with respect α_i^t and $x_{ic}^t(\theta)$ do not appear anymore. $\square$

(ii) *Openness*

Let $\mathcal{P}_{\bar{u}} = \{(\xi, \varepsilon) : \mathcal{F}(\xi, \varepsilon, \bar{u}) = 0\}$ denote the Pareto optimal manifold for $u = \bar{u}$, and consider the natural projection $\pi : \mathcal{P}_{\bar{u}} \rightarrow \mathcal{E}$, $\pi(\xi, \varepsilon, \bar{u}) = \varepsilon$. As proper mappings take closed sets into closed sets, $\mathcal{S}_{\bar{u}}$ is open if the natural projection is *proper*. Hence we need to prove that for any sequence $(\xi_k(\bar{u}), \varepsilon_k(\bar{u}))_{k=1}^{\infty}$ such that $\mathcal{F}(\xi_k, \varepsilon_k, \bar{u}) = 0$ for all k, and $\varepsilon_k \rightarrow \varepsilon$ as $k \rightarrow \infty$, there exists a converging subsequence of $(\xi_k(\bar{u}))_{k=1}^{\infty}$ with limit $\xi(\bar{u})$ such that $\mathcal{F}(\xi, \varepsilon, \bar{u}) = 0$. To this end, note first that $\{\alpha_k(\bar{u})\}_{k=1}^{\infty}$ must converge, say to α, as it belongs to the compact set $[0, 1]^{T \times I}$. Moreover, $D_c U_i(x, \theta) > K$ for all $c \in \hat{C}(\theta)$ as $x_c \rightarrow 0$,with K large, imply $\{x_{ik}(\bar{u})\}_{k=1}^{\infty} \gg 0$ for all i; while since $D_c U_i(x, \theta) < k$, with k small, as $x_c \rightarrow \infty$, there exists a positive vector G such that $x_{ik}(\bar{u}) < G$, hence $\{x_k(\bar{u})\}_{k=1}^{\infty}$ must converge, say to x. Given the assumptions on U_i, $U_{ik}(x_i, \theta) \rightarrow U_i(x_i, \theta)$ implies $DU_{ik}(x_i, \theta) \rightarrow DU_i(x_i, \theta)$ uniformly on compact sets for all (x_i, θ), then this must also hold at $x_i = x_{ik}(\bar{u})$ for all i. Finally, from (15)-(17) one gets $(\lambda_k(\bar{u}), \eta_k(\bar{u})) \rightarrow (\lambda(\bar{u}), \eta(\bar{u}))$. $\square$

Proof of Proposition 2

For the sake of brevity we provide the proof only for the case where $U_\theta > 0$ at least in some interval $d\theta$. The proof for the case where $U_\theta = 0$ for all θ and $\partial L(\theta)/\partial\theta > 0$ in some interval $d\theta$ follows exactly the same lines; while the result for the case where $U_\theta = 0$, $\partial L(\theta)/\partial\theta = 0$ and $\partial a_i^t(\theta)/\partial\theta > 0$ simply follows from algebraic manipulations of first-order conditions of the Pareto program.

Assume without loss of generality that $\langle p_i^1, \Theta \rangle$ FOSD $\langle p_i^2, \Theta \rangle$ and that $\alpha_i^1 > 0$, $\alpha_i^2 > 0$ and let

$\langle x^P, \alpha^P \rangle$ a generic Pareto optimal allocation. Let Π be a stochastic $N \times N$ matrix, and denote $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m)$ be the element in the n-th row and the m-th column of Π. Assume $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m)$ $= 0$ for all pairs (n, m) such that $n > m$; $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m)$ for all pairs (n, m) with $n = m$, and:

$$\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = \min \left\{ [p_i^1(\theta_m) - \sum_{l=1}^{n-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m)],\ [p_i^2(\theta_n) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_k)] \right\}$$

for all (n, m) with $n < m$. As a preliminary step, we show that Π satisfies the following properties:

(i) $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) \leq p_i^2(\theta_n)$ for $n = m$;

(ii) $\sum_{n=1}^N \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^1(\theta_m)$, and $\sum_{m=1}^N \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^2(\theta_n)$;

(iii) $u_i^2(x_i^{2P}) < \sum_{m=1}^N \sum_{n=1}^N \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) U_i(x_i^{2P}(\theta_n), \theta_m)$.

Part (i) We can obviously restrict to the case where $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) > 0$ for $n = m$. By construction, in this case $\tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) = p_i^2(\theta_l) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_k)$ for all $l < n$. Hence, for all (n, m) such that $n = m$ one must have $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^1(\theta_m) - \sum_{l=1}^{n-1} \left(p_i^2(\theta_l) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_k) \right)$ $= p_i^1(\theta_m) - \sum_{l=1}^{n-1} p_i^2(\theta_l) + \sum_{k=1}^{m-1} p_i^1(\theta_k)$, which implies $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) < p_i^2(\theta_n)$ by FOSD.

