Non-Exclusive Competition in the Market for Lemons*

Andrea Attar[†] Thomas Mariotti[‡] François Salanié[§]

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Abstract

In order to check the impact of the exclusivity regime on equilibrium allocations, we set up a simple Akerlof-like model in which buyers may use arbitrary tariffs. Under exclusivity, we obtain the (zero-profit, separating) Riley-Rothschilds-Stiglitz allocation. Under non-exclusivity, there is also a unique equilibrium allocation that involves a unique price, as in Akerlof (1970). These results can be applied to insurance (in the dual model in Yaari, 1987), and have consequences for empirical tests of the existence of asymmetric information.

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[†]Toulouse School of Economics (GREMAQ).
[‡]Toulouse School of Economics (GREMAQ/CNRS, IDEI).
[§]Toulouse School of Economics (LERNA/INRA).

1. Introduction

Adverse selection is widely recognized as a major obstacle to the efficient functioning of markets. This is especially true on financial markets, where buyers care about the quality of the assets they purchase, and fear that sellers have superior information about it. The same difficulties impede trade on second-hand markets and insurance markets. Theory confirms that adverse selection may indeed have a dramatic impact on economic outcomes. First, all mutually beneficial trades need not take place in equilibrium. For instance, in Akerlof's (1970) model of second-hand markets, only the lowest quality goods are traded at the equilibrium price. Second, there may be difficulties with the very existence of equilibrium. For instance, in Rothschild and Stiglitz's (1976) model of insurance markets, an equilibrium fails to exist whenever the proportion of low-risk agents is too high.

Most contributions to the theory of competition under adverse selection have considered frameworks in which competitors are restricted to make exclusive offers. This assumption is for instance appropriate in the case of car insurance, since law forbids to take out multiple policies on a single vehicle. By contrast, competition on financial markets is typically non-exclusive, as each agent can trade with multiple partners who cannot monitor each others' trades with the agent. This paper supports the view that this difference in the nature of competition may have a significant impact on the way adverse selection affects market outcomes. This has two consequences. First, empirical studies that test for the presence of adverse selection should use different methods depending on whether competition is exclusive or not. Second, the regulation of markets plagued by adverse selection should be adjusted to the type of competition that prevails on them.

To illustrate these points, we consider a stylized model of trade under adverse selection. In our model, a seller endowed with some quantity of a good attempts to trade it with a finite number of buyers. The seller and the buyers have linear preferences over quantities and transfers exchanged. In line with Akerlof (1970), the quality of the good is the seller's private information. Unlike in his model, the good is assumed to be perfectly divisible, so that any fraction of the seller's endowment can potentially be traded. An example that fits these assumptions is that of a firm which floats a security issue by relying on the intermediation services of several investment banks. Buyers compete by simultaneously offering menus of contracts, or, equivalently, price schedules.[1] After observing the menus offered, the seller decides of her trade(s). Competition is exclusive if the seller can trade with at most one buyer, and non-exclusive if trades with several buyers are allowed.

Under exclusive competition, our conclusions are qualitatively similar to Rothschild and Stiglitz's (1976). In a simple version of the model with two possible levels of quality, pure strategy equilibria exist if and only if the probability that the good is of high quality is low enough. Equilibria are separating: the seller trades her whole endowment when quality is low, while she only trades part of it when quality is high.

The analysis of the non-exclusive competition game yields strikingly different results. Pure strategy equilibria always exist, both for binary and continuous quality distributions. Aggregate equilibrium allocations are generically unique, and have an all-or-nothing feature: depending of whether quality is low or high, the seller either trades her whole endowment or does not trade at all. Buyers earn zero profit on average in any equilibrium. These

[1] As established by Peters (2001) and Martimort and Stole (2002), there is no need to consider more general mechanisms in this multiple-principal single-agent setting.

allocations can be supported by simple menu offers. For instance, one can construct linear price equilibria in which buyers offer to purchase any quantity of the good at a constant unit price equal to the expectation of their valuation of the good conditional on the seller accepting to trade at that price. While other menu offers are consistent with equilibrium, corresponding to non-linear price schedules, an important insight of our analysis is that this is also the unit price at which all trades take place in any equilibrium.

These results are of course in line with Akerlof's (1970) classic analysis of the market for lemons, for which they provide a fully strategic foundation. It is worth stressing the differences between his model and ours. Akerlof (1970) considers a market for a non-divisible good of uncertain quality, in which all agents are price-takers. Thus, by assumption, all trades must take place at the same price, in the spirit of competitive equilibrium models. Equality of supply and demand determines the equilibrium price level, which is equal to the average quality of the goods that are effectively traded. Multiple equilibria may occur in a generic way.[2] By contrast, we allow agents to trade any fraction of the seller's endowment. Moreover, our model is one of imperfect competition, in which a fixed number of buyers choose their offers strategically. In particular, our analysis does not rely on free entry arguments. Finally, buyers can offer arbitrary menus of contracts, including for instance non-linear price schedules. That is, we avoid any a priori restrictions on instruments. The fact that all trades take place at a constant unit price in equilibrium is therefore no longer an assumption, but rather a consequence of our analysis.

A key to our results is that non-exclusive competition expands the set of deviations that are available to the buyers. Indeed, each buyer can strategically use the offers of his competitors to propose additional trades to the seller. Such deviations are blocked by *latent* contracts, that is, contracts that are not traded in equilibrium but which the seller finds it profitable to trade at the deviation stage. These latent contracts are not necessarily complex or exotic. For instance, in a linear price equilibrium, all the buyers offer to purchase any quantity of the good at a constant unit price, but only a finite number of contracts can end up being traded as long as the seller does not randomize on the equilibrium path. The purpose of the other contracts, which are not traded in equilibrium, is only to deter cream-skimming deviations that aim at attracting the seller when quality is high. The use of latent contracts has been criticized on several grounds. First, they may allow one to support multiple equilibrium allocations, and even induce an indeterminacy of equilibrium.[3] This is not the case in our model, since aggregate equilibrium allocations are generically unique. Second, a latent contract may appear as a non-credible threat, if the buyer who issues it would make losses in the hypothetical case where the seller were to trade it.[4] Again, this need not be the case in our model. In fact, we construct examples of equilibria in which latent contracts would be strictly profitable if traded.

This paper is closely related to the literature on common agency between competing principals dealing with a privately informed agent. To use the terminology of Bernheim and Whinston (1986), our non-exclusive competition game is a *delegated* common agency game, as the seller can choose a strict subset of buyers with whom she wants to trade. In the specific

[2] This potential multiplicity of equilibria arises because buyers are assumed to be price-takers. Mas-Colell, Whinston and Green (1995, Proposition 13.B.1) allow buyers to strategically set prices in a market for a non-divisible good where trades are restricted to be zero-one. The equilibrium is then generically unique.

[3] Martimort and Stole (2003, Proposition 5) show that, in a complete information setting, latent contracts can be used to support any level of trade between the perfectly competitive outcome and the Cournot outcome.

[4] Latent contracts with negative virtual profits have been for example considered in Hellwig (1983).

context of incomplete information, a number of recent contributions use standard mechanism design techniques to characterize equilibrium allocations. The basic idea is that, given a profile of mechanisms proposed by his competitors, the best response of any single principal can be fully determined by focusing on simple menu offers corresponding to direct revelation mechanisms. This allows one to construct equilibria that satisfy certain regularity conditions. This approach has been successfully applied in various delegated agency contexts.[5] Closest to this paper is Biais, Martimort and Rochet (2000), who study competition among principals in a common value environment. In their model, uninformed market-makers supply liquidity to an informed insider. The insider's preferences are quasi-linear, and quadratic with respect to quantities exchanged. Unlike in our model, the insider has no capacity constraint. Variational techniques are used to construct an equilibrium in which market-makers post convex price schedules. Such techniques do not apply in our model, as all agents have linear preferences, and the seller cannot trade more than her endowment. Instead, we allow for arbitrary menu offers, and we characterize candidate equilibrium allocations in the usual way, that is by checking whether they survive to possible deviations. While this approach may be difficult to apply in more complex settings, it delivers interesting new insights, in particular on the role of latent contracts.

The paper is organized as follows. Section 2 focuses on a simple two-type example. We show that the aggregate equilibrium allocations that survive a weak perfection refinement are generically unique. We also characterize equilibrium menu offers, with special emphasis on latent contracts. Section 3 concludes with some possible applications of our results.

2. A Two-Type Example

In this section, we set the stage for the general model of Section 3 by considering a simple two-type example of non-exclusive competition inspired by the security design models of DeMarzo and Duffie (1999) and Biais and Mariotti (2005).

2.1. The Non-Exclusive Competition Game

There are two kinds of agents: a single seller, and a finite number of buyers indexed by $i = 1, \ldots, n$, where $n \geq 2$. At date 1, the seller receives an endowment consisting of one unit of a perfectly divisible good that she can trade at date 0 with the buyers. Transfers are made upfront at date 0 and quantities are delivered at date 1.

The seller has preferences represented by

$$T - \delta\theta Q, \tag{1}$$

where $Q \in [0, 1]$ is the total quantity of the good she sells, and $T \in \mathbb{R}_+$ is the total transfer she receives in return. The parameter $\delta \in (0, 1)$ represents the discount factor of the seller, while θ is a random variable that stands for the quality of the good. Quality can be either low, $\theta = \underline{\theta}$, or high, $\theta = \overline{\theta}$, for some $\overline{\theta} > \underline{\theta} > 0$. We denote by $\nu \in (0, 1)$ the probability that quality is high and by $E[\theta]$ the average quality level.

Each buyer i has preferences represented by

$$\theta q^i - t^i, \tag{2}$$

[5] See for instance Khalil, Martimort and Parigi (2007) or Martimort and Stole (2007).

where $q^i \in [0,1]$ is the quantity of the good he purchases, and $t^i \in \mathbb{R}_+$ is the transfer he makes in return.

Both the seller and the buyers care about quality θ. Gains from trade nevertheless arise in this common value environment because, as is apparent from (1) and (2), the buyers are more patient than the seller.[6] Since the ex ante private value of the good for the seller, $\delta E[\theta]$, is less than the value it has for the buyers, $E[\theta]$, there are gains of transferring the good at date 1 from the former to the latter in exchange for date 0 transfers. However, in line with Akerlof (1970), such trades are potentially impeded because the seller has private information about quality at the trading stage. Following common usage, we shall thereafter refer to quality as the type of the seller.

Buyers compete in menus for the good offered by the seller. As in Biais, Martimort and Rochet (2000), trading is non-exclusive in the sense that the seller can pick or reject any of the offers made to her, and can simultaneously trade with several buyers.

The following timing of events at date 0 characterizes our non-exclusive competition game:

1. Each buyer i proposes a menu of contracts, that is, a non-empty set C^i of quantity-transfer pairs $(q^i, t^i) \in [0,1] \times \mathbb{R}_+$ that contains at least the no-trade contract $(0,0)$.[7]

2. After privately learning the quality θ, the seller selects one contract (q^i, t^i) from each of the menus C^i's offered by the buyers, subject to the constraint that $\sum_i q^i \leq 1$.

Finally, at date 1, quantities are delivered and any remaining consumption takes place. A pure strategy for the seller is a mapping s that associates to each type θ and each menu profile $(C^1, \ldots, C^n)$ a vector $((q^1, t^1), \ldots, (q^n, t^n)) \in ([0,1] \times \mathbb{R}_+)^n$ such that $(q^i, t^i) \in C^i$ for each i and $\sum_i q^i \leq 1$. We accordingly denote by $s^i(\theta, C^1, \ldots, C^n)$ the contract traded by type θ of the seller with buyer i. To ensure that the seller's problem

$$U(\theta, C^1, \ldots, C^n) = \sup \left\{ \sum_i t^i - \delta\theta \sum_i q^i : \sum_i q^i \leq 1 \text{ and } (q^i, t^i) \in C^i \text{ for all } i \right\}$$

has a solution for any type θ and menu profile $(C^1, \ldots, C^n)$, which must be the case in a perfect Bayesian equilibrium, we shall require the buyers' menus to be compact sets. This restriction is however not needed to characterize possible equilibrium outcomes.

2.2. A Weaker Equilibrium Concept for Characterizing Aggregate Equilibrium Allocations

One of our objectives is to show that the non-exclusive competition game always admits a perfect Bayesian equilibrium. However, while imposing strong restrictions on the seller's behavior off the equilibrium path is of course desirable when it comes to explicitly construct an equilibrium, much weaker restrictions are in fact sufficient to characterize possible aggregate equilibrium allocations. To formulate these restrictions, we start by defining a local best response property for the seller's strategy.

Definition 1. *A strategy s for the seller is* locally sequentially rational *at the menu profile $(C^1, \ldots, C^n)$ if and only if, for each buyer i, and for each menu $\tilde{C}^i$ containing C^i and differing*

[6] The fact that the discount factor of the buyers is equal to 1 is just a convenient normalization.

