Competing for Ownership*

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Abstract

We develop a tractable model of the allocation of control in firms in competitive markets, which permits us to study how changes in the scarcity of assets, skills or liquidity in the market translate into control inside the organization. Firms will be more integrated when the terms of trade are more favorable to the short side of the market, when liquidity is unequally distributed among existing firms and following a uniform increase in productivity. The model identifies a price-like mechanism whereby local liquidity or productivity shocks propagate and lead to widespread organizational restructuring.

1 Introduction

The modern theory of the firm emphasizes contractual frictions and the resulting importance of organizational design elements such as task allocations, asset ownership, and the assignment of authority and control. Having introduced a rich set of new variables into economic analysis, it has moved us

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well beyond the neoclassical "black box" model of the firm, making breakthroughs in our comprehension of economic institutions as different as the modern corporation and the sharecropped farm. But a full understanding of firms that operate in market economies also requires a framework that, like the neoclassical one, can track the external influences of the market on organizational decisions as well as the feedback of those decisions to the market and to other firms.

The purpose of this paper is to provide a tractable model for the analysis of this kind of interaction. We focus on the structure of ownership and control, understood here, as in Grossman-Hart (1986), as an allocation among a firm's stakeholders of decision rights. Our main interest is how the scarcity of different types of managers in the market as a whole affects the choice of ownership structure and with how changes in the fundamentals of some firms can spill over to economy-wide reorganizations.

As in their model, pairs of production units – each consisting of a manager and a collection of assets – must produce together in order to generate marketable output. Firms comprising a pair of units and the contracts specifying the ownership structure are formed through a competitive matching process. A key attribute of the environment we analyze is that liquidity – instruments such as cash that can be transferred costlessly and without any incentive distortions – is scarce. This feature renders the competitive analysis of organization nontrivial, because one can no longer simply identify equilibrium organizational outcomes with the surplus maximizing ones.

The model highlights two distinct effects that arise from a change in fundamentals such as liquidity endowments or technology. The first, "internal effect," is familiar enough from the literature: the surplus that each partner obtains from a given contract is a function of the characteristics of the relationship, in particular the production technology and the liquidity available. Higher productivity or more liquidity in the firm not only enlarges the set of feasible payoffs for the two managers, but also "flattens" its frontier, that is, it increases the transferability of surplus. If productivity is high, managers have a high opportunity cost of failing to maximize profit; if firms have more liquidity, they can avoid using inefficient contractual instruments. Hence, a firm that receives a positive shock to its productivity or liquidity endowment will be able to accomplish surplus division more efficiently and reduce organizational distortions.

But such a shock may have much wider effects than on the firm that experiences it. The internal effect implies that a manager has effectively a higher "ability to pay" for a partner after a positive shock than before. He may therefore bid up the terms of trade in the supplier market: in order to meet the new price, firms which have not benefited from the shock will have to refinance and/or restructure. Thus the shock may have an *external effect*: "local" shocks may propagate via the market mechanism, leading to widespread reorganization.

The market equilibrium of our model turns out to be amenable to a Marshallian supply-demand style of analysis, making the role of the external effect especially transparent. Suppose, for instance, that the short side of the market represents downstream producers and the long side their upstream suppliers. An increase in the supply of downstream units will raise the share of surplus accruing to upstream units: downstream managers will find shares of profit lowered and upstream managers will own more assets. More interestingly, while the internal effects of positive shocks to liquidity and technology are similar – they both decrease integration – the external effects are quite different. For instance, a uniform increase in the liquidity level of all agents lowers the degree of integration in all firms (the internal effect dominates the external effect). By contrast, a uniform shock to productivity increases the degree of integration in all firms (the external effect dominates the internal effect). As we show in Section 3, the model can capture the effects of more complex changes in the liquidity endowments or in productivity.

1.1 Literature

Our modeling of ownership borrows heavily from Grossman and Hart (1986) and Hart and Moore (1990). However, we depart from their analysis in three respects. First, we assume that the set of assets may be divided between the managers in any way, thereby yielding a continuum of ownership structures.

This feature yields a tractable model amenable to competitive analysis. Second, to make the surplus transfer role of ownership especially transparent, we abstract away from the hold up problem. Under this assumption, the set of feasible decsions is unaffected by who owns an asset, and therefore awarding ownership of more assets to one manager raises his payoff, since it ensures that more of the ensuing planning decisions will go in his preferred direction. The downside for the relationship is that these decisions may pose significant costs on the other members of the firm.