Part (ii) By construction, $\sum_{n=1}^N \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^1(\theta_m)$. The proof that $\sum_{m=1}^N \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m)$ $= p_i^2(\theta_n)$ is by induction, we show first that the equality holds for $n = 1$. This amount to show that $\exists\ m \leq N$ such that $p_i^1(\theta_m) > p_i^2(\theta_1) - \sum_{k=1}^{m-1} \tilde{\pi}(x_i^{2P}(\theta_1), \theta_k)$. This is true since otherwise it should be $p_i^1(\theta_m) \leq \max \left\{ 0, p_i^2(\theta_1) - \sum_{k=1}^{m-1} p_i^1(\theta_k) \right\}$, for all $m = 1, ..., N+1$, which is impossible as $\max \left\{ 0, p_i^2(\theta_1) - \sum_{k=1}^{m-1} p_i^1(\theta_k) \right\} = 0$ for some $m \leq N$ since $\sum_{k=1}^N p_i^1(\theta_k) = 1 > p_i^2(\theta_1)$.

Suppose now that $\sum_{m=1}^N \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = p_i^2(\theta_n)$ for $n = 1, 2, ...\bar{n}$, but $\sum_{m=1}^N \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m)$ $< p_i^2(\theta_{\bar{n}+1})$. Then by construction one must have $\tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) = p_i^1(\theta_m) - \sum_{l=1}^{\bar{n}} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m)$ $\forall\ m > n$. By summing over m it follows:

$$\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) = \sum_{m=\bar{n}+1}^{N} \left(p_i^1(\theta_m) - \sum_{l=1}^{\bar{n}} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) \right)$$

which, in turn, implies:

$$\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) = 1 - \sum_{m=1}^{\bar{n}} p_i^1(\theta_m) - \sum_{m=1}^{N} \sum_{l=1}^{\bar{n}} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m) + \sum_{m=1}^{\bar{n}} \sum_{l=1}^{\bar{n}} \tilde{\pi}(x_i^{2P}(\theta_l), \theta_m)$$

where the right-hand-side of this expression is equal to $1 - \sum_{m=1}^{\bar{n}} p_i^1(\theta_m) - \sum_{n=1}^{\bar{n}} p_i^2(\theta_n) +$ $\sum_{m=1}^{\bar{n}} p_i^1(\theta_m) = 1 - \sum_{n=1}^{n} p_i^2(\theta_n)$. This proves the claim since $\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) = 1-$

$\sum_{n=1}^{n} p_i^2(\theta_n) > p_i^2(\theta_{\bar{n}+1})$, contradicting $\sum_{m=\bar{n}+1}^{N} \tilde{\pi}(x_i^{2P}(\theta_{\bar{n}+1}), \theta_m) \leq p_i^2(\theta_n)$.

Part (iii) As $\tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) = 0$ for all (n, m) with $n > m$ and $U_\theta > 0$, for all n, we have:

$$\sum_{m=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) U_i(x_i^{2P}(\theta_n), \theta_m) > \sum_{m=1}^{N} \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) U(x_i^{2P}(\theta_n), \theta_n) = p_i^2(\theta_n) U_i(x_i^{2P}(\theta_n), \theta_n)$$

by summing over n one obtains (iii).

We can now prove that $\eta_1^P < \eta_2^P$. The proof is by contradiction. Assume first $\eta_1^P > \eta_2^P$. Consider an allocation such that all *type i'* agents with $i' \neq i$ receive $(x_{i'}^P, \alpha_{i'}^P)$; a measure $\alpha_i^1 = \alpha_i^{1P} + d\alpha_i$ of the set of *type i* workers is assigned to *sector 1*, with $d\alpha_i$ sufficiently small, while a measure $\alpha_i^2 = \alpha_i^{2P} - d\alpha_i$ is assigned to *sector 2*. Moreover, assume that all *type i* workers in *sector* 2 and a measure $\alpha_i^{1P} - d\alpha_i$ of *type i* workers obtain x_i^{2P} and x_i^{1P}, respectively; a set of measure $d\alpha_i$ of *type i* workers obtain $\tilde{x}_i^1 = (..., \tilde{x}_i^1(\theta_m), ...)$ with $\tilde{x}_i^1(\theta_m) = \sum_n \tilde{\pi}(x_i^{2P}(\theta_n), \theta_m) x_i^{2P}(\theta_n)$, while another set of measure $d\alpha_i$ of *type i* workers, obtains the allocation $x_i^{1P} + \varepsilon = (..., x_i^{1P}(\theta_n) + \varepsilon(\theta_n), ...)$ where, for all n, $\varepsilon(\theta_n)$ is such that $\varepsilon_1(\theta_n) = \varepsilon$, $\varepsilon_2(\theta_n) = -\varepsilon$, with $\varepsilon > 0$ and sufficiently small, and $\varepsilon_c(\theta_n) = 0$ for all $c > 2$. By construction this allocation is feasible; moreover $\tilde{x}_i^1$ and $x_i^{1P} + \varepsilon$ are strictly preferred to x_i^{2P} and x_i^{1P}, respectively. This contradicts the optimality of $\langle x^P, \alpha^P \rangle$; thus, $\eta_1^P \leq \eta_2^P$. Finally, a standard continuity argument implies $\eta_1^P \neq \eta_2^P$. $\square$

Proof of Proposition 4

As a preliminarily step we state without proof the following well known lemma which will be used several times subsequently. Let $P^t(\theta_n) = \sum_{\theta \leq \theta_n} p^t(\theta_n)$ for $n \in N$ and $t = 1, 2$,