[7] As usual, the assumption that each menu must contain the no-trade contract allows one to deal with participation in a simple way.

from C^i by a finite number of contracts,

$$U(\theta, C^1, \ldots, C^{i-1}, \tilde{C}^i, C^{i+1}, \ldots, C^n) > U(\theta, C^1, \ldots, C^n)$$

implies that

$$s^i(\theta, C^1, \ldots, C^{i-1}, \tilde{C}^i, C^{i+1}, \ldots, C^n) \in \arg\max_{c^i \in \tilde{C}^i} \{U(\theta, C^1, \ldots, C^{i-1}, \{c^i\}, C^{i+1}, \ldots, C^n)\}$$

for any type θ of the seller.

That s is locally sequentially rational at $(C^1, \ldots, C^n)$ thus means that, if a buyer deviates from his menu by adding a finite number of contracts to it, and if the seller can secure a strictly higher payoff by purchasing one of these new contracts instead of any the contracts initially proposed by this buyer, she will select one of the new contracts that give her the highest payoff given the menus offered by the other buyers. It should be noted that this criterion imposes no restriction on the seller's behavior when a buyer deviates by offering new contracts that do not increase the seller's payoff.

Definition 2. *A (Bayes–Nash) equilibrium $(C^1, \ldots, C^n, s)$ of the non-exclusive competition game is* locally perfect *if and only if s is locally sequentially rational at $(C^1, \ldots, C^n)$.*

Thus an equilibrium is locally perfect if no buyer can deviate by adding a finite number of contracts to his equilibrium menu, assuming that those types of the seller that would achieve a strictly higher payoff at the deviation stage by optimally selecting one of the new contracts indeed select it. It is immediate that a perfect Bayesian equilibrium is locally perfect, while the converse does not hold in general. While local perfection is a rather weak equilibrium refinement, we now show that it generically leads to a unique prediction for aggregate equilibrium allocations.

2.3. Equilibrium Analysis

In this subsection, we first characterize the restrictions imposed by local perfection on the possible outcomes of the non-exclusive competition game. Next, we show that there always exists a perfect Bayesian equilibrium in which buyers post linear prices. Finally, we offer a characterization of perfect Bayesian equilibria, both in terms of issued and traded contracts, emphasizing in particular the role of latent contracts.

2.3.1. Aggregate Equilibrium Allocations

Let $\underline{c}^i = (\underline{q}^i, \underline{t}^i)$ and $\overline{c}^i = (\overline{q}^i, \overline{t}^i)$ be the contracts traded by the two types of the seller with buyer i in equilibrium, and let $(\underline{Q}, \underline{T}) = \sum_i \underline{c}^i$ and $(\overline{Q}, \overline{T}) = \sum_i \overline{c}^i$ be the corresponding aggregate equilibrium allocations. To characterize these allocations, one needs only to require that three types of deviations by a buyer be blocked in equilibrium. In each case, the deviating buyer uses the offers of his competitors as a support for his own deviation. This intuitively amounts to pivoting around the aggregate equilibrium allocation points $(\underline{Q}, \underline{T})$ and $(\overline{Q}, \overline{T})$ in the (Q, T) space. We now consider each deviation in turn.

Attracting type $\underline{\theta}$ by pivoting around $(\underline{Q}, \underline{T})$. The first type of deviations allows one to prove that type $\underline{\theta}$ trades efficiently in any equilibrium.

Lemma 1. $\underline{Q} = 1$ *in any locally perfect equilibrium.*

Proof. Suppose instead that $\underline{Q} < 1$, and consider some buyer i. A deviation for this buyer consists in offering the same menu as before, plus two new contracts. The first one is

$$\underline{c}^i(\varepsilon) = (\underline{q}^i + 1 - \underline{Q}, \underline{t}^i + (\delta\underline{\theta} + \varepsilon)(1 - \underline{Q})),$$

where ε is some positive number, and is designed to attract type $\underline{\theta}$. The second one is

$$\overline{c}^i(\varepsilon) = (\overline{q}^i, \overline{t}^i + \varepsilon^2),$$

and is designed to attract type $\overline{\theta}$. The key feature of this deviation is that type $\underline{\theta}$ can sell her whole endowment by trading $\underline{c}^i(\varepsilon)$ together with the contracts $\underline{c}^j$, $j \neq i$. Since the unit price at which buyer i offers to purchase the quantity increment $1 - \underline{Q}$ in $\underline{c}^i(\varepsilon)$ is $\delta\underline{\theta} + \varepsilon$, this guarantees her a payoff increase $(1 - \underline{Q})\varepsilon$ compared to what she obtains in equilibrium. When ε is close enough to zero, she cannot obtain as much by trading $\overline{c}^i(\varepsilon)$ instead. Indeed, even if this were to increase her payoff compared to what she obtains in equilibrium, the corresponding increase would be at most $\varepsilon^2 < (1 - \underline{Q})\varepsilon$. Hence, by local perfection, type $\underline{\theta}$ trades $\underline{c}^i(\varepsilon)$ following buyer i's deviation. Consider now type $\overline{\theta}$. By trading $\overline{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$, she can increase her payoff by ε^2 compared to what she obtains in equilibrium. By trading $\underline{c}^i(\varepsilon)$ instead, the most she can obtain is her equilibrium payoff, plus the payoff from selling the quantity increment $1 - \underline{Q}$ at unit price $\delta\underline{\theta} + \varepsilon$. For ε close enough to zero, $\delta\underline{\theta} + \varepsilon < \delta\overline{\theta}$ so that this unit price is too low from the point of view of type $\overline{\theta}$. Hence, by local perfection, type $\overline{\theta}$ trades $\overline{c}^i(\varepsilon)$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is

$$-\nu\varepsilon^2 + (1 - \nu)[(1 - \delta)\underline{\theta} - \varepsilon](1 - \underline{Q})$$

which is strictly positive for ε close enough to zero if $\underline{Q} < 1$. Thus $\underline{Q} = 1$, as claimed. ■

One can illustrate the deviation used in Lemma 1 as follows. Observe first that a basic implication of incentive compatibility is that, in any equilibrium, $\overline{Q}$ cannot be higher than $\underline{Q}$. Suppose then that $\underline{Q} < 1$ in a candidate equilibrium. This situation is depicted on Figure 1. Point $\underline{A}$ corresponds to the aggregate equilibrium allocation $(\underline{Q}, \underline{T})$ of type $\underline{\theta}$, while point $\overline{A}$ corresponds to the aggregate equilibrium allocation $(\overline{Q}, \overline{T})$ of type $\overline{\theta}$. The two solid lines passing through these points are the equilibrium indifference curves of type $\underline{\theta}$ and type $\overline{\theta}$, with slopes $\delta\underline{\theta}$ and $\delta\overline{\theta}$. The dotted line passing through $\underline{A}$ is an indifference curve for the buyers, with slope $\underline{\theta}$.

—Insert Figure 1 here—

Suppose now that some buyer deviates and includes in his menu an additional contract that makes available the further trade $\underline{AA'}$. This leaves type $\underline{\theta}$ indifferent, since she obtains the same payoff as in equilibrium. Type $\overline{\theta}$, by contrast, cannot gain by trading this new contract. Assuming that the deviating buyer can break the indifference of type $\underline{\theta}$ in his favor, he strictly gains from trading the new contract with type $\underline{\theta}$, as the slope $\delta\underline{\theta}$ of the line segment $\underline{AA'}$ is strictly less than $\underline{\theta}$. This contradiction shows that one must have $\underline{Q} = 1$ in any equilibrium. The assumption on indifference breaking is relaxed in the proof of Lemma 1, which relies only on the local perfection of the seller's strategy.

Attracting type $\underline{\theta}$ by pivoting around $(\overline{Q},\overline{T})$. Having established that $\underline{Q}=1$, we now investigate the aggregate quantity $\overline{Q}$ traded by type $\overline{\theta}$ in equilibrium. The second type of deviations allows one to partially characterize the circumstances in which the two types of the seller trade different aggregate allocations in equilibrium. We say in this case that the equilibrium is *separating*. An immediate implication of Lemma 1 is that $\overline{Q}<1$ in any separating equilibrium. Let then $p=(\underline{T}-\overline{T})/(1-\overline{Q})$ be the slope of the line connecting the points $(\overline{Q},\overline{T})$ and $(1,\underline{T})$ in the (Q,T) space. Thus p is the implicit unit price at which the quantity $1-\overline{Q}$ can be sold to move from $(\overline{Q},\overline{T})$ to $(1,\underline{T})$. By incentive compatibility, p must lie in the interval $[\delta\underline{\theta},\delta\overline{\theta}]$ in any separating equilibrium. The strategic analysis of the buyers' behavior induces further restrictions on p.

Lemma 2. *In a separating locally perfect equilibrium, $p<\delta\overline{\theta}$ implies that $p\geq\underline{\theta}$.*

Proof. Suppose that $p<\delta\overline{\theta}$ in a separating equilibrium, and consider some buyer i. A deviation for this buyer consists in offering the same menu as before, plus two new contracts. The first one is

$$\underline{c}^i(\varepsilon)=(\overline{q}^i+1-\overline{Q},\overline{t}^i+(p+\varepsilon)(1-\overline{Q})),$$

where ε is some positive number, and is designed to attract type $\underline{\theta}$. The second one is

$$\overline{c}^i(\varepsilon)=(\overline{q}^i,\overline{t}^i+\varepsilon^2),$$

and is designed to attract type $\overline{\theta}$. The key feature of this deviation is that type $\underline{\theta}$ can sell her whole endowment by trading $\underline{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j\neq i$. Since the unit price at which buyer i offers to purchase the quantity increment $1-\overline{Q}$ in $\underline{c}^i(\varepsilon)$ is $p+\varepsilon$, this guarantees her a payoff increase $(1-\overline{Q})\varepsilon$ compared to what she obtains in equilibrium. As in the proof of Lemma 1, it is easy to check that when ε is close enough to zero, she cannot obtain as much by trading $\overline{c}^i(\varepsilon)$ instead. Hence, by local perfection, type $\underline{\theta}$ trades $\underline{c}^i(\varepsilon)$ following buyer i's deviation. Consider now type $\overline{\theta}$. By trading $\overline{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j\neq i$, she can increase her payoff by ε^2 compared to what she obtains in equilibrium. As in the proof of Lemma 1, it is easy to check that when $p+\varepsilon<\delta\overline{\theta}$, she cannot obtain as much by trading $\underline{c}^i(\varepsilon)$ instead. Hence, by local perfection, type $\overline{\theta}$ trades $\overline{c}^i(\varepsilon)$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is

$$-\nu\varepsilon^2+(1-\nu)[\underline{\theta}(\overline{q}^i-\underline{q}^i)-\overline{t}^i+\underline{t}^i+(\underline{\theta}-p-\varepsilon)(1-\overline{Q})],$$

which must be at most zero for any ε close enough to zero. Summing over the i's and letting ε go to zero then yields

$$\underline{\theta}(\overline{Q}-1)-\overline{T}+\underline{T}+n(\underline{\theta}-p)(1-\overline{Q})\leq 0,$$

which, from the definition of p and the fact that $\overline{Q}<1$, implies that

$$(n-1)(\underline{\theta}-p)\leq 0.$$

Since $n\geq 2$, it follows that $p\geq\underline{\theta}$, as claimed. ∎

In the proof of Lemma 1, we showed that, if $\underline{Q}<1$, then each buyer has an incentive to deviate. By contrast, in the proof of Lemma 2, we only show that if $p<\min\{\delta\overline{\theta},\underline{\theta}\}$ in a candidate separating equilibrium, then at least one buyer has an incentive to deviate. This makes it more difficult to illustrate why the deviation used in Lemma 2 might be profitable. It is however easy to see why this deviation would be profitable to an entrant or, equivalently, to an inactive buyer that would not trade in equilibrium. This situation is depicted on Figure 2. The dotted line passing through $\overline{A}$ is an indifference curve for the buyers, with slope $\underline{\theta}$. Contrary to the conclusion of Lemma 2, the figure is drawn in such a way that this indifference curve is strictly steeper than the line segment $\overline{A}\underline{A}$.

—Insert Figure 2 here—

Suppose now that the entrant offers a contract that makes available the trade $\overline{A}\underline{A}$. This leaves type $\underline{\theta}$ indifferent, since she obtains the same payoff as in equilibrium by trading the aggregate allocation $(\overline{Q}, \overline{T})$ together with the new contract. Type $\overline{\theta}$, by contrast, cannot gain by trading this new contract. Assuming that the entrant can break the indifference of type $\underline{\theta}$ in his favor, he earns a strictly positive payoff from trading the new contract with type $\underline{\theta}$, as the slope p of the line segment $\overline{A}\underline{A}$ is strictly less than $\underline{\theta}$. This shows that, unless $p \geq \underline{\theta}$, the candidate separating equilibrium is not robust to entry. The assumption on indifference breaking is relaxed in the proof of Lemma 2, which further shows that the proposed deviation is profitable to at least one active buyer.