Third and most important, we assume that liquidity – instruments such as cash that can be transferred costlessly and without any incentive distortions – is scarce. In this case both decision right allocations and financial contracting will be a non trivial function of the liquidity available in the relationship. This internal effect of limited liquidity is well understood in the corporate finance literature (Jensen and Meckling 1976, Aghion and Bolton 1992): for a given surplus required by a contracting party, more liquidity will make contracting more efficient. We add to that literature by pointing out an external effect of liquidity, whereby changes to the liquidity of *other* firms may lead a firm to modify its own control right allocation, possibly at the cost of efficiency.

Limitations on liquidity also imply that heterogenity in or changes to other variables, such as the joint productivity or relative scarcities of production units, have significant and possibly unexpected impact on organizational choices. This differentiates our model from McLaren (2000) which considers market thickness effects on hold-up and specific investments to explain vertical integration or Grossman and Helpman (2002), which explains differences in the choice between outsourcing and integration by trading off exogenous differences in costs of contracting and search.

2 Model

We consider an economy in which production requires the cooperation of two production units, indexed by $1, 2$. Each unit consists of a risk-neutral manager and a collection of assets that he will have to work with in order to produce. We have in mind competitive outcomes, and so we suppose that there is a large number of production units: each side of the market is a continuum with Lebesgue measure. The type 1's are represented by $i \in I = [0, 1]$ while the type 2's are represented by $j \in J = [0, n]$, where $n < 1$; thus, the 2's are relatively scarce.

Many interpretations are possible: the two types of manager might be supplier and manufacturer, and the assets plant and equipment; a chain restaurateur and franchising corporation (in which case some of the assets are reputational); or as a firm and its workforce, for which the assets might be interpreted as tasks.

In an individual production unit, an asset's contribution to profit depends on a planning decision made by one of the managers, not necessarily the one who will have to operate it. Planning decisions are not contractible, but the right to make them can be allocated via contract to either manager. For simplicity we assume that planning choices (e.g., choosing the background music for a retail store) are costless. But while potentially beneficial for profits (some music is likely to induce consumers to make impulse purchases), those choices affect the private cost of later operations (such music may be unpleasant for the store's floor manager).

The i-th type-1 manager will have at her disposal a quantity $l_1(i) \geq 0$ of cash (or "liquidity") which may be consumed at the end of the period and which may be useful in contracting with managers of the opposite type; for the type 2's, the liquidity endowment is $l_2(j)$. The indices i and j have been chosen in order of increasing liquidity.

When discussing a generic production unit or its manager, we shall usually drop the indices.

2.1 The Basic Organizational Design Problem

2.1.1 Technology and Preferences

Managers seek to maximize their expected income (including the initial liquidity) less the private costs of operating the enterprise.

The collection of assets in the type-1 production unit is represented by a continuum indexed by $k \in [0, 1)$; the type-2 assets are indexed by $k \in [1, 2)$.

An asset's contribution to profit is proportional to the planning level $q(k)$, where $q(k) \in [0,1]$.

Planning decisions contribute to the firm's performance as follows. The firm either succeeds, generating profit $R > 0$, with probability $p(q)$; or it fails, generating 0, with probability $1 - p(q)$, where $q : [0,2) \to [0,1]$ are the planning decisions. The success probability functional is

$$p(q) = \frac{A}{R} \int_0^2 q(k) dk,$$

where A is a technological parameter. Obviously, for $p(\cdot)$ to be well-defined, we also need $A/R < 1/2$.

Either manager is capable of making planning decisions. There is no cost to making a plan, but there is a (private) operating cost to the manager who subsequently works with an asset: the 1-manager bears cost $c(q(k)) = \frac{1}{2} q(k)^2$ for $k \in [0,1)$, and zero for $k \in [1,2]$; similarly for 2, the cost is $c(q(k))$ on $[1,2)$ and zero on $[0,1)$. For brevity we write

$$C_1(q) = \int_0^1 c(q(k))\, dk, \qquad C_2(q) = \int_1^2 c(q(k))\, dk.$$

This is the cost externality we alluded to: the cost to the manager operating the asset is increasing in $q(k)$, whether or not he has chosen it. For instance in a manufacturing enterprise, q could index choices of possible parts or material inputs, ordered by the value they contribute to the final product, while $c(q)$ could represent the cost of managerial attention devoted to overseeing assembly, supervising workers, and so on. Each input choice requires solving a number of manufacturing process problems; we are supposing that higher value inputs require greater learning and adaptation effort on the part of the manufacturer's management.[1]