Lemma 18 *For any map* $g : \Theta \rightarrow \Re^+$, $\theta \rightarrow g(\theta)$, *with* $dg(\theta_{n+1}) = g(\theta_{n+1}) - g(\theta_n)$, *the following identity holds:*

$$\sum_{\theta \in \Theta} \left(p^1(\theta) - p^2(\theta)\right) g(\theta) := \sum_{n \in N} \left(P^2(\theta_n) - P^1(\theta_n)\right) dg(\theta_{n+1})$$

We can now prove the statement of the proposition. The first-order conditions with respect to x of the Pareto program, together with strict concavity of $U(x, \theta)$ imply $x^{1P}(\theta) = x^{2P}(\theta) = x^P(\theta)$ for all θ. Let $x^P : \Theta \rightarrow \Re_+^2$, $\theta \rightarrow x^P(\theta)$, be the map associating to each $\theta \in \Theta$ the optimal consumption vector $x^P(\theta)$. Assume $\theta_{n+1} - \theta_n = d\theta$ for all n, with $d\theta$ sufficiently small, and let $dU\left(x^P(\theta_n), \theta_n\right) = U\left(x^P(\theta_{n+1}), \theta_{n+1}\right) - U\left(x^P(\theta_n), \theta_n\right)$, one then obtains:

$$dU\left(x^P(\theta_n), \theta_n\right) \approx \sum_{c=1,2} dx_c^P(\theta_n) \eta_c^P + U_\theta\left(x^P(\theta_n), \theta_n\right) d\theta \tag{18}$$

By Lemma (18), $u^1(x^{1P}) \gtreqless u^2(x^{2P})$ if $dU\left(x^P(\theta),\theta\right) \gtreqless 0$; hence (18) implies $u^1(x^{1P}) \gtreqless u^2(x^{2P})$ if $\sum_{c=1,2} dx^P(\theta_n)\eta_c^P + U_\theta\left(x^P(\theta_n),\theta_n\right)d\theta \gtreqless 0$. For $d\theta$ small, $dx_1^P(\theta_n) \approx (U_{1\theta}\,|U_{22}| + U_{2\theta}U_{21})/\Lambda)d\theta$ and $dx_2^P(\theta_n) \approx (U_{2\theta}\,|U_{11}|+U_{1\theta}U_{12})/\Lambda)d\theta$, where $\Lambda = U_{11}U_{22}-(U_{12})^2 > 0$. Summing up, we obtain:

$$\sum_{c=1,2} dx_c^P(\theta_n)\eta_c \approx U_1\frac{U_{1\theta}\,|U_{22}| + U_{2\theta}U_{12}}{\Lambda}d\theta + U_2\frac{U_{2\theta}\,|U_{11}| + U_{1\theta}U_{12}}{\Lambda}d\theta \tag{19}$$

(18) and (19) then imply that $dU\left(x^P(\theta_n),\theta_n\right) \gtreqless 0$ if:

$$\frac{U_{1\theta}}{\Lambda}\left(U_1|U_{22}| + U_2U_{12}\right) + \frac{U_{2\theta}}{\Lambda}(U_2|U_{11}| + U_1U_{12}) + U_\theta \gtreqless 0 \tag{20}$$

(20) together with supermodularity in x, increasing differences in (x,θ) and $U_\theta > 0$, imply *(i)*; *(ii)* follows from (20) and decreasing differences in (x,θ). Finally *(iii)* also follows from simple algebraic manipulations of (20). □

Proof of Proposition 7

Part (i). We first prove the claim assuming separability, i.e., $U(x, L-l,\theta) = f(\hat{x}) - \psi(l,\theta)$. Let $\Delta\tilde{u}^P = \sum_{\theta\in\Theta} p^2(\theta)\psi(l^{2P}(\theta),\theta) - \sum_{\theta\in\Theta} p^1(\theta)\psi(l^{1P}(\theta),\theta)$ and define $\sigma_\psi(l,\theta) \equiv \psi_{ll}(l,\theta)/\psi_l(l,\theta)$. Summing by parts,

$$\Delta\tilde{u}^P = -(\sum_{\theta\in\Theta}(p^1(\theta) - p^2(\theta))\psi(l^{1P}(\theta),\theta) + \sum_{\theta\in\Theta} p^2(\theta)(\psi(l^{2P}(\theta),\theta) - \psi(l^{1P}(\theta),\theta))$$

Now, let η_t^P the value of the Lagrange multiplier, calculated in the optimum, and denote $l_1(\theta)$ be the function implicitly defined by $\psi_l(l(\theta),\theta) = \eta_1^P$; and $l(\theta,\eta)$ that defined by $\psi_l(l(\theta),\theta) = \eta$. Let $\Delta P(\theta_n) = P^1(\theta_n) - P^2(\theta_n)$, we then have:

$$\Delta\tilde{u}^P \approx \sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_n+\Delta\theta}\left(\frac{d\psi(l_1(\theta),\theta)}{d\theta}\right)d\theta + \sum_{\theta\in\Theta} p^2(\theta)\int_{\eta_1^P}^{\eta_2^P}(\sigma_\psi(l(\theta,\eta),\theta))^{-1}d\eta$$

and hence:

$$\Delta\tilde{u}^P \approx \sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_n+\Delta\theta}\left(\frac{|\psi_{l\theta}(l_1(\theta),\theta)|}{\sigma_\psi(l_1(\theta),\theta)} + \psi_\theta(l_1(\theta),\theta)\right)d\theta + \sum_{\theta\in\Theta} p^2(\theta)\int_{\eta_1^P}^{\eta_2^P}\left(\sigma_\psi(l(\theta,\eta),\theta)\right)^{-1}d\eta \tag{21}$$