Attracting both types by pivoting around $(\overline{Q}, \overline{T})$. A separating equilibrium must be robust to deviations that attract both types of the seller. This third type of deviations allows one to find a necessary condition for the existence of a separating equilibrium. When this condition fails, both types of the seller must trade the same aggregate allocations in equilibrium. We say in this case that the equilibrium is *pooling*.

Lemma 3. *If* $E[\theta] > \delta\overline{\theta}$, *any locally perfect equilibrium is pooling, and*

$$(\underline{Q}, \underline{T}) = (\overline{Q}, \overline{T}) = (1, E[\theta]).$$

Proof. Suppose that a separating equilibrium exists, and consider some buyer i. A deviation for this buyer consists in offering the same menu as before, plus one new contract,

$$\tilde{c}^i(\varepsilon) = (\overline{q}^i + 1 - \overline{Q}, \overline{t}^i + (\delta\overline{\theta} + \varepsilon)(1 - \overline{Q})),$$

where ε is some positive number, that is designed to attract both types of the seller. The key feature of this deviation is that both types can sell their whole endowment by trading $\tilde{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$. Since the unit price at which buyer i offers to purchase the quantity increment $1 - \overline{Q}$ in $\tilde{c}^i(\varepsilon)$ is $\delta\overline{\theta} + \varepsilon$, and since $\delta\overline{\theta} \geq p$, this guarantees both types of the seller a payoff increase $(1 - \overline{Q})\varepsilon$ compared to what they obtain in equilibrium. Hence, by local perfection, both types trade $\tilde{c}^i(\varepsilon)$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is

$$(E[\theta] - \delta\overline{\theta} - \varepsilon)(1 - \overline{Q}) + (1 - \nu)[\underline{\theta}(\overline{q}^i - \underline{q}^i) - \overline{t}^i + \underline{t}^i],$$

which must be at most zero for any ε. Summing over the i's and letting ε go to zero then yields

$$n(E[\theta] - \delta\overline{\theta})(1 - \overline{Q}) + (1 - \nu)[\underline{\theta}(\overline{Q} - 1) - \overline{T} + \underline{T}] \leq 0,$$

which, from the definition of p and the fact that $\overline{Q} < 1$, implies that

$$n(E[\theta] - \delta\overline{\theta}) + (1 - \nu)(p - \underline{\theta}) \leq 0.$$

Starting from this inequality, two cases must be distinguished. If $p < \delta\overline{\theta}$, then Lemma 2 applies, and therefore $p \geq \underline{\theta}$. It then follows that $E[\theta] \leq \delta\overline{\theta}$. If $p = \delta\overline{\theta}$, the inequality can be rearranged so as to yield

$$(n - 1)(E[\theta] - \delta\overline{\theta}) + \nu\overline{\theta}(1 - \delta) \leq 0.$$

Since $n \geq 2$, it follows again that $E[\theta] \leq \delta\overline{\theta}$, which shows the first part of the result. Consider next some pooling equilibrium, and denote by $(1, T)$ the corresponding aggregate equilibrium allocation. To show that $T = E[\theta]$, one needs to establish that the buyers' aggregate payoff is zero in equilibrium. Let B^i be buyer i's equilibrium payoff, which must be at least zero since each buyer always has the option not to trade. A deviation for this buyer consists in offering the same menu as before, plus one new contract,

$$\hat{c}^i(\varepsilon) = (1, T + \varepsilon),$$

where ε is some positive number. It is immediate that both types trade $\hat{c}^i(\varepsilon)$ following buyer i's deviation. The change in payoff for buyer i induced by this deviation is

$$E[\theta] - T - \varepsilon - B^i,$$

which must be at most zero for any ε. Letting ε go to zero yields

$$B^i \geq E[\theta] - T = \sum_j B^j$$

where the equality follows from the fact that each type of the seller sells her whole endowment in a pooling equilibrium. Since this inequality holds for each i and all the B^i's are at least zero, they must all in fact be equal to zero. Hence $T = E[\theta]$, as claimed. ■

In the proof of Lemma 3, we show that if $E[\theta] > \delta\overline{\theta}$ in a candidate separating equilibrium, then at least one buyer has an incentive to deviate. As for Lemma 2, this makes it difficult to illustrate why this deviation might be profitable. It is however easy to see why this deviation would be profitable to an entrant or, equivalently, to an inactive buyer that would not trade in equilibrium. This situation is depicted on Figure 3. The dotted line passing through $\overline{A}$ is an indifference curve for the buyers, with slope $E[\theta]$. Contrary to the conclusion of Lemma 3, the figure is drawn in such a way that this indifference curve is strictly steeper than the indifference curves of type $\overline{\theta}$.

—Insert Figure 3 here—

Suppose now that the entrant offers a contract that makes available the trade $\overline{A}\underline{A}'$. This leaves type $\overline{\theta}$ indifferent, since she obtains the same payoff as in equilibrium by trading the aggregate allocation $(\overline{Q}, \overline{T})$ together with the new contract. Type $\underline{\theta}$ strictly gains by trading this new contract. Assuming that the entrant can break the indifference of type $\overline{\theta}$ in his favor, he earns a strictly positive payoff from trading the new contract with both types as the slope $\delta\overline{\theta}$ of the line segment $\overline{A}\underline{A}'$ is strictly less than $E[\theta]$. This shows that, unless $E[\theta] \leq \delta\overline{\theta}$, the candidate equilibrium is not robust to entry. Once again, the assumption on indifference breaking is relaxed in the proof of Lemma 3, which further shows that the proposed deviation is profitable to at least one active buyer.

The following result provides a partial converse to Lemma 3.

Lemma 4. *If $E[\theta] < \delta\overline{\theta}$, any locally perfect equilibrium is separating, and*

$$(\overline{Q}, \overline{T}) = (0, 0) \text{ and } (\underline{Q}, \underline{T}) = (1, \underline{\theta}).$$

Proof. Suppose first that a pooling equilibrium exists, and denote by $(1, T)$ the aggregate allocation traded by both types in this equilibrium. Then the buyers' aggregate payoff is $E[\theta] - T$. One must have $T - \delta\overline{\theta} \geq 0$ otherwise type $\overline{\theta}$ would not trade. Since the buyers' aggregate payoff must be at least zero in equilibrium, it follows that $E[\theta] \geq \delta\overline{\theta}$, which shows the first part of the result. Next, observe that in any separating equilibrium, the buyers' aggregate payoff is equal to

$$(1 - \nu)(\underline{\theta} - \underline{T}) + \nu(\overline{\theta}\overline{Q} - \overline{T}) = (1 - \nu)[\underline{\theta} - p(1 - \overline{Q})] + \nu\overline{\theta}\overline{Q} - \overline{T}$$

by definition of p. We claim that $p \geq \underline{\theta}$ in any such equilibrium. If $p < \delta\overline{\theta}$, this follows at once from Lemma 2. If $p = \delta\overline{\theta}$, this follows from Lemma 3, which implies that $\delta\overline{\theta} \geq E[\theta] > \underline{\theta}$ whenever a separating equilibrium exists. Using this claim along with the fact that $\overline{T} \geq \delta\overline{\theta}\overline{Q}$, one obtains that the buyers' aggregate payoff is at most $(E[\theta] - \delta\overline{\theta})\overline{Q}$. Since this must be at least zero, one necessarily has $(\overline{Q}, \overline{T}) = (0, 0)$ whenever $E[\theta] < \delta\overline{\theta}$. In particular, the buyers' aggregate payoff $(1 - \nu)(\underline{\theta} - p)$ is then equal to zero. It follows that $p = \underline{\theta}$ and thus $\underline{T} = \underline{\theta}$, which shows the second part of the result. ■

The following is an important corollary of our analysis.

Corollary 1. *Each buyer's payoff is zero in any locally perfect equilibrium.*

Proof. In the case of a pooling equilibrium, the result has been established in the proof of Lemma 3. In the case of a separating equilibrium, it has been shown in the proof of Lemma 4 that the buyers' aggregate payoff is at most $(E[\theta] - \delta\overline{\theta})\overline{Q}$. As a separating equilibrium exists only if $E[\theta] \leq \delta\overline{\theta}$, it follows that the buyers' aggregate payoff is at most zero in any such equilibrium. Since each buyer always has the option not to trade, the result follows. ■

Based on a weak equilibrium refinement, Lemmas 1 to 4 provide a full characterization of the aggregate allocations that can be sustained in a pure strategy equilibrium of the non-exclusive competition game. While each buyer always receives a zero payoff in equilibrium, the structure of equilibrium allocations is directly affected by the severity of the adverse selection problem.

We shall say that adverse selection is *mild* whenever $E[\theta] > \delta\overline{\theta}$. Separating equilibria are ruled out in these circumstances. Indeed, if the aggregate allocation $(\overline{Q}, \overline{T})$ traded by type $\overline{\theta}$ were such that $\overline{Q} < 1$, some buyer would have an incentive to induce both types of the seller to trade this allocation, together with the additional quantity $1 - \overline{Q}$ at a unit price between $\delta\overline{\theta}$ and $E[\theta]$. Competition among buyers then bids up the price of the seller's endowment to its average value $E[\theta]$, a price at which both types of the seller are ready to trade. This situation is depicted on Figure 4. The dotted line passing through the origin is the equilibrium indifference curve of the buyers, with slope $E[\theta]$.

—Insert Figure 4 here—

We shall say that adverse selection is *strong* whenever $E[\theta] < \delta\overline{\theta}$. Pooling equilibria are ruled out in these circumstances, as type $\overline{\theta}$ is no longer ready to trade her endowment at price $E[\theta]$. However, non-exclusive competition induces a specific cost of screening the seller's type in equilibrium. Indeed, any separating equilibrium must be such that no buyer has an incentive to deviate and induce type $\underline{\theta}$ to trade the aggregate allocation $(\overline{Q}, \overline{T})$, together with the additional quantity $1 - \overline{Q}$ at some mutually advantageous price. To eliminate any incentive for buyers to engage in such trades with type $\underline{\theta}$, the implicit unit price at which this additional quantity $1 - \overline{Q}$ can be sold in equilibrium must be relatively high, implying at most an aggregate payoff $(E[\theta] - \delta\overline{\theta})\overline{Q}$ for the buyers. Hence type $\overline{\theta}$ can trade actively in a separating equilibrium only in the non-generic case $E[\theta] = \delta\overline{\theta}$, while type $\overline{\theta}$ does not trade at all under strong adverse selection. This situation is depicted on Figure 5. The dotted line passing through the origin is the equilibrium indifference curve of the buyers, with slope $\underline{\theta}$.

—Insert Figure 5 here—

Our analysis provides a fully strategic foundation for Akerlof's (1970) original intuition: if adverse selection is severe enough, only goods of low quality are traded in any market equilibrium. This contrasts sharply with the predictions of standard models of competition under adverse selection, in which exclusivity clauses are typically assumed to be enforceable. Indeed, when the rules of the competition game are such that the seller can trade with at most one buyer, the structure of market equilibria is formally analogous to that obtaining in the competitive insurance model of Rothschild and Stiglitz (1976). First, any pure strategy equilibrium must be separating, with type $\underline{\theta}$ selling her whole endowment, $\underline{Q}^e = 1$, and type $\overline{\theta}$ selling less than her whole endowment, $\overline{Q}^e < 1$. The corresponding contracts trade at unit prices $\underline{\theta}$ and $\overline{\theta}$ respectively, yielding both a zero payoff to the buyers. Second, type $\underline{\theta}$ must be indifferent between her equilibrium contract and that of type $\overline{\theta}$, implying that

$$\overline{Q}^e = \frac{(1-\delta)\underline{\theta}}{\overline{\theta} - \delta\underline{\theta}}.$$

This contrasts with the separating outcome that prevails under non-exclusivity whenever $E[\theta] < \delta\overline{\theta}$, in which type $\underline{\theta}$ strictly prefers her aggregate equilibrium allocation to that of type $\overline{\theta}$, who in turn does not trade at all. An immediate implication of our analysis is thus that the equilibrium allocations under exclusivity cannot be sustained in equilibrium under non-exclusivity. These allocations are depicted on Figure 6. Point $\underline{A}^e$ corresponds to the equilibrium contract of type $\underline{\theta}$, while point $\overline{A}^e$ corresponds to the equilibrium contract of type $\overline{\theta}$. The two solid lines passing through these points are the equilibrium indifference curves of type $\underline{\theta}$ and type $\overline{\theta}$. The dotted line passing through the origin are indifference curves for the buyers, with slope $\underline{\theta}$ and $\overline{\theta}$.