[1] In the 1960's, W. Corporation owned an electronic systems division that manufactured airplane cockpit voice recorders, and a composite materials division that made various compounds suitable for heat-resistant recording tape, a critical input for recorders. The electronic systems division had perfected a manufacturing process that used mylar tape, but W. ordered them to use a new metal-oxide tape developed by its materials division. The new tape was less flexible than mylar, and therefore subject to kinking and breakage,

Or, in a relationship between a fast food chain and a franchisee, numerous operating decisions, from the content of an advertising campaign, color scheme in the restaurant, participation in discount coupon promotions, or franchise relocation could in principle be delegated to the franchisee or retained by the company. Most of these decisions will have significant effects on the franchisee's welfare, though it is hard to see most of them having much impact on the chain apart from their influence on expected profit. And while one might imagine a complete contingent contract on relocation might be drawn up (for instance, relocation might be dependent on location of a competing chain's stores, local income levels and crime rates, etc.), in practice this remains a residual decision of the chain (Blair and Lafontaine, 2005).

2.1.2 Contracts

We have already made the following contractibility assumptions:[2]

Assumption (i) The *right to decide* $q(k)$ is both alienable and contractible.
(ii) The decisions q are never contractible.
(iii) The costs $C_i(q)$ are private and noncontractible.

A contract (ω, t) specifies the allocation of ownership ω and liquidity transfers t made from 1 to 2 before any planning or production takes place. The liquidity levels of the two types being l_1 and l_2 respectively, we must have $t \in [-l_2, l_1]$. The ownership allocation ω is the fraction of assets re-assigned to one of the managers. The type-1 manager owns assets in $[0, 1-\omega)$, where $-1 < \omega \leq 1$, and the type-2 owns $[1-\omega, 2)$.

Since we want to focus here on allocations of control rights, we will simplify matters by ignoring the usual effects related to variations in the sharing

which raised manufacturing problems that required nearly a year of process redevelopment to resolve. A former manager of the systems division admits that had it been up to him, his division would have stuck with the mylar tape, simply because the experimentation and retooling costs were not (and because of verifiability and incentive problems likely could never be) appropriately reimbursed, even if the metal oxide tape arguably had slightly better heat-resistance and recording properties.

[2] See Aghion et al. (2004) for similar assumptions.

of profits. Instead, we simply assume that each manager gets half of the realized output, that is he gets $R/2$ if output is R and 0 if output is 0. Obviously, this is a rather stark representation of the constraints faced by real firms in the use of incentive pay. Similar kinds of assumptions have been used elsewhere in the literature (Aghion and Tirole, 1997; Holmström and Tirole, 1998). In Appendix I, we show that this restriction can be derived as a consequence of a simple moral hazard problem. It is straightforward, albeit algebra-intensive, to relax this assumption and allow for a rich set of budget-balancing sharing rules. The yield is some predictions on the interplay between ownership allocations and profit shares; the modified model of the firm can easily be embedded in our framework. The results in Section 3 will go through with only minor modification.[3]

This leaves out a logical possibility:[4] the managers might use a third party "budget breaker" who will pay the firm if there is success and will be paid out of the liquidity available in the firm if there is failure. Using third parties in this way may improve efficiency, but only if the third party gets more when the firm fails than when it succeeds. Apart from the undesirable incentive problems this creates (the third party may want the firm to fail), this modification would not change the basic message of this paper.[5]

When $\omega = 0$, each manager retains ownership of his original assets, and, following the literature, we refer to this situation as *non-integration.* As ω increases beyond 0, we have an increasing degree of integration (a growing fraction of the assets are owned by 2), until with $\omega = 1$ we have full integration. (The symmetric cases with $\omega < 0$ correspond to 1-ownership, but

[3] See our compagnion paper Legros-Newman (2007).

[4] There are three others. First, that the managers "swap" assets: in additon to ω, which indicates how many of 1's assets are shifted to 2, the contract would have an additional variable ψ indicating how many of 2's assets are shifted to 1. Second, that the managers pledge their liquidity to increase the total revenue available *after* the output is realized. Third that agents use external finance, i.e., sign debt contracts. We show in Appendix I that none of these possibilities can improve on contracts as we define them.