Since $\eta_2^P > \eta_1^P$ by Proposition 2, and $\sigma_\psi \geq 0$, the second addendum in (21) is positive. By

FOSD the first addendum is also positive for $|\psi_\theta|$ large, hence $\Delta\tilde{u}^P > 0$.

Let now consider the non-separability case. Define an auxiliary program which maximizes $\sum_{t=1,2}\alpha^t u^t(x^t)$ under the feasibility constraints and the additional constraints $\hat{x}^1 = \hat{x}^2 = \hat{x}^{1P}$, where $\hat{x}^{1P}$ is part of the solution of the Pareto program. Let $(\alpha^F, x_L^{1F}, x_L^{2F})$ be the solution of this program and define $\Delta u^F = \sum_{\theta\in\Theta} p^1(\theta)U(\hat{x}^{1P}(\theta), x_L^{1F}(\theta), \theta) - \sum_{\theta\in\Theta} p^2(\theta)U(\hat{x}^{1P}(\theta), x_L^{2F}(\theta), \theta)$.

As an intermediate result, we show that, for any ε, there exists σ_{x_L} sufficiently large such that $|x_L^F - x_L^P| < \varepsilon$ and $|\Delta u^P - \Delta u^F| < \varepsilon$, with ε arbitrarily small. In order to prove this result, let $\pi(\theta) = \alpha^P p^1(\theta)/(\alpha^P p^1(\theta) + (1-\alpha^P)p^2(\theta))$ for all θ, and consider the allocation $(\alpha^P, \tilde{x})$ such that $\tilde{x}_L(\theta) = \pi(\theta)x_L^{1P}(\theta) + (1-\pi(\theta))x_L^{2P}(\theta)$, and $\tilde{x}_c^1 = \tilde{x}_c^2 = \tilde{x}_c$ for $c = 1, 2$, where $\tilde{x}_c$ is assumed sufficiently small so as to satisfy the feasibility constraints. $\Delta EU = EU(\alpha^P, x^P) - EU(\alpha^P, \tilde{x})$ can be rewritten as:

$$\Delta EU = \alpha^P \sum_{\theta\in\Theta} p^1(\theta)A(\theta) + (1-\alpha^P)\sum_{\theta\in\Theta} p^2(\theta)B(\theta) + \sum_{\theta\in\Theta}(\alpha^P p^1(\theta) + (1-\alpha^P)p^2(\theta))C(\theta)$$

where $A(\theta) = U(x^{1P}(\theta), \theta) - U(\tilde{x}_1(\theta), \tilde{x}_2(\theta), x_L^{1P}(\theta), \theta)$, and where $B(\theta) = U(x^{2P}(\theta), \theta) - U(\tilde{x}_1(\theta), \tilde{x}_2(\theta), x_L^{2P}(\theta), \theta)$, and

$$C(\theta) = \pi(\theta)U(\tilde{x}_1(\theta), \tilde{x}_2(\theta), x_L^{1P}(\theta), \theta) + (1-\pi(\theta))U(\tilde{x}_1(\theta), \tilde{x}_2(\theta), x_L^{2P}(\theta), \theta) - U(\tilde{x}(\theta), \theta)$$

$C(\theta)$ is proportional to σ_{x_L}; and for all θ both $A(\theta)$ and $B(\theta)$ are bounded above and below because the aggregate endowment is strictly positive and finite and $D_c U(x,\theta) > K$ as $x_c \to 0$, with K large. As a consequence, an optimality argument implies that for any $\varepsilon > 0$ there exists σ_{x_L} sufficiently large such that $\left|x_L^F - x_L^P\right| < \varepsilon$. Since $|\hat{x}^{1P} - \hat{x}^{2P}| \to 0$ as $\left|x_L^F - x_L^P\right| \to 0$ the signs of Δu^P and Δu^F must coincide for σ_{x_L} sufficiently large.

Summing by parts, after some algebraic manipulations one obtains $\Delta u^F = \Delta\tilde{u}^P - G$, where the extra term G is:[41]

$$G = \sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_n+\Delta\theta}[\sum_{c=1,2}(U_c^F + \frac{U_{cx_L}^F}{\sigma_{x_L}^F}) \times \frac{U_{x_L\theta}^P(U_{12}^P U_{c'x_L}^P + |U_{c'c'}^P| U_{cx_L}^P)}{|\Lambda^P|}]d\theta$$

By using the result of the separability case the claim of part *(i)* then follows as G is bounded under **A1**. $\square$

Part (ii) We begin again with the separability case. Observe that $|\psi_{l\theta}/\sigma_\psi| > \delta$ im-

[41] In order to simplify the notation, hereafter the superscripts F and P will indicate that a function is evaluated at x^F and x^P, respectively.