—Insert Figure 6 here—

As in Rothschild and Stiglitz (1976), a pure strategy equilibrium exists under exclusivity only under certain parameter restrictions. This contrasts with the non-exclusive competition game, which, as shown below, always admits an equilibrium. Specifically, the equilibrium indifference curve of type $\overline{\theta}$ must lie above the indifference curve for the buyers with slope $E[\theta]$

passing through the origin, for otherwise there would exist a profitable deviation attracting both types of the seller. This is the case if and only if the probability ν that the good is of high quality is low enough. Simple computations show that the corresponding threshold

$$\nu^e = \frac{\delta(\overline{\theta} - \underline{\theta})}{\overline{\theta} - \delta\underline{\theta}},$$

for ν below which an equilibrium exists under exclusivity is strictly above the threshold

$$\nu^{ne} = \max\left\{0, \frac{\delta\overline{\theta} - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right\}$$

for ν below which the equilibrium is separating under non-exclusivity. Whenever $0 < \nu < \nu^{ne}$, the equilibrium is separating under both exclusivity and non-exclusivity, and more trade takes place in the former case. By contrast, whenever $\nu^{ne} < \nu < \nu^e$, the equilibrium is separating under exclusivity and pooling under non-exclusivity, and more trade takes place in the latter case. From an ex-ante viewpoint, exclusive competition leads to a more efficient outcome under strong adverse selection, while non-exclusive competition leads to a more efficient outcome under mild adverse selection.

2.3.2. Equilibrium Existence

We now establish that a perfect Bayesian equilibrium always exists in the non-exclusive competition game. Specifically, we show that there always exists an equilibrium in which all buyers post linear prices. In such an equilibrium, the unit price at which any quantity can be traded is equal to the expected quality of the goods that are actively traded.

Proposition 1. *The following holds*:

(i) Under mild adverse selection, there exists a perfect Bayesian equilibrium of the non-exclusive competition game in which each buyer offers the menu

$$\{(q,t) \in [0,1] \times \mathbb{R}_+ : t = E[\theta]q\},$$

and thus stands ready to buy any quantity of the good at a constant unit price $E[\theta]$.

(ii) Under strong adverse selection, there exists a perfect Bayesian equilibrium of the non-exclusive competition game in which each buyer offers the menu

$$\{(q,t) \in [0,1] \times \mathbb{R}_+ : t = \underline{\theta}q\},$$

and thus stands ready to buy any quantity of the good at a constant unit price $\underline{\theta}$.

In the non-generic case $E[\theta] = \delta\overline{\theta}$, it is easy to check that there exist two linear price equilibria, a pooling equilibrium with constant unit price $E[\theta]$ and a separating equilibrium with constant unit price $\underline{\theta}$. In addition, there exists in this case a continuum of separating equilibria in which type $\overline{\theta}$ trades actively. Indeed, to sustain an equilibrium trade level $\overline{Q} \in (0,1)$ for type $\overline{\theta}$, it is enough that all buyers offer to buy any quantity of the good at unit price $\underline{\theta}$, and that one buyer offers in addition to buy any quantity of the good up to $\overline{Q}$ at unit price $E[\theta]$. Both types $\underline{\theta}$ and $\overline{\theta}$ then sell a fraction $\overline{Q}$ of their endowment at unit price $E[\theta]$, while type $\underline{\theta}$ sells the remaining fraction of her endowment at unit price $\underline{\theta}$.

2.3.3. Equilibrium Characterization

We now explore in more depth the structure of the menus offered by the buyers in equilibrium. Our first result provides equilibrium restrictions on the price of all issued contracts.

Proposition 2. *The following holds*:

(i) Under mild adverse selection, the unit price of any contract issued in a perfect Bayesian equilibrium of the non-exclusive competition game is at most $E[\theta]$.

(ii) Under strong adverse selection, the unit price of any contract issued in a perfect Bayesian equilibrium of the non-exclusive competition game is at most $\underline{\theta}$.

The intuition for this result is as follows. If some buyer offered to purchase some quantity at a unit price above $E[\theta]$ under mild adverse selection, then any other buyer would have an incentive to induce both types of the seller to trade this contract and to sell him the remaining fraction of their endowment at a unit price slightly below $E[\theta]$. Similarly, if some buyer offered to purchase some quantity at a unit price above $\underline{\theta}$ under strong adverse selection, then any other buyer would have an incentive to induce type $\underline{\theta}$ to trade this contract and to sell him the remaining fraction of her endowment at a unit price slightly below $\underline{\theta}$. As a corollary, one obtains a simple characterization of the price of traded contracts.

Corollary 2. *The following holds*:

(i) Under mild adverse selection, the unit price of any contract traded in a perfect Bayesian equilibrium of the non-exclusive competition game is $E[\theta]$.

(ii) Under strong adverse selection, the unit price of any contract traded in a perfect Bayesian equilibrium of the non-exclusive competition game is $\underline{\theta}$.

With these preliminaries at hand, we can investigate which contracts need to be issued to sustain the aggregate equilibrium allocations. From a strategic viewpoint, what matters for each buyer is the outside option of the seller, that is, what aggregate allocations she can achieve by trading with the other buyers only. For each buyer i, and for each menu profile $(C^1, \ldots, C^n)$, this is described by the set of aggregate allocations that remain available if buyer i withdraws his menu offer C^i. One has the following result.

Proposition 3. *The following holds*:

(i) Under mild adverse selection, and in any perfect Bayesian equilibrium, the aggregate allocation $(1, E[\theta])$ traded by both types of the seller remains available if any buyer withdraws his menu offer.

(ii) Under strong adverse selection, and in any perfect Bayesian equilibrium, the aggregate allocation $(1, \underline{\theta})$ traded by type $\underline{\theta}$ of the seller remains available if any buyer withdraws his menu offer.

The aggregate equilibrium allocations must therefore remain available even if a buyer deviates from his equilibrium menu offer. The reason is that this buyer would otherwise have an incentive to offer both types to sell their whole endowment at a price slightly below $E[\theta]$ (in the mild adverse selection case), or to offer type $\underline{\theta}$ to sell her whole endowment at price

$\underline{\theta}$ while offering type $\overline{\theta}$ to sell a smaller part of her endowment on more advantageous terms (in the strong adverse selection case). The flip side of this observation is that no buyer is essential in providing the seller with her aggregate equilibrium allocation. This rules out standard Cournot outcomes in which the buyers would simply share the market and in which all issued contracts would actively be traded by some type of the seller. As an illustration, when there are two buyers, there is no equilibrium in which each buyer would only offer to purchase half of the seller's endowment.

Equilibrium typically involves more restrictions on menus offers than those prescribed by Proposition 3. For instance, in the mild adverse selection case, there is no equilibrium in which each buyer only offers the allocation $(1, E[\theta])$ besides the no-trade contract. Indeed, any buyer could otherwise deviate by offering to purchase a quantity $\overline{q} < 1$ at some price $\overline{t} \in (E[\theta] - \delta\overline{\theta}(1-\overline{q}), E[\theta] - \delta\underline{\theta}(1-\overline{q}))$. By construction, this is a cream-skimming deviation that attracts only type $\overline{\theta}$, and that yields the deviating buyer a payoff

$$\nu(\overline{\theta}\overline{q} - \overline{t}) > \nu[\overline{\theta}\overline{q} - E[\theta] + \delta\underline{\theta}(1-\overline{q})],$$

which is strictly positive if $\overline{q}$ is close enough to 1. To block such deviations, latent contracts must be issued that are not actively traded in equilibrium but which the seller has an incentive to trade if some buyer attempts to break the equilibrium. In order to play this deterrence role, the corresponding latent allocations must remain available if any buyer withdraws his menu offer. For instance, in the mild adverse selection case, the cream-skimming deviation described above is blocked if the quantity $1 - \overline{q}$ can always be sold at unit price $E[\theta]$ at the deviation stage, since both types of the seller then have the same incentives to trade the contract proposed by the deviating buyer. Generalizing this logic leads to the linear price equilibria described in Proposition 2.

It is difficult to provide a full characterization of the latent contracts that are needed in equilibrium. Partial results can be obtained along the lines of Proposition 3. For instance, one can check from the proof of Proposition 3(ii) that, in the strong adverse selection case, the aggregate allocation $(1, \underline{\theta})$ traded by type $\underline{\theta}$ is in fact a limit point of the set of aggregate allocations that remain available if any buyer withdraws his menu offer. Indeed, allocations arbitrarily close to $(1, \underline{\theta})$ need to be available in order to block cream-skimming deviations. A more interesting question, however, is whether one can construct non-linear equilibria in which latent contracts are issued at a unit price different from that of the aggregate allocation that is traded in equilibrium. One has the following result.

Proposition 4. *The following holds*:

(i) Under mild adverse selection, there exists for each $\phi \in [\delta\overline{\theta}, E[\theta])$ a perfect Bayesian equilibrium of the non-exclusive competition game in which each buyer offers the menu

$$\left\{(q,t) \in \left[0, \frac{\overline{\theta} - E[\theta]}{\overline{\theta} - \phi}\right] \times \mathbb{R}_+ : t = \phi q\right\} \cup \{(1, E[\theta])\}.$$

(ii) Under strong adverse selection, there exists for each $\psi \in (\underline{\theta}, \underline{\theta} + (\delta\overline{\theta} - E[\theta])/(1-\nu)]$ a perfect Bayesian equilibrium of the non-exclusive competition game in which each buyer offers the menu

$$\{(0,0)\} \cup \left\{(q,t) \in \left[\frac{\psi - \underline{\theta}}{\psi}, 1\right] \times \mathbb{R}_+ : t = \psi q - \psi + \underline{\theta}\right\}.$$

This results shows that equilibrium allocations can also be supported through non-linear prices. In such equilibria, the price each buyer is willing to pay for an additional unit of the good is not the same for all quantities purchased. For instance, in the equilibrium for the strong adverse selection case described in Proposition 4(ii), buyers are not ready to pay anything for all quantities up to the level $(\psi - \underline{\theta})/\psi$, while they are ready to pay ψ for each additional unit of the good above this level. The price schedule posted by each buyer is such that, for any $q < 1$, the unit price $\max\{0, \psi - (\psi - \underline{\theta})/q\}$ at which he offers to purchase the quantity q is strictly below $\underline{\theta}$, while the marginal price ψ at which he offers to purchase an additional unit given that he has already purchased a quantity $q \geq (\psi - \underline{\theta})/\psi$ is strictly above $\underline{\theta}$. As a result of this, the equilibrium budget set of the seller, that is,

$$\left\{(Q,T) \in [0,1] \times \mathbb{R}_+ : Q = \sum_i q^i \text{ and } T \leq \sum_i t^i \text{ where } (q^i, t^i) \in C^i \text{ for all } i\right\},$$

is not convex in this equilibrium. In particular, the seller has a strict incentive to deal with a single buyer in equilibrium. This contrasts with recent work on competition in non-exclusive mechanisms under incomplete information, in which attention is typically restricted to equilibria in which the informed agent has a convex budget set in equilibrium, or, what amounts to the same thing, where the set of allocations available to her is the frontier of a convex budget set.[8] In our model, this would for instance arise if all buyers posted concave price schedules. It is therefore interesting to notice that, as a matter of fact, our non-exclusive competition game admits no equilibrium in which each buyer i posts a strictly concave price schedule T^i. The reason is that the aggregate price schedule $\mathcal{T}$ defined by $\mathcal{T}(Q) = \sup\{\sum_i T^i(q^i) : \sum_i q^i = Q\}$ for all aggregate trades Q would otherwise be strictly concave as well. This would in turn imply that contracts are issued at a unit price strictly above $\mathcal{T}(1)$, which, as shown by Proposition 2, is impossible in equilibrium.

A further implication of Proposition 5 is that latent contracts supporting the equilibrium allocations can be issued at a profitable price. For instance, in the strong adverse selection case, any contract in the set $\{[(\psi - \underline{\theta})/\psi, 1) \times \mathbb{R}_+ : t = \psi q - \psi + \underline{\theta}\}$ would yield its issuer a strictly positive payoff, even if it were traded by type $\underline{\theta}$ only. In equilibrium, no mistakes occur, and buyers correctly anticipate that none of these contracts will be traded. Nonetheless, removing these contracts would break the equilibrium. One should notice in that respect that the role of latent contracts in non-exclusive markets has usually been emphasized in complete information environments in which the agent does not trade efficiently in equilibrium.[9] In these contexts, latent contracts can never be profitable. Indeed, if they were, there would always be room for proposing an additional latent contract at a less profitable price and induce the agent to accept it. In our model, by contrast, type $\underline{\theta}$ sells her whole endowment in equilibrium. It follows from Proposition 2 that there cannot be any latent contract inducing a negative profit to the issuer. In addition, there is no incentive for any single buyer to raise the price of these contracts and make the seller willing to trade them.