[5] It would, however, make the analysis more complex; in particular, we would lose the simple supply-demand analysis that we perform here. For an example of the use of third parties in the formation of firms when there are liquidity constraints see Legros and Newman (1996).

will not occur in any competitive equilibrium of our model, given the greater scarcity of 2's.) Since ω not only describes the ownership structure but also provides a scalar measure of the fraction owned by one party, we shall often refer to its (absolute) value as the *degree of integration* of the firm.

2.1.3 The Feasible Set for a Firm

Given the incentive problems arising from contractual incompleteness, it should come as no surprise that the first-best solution (in which $q(k) = A$ for all k) cannot be attained. For tasks $k \in [0, 1)$, when manager 1 makes the planning decision, he will underprovide q since he bears the full cost of the decision but gets only half of the revenue benefit. By contrast, if the plan is made by manager 2, that manager will overprovide q since by increasing q, expected output increases and 2 bears no cost.

Since the profit shares are fixed, without liquidity, the only remaining way to allocate surplus is to modify the degree of integration ω. Given a contract (ω, t) the two managers subsequently choose q noncooperatively to maximize their corresponding objectives:

$$
\begin{aligned}
u_1(\omega, t) &= \max_{q(k)\in[0,1],k\in[0,1-\omega]} \frac{A}{2}\int_0^2 q(k)dk - \frac{1}{2}\int_0^1 q(k)^2 dk - t \\
u_2(\omega, t) &= \max_{q(k)\in[0,1],k\in(1-\omega,2]} \frac{A}{2}\int_0^2 q(k)dk - \frac{1}{2}\int_1^2 q(k)^2 dk + t.
\end{aligned}
$$

It is straightforward to see that manager 1 will choose the same level of $q(k)$, namely $q(k) = \frac{A}{2}$ on the assets he controls, and that manager 2 will set $q(k) = 1$ for $k \in [1 - \omega, 1)$ and $q(k) = \frac{A}{2}$ for $k \in [1, 2)$. Then, the payoffs associated to a contract (ω, t) are,

$$u_1(\omega, t) = \frac{3}{8}A^2 - \omega\frac{(2-A)^2}{8} - t \tag{1}$$

and

$$u_2(\omega, t) = \frac{3}{8}A^2 + \omega\frac{A(2-A)}{4} + t \tag{2}$$

Because reallocating control rights does not affect the feasible set of planning decisions, a manager gaining control of additional assets cannot be worse off.[6]

Proposition 1 *A manager's payoff is nondecreasing in the fraction of assets he owns.*

Observe that the total surplus generated by a contract ω, $u_1(\omega, t) + u_2(\omega, t)$, is maximal at $\omega = 0$ (nonintegration) provided

$$A < 2/3. \tag{3}$$

To focus on the interesting issues, we shall always assume that A is in this range; otherwise, there is no tradeoff between surplus generation and surplus division: full integration is second-best optimal and is always the equilibrium outcome.

When agents of types 1 and 2 have liquidity l_1 and l_2, the set of feasible payoffs they can attain via contracting is defined by (1) and (2), along with uncontingent transfers that do not exceed the initial liquidities. Specifically, the feasible payoff set is

$$U(l_1, l_2) = \{(u_1, u_2) : \exists (\omega, t) \in [0,1] \times [-l_2, l_1], u_i = u_i(\omega, t)\}.$$

Given the risk neutrality of the managers, ex-ante transfers do not affect total surplus; in particular we have $u_1(\omega, t) = u_1(\omega, 0) - t$ and $u_2(\omega, t) = u_2(\omega, 0) + t$. Figure 1 illustrates a typical feasible set when agents have liquidity l_1 and l_2.

Note that the Pareto frontier when there is no liquidity and when $v_2 \geq 3A^2/8$ is

$$v_2 = -\alpha v_1 + (\alpha + 1)\frac{3}{8}A^2, \tag{4}$$

where $\alpha = 2A/(2 - A) < 1$.

When managers have no liquidity, $t = 0$ and as 1's payoff decreases, the number of assets 2 owns (weakly) increases. At the same time total surplus is

[6] This invariance of the feasible set to transfers of control stems from the absence of investments made before q is chosen; in particular it extends to cases in which there are noncontractible investments ex post and/or in which sharing rules are flexible. See Legros-Newman (2005).