plies σ_ψ finite. This together with Lemma 18, and the continuity of preferences, imply that there exists $d \in \Re_{++}$ such that if $dl^P(\theta) = l_2^P(\theta) - l_1^P(\theta) \leq d$ for all θ, $\Delta\tilde{u}^P < 0$. It then remains to prove that $\Delta\tilde{u}^P < 0$ whenever $dl(\theta) > d$ for some θ. From (21) we have $\sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} (d\psi(l_1(\theta),\theta)/d\theta)d\theta < 0$ for ψ_θ sufficiently large (i.e., U_θ small). Then since $|\psi_{l\theta}/\sigma_\psi| > 0$ and σ_ψ is large, $\Delta\tilde{u}^P < 0$ if there exists a strictly positive h such that $\Delta\eta^P = \eta_2^P - \eta_1^P < h$. In the following, we use an optimality argument to prove the existence of an upper bound on $\Delta\eta^P$. By definition $EU(x^P, \alpha^P) \geq EU(x', \alpha')$, for all feasible (x', α'). In particular, consider the consumption allocation $\check{x}$ such that $\check{x}_c^t = x_c^{tP}$ for $c = 1, 2$; $\check{l} = \beta l^{1P} + (1-\beta) l^{2P}$, with $\beta \in (0, 1)$. Since $l^{1P} < l^{2P}$ by Proposition 2, a continuity argument implies that for any β sufficiently small there exists a real number k such that $0 < \check{\alpha} = \alpha^P + k < 1$, and $(\check{x}, \check{\alpha})$ satisfies the feasibility constraints (possibly as inequality).

Let $\Delta EU = EU(x^P, \alpha^P) - EU(\check{x}, \check{\alpha}) \geq 0$. By adding and subtracting $EU(\check{x}, \alpha^P)$ to ΔEU, and then using the first-order conditions of the Pareto program one gets $\Delta EU = \tilde{A}^P + \tilde{B}^P$ where $\tilde{A}^P = \sum_{t\in T,\theta\in\Theta} \alpha_t^P p^t(\theta) \Delta\psi(l^t(\theta))$, with $\Delta\psi(l^t(\theta)) = (\psi(\check{l}(\theta), \theta) - \psi(l^{tP}(\theta), \theta))$, and where:

$$\tilde{B}^P \approx \Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\psi_l(\check{l}(\theta), \theta)(\beta \frac{|\psi_{l\theta}(l_1(\theta), \theta)|}{\psi_{ll}(l_1(\theta), \theta)} + (1-\beta) \frac{|\psi_{l\theta}(l_2(\theta), \theta)|}{\psi_{ll}(l_2(\theta), \theta)}) + \psi_\theta(\check{l}(\theta), \theta) \right) d\theta$$

with $\Delta\alpha = (\check{\alpha} - \alpha^P)$. For β sufficiently close to 0,

$$\tilde{B}^P \approx \Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\psi_l(\check{l}(\theta), \theta) \frac{|\psi_{l\theta}(l_2(\theta), \theta)|}{\psi_{ll}(l_2(\theta), \theta)} + \psi_\theta(\check{l}(\theta), \theta) \right) d\theta \leq$$

$$\Delta\alpha \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(\frac{|\psi_{l\theta}(l_2(\theta), \theta)|}{\sigma_\psi(l_2(\theta), \theta)} + \psi_\theta(\check{l}(\theta), \theta) \right) d\theta$$

since $\check{l}(\theta) \leq l_2(\theta)$ for all θ and $\psi_{ll} > 0$.

$\tilde{B}^P$ is bounded above as all derivatives of ψ are bounded by **A1**. Moreover, from the first-order conditions of the Pareto program and the convexity of ψ it follows $\tilde{A}^P < A' = \sum_{t\in T} \alpha_t^P \eta_t^P \sum_{\theta\in\Theta} p^t(\theta) \Delta l^t(\theta)$ where $\Delta l^t(\theta) = (\check{l}(\theta) - l^{tP}(\theta))$. Using the definition of $\hat{l}(\theta)$ we then get:

$$A' = \alpha_1^P \eta_1^P (1-\beta) \sum_{\theta\in\Theta} p^1(\theta)(l^{2P}(\theta) - l^{1P}(\theta)) - \alpha_2^P \eta_2^P \beta \sum_{\theta\in\Theta} p^2(\theta)(l^{2P}(\theta) - l^{1P}(\theta))$$

As $(l^{2P}(\theta) - l^{1P}(\theta)) > d$, the above expression implies $\tilde{A}^P \to -\infty$ as $\eta_2^P - \eta_1^P \to +\infty$. We can conclude that $\Delta EU = \tilde{A}^P + \tilde{B}^P \geq 0$ implies $\eta_2^P - \eta_1^P < h$ for some positive h and hence

$\Delta\tilde{u}^P < 0$.

By using the same type of argument developed for part *(i)* the proof extends to the case of nonseparability. $\square$

Proof of Proposition 8

To avoid introducing further notation it is convenient to prove the claim for the particular case $\psi_l(0,\theta) = 0$ for all θ. A continuity argument will then immediately imply the proof holds also for the case where $\psi_l(0,\theta) \leq \varepsilon$, with ε sufficiently small for all θ.