Finally, Proposition 5 shows that market equilibria can always be supported with only one active buyer, provided that the other buyers coordinate by offering appropriate latent

[8] See for instance Biais, Martimort and Rochet (2000), Khalil, Martimort and Parigi (2007) or Martimort and Stole (2007).

[9] See for instance Hellwig (1983), Martimort and Stole (2003), Bisin and Guaitoli (2004) or Attar and Chassagnon (2006).

contracts. Hence non-exclusive competition does not necessarily entail that the seller will enter into multiple contracting relationships.

3. The continuous-type case

In order to check the robustness of the intuitions derived until now, we switch to the case in which the seller's type is continuously distributed over an interval. The model remains essentially the same, with some changes that we comment below. We derive results that confirm those obtained in the two-type case; in particular equilibria exist, and all contracts traded must be traded at the same price.

3.1. The model and the monopsony case

The seller's preferences now write

$$T - \theta Q$$

where T is the total transfer obtained when selling a quantity $Q \in [0,1]$. The quality θ is the seller's private information, and its distribution is characterized by a c.d.f. F, with a p.d.f. f assumed strictly positive on a bounded interval $[\underline{\theta}, \bar{\theta}]$.

Each buyer $i = 1..n$ ($n \geq 2$) gets

$$v(\theta)q_i - t_i$$

when he buys a quantity q_i to the seller of type θ, for a transfer t_i. The valuation function $v(.)$ is assumed continuous, but not necessarily monotonic. For convenience, we assume that it is defined for all real numbers, even outside $[\underline{\theta}, \bar{\theta}]$.

Let the set of feasible contracts be $\mathcal{C} = [0,1] \times R$, with typical element $(q,\, t)$. The game we consider is the same as in the previous section. In the first period, buyers simultaneously post menus of contracts, that are compact[10] subsets $\mathcal{C}_i$ of $\mathcal{C}$ including the null contract (0, 0). In the second period, the seller chooses one contract in each subset $\mathcal{C}_i$, and trades take place accordingly.

For given subsets $(\mathcal{C}_1,\, ..,\, \mathcal{C}_n)$, the seller's payoff is

$$U(\theta) \equiv \sup\{\sum_i t_i - \theta \sum_i q_i; \sum_i q_i \leq 1, (q_i, t_i) \in \mathcal{C}_i \qquad \forall\ i\} \tag{3}$$

$U(\theta)$ is convex and weakly decreasing. Its derivative is well-defined almost everywhere, and wherever it exists it is equal to $(-Q(\theta))$, that is minus the total quantity sold by type θ.

Because it will be useful in the sequel, we provide additional definitions. First consider the function

$$w(p) \equiv \int_{-\infty}^{p} [v(\theta') - p] dF(\theta')$$

From our assumptions, w is continuous, is zero below $\underline{\theta}$, and is strictly decreasing above $\bar{\theta}$. It is bounded, and admits a maximum value $w^m \geq 0$. Let p^m be the highest p such that

[10]The only reason for introducing this compactness requirement is to be able to properly define Perfect Bayesian Equilibria, whenever this concept is used.

$w(p) = w^m$. Notice that $p^m \in [\underline{\theta}, \bar{\theta}]$. Let us also define p^* as the supremum of those p such that $w(p) > 0$ (set $p^* = \underline{\theta}$ if this set is empty). In the following we assume that

Assumption 1 $w(p) < 0$ *for* $p > p^*$.

This assumption has some bite both in the case $w^m = 0$ (in which we have set $p^* = p^m = \underline{\theta}$) and in the case $w^m > 0$ (and then $p^* > p^m > \underline{\theta}$). It essentially allows to avoid discussing multiple equilibria; since by definition we know that $w(p) \leq 0$ for $p > p^*$, the assumption is weak, and holds for generic choices of both the valuation function v and the distribution F.

To give some intuitive content to these definitions, consider the monopsony case when there is a single buyer ($n = 1$). Suppose that this buyer is restricted to offer to buy one unit at the price p. Then $w(p)$ is its profit, p^m is the monopsony price, and p^* is the highest price at which one unit can be profitably bought, and is thus the competitive price.

The monopsony case seems more complex when the monopsony is allowed to offer arbitrary sets of contracts. Fortunately, and as is well-known from the Revelation Principle, one only has to maximize

$$\int [(v(\theta) - \theta)Q(\theta) - U(\theta)]dF(\theta)$$

under the incentive-compatibility (IC) constraints

$$\forall\, \theta \qquad U'(\theta) = -Q(\theta) \;\text{ a.e.} \qquad\qquad Q(.) \text{ is weakly decreasing}$$

and the individual rationality (IR) constraint

$$\forall\, \theta \qquad U(\theta) \geq 0$$

The following result is due to Samuelson (1984) :

Lemma 1 ***(Samuelson, 1984)*** *The monopsony profit is exactly w^m. The monopsony can obtain this profit by offering to buy one unit at the price p^m.*

3.2. Robust equilibria

Let us now define our equilibrium concept. As in most of the literature, we restrict attention to pure strategies for the buyers, but we allow the seller to randomize. Second we look for Bayes-Nash equilibria that verify a simple refinement called robustness.

Definition 1 *A Bayes-Nash equilibrium of the two-stage game is moreover* **robust** *if a buyer cannot profitably deviate by adding one contract to its equilibrium subset of offers, assuming that*

i) those types of sellers that would strictly gain by trading the new contract indeed trade it;

ii) those types of sellers that would strictly loose from trading the new contract do not change their behaviour compared to the equilibrium path.

Requirement i) is similar to the local perfection requirement we introduced in the two-type case, and is thus a weak form of Nash perfection. Requirement ii) is new, and expresses that sellers do not play an active role in deterring deviations by buyers if they do not profit from doing so. This requirement was not needed in the study of the two-type case, because we were able to perfectly control the behaviour of all types following a deviation. This is more difficult with a continuum of types, and for the sake of simplicity we choose to reinforce the equilibrium concept.

3.3. Non-Exclusive competition

We now turn to the main result of this section.

Proposition 1 *Under non-exclusive competition, all robust Bayes-Nash equilibria are such that the aggregate quantity traded is $Q(\theta) = 1$ if $\theta < p^*$, and $Q(\theta) = 0$ if $\theta > p^*$. Buyers get zero-profits.*

One robust Perfect Bayes-Nash equilibrium obtains for example when each buyer proposes to buy any quantity at a unit price p^*, and the seller sells one unit to a randomly chosen buyer.

Therefore robust equilibria exist. Moreover the equilibrium aggregate quantities and transfers are unique. Because $p^m \leq p^*$, there is more trade than in the monopsony case, which does not come as a surprise. Recall that p^* verifies $w(p^*) = 0$, or equivalently

$$p^* = E[v(\theta)|\theta \leq p^*]$$

Hence the equilibrium trades correspond to those that would obtain in the classical Akerlof model. Recall though that our model allows for a divisible good, together with arbitrary tariffs, in an imperfect competition framework. This result thus provides solid game-theoretic foundations to Akerlof's predictions.

Finally, as in the two-type case define the unit price of a contract (q, t) as the ratio t/q, whenever q is positive. One gets

Proposition 2 *Under non-exclusive competition, in any robust Bayes-Nash equilibrium, all contracts issued have a unit price below p^*, and all contracts traded have a unit price equal to p^*.*

This result illustrates how competition disciplines buyers; even though they are allowed to use arbitrary tariffs, at equilibrium they end up trading at a unique price.

It is useful to compare these results to those obtained under *exclusive* competition. The game remains the same, but the seller now can only trade with one buyer; the only change is that now

$$U(\theta) \equiv \sup\{t_i - \theta q_i; i = 1..n, (q_i, t_i) \in \mathcal{C}_i\}$$

Recall that in the two-type case results were similar to those derived in the Rotschild-Stiglitz model : in particular equilibria need not exist. In the continuous-type case, define the allocation (U^*, Q^*) as the unique solution to

$$U(\theta) = (v(\theta) - \theta)Q(\theta) \qquad U'(\theta) = -Q(\theta) \qquad Q(\underline{\theta}) = 1$$

Proposition 3 *Under exclusive competition, suppose that a robust Bayes-Nash equilibrium exists. Then the equilibrium allocation is* (U^*, Q^*)*, as defined above. Moreover* $v(.)$ *must be non-increasing; and if the right-derivative of* v *exists at* $\underline{\theta}$*, then it must be equal to zero.*

The first part of the Proposition is established in Appendix. The second part is required so as to ensure that Q^* is non-increasing. The last part is established in Riley (2001, p. 446) (see also Riley, 1985). Overall the Proposition shows that under exclusivity non-existence is the rule rather than the exception. Ths is in contrast with the existence result that obtains under non-exclusivity. Moreover this result illustrates how the exclusivity regime determines the equilibrium allocations. These allocations correspond to thopse obtained in the Akerlof model when competition is non-exclusive; they share the properties of Riley-Rothschild-Stiglitz allocations when competition is exclusive.

4. Conclusion

In this paper, we have studied a simple imperfect competition model of trade under adverse selection. When competition is exclusive, the existence of equilibria is problematic, while equilibria always exist when competition is non-exclusive. In this latter case, aggregate quantities and transfers are generically unique, and correspond to the allocations that obtain in Akerlof's (1970) model. Linear price equilibria can be constructed in which buyers stand ready to purchase any quantity at a constant unit price.

The fact that possible market outcomes tightly depend on the nature of competition suggests that the testable implications of competitive models of adverse selection should be evaluated with care. Indeed, these implications are typically derived from the study of exclusive competition models, such as Rothschild and Stiglitz's (1976) two-type model of insurance markets. By contrast, our analysis shows that more competitive outcomes can be sustained in equilibrium under non-exclusive competition, and that these outcomes can involve a substantial amount of pooling.

These results offer new insights into the empirical literature on adverse selection. For instance, several studies have taken to the data the predictions of theoretical models of insurance provision, without reaching clear conclusions.[11] Cawley and Philipson (1999) argue that there is little empirical support for the adverse selection hypothesis in life insurance. In particular, they find no evidence that marginal prices raise with coverage. Similarly, Finkelstein and Poterba (2004) find that marginal prices do not significantly differ across annuities with different initial annual payments. The theoretical predictions tested by these authors are however derived from models of exclusive competition,[12] while our results clearly indicate that they do not hold when competition is non-exclusive, as in the case of life insurance or annuities. Indeed, non-exclusive competition might be one explanation for the limited evidence of screening and the prevalence of nearly linear pricing schemes on these markets. As a result, more sophisticated procedures need to be designed in order to test for the presence of adverse selection in markets where competition is non-exclusive.

[11] See Chiappori and Salanié (2003) for a survey of this literature.

[12] Chiappori, Jullien, Salanié and Salanié (2006) have derived general tests based on a model of exclusive competition, that they apply to the case of car insurance.

Appendix

Proof of Proposition 1. (i) Consider first the mild adverse selection case. The proof goes through a series of steps.

Step 1. Given the menus offered, any best response of the seller leads to an aggregate trade $(1, E[\theta])$ irrespective of her type. Assuming that each buyer trades the same quantity with both types of the seller, all buyers obtain a zero payoff.

Step 2. No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. Indeed, such a deviation is profitable only if $E[\theta]q > t$. However, given the menus offered by the other buyers, the seller always has the option to trade quantity q at unit price $E[\theta]$. She would therefore be strictly worse off trading the contract (q, t) no matter her type. Such a deviation is thus infeasible.

Step 3. No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, an additional contract $(\underline{q}, \underline{t})$ attracts type $\underline{\theta}$ only if $\underline{t} \geq E[\theta]\underline{q}$, since she has the option to trade any quantity at unit price $E[\theta]$. The corresponding payoff for the deviating buyer is then at most $(\underline{\theta} - E[\theta])\underline{q}$ which is at most zero.

Step 4. It follows from Step 3 that a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q}, \overline{t})$ attracts type $\overline{\theta}$ only if $\overline{t} \geq E[\theta]\overline{q}$, since she has the option to trade any quantity at unit price $E[\theta]$. However, type $\underline{\theta}$ can then also weakly increase her payoff by mimicking type $\overline{\theta}$'s behavior. One can therefore construct the seller's strategy in such a way that it is impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5. It follows from Steps 3 and 4 that a profitable deviation must involve trading with both types. Whatever the contract traded by the seller with the deviating buyer, and no matter her type, she can sell to the other buyers the remaining fraction of her endowment at unit price $E[\theta]$. Hence each type of the seller faces the same problem, namely to use optimally the deviating buyer's and the other buyers' offers to sell her whole endowment at the maximum price. One can therefore construct the seller's strategy in such a way that each type selects the same contract from the deviating buyer's menu. By Step 2, this makes such a deviation non profitable. The result follows.

(ii) Consider next the strong adverse selection case. The proof goes through a series of steps.

Step 1. Given the menus offered, any best response of the seller leads to an aggregate trade $(1, \underline{\theta})$ for type $\underline{\theta}$ and $(0, 0)$ for type $\overline{\theta}$, and all buyers obtain a zero payoff.

Step 2. No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. Indeed, such a deviation is profitable only if $E[\theta]q > t$. Under strong adverse selection, this however implies that $t - \delta\overline{\theta}q < 0$, so that type $\overline{\theta}$ would be strictly worse off trading the contract (q, t). Such a deviation is thus infeasible.

Step 3. No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, an additional contract $(\underline{q}, \underline{t})$ attracts type $\underline{\theta}$ only if $\underline{t} \geq \underline{\theta}\underline{q}$, since she always has the option to trade quantity $\underline{q}$ at unit price $\underline{\theta}$. The corresponding payoff for the deviating buyer is then at most zero.

Step 4. It follows from Step 3 that a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q}, \overline{t})$ attracts type $\overline{\theta}$ only if $\overline{t} \geq \delta\overline{\theta}\overline{q}$. However, since $\delta\overline{\theta} > E[\theta] > \underline{\theta}$ under strong adverse selection, type $\underline{\theta}$ can then strictly increase her payoff by trading the contract $(\overline{q}, \overline{t})$ and selling to the other buyers the remaining fraction of her endowment at unit price $\underline{\theta}$. It is therefore impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5. It follows from Steps 3 and 4 that a profitable deviation must involve trading with both types. Given the offer of the other buyers, the most profitable deviations lead to trading some quantity $\overline{q}$ at unit price $\delta\overline{\theta}$ with type $\overline{\theta}$, and trading a quantity 1 at unit price $\delta\overline{\theta}\overline{q} + \underline{\theta}(1-\overline{q})$ with type $\underline{\theta}$. By construction, type $\underline{\theta}$ is indifferent between trading the contract $(1, \delta\overline{\theta}\overline{q} + \underline{\theta}(1-\overline{q}))$ and trading the contract $(\overline{q}, \delta\overline{\theta}\overline{q})$ while selling to the other buyers the remaining fraction of her endowment at unit price $\underline{\theta}$. As for type $\overline{\theta}$, she is indifferent between trading the contract $(\overline{q}, \delta\overline{\theta}\overline{q})$ and not trading at all. The corresponding payoff for the deviating buyer is then

$$\nu\overline{\theta}(1-\delta)\overline{q} + (1-\nu)[\underline{\theta} - \delta\overline{\theta}\overline{q} - \underline{\theta}(1-\overline{q})] = (E[\theta] - \delta\overline{\theta})\overline{q},$$

which is at most zero under strong adverse selection. The result follows. ■

Proof of Proposition 2. (i) Consider first the mild adverse selection case. Suppose that there exists an equilibrium in which some buyer i offers a contract $c^i = (q^i, t^i)$ at unit price $t^i/q^i > E[\theta]$. Notice that one must have $E[\theta] - t^i \geq \delta\overline{\theta}(1-q^i)$ otherwise c^i would give type $\overline{\theta}$ more than her equilibrium payoff. Similarly, one must have $q^i < 1$ otherwise c^i would give both types more than their equilibrium payoff. Any other buyer j could offer a menu consisting of the no-trade contract and of the contract

$$c^j(\varepsilon) = (1-q^i, E[\theta] - t^i + \varepsilon),$$

where $0 < \varepsilon < t^i - q^i E[\theta]$. If both c^i and $c^j(\varepsilon)$ were available, both types of the seller would sell their whole endowment at price $E[\theta] + \varepsilon$ by trading c^i with buyer i and $c^j(\varepsilon)$ with buyer j, thereby increasing their payoff by ε compared to what they obtain in equilibrium. Buyer j's equilibrium payoff is thus at least

$$E[\theta](1-q^i) - (E[\theta] - t^i + \varepsilon) = t^i - q^i E[\theta] - \varepsilon > 0,$$

which is impossible since each buyer's payoff is zero in any equilibrium. Hence, no contract can be issued at a price strictly above $E[\theta]$.

(ii) Consider next the strong adverse selection case. Suppose that there exists an equilibrium in which some buyer i offers a contract $c^i = (q^i, t^i)$ at unit price $t^i/q^i > \underline{\theta}$. Notice that one must have $t^i/q^i \leq \delta\overline{\theta}$ otherwise c^i would give type $\overline{\theta}$ more than her equilibrium payoff. Similarly, one must have $\underline{\theta} - t^i \geq \delta\underline{\theta}(1-q^i)$ and $q^i < 1$ otherwise c^i would give type $\underline{\theta}$ more than her equilibrium payoff. Any other buyer j could offer a menu consisting of the no-trade contract and of the contract

$$c^j(\varepsilon) = (1-q^i, \underline{\theta} - t^i + \varepsilon),$$

where $0 < \varepsilon < \min\{t^i - q^i\underline{\theta}, \delta\overline{\theta} - \overline{\theta}\}$. If both c^i and $c^j(\varepsilon)$ were available, type $\underline{\theta}$ would sell her whole endowment at price $\underline{\theta} + \varepsilon$ by trading c^i with buyer i and $c^j(\varepsilon)$ with buyer j, thereby increasing her payoff by ε compared to what she obtains in equilibrium. Since $\underline{\theta} + \varepsilon < \delta\overline{\theta}$, type $\overline{\theta}$ would strictly lose from trading $c^j(\varepsilon)$ with buyer j. Buyer j's equilibrium payoff is thus at least

$$(1-\nu)[\underline{\theta}(1-q^i) - (\underline{\theta} - t^i + \varepsilon)] = (1-\nu)(t^i - q^i\underline{\theta} - \varepsilon) > 0,$$

which is impossible since each buyer's payoff is zero in any equilibrium. Hence, no contract can be issued at a price strictly above $\underline{\theta}$. ■

Proof of Corollary 2. (i) Consider first the mild adverse selection case. We know that no contract is issued, and a fortiori traded, at a unit price strictly above $E[\theta]$. Suppose now that a contract with unit price strictly below $E[\theta]$ is traded in equilibrium. Since the aggregate allocation traded by both types is $(1, E[\theta])$, this implies that at least one buyer must be trading a contract at a unit price strictly above $E[\theta]$. But, as observed in the first part of the proof, this is impossible. The result follows.

(ii) Consider next the strong adverse selection case. We know that no contract is issued, and a fortiori traded, at a unit price strictly above $\underline{\theta}$. Suppose now that a contract with unit price strictly below $\underline{\theta}$ is traded in equilibrium. Since the aggregate allocation traded by type $\underline{\theta}$ is $(1, \underline{\theta})$, this implies that at least one buyer must be trading a contract at a unit price strictly above $\underline{\theta}$. But, as observed in the first part of the proof, this is impossible. The result follows. ■

Proof of Proposition 3. Fix some equilibrium with menu offers $(C^1, \ldots, C^n)$, and let

$$\mathcal{A}^{-i} = \left\{ \sum_{j \neq i} (q^j, t^j) : \sum_{j \neq i} q^j \leq 1 \text{ and } (q^j, t^j) \in C^j \text{ for all } j \neq i \right\}$$

be the set of aggregate allocations that remain available if buyer i withdraws his menu offer C^i. It should be noted that, by construction, $\mathcal{A}^{-i}$ is a compact set.

(i) Consider first the mild adverse selection case, and suppose that the aggregate allocation $(1, E[\theta])$ traded by both types does not belong to $\mathcal{A}^{-i}$. Since $\mathcal{A}^{-i}$ is compact, there exists an open set of $[0, 1] \times \mathbb{R}_+$ that contains $(1, E[\theta])$ and that does not intersect $\mathcal{A}^{-i}$. Moreover, by Proposition 2(i), any allocation (Q^{-i}, T^{-i}) in $\mathcal{A}^{-i}$ is such that $T^{-i} \leq E[\theta]Q^{-i}$. It follows that the allocation $(1, E[\theta])$ is strictly preferred by each type to any allocation in $\mathcal{A}^{-i}$. By continuity of preferences, so is the contract $(1, E[\theta] - \varepsilon)$ for some ε positive and close enough to zero. Thus, if $(1, E[\theta] - \varepsilon)$ were available, both types would trade it. This implies that buyer i's equilibrium payoff is at least ε, which is impossible since each buyer's payoff is zero in any equilibrium. The result follows.

(ii) Consider next the strong adverse selection case, and suppose that the aggregate allocation $(1, \underline{\theta})$ traded by type $\underline{\theta}$ does not belong to $\mathcal{A}^{-i}$. Since $\mathcal{A}^{-i}$ is compact, there exists an open set of $[0, 1] \times \mathbb{R}_+$ that contains $(1, \underline{\theta})$ and that does not intersect $\mathcal{A}^{-i}$. Moreover, by Proposition 2(ii), any allocation (Q^{-i}, T^{-i}) in $\mathcal{A}^{-i}$ is such that $T^{-i} \leq \underline{\theta}Q^{-i}$. It follows that there exists a contract $(\overline{q}^i, \overline{t}^i)$ with unit price $\overline{t}^i / \overline{q}^i \in (\delta\overline{\theta}, \overline{\theta})$ such that the allocation $(1, \underline{\theta})$ is strictly preferred by type $\underline{\theta}$ to any allocation obtained by trading the contract $(\overline{q}^i, \overline{t}^i)$ together with some allocation (Q^{-i}, T^{-i}) in $\mathcal{A}^{-i}$ such that $\overline{q}^i + Q^{-i} \leq 1$. Moreover, since $\overline{t}^i / \overline{q}^i > \delta\overline{\theta}$, the contract $(\overline{q}^i, \overline{t}^i)$ guarantees a strictly positive payoff to type $\overline{\theta}$. Thus, if both $(1, \underline{\theta})$ and $(\overline{q}^i, \overline{t}^i)$ were available, type $\underline{\theta}$ would trade $(1, \underline{\theta})$ and type $\overline{\theta}$ would trade $(\overline{q}^i, \overline{t}^i)$. This implies that buyer i's equilibrium payoff is at least $\nu(\overline{\theta}\overline{q}^i - \overline{t}^i) > 0$, which is impossible since each buyer's payoff is zero in any equilibrium. The result follows. ■

Proof of Proposition 4. (i) Consider first the mild adverse selection case. The proof goes through a series of steps.

Step 1. Given the menus offered, any best response of the seller leads to an aggregate trade $(1, E[\theta])$ irrespective of her type. Since $\phi < E[\theta]$, it is optimal for each type of the seller to trade her whole endowment with a single buyer. Assuming that each type of the seller trades with the same buyer, all buyers obtain a zero payoff. Note also that if any buyer withdraws his menu offer, the most the seller can achieve by trading with the other buyers consists in trading with a single buyer.

Step 2. No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. Indeed, such a deviation is profitable only if $E[\theta]q > t$. Since $\phi < E[\theta]$, the highest payoff the seller can achieve by purchasing the contract (q, t) together with some contract in the menu offered by the other buyers is less than the payoff from trading the contract $(1, E[\theta])$, which remains available at the deviation stage. She would therefore be strictly worse off trading the contract (q, t) no matter her type. Such a deviation is thus infeasible.

Step 3. No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, trading an additional contract $(\underline{q}, \underline{t})$ with type $\underline{\theta}$ is profitable only if $\underline{\theta}\underline{q} > \underline{t}$. The same argument as in Step 2 then shows that type $\underline{\theta}$ would be strictly worse off trading the contract $(\underline{q}, \underline{t})$ rather than the contract $(1, E[\theta])$, which remains available at the deviation stage. Such a deviation is thus infeasible.

Step 4. It follows from Step 3 that a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q}, \overline{t})$ that is profitable when traded with type $\overline{\theta}$ attracts her only if $\overline{t} + \phi(1 - \overline{q}) \geq E[\theta]$, that is, only if she can weakly increase her payoff by trading the contract $(\overline{q}, \overline{t})$ and selling to the other buyers the remaining fraction of her endowment at unit price ϕ. That this is feasible follows from the fact that, when $\overline{\theta}\overline{q} > \overline{t}$ and $\overline{t} + \phi(1 - \overline{q}) \geq E[\theta]$, the quantity $1 - \overline{q}$ is less than the maximal quantity $(\overline{\theta} - E[\theta])/(\overline{\theta} - \phi)$ that can be traded at unit price ϕ with the other buyers. Moreover, the fact that $\phi \geq \delta\overline{\theta}$ guarantees that it is indeed optimal for type $\overline{\theta}$ to behave in this way at the deviation stage. However, type $\underline{\theta}$ can then also weakly increase her payoff by mimicking type $\overline{\theta}$'s behavior. One can therefore construct the seller's strategy in such a way that it is impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5. It follows from Steps 3 and 4 that a profitable deviation must involve trading with both types. Whatever the contract traded by the seller with the deviating buyer, and no matter her type, she will sell to the other buyers the remaining fraction of her endowment at unit price ϕ. Hence, each type of the seller faces the same problem, namely to use optimally the deviating buyer's and the other buyers' offers to sell her whole endowment at the maximum price. One can therefore construct the seller's strategy in such a way that each type selects the same contract from the deviating buyer's menu. By Step 2, this makes such a deviation non profitable. The result follows.