Let $l(\eta,\theta)$ be the function implicitly defined by $\psi_l(l(\theta),\theta) = \eta$ as before, and let $T(\eta,\theta) \equiv (1/\sigma_\psi(l(\eta,\theta),\theta)) - l(\eta,\theta)$, $\psi_l(0,\theta) = 0$ implies $T(0,\theta) = 0$ for all θ. Since $\psi_{lll}(l,\theta) > 0$ implies $\partial T(\eta,\theta)/\partial\eta \leq 0$, $l(\eta,\theta) \geq (1/\sigma_\psi(l(\eta,\theta),\theta))$ for all (η,θ). Given the definition of $\Delta\tilde{u}^P$ in equation (21) (appearing in the proof of Proposition 7), this inequality implies:

$$\text{(22)}\quad \Delta\tilde{u}^P \leq \sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_{n+1}}\left(\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_\psi(l_2(\theta),\theta)} + \psi_\theta(l_2(\theta),\theta)\right)d\theta + \sum_{\theta\in\Theta}p^1(\theta)\left(\int_{\eta_1^P}^{\eta_2^P} l(\eta,\theta)d\eta\right)$$

Let $h(l,\theta) \equiv \psi_l(l,\theta)l$ and let $l(\theta,\eta)$ be the function implicitly defined by $\psi_l(l(\theta),\theta) = \eta$, by adding and subtracting $\sum_{\theta\in\Theta}p^1(\theta)\psi(l^{2P}(\theta),\theta)$ to the left-hand-side and $\sum_{\theta\in\Theta}p^1(\theta)h(l^{2P}(\theta),\theta)$ to the right-hand-side of (17), and using Lemma 18 one gets:

$$\sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_{n+1}}\left(\frac{d\psi(l_2(\theta),\theta)}{d\theta}\right)d\theta + \sum_{\theta\in\Theta}p^1(\theta)\int_{\eta_1^P}^{\eta_2^P}\left(\psi_l(l(\theta,\eta),\theta)\frac{\partial l(\theta,\eta)}{\partial\eta}\right)d\eta =$$
$$\sum_{\theta\in\Theta}p^1(\theta)\int_{\eta_1^P}^{\eta_2^P}\left(h_l(l(\theta,\eta),\theta)\frac{\partial l(\theta,\eta)}{\partial\eta}\right)d\eta + \sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_{n+1}}\left(\frac{dh(l_2(\theta)\,\theta)}{d\theta}\right)d\theta$$

Simple algebraic manipulations allow to rewrite the above equality as:

$$\text{(23)}\qquad \sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_{n+1}}\psi_\theta(l_2(\theta),\theta)d\theta = \sum_{\theta\in\Theta}p^1(\theta)\int_{\eta_1^P}^{\eta_2^P}l(\eta,\theta)d\eta$$

where $l_2(\theta)$ denotes the function implicitly defined by $\psi_l(l_2(\theta),\theta) = \eta_2^P$. Equations (22) and (23) then imply:

$$\Delta\tilde{u}^P \leq \sum_{n\in N}\Delta P(\theta_n)\int_{\theta_n}^{\theta_{n+1}}\left(\frac{|\psi_{l\theta}(l_2(\theta),\theta)|}{\sigma_\psi(l_2(\theta),\theta)} + 2\psi_\theta(l_2(\theta),\theta)\right)d\theta$$

Then since by FOSD $\Delta P(\theta_n) \leq 0$ with at least one strict inequality, the integrand function is positive, so that $\Delta u^P \leq 0$ because $\Delta u^P = \Delta \tilde{u}^P$. $\square$

Proof of Proposition 11

Let $h(l) = \psi'(l)l$ as before, $\Delta h = \sum_{\theta\in\Theta} p^2(\theta) h\left(l^2(\theta)\right) - h(l^1(\theta_N))$, and $\Delta\sigma = (\sigma_\psi - \sigma_h)$, as a preliminary result we prove that $\Delta u^P = 0$ implies $sign\Delta\sigma = -sign\Delta h$. Denote $l^2(h) = \sum_{\theta\in\Theta} p^2(\theta) h\left(l^2(\theta)\right)$ the certainty equivalent of the distribution $\langle p^2, l^2\rangle$ under h. Since $\hat{x}^{1P} = \hat{x}^{2P}$, $\Delta u^P = 0$ implies $\psi\left(l^1(\theta_N)\right) = \sum_{\theta\in\Theta} p^2(\theta)\psi\left(l^2(\theta)\right)$; $l^1(\theta_N)$ is then the certainty equivalent, under ψ, of the distribution $\left\langle p^2, l^{2P}\right\rangle$. Since $h(l)$ is increasing, and $l^1(\theta_N) \gtreqless l^2(h)$ whenever $\Delta\sigma \gtreqless 0$, it follows that $\Delta h \lesseqgtr 0$ if $\Delta\sigma \gtreqless 0$.