(ii) Consider next the strong adverse selection case. The proof goes through a series of steps.

Step 1. Given the menus offered, any best response of the seller leads to an aggregate trade $(1, \underline{\theta})$ for type $\underline{\theta}$ and $(0, 0)$ for type $\overline{\theta}$. Since each buyer is not ready to pay anything for quantities up to $(\psi - \underline{\theta})/\psi$ and offers to purchase each additional unit at a constant marginal price ψ above this level, it is optimal for type $\underline{\theta}$ to trade her whole endowment with a single buyer, and all buyers obtain a zero payoff. Note also that if any buyer withdraws his menu offer, the most the seller can achieve by trading with the other buyers consists in trading with a single buyer.

Step 2. No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. This can be shown as in Step 2 of the proof of Proposition 1(ii).

Step 3. No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, trading an additional contract $(\underline{q}, \underline{t})$ with type $\underline{\theta}$ is profitable only if $\underline{\theta}\underline{q} > \underline{t}$. Since $\psi > \underline{\theta}$, the highest payoff type $\underline{\theta}$ can achieve by purchasing the contract (q, t) together with some contract in the menu offered by the other buyers is less than the payoff from trading the contract $(1, \underline{\theta})$, which remains available at the deviation stage. She would therefore be strictly worse off trading the contract (q, t). Such a deviation is thus infeasible.

Step 4. It follows from Step 3 that a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q}, \overline{t})$ attracts type $\overline{\theta}$ only if $\overline{t} \geq \delta\overline{\theta}\overline{q}$. Two cases must be distinguished. If $\overline{q} \leq \underline{\theta}/\psi$, then type $\underline{\theta}$ can trade the contract $(\overline{q}, \overline{t})$ and sell to some other buyer the remaining fraction of her endowment at price $\psi(1 - \overline{q}) - \psi + \underline{\theta}$. The price at which she can sell her whole endowment is therefore at least $(\delta\overline{\theta} - \psi)\overline{q} + \underline{\theta}$, which is strictly higher than the price $\underline{\theta}$ that she obtains in equilibrium since $\delta\overline{\theta} > \underline{\theta} + (\delta\overline{\theta} - E[\theta])/(1 - \nu) \geq \psi$. If $\overline{q} \geq \underline{\theta}/\psi$, then by trading the contract $(\overline{q}, \overline{t})$, type $\underline{\theta}$ obtains at least a payoff $\delta(\overline{\theta} - \underline{\theta})\underline{\theta}/\psi$, which, since $\delta\overline{\theta} > \psi > \underline{\theta}$, is more than her equilibrium payoff $(1 - \delta)\underline{\theta}$. Thus type $\underline{\theta}$ can always strictly increase her payoff by trading the contract $(\overline{q}, \overline{t})$. It is therefore impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5. It follows from Steps 3 and 4 that a profitable deviation must involve trading with both types. Given the offer of the other buyers, the most profitable deviations lead to trading some quantity $\overline{q} \leq \underline{\theta}/\psi$ at unit price $\delta\overline{\theta}$ with type $\overline{\theta}$, and trading a quantity 1 at unit price $\delta\overline{\theta}\overline{q} + \underline{\theta} - \psi\overline{q}$ with type $\underline{\theta}$. By construction, type $\underline{\theta}$ is indifferent between trading the contract $(1, \delta\overline{\theta}\overline{q} + \underline{\theta} - \psi\overline{q})$ and trading the contract $(\overline{q}, \delta\overline{\theta}\overline{q})$ while selling to the other buyers the remaining fraction of her endowment at price $\psi(1-\overline{q}) - \psi + \underline{\theta}$. As for type $\overline{\theta}$, she is indifferent between trading the contract $(\overline{q}, \delta\overline{\theta}\overline{q})$ and not trading at all. The corresponding payoff for the deviating buyer is then

$$\nu\overline{\theta}(1-\delta)\overline{q} + (1-\nu)(\underline{\theta} - \delta\overline{\theta}\overline{q} - \underline{\theta} + \psi\overline{q}) = [\nu\overline{\theta} + (1-\nu)\psi - \delta\overline{\theta}]\overline{q},$$

which is at most zero since $\psi \leq \underline{\theta} + (\delta\overline{\theta} - E[\theta])/(1-\nu)$. The result follows. ■

Proof of Lemma 1 : for further reference, we solve here a slightly more general problem, that is paramaterized by three elements $(Q_1, \theta_0, \theta_1)$, with $\theta_0 \leq \theta_1$ and $0 \leq Q_1 \leq 1$. This problem consists in maximizing

$$\int_{-\infty}^{\theta_1} [(v(\theta) - \theta)Q(\theta) - U(\theta)]dF(\theta)$$

under the IC and (IR) constraints, and two additional constraints that we now spell. The first constraint imposes that $Q(\theta) = 1$ if $\theta \leq \theta_0$. The second constraint imposes that $Q(\theta)$ is at least equal to Q_1. Notice that the monopsony problem corresponds to $Q_1 = 0$, $\theta_0 = -\infty$, $\theta_1 = +\infty$.

Using standard techniques, the problem reduces to

$$\int_{-\infty}^{\theta_1} [v(\theta) - \theta]Q(x)dF(x) - \int_{-\infty}^{\theta_1} Q(\theta)F(\theta)d\theta$$

under the constraint that Q is weakly decreasing, and our two additional constraints. The objective is linear in Q. Moreover any Q verifying the constraints is a convex combination of functions indexed by $\theta' \geq \theta_0$, such that $Q(\theta) = 1$ if $\theta \leq \theta'$, and $Q(\theta) = Q_1$ if $\theta > \theta'$. Therefore the monopoly cannot loose anything by using such functions; each function corresponds to offering to buy one unit for a transfer θ'.

Hence the problem reduces to maximizing on $\theta' \geq \theta_0$

$$\int_{-\infty}^{\theta'} [v(\theta) - \theta]dF(x) - \int_{-\infty}^{\theta'} F(\theta)d\theta + Q_1 \int_{\theta'}^{\theta_1} [v(\theta) - \theta]dF(x) - Q_1 \int_{\theta'}^{\theta_1} F(\theta)d\theta$$

$$= \int_{-\infty}^{\theta'} v(\theta)dF(x) - \theta' F(\theta') + Q_1 \int_{\theta'}^{\theta_1} v(\theta)dF(x) + Q_1[\theta' F(\theta') - \theta_1 F(\theta_1)]$$

$$= w(\theta') + Q_1[w(\theta_1) - w(\theta')]$$

and thus the monopoly's payoff is equal to

$$Q_1 w(\theta_1) + (1 - Q_1) \sup_{\theta' \in [\theta_0, \theta_1]} w(\theta') \tag{A.1}$$

In the monopoly problem under study, we have $Q_1 = 0$, $\theta_0 = -\infty$, $\theta_1 = +\infty$, which gives the result.

Another consequence of (A.1) will be used when dealing with competition : if aggregate profits are zero or above zero, and if $Q(\theta) = 1$ for $\theta < p^*$, then $Q(\theta) = 0$ for $\theta > p^*$ (apply the above formula with $\theta_1 = +\infty$, $Q_1 = 0$ and $\theta_0 = p^*$, together with Assumption 1). QED.

Proof of Proposition 1 : let us first study necessity. Suppose we are given a robust equilibrium, whose outcome include the set $\mathcal{C}_i$ of contracts offered by each buyer $i = 1..n$, and payoffs $U(\theta)$ and total quantity traded $Q(\theta)$ (possibly random) for each type θ of the agent. Let us define $b^i(\theta)$ as the equilibrium expected profit obtained by buyer i from the seller of type θ. Define also θ_0 as the supremum of those types that sell a quantity one (set $\theta_0 = -\infty$ if this set is empty).

Let $\theta_1 > \theta_0$, and let $Q_1 < 1$ be a quantity that this stype sells with strictly positive probability. Because Q_1 is possibly traded at equilibrium, there exists $(q_i,t_i)_{i=1..n}$ in $\mathcal{C}_1 \times .. \times \mathcal{C}_n$ such that

$$Q_1 = \sum_i q_i \qquad U(\theta_1) = \sum_i t_i - \theta_1 Q_1 \tag{A.2}$$

Choose any buyer i, and consider the following deviation : offer the same subset of contracts $\mathcal{C}_i$ as before, plus the contract $(q_i + 1 - Q_1,\, t_i + \theta_1(1 - Q_1))$. The seller reacts to this deviation depending on his type θ.

If $\theta > \theta_1$, then θ strictly prefers $(q_i,\, t_i)$ to the new contract, because its unit price is too low. We can then apply part i) of our robustness refinement to conclude that θ does not change its behavior.

If $\theta < \theta_1$, then θ can choose to trade the new contract, together with the contracts $(q_j,\, t_j)_{j \neq i}$ that are defined in (A.2). Then θ would sell exactly one unit, and would get a payoff

$$\theta_1(1 - Q_1) + t_i + \sum_{j \neq i} t_j - \theta = U(\theta_1) + \theta_1 - \theta > U(\theta)$$

because $U(\theta) + \theta$ is strictly increasing on $[\theta_0, \theta_1]$. Since $U(\theta)$ is the best payoff θ can get by rejecting the new contract, we have shown that θ strictly gains by trading the new contract compared to not trading it, and from part ii) of our robustness requirement he must do so.

Now we can compute the change in profits for buyer i, following the deviation. For $\theta \leq \theta_1$, now buyer i gets

$$(q_i + 1 - Q_1)v(\theta) - t_i - \theta_1(1 - Q_1)$$

while at equilibrium i was getting an expected profit $b^i(\theta)$. Therefore the variation in profits can be written

$$\int_{-\infty}^{\theta_1} [(q_i + 1 - Q_1)(v(\theta) - \theta_1) + \theta_1 q_i - t_i - b^i(\theta)]dF(\theta)$$

and this must be weakly negative (otherwise the deviation would be strictly profitable). Using the definition of w, we obtain

$$(q_i + 1 - Q_1)w(\theta_1) \leq \int_{-\infty}^{\theta_1} [t_i - \theta_1 q_i + b^i(\theta)]dF(\theta)0$$

Now we can sum over i. Notice that at equilibrium the total profits from θ are a.e.

$$\sum_i b^i(\theta) = (v(\theta) - \theta)Q(\theta) - U(\theta)$$

Using also (A.2), we get

$$(Q_1 + n(1 - Q_1))w(\theta_1) \le \int_{-\infty}^{\theta_1} [(v(\theta) - \theta)Q(\theta) - (U(\theta) - U(\theta_1))]dF(\theta)$$

where $n \ge 2$ is the number of buyers. Let us study the right-hand side integral. We know that $(Q,\, U - U(\theta_1))$ must satisfy the IC and IR constraints, that moreover $Q(\theta) \ge Q_1$, and finally that $Q(\theta) = 1$ for $\theta < \theta_0$. Using the expression for the monopoly profits derived in (A.1), we get that the right-hand-side integral must lie below

$$Q_1 w(\theta_1) + (1 - Q_1) \sup_{\theta \in [\theta_0, \theta_1]} w(\theta)$$

Replacing and simplifying since $Q_1 < 1$, we finally get

$$nw(\theta_1) \le \sup_{\theta \in [\theta_0, \theta_1]} w(\theta) \tag{A.3}$$

This must hold for all $\theta_1 > \theta_0$, by definition of θ_0. We can take supremums to get

$$n \sup_{\theta_1 > \theta_0} w(\theta_1) \le \sup_{\theta_1 > \theta_0} \sup_{\theta \in [\theta_0, \theta_1]} w(\theta) = \sup_{\theta \ge \theta_0} w(\theta)$$

and by continuity of w, and because $n \ge 2$, we get

$$\sup_{\theta \ge \theta_0} w(\theta) \le 0$$

From Assumption 1, this implies that $\theta_0 \ge p^*$, so that $Q(\theta) = 1$ for $\theta < p^*$. Applying the result stated in the last paragraph of the proof to Lemma 1, we get that $Q(\theta)$ is equal to one for all $\theta < p^*$, and $Q(\theta)$ is zero above p^*.

There only remains to show that the proposed candidate is an equilibrium, which is easy and omitted. QED.

Proof of Proposition 2 : it is enough to show that all contracts issued have a unit price below p^*. Suppose otherwise, and consider a contract $(q,\, t)$ with a unit price strictly above p^*, offered by one of the buyers. Another buyer could then deviate by adding the contract $C' = (1 - q,\, (p^* - \varepsilon)(1 - q))$ to its equilibrium offer, where ε is such that $t - p^* q > \varepsilon(1 - q)$. Then clearly types θ above $p^* - \varepsilon$ do not trade this contract, since the unit price is too low. Types below $p^* - \varepsilon$ could trade this contract together with contract $(q,\, t)$ and get

$$t + (p^* - \varepsilon)(1 - q) - \theta = p^* - \theta + t - p^* q - \varepsilon(1 - q) > p^* - \theta = U(\theta)$$

so that under robustness these types should accept to trade C'. Overall our buyer gets

$$\int^{p^* - \varepsilon} [v(\theta) - p^* + \varepsilon][1 - q]dF(\theta) = (1 - q)w(p^* - \varepsilon)$$

which is positive for ε small and well-chosen, by definition of p^*. Q.E.D.

Proof of Proposition 3 : Suppose that a robust equilibrium exists, with outcome $(U,\, Q)$. Let us first prove the following result, that is used repeatedly in this proof :

Lemma 2 *Choose $\theta_a < \theta_b$, and suppose that the following property holds at (θ_a, θ_b) :*

$$\exists\, \theta, \theta' \qquad \theta_a < \theta < \theta' < \theta_b \qquad \textit{and} \qquad Q(\theta) > Q(\theta') \tag{A.4}$$

Define

$$q_0 = \frac{U(\theta_a) - U(\theta_b)}{\theta_b - \theta_a} \qquad t_0 = \frac{\theta_b U(\theta_a) - \theta_a U(\theta_b)}{\theta_b - \theta_a}$$

Then $Q(\theta_b) < q_0 < Q(\theta_a)$, and at equilibrium one must have

$$n \int_{\theta_a}^{\theta_b} [v(\theta) q_0 - t_0] dF(\theta) \leq \int_{\theta_a}^{\theta_b} [(v(\theta) - \theta) Q(\theta) - U(\theta)] dF(\theta)$$

Proof of Lemma 2 : since $U'(\theta)$ is equal to $(-Q(\theta))$ almost everywhere, q_0 is computed as an average of the quantities traded; under (A.4) it must be that $Q(\theta_b) < q_0 < Q(\theta_a)$. Notice moreover that

$$t_0 = U(\theta_a) + \theta_a q_0 = U(\theta_b) + \theta_b q_0$$

Now suppose that principal i deviates by adding this contract to his equilibrium offer. For $\theta > \theta_b$, convexity of U implies first

$$U(\theta) \geq U(\theta_b) + (\theta - \theta_b)(-Q(\theta_b))$$

and using the definitions of q_0 and t_0 we get

$$U(\theta) \geq t_0 - \theta q_0 + (\theta - \theta_b)(q_0 - Q(\theta_b))$$

which is strictly greater than $t_0 - \theta q_0$. Thus θ strictly prefers his equilibrium trade to trading $(q_0,\, t_0)$; from robustness this implies that following the buyer's deviation θ does not trade $(q_0,\, t_0)$, and does not change his behaviour. The same properties can be shown similarly for all types $\theta < \theta_a$. Finally consider types such that $\theta_a < \theta < \theta_b$. By convexity we have

$$U(\theta_a) \geq U(\theta) + (\theta - \theta_a) Q(\theta)$$
$$U(\theta_b) \geq U(\theta) + (\theta - \theta_b) Q(\theta)$$

and from (A.4) at least one of these inequalities is strict. Multiplying by well-chosen positive constants and summing, we get

$$U(\theta_a)(\theta_b - \theta) + U(\theta_b)(\theta - \theta_a) > U(\theta)(\theta_b - \theta_a)$$

which reduces to $t_0 - \theta q_0 > U(\theta)$. Hence under robustness all types in $]\theta_a, \theta_b[$ choose to trade $(q_0,\, t_0)$. This establishes that for any principal i the variation in profits is

$$\int_{\theta_a}^{\theta_b} [v(\theta) q_0 - t_0 - b^i(\theta)] dF(\theta)$$

where $b^i(\theta)$ is the expected profit that principal i gets from type θ on the equilibrium path. We get the result by summing over $i = 1..n$, because $\sum_i b^i(\theta) = (v(\theta) - \theta) Q(\theta) - U(\theta)$. *QED.*

Now choose some type θ_0 in $]\underline{\theta}, \bar{\theta}[$. We know that $Q_0 \equiv Q(\theta_0)$ is well-defined a.e., so that we concentrate on this case. Suppose that Q_0 is positive. We distinguish two cases, in order to show that the buyers' aggregate profits are zero or below zero when Q_0 is traded.

First case : suppose first that θ_0 is the only type to sell Q_0. Then for any $\theta_1 < \theta_0 < \theta_2$, (A.4) holds at (θ_1, θ_2). We can thus apply Lemma 2, and because $t_0 = U(\theta_1) + \theta_1 q_0$ we obtain

$$n \int_{\theta_1}^{\theta_2} [v(\theta)q_0 - U(\theta_1) - \theta_1 q_0] dF(\theta) \leq \int_{\theta_1}^{\theta_2} [(v(\theta) - \theta)Q(\theta) - U(\theta)] dF(\theta)$$

Since this is valid for any $\theta_1 < \theta_0 < \theta_2$, one can divide this inequality by $(F(\theta_2) - F(\theta_1))$ and compute the limit when both bounds go to θ_0, to get

$$n[v(\theta_0)Q_0 - U(\theta_0) - \theta_0 Q_0] \leq (v(\theta_0) - \theta_0)Q_0 - U(\theta_0)$$

or equivalently

$$(v(\theta_0) - \theta_0)Q_0 - U(\theta_0) \leq 0$$

which indicates that the buyers' aggregate profits from trading Q_0 cannot be positive.

Second case : the only other case is when $Q(\theta) = Q_0$ on some maximum interval (θ_2, θ_3) containing θ_0. Choose (θ_1, θ_4) such that $\theta_1 < \theta_2 < \theta_3 < \theta_4$. One can now apply Lemma 2 at, say, (θ_1, θ_3), and take limits as above when θ_1 goes to θ_2, to obtain

$$n \int_{\theta_2}^{\theta_3} [v(\theta)Q_0 - U(\theta) - \theta Q_0] dF(\theta) \leq \int_{\theta_3}^{\theta_4} [(v(\theta) - \theta)Q_0 - U(\theta)] dF(\theta)$$

or equivalently

$$\int_{\theta_2}^{\theta_3} [(v(\theta) - \theta)Q_0 - U(\theta)] dF(\theta) \leq 0$$

Hence we have established that whatever the quantity traded the buyers' aggregate profits are zero or below zero. Because aggregate profits must be at least zero, this implies that profits are exactly zero for all quantities traded (apart for a negligible subset of θ), as announced.

We can now extend our analysis of the second case : choose some θ' such that $\theta_2 < \theta' < \theta_3$, and apply Lemma 2 at (θ_1, θ'), and take the limit when θ_1 goes to θ_2 to get

$$\int_{\theta_2}^{\theta'} [(v(\theta) - \theta)Q_0 - U(\theta)] dF(\theta) \leq 0$$

Similarly apply Lemma 1 at (θ', θ_4), and take the limit when θ_4 goes to θ_3 to get

$$\int_{\theta'}^{\theta_3} [(v(\theta) - \theta)Q_0 - U(\theta)] dF(\theta) \leq 0$$

Because these two functions of θ' add up to zero, these inequalities imply that they are identically equal to zero. Summarizing, we have shown that $U(\theta) = (v(\theta) - \theta)Q_0$ for $\theta \in [\underline{\theta}, \bar{\theta}]$.

When Q is a constant on some interval, this means that each θ sells the same quantity for a unit price $v(\theta)$. For clear incentive-compatibility reasons, we get that on any interval on which $Q(.)$ is a constant then $v(.)$ must also be a constant.

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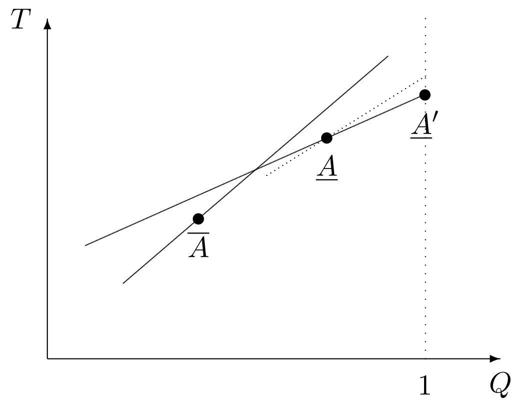

FIGURE 1.—Attracting type $\underline{\theta}$ by pivoting around $(\underline{Q}, \underline{T})$.

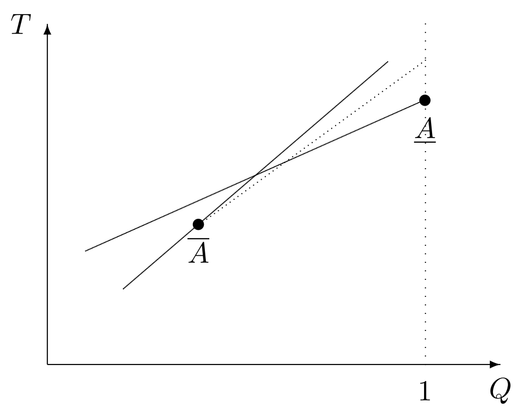

FIGURE 2.—Attracting type $\underline{\theta}$ by pivoting around $(\overline{Q}, \overline{T})$.

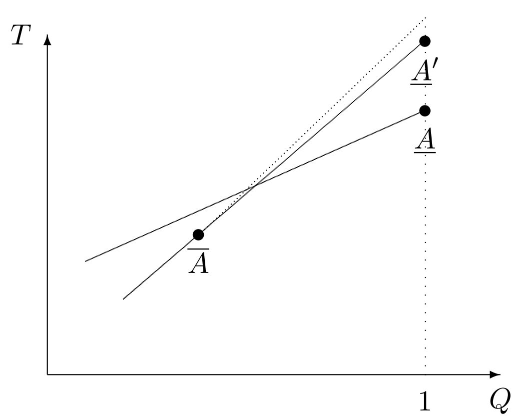

FIGURE 3.—Attracting both types by pivoting around $(\overline{Q}, \overline{T})$.

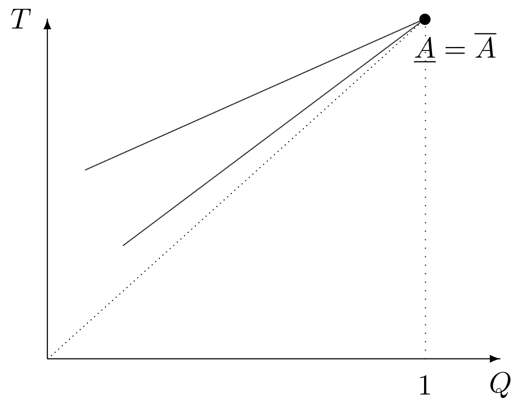

Figure 4.—Aggregate equilibrium allocations in the mild adverse selection case.

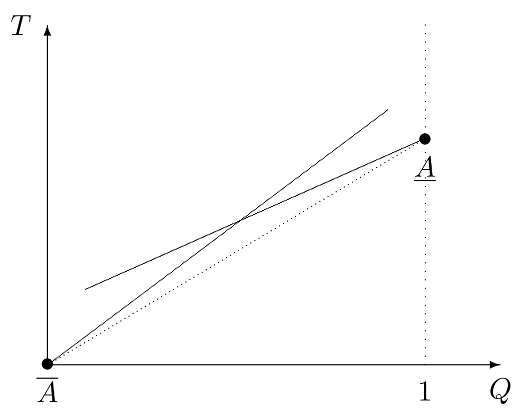

Figure 5.—Aggregate equilibrium allocations in the strong adverse selection case.

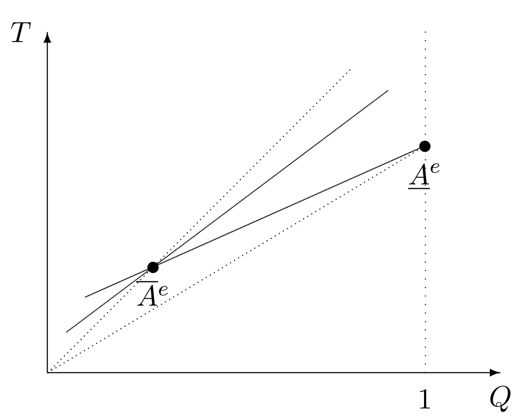

Figure 6.—Equilibrium allocations under exclusive competition.