We can now prove the claim, beginning with the case $\sigma_\psi < \sigma_h$, and hence by Lemma 10 $\partial\zeta_{l,w}/\partial w < 0$. To this end, we introduce an auxiliary program which maximizing $\sum_{t=1,2}\alpha^t u^t(x^t)$ under the feasibility constraints and the additional constraint:

$$\Delta u = \sum_{\theta\in\Theta} p^2(\theta)\psi(l^2(\theta)) - \psi(l^1(\theta_N)) \leq 0 \tag{24}$$

The FOCs with respect to $l^t(\theta)$, $t = 1, 2$, and α of this program are:

$$\psi'(l^1(\theta_N)) = \hat{\eta}_1 a(\theta_N) + \frac{\varkappa}{\alpha}\psi'(l^1(\theta_N)) \tag{25}$$

$$\psi'(l^2(\theta)) = \hat{\eta}_2 a(\theta) - \frac{\varkappa}{1-\alpha}\psi'(l^2(\theta)), \quad \forall\, \theta \in \Theta \tag{26}$$

$$\sum_{\theta\in\Theta} p^2(\theta)\psi(l^2(\theta)) - \psi(l^1(\theta_N)) = \hat{\eta}_2 \sum_{\theta\in\Theta} p^2(\theta) a(\theta) l^2(\theta) - \hat{\eta}_1 a(\theta_N) l^1(\theta_N) \tag{27}$$

where $\hat{\eta}_t$ for $t = 1, 2$ are the Lagrangian multipliers associated to the feasibility constraints of the auxiliary program, and $\varkappa$ is the multiplier associated with (24). Substituting (25) and (26) into (27) one gets:

$$\Delta u = \Delta h + \varkappa\left(\frac{\sum_{\theta\in\Theta} p^2(\theta) h(l^2(\theta))}{1-\alpha} + \frac{h(l^1(\theta_N))}{\alpha}\right) \tag{28}$$

We now prove that $\varkappa = 0$ and (24) holds as inequality whenever $\Delta\sigma < 0$. This immediately implies that $\Delta u^P < 0$ for $\Delta\sigma < 0$.

First we must have $\varkappa = 0$; indeed by (28) $\Delta h < 0$ for $\varkappa > 0$ and $\Delta u = 0$, but this is impossible since, as we showed above, $\Delta h > 0$ whenever $\Delta u = 0$ and $\Delta\sigma < 0$. Moreover, (24) must hold as inequality. Otherwise one would have $\Delta u = 0$, and hence $\Delta h > 0$ whenever $\Delta\sigma < 0$, which contradicts (28).

Proving that $\Delta\sigma > 0$ implies $\Delta u^P > 0$ and that $\Delta\sigma = 0$ implies $\Delta u^P = 0$, requires the same type of argument developed above. □

Proof of Proposition 12

We begin with the case where lottery contracts are unenforceable. Consider the *auxiliary* program which maximizes $\sum_{t\in T} u_i^t(x_i^t)\varphi_i^t$ within the compact set defined by the agents' budget constraints and the additional constraint $x_i^t(\theta) \in \bar{X} \subset \Re^C \times [0, L]$, with $\bar{X}$ finite but sufficiently large. Since the endowment of the economy is finite, the set of solutions of program (3)-(5) and that of the auxiliary program coincide for $\bar{X}$ sufficiently large. As both production and intermediation technologies are linear, equilibrium prices satisfy: $\phi_i^t(\theta) = g_i^t p_i^t(\theta)$ for some $g_i^t \in \Re_+$, and $w_i^t(\theta) = q_t a_i^t(\theta)$ for $i \in I$, $t \in T$ and $\theta \in \Theta$; and at these prices assets' supply and labor demands are indeterminate. Using these conditions and normalizing prices appropriately, the budget correspondence can be rewritten as:

$$B_i^t(q) = \left\{ (x_i^t, \varphi_i^t) : \sum_{\theta\in\Theta, c\in C} p_i^t(\theta)\, q_c(x_{ic}^t(\theta) - e_{ic}) - q_t \sum_{\theta\in\Theta} p_i^t(\theta)\, a_i^t(\theta)(L - x_{iL}^t(\theta)) \leq 0,\ \varphi_i^t \in \Delta \right\}$$

$B_i^t(q)$ is continuous for all $q \gg 0$. As a consequence, the individual demand correspondences for commodities and occupations, $(\zeta_i^t(q), \varphi_i^t(q))$, are upper- hemicontinous. Only $\varphi_i^t(q)$ but not $\zeta_i^t(q)$, tough, is convex valued. By construction, however, the per capita demand correspondence $\xi_i^t(q) = \sum_{t\in T} \varphi_i^t(q)\zeta_i^t(q)$ is upperhemicontinuous and convex valued. Hence, a standard application of the Kakutani fixed point theorem, in the space $\bar{X} \times \Delta^{(C-1)\times I}$ implies the existence result. The existence proof for the case of enforceable lottery contracts is basically the same. However, it requires a slightly different definition of the *auxiliary* program. In order to show that the set of feasible solutions is bounded, in the space of lotteries the set of constraints of the *auxiliary* program must be defined by imposing both the conditions $x \in \bar{X}$ and $\gamma^i \geq \varepsilon > 0$, with ε sufficiently small. A standard optimality argument then implies that $\gamma^i \geq \varepsilon$ hold with slack if $D_c U_i(x, \theta) < k$, with k sufficiently small for x_c sufficiently large. The rest of the proof then follows exactly the same lines as before.□

Proof of Proposition 14

Competitive equilibria satisfy the fair treatment condition, so that if $(\varphi_i^t, \varphi_i^{t'}) >> 0$, then $u_i^t(x_i^t) = u_i^{t'}(x_i^{t'})$. Indeed, if $u_i^t(x_i^t) > u_i^{t'}(x_i^{t'})$, $\varphi_i^{t'} > 0$ would not be optimal.

Now let $(x^*, \varphi^*, q^*, z^*, \phi^*)$ a competitive equilibrium such that $(\varphi_i^{*t}, \varphi_i^{*t'}) >> 0$. Suppose it is not interim efficient, there must exist a feasible allocation $(\widehat{x},\widehat{\varphi}) \neq (x^*,\varphi^*)$ such that $u_i^t(\widehat{x}_i^t) = u_i^{t'}(\widehat{x}_i^{t'})$ for all i, t and t' with $(\widehat{\varphi}_i^t,\widehat{\varphi}_i^{t'}) >> 0$, and $(\widehat{x}_i,\widehat{\varphi}_i) \succeq_i (x_i^*,\varphi_i^*)$ with $(\widehat{x}_i,\widehat{\varphi}_i) \succ_i (x_i^*,\varphi_i^*)$ for at least one i. Then:

$$\sum_{t\in T}\widehat{\varphi}_i^t\sum_{\theta\in\Theta}p_i^t(\theta)\sum_{c\in C}q_c^*(\widehat{x}_{ic}^t(\theta)-e_{ic}) \geq \sum_{t\in T}\widehat{\varphi}_i^t\sum_{\theta\in\Theta}p_i^t(\theta)q_t^*a_i^t(\theta)(L-\widehat{x}_{Li}^t(\theta)), \quad \forall\, i\in I$$

where the inequality must be strict for at least one i. Multiplying both sides by μ_i and adding up, one obtains:

$$\sum_{c\in C}q_c^*\left(\sum_{i\in I}\mu_i\left(\sum_{t\in T,\theta\in\Theta}\widehat{\varphi}_i^t p_i^t(\theta)\sum_{c\in C}\widehat{x}_{ic}^t(\theta)-e_{ic}-\sum_{t\in T,\theta\in\Theta}\widehat{\varphi}_i^t p_i^t(\theta)a_i^t(\theta)(L-\widehat{x}_{Li}^t(\theta))\right)\right)>0$$

which implies that $(\widehat{x}_i,\widehat{\varphi}_i)$ violates feasibility. $\square$

Proof of Proposition 17

Let $\langle\alpha^P(\bar{u}),x^P(\bar{u})\rangle$ be the Pareto optimal allocation associated to $\bar{u}$. We show that there exists an equilibrium with transfer policy $\tilde{\wp}$ with $w_i^t=\hat{w}_i^t=\eta_t^P a_i^t$, $f_i^t=0$ and

$$s_i^t=\sum_{\theta\in\Theta,c\in C}p_i^t(\theta)(\eta_c^P(x_{ic}^{tP}(\theta)-e_{ic})-\eta_t^P\sum_{\theta\in\Theta}p_i^t(\theta)a_i^t(\theta)l_i^{tP}(\theta)$$

such that $\varphi_i^t=\alpha_i^{tP}$, $x_i=x_i^P$, $q_c/q_1=\eta_c^P/\eta_1^P$, $\phi_i^t=p_i^t$, for $c\in C$, $t\in T$ and $i\in I$.

First $\tilde{\wp}$ is budget balancing by construction. Moreover, $\langle\alpha^P(\bar{u}),x^P(\bar{u})\rangle$ satisfies as equality all the budget constraints at the prices, wages and subsidies vectors defined above. Hence $(\alpha_i^P(\bar{u}),x_i^P(\bar{u}))$ must solve the *type* i agents' maximization program. Finally, all the market clearing conditions are satisfied at $\phi_i^t=p_i^t$ and $w_i^t=\hat{w}_i^t=\eta_t^P a_i^t$ for all $t\in T$. Indeed, at these prices the supply of all state contingent assets, as well as labor demand, are indeterminate .

Consider now an economy where $x_{iL}^t(\theta)=L-L(\theta)$ for all $w_i^t(\theta)>0$, with $\theta\in\Theta$, $t\in T$ and $i\in I$. Take a Pareto optimum $\langle\alpha^P(\bar{u}),x^P(\bar{u})\rangle$ of this economy and consider a policy $\tilde{\wp}$ such that: $\hat{w}_i^t(\theta)=\hat{w}_i=\max_{t\in T}\{\eta_t^P a_i^t(\theta_N)\}$; $s_i^t=\sum_{c\in C,\theta\in\Theta}p_i^t(\theta)\eta_c^P(x_{ic}^{tP}(\theta)-e_{ic})-\hat{w}_i^t\sum_{\theta\in\Theta}p_i^t(\theta)L(\theta)$; and $f_i^t(\theta)=\hat{w}_i-\eta_t^P a_i^t(\theta)$ for all θ. By the same argument developed above, one verifies that $(\alpha_i^P(\bar{u}),x_i^P(\bar{u}))$ solves the *type* i agents' maximization program for $\phi_i^t=p_i^t$ given the transfer policy just defined, that the market clearing conditions are satisfied, and that $\tilde{\wp}$ is budget balancing. $\square$