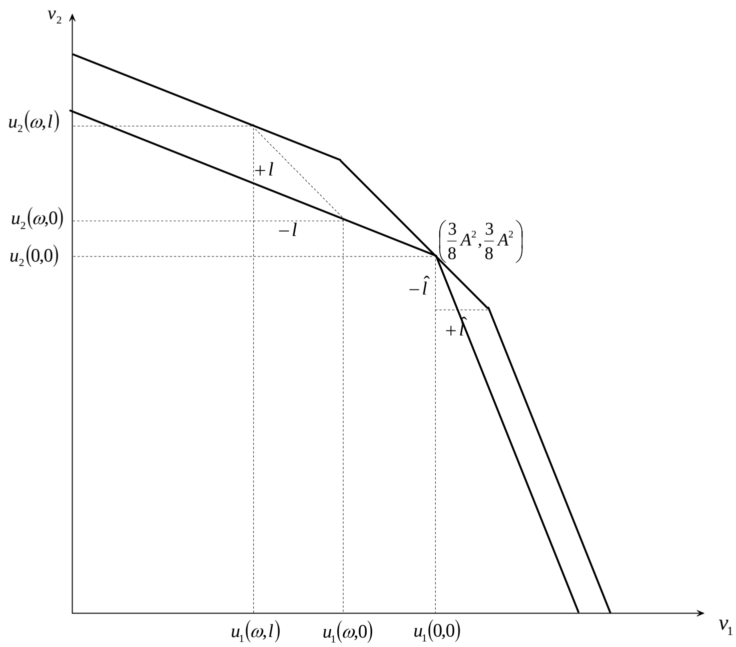

Figure 1: Feasible Set

decreasing; thus it is fair to say that here *reallocations of ownership are used to transfer surplus, not merely to generate it.* Notice as well that this mode of surplus transfer is less efficient than transferring cash; thus any liquidity that the managers have to spare will be used first to meet the surplus division demanded by the market before they transfer ownership.

2.2 Market Equilibrium

Market equilibrium is a partition of the set of agents into coalitions that share surplus on the Pareto frontier; the partition is stable in the sense that no new firm could form and strictly improve the payoffs to its members. The only coalitions that matter are singletons and pairs (which we call "firms") consisting of one type 1 production unit $i \in I$ and one type 2 production unit $j \in J$. Since there is excess supply of type 1 production units, there is at least a measure $1 - n$ of type 1 managers who do not find a match and who therefore obtain a surplus of zero. Stability requires that no unmatched type 1 manager can bid up the surplus of a type-2 manager while getting a positive surplus. Necessary conditions for this are that all type 2 managers are matched and that they have a surplus not smaller than $u_2(0,0) = \frac{3}{8}A^2$. As is apparent from the construction of the feasible set, when $v_2 > u_2(0,0)$, payoffs on the Pareto frontier are achieved by transferring the liquidity of type 1 only, that is, the 2's liquidity does not matter. Thus all 2's are equally good as far as a 1 is concerned and they must therefore receive the same surplus.[7]

This "equal treatment" property for the 2's is an important simplification relative to most assignment models in which there is heterogeneity on both sides of the market.[8] Identify the set of firms F with the index of the type 1 manager in the firm "firm i" indicates that the firm consists of the i-th type 1 production unit and a type 2 manager.

Definition 1 *An equilibrium consists of a set of firms $F \subset I$ with Lebesgue measure n, a surplus v_2^* received by the type 2 managers, and a surplus function $v_1^*(i)$ for type 1 managers such that:*

(i) (feasibility) For all $i \in F$, $(v_1^*(i), v_2^*) \in U(l_1(i), 0)$. For all $i \notin F$, $v_1^*(i) = 0$.

(ii) (stability) For all $i \in I$, for all $j \in J$, for all $(v_1, v_2) \in U(l_1(i), l_2(j))$, either $v_1 \leq v_1^*(i)$ or $v_2 \leq v_2^*$.

2.2.1 Characterizing Market Equilibrium

Since the type-2 managers have the same equilibrium payoff, we can reason in a straightforward demand-and-supply style by analyzing a market in which the traded commodity is the type 2's. We construct the demand as follows. The amount of surplus a 1 is willing *and able* to transfer to a 2 depends on how much liquidity he has. The willingness to pay of type 1 is the value of

[7] If in firm (i,j) type 2 j has a strictly larger surplus than type 2 j' in the firm (i',j'), the firm (i,j') could form and both i and j' could be better off since the Pareto frontier is strictly decreasing.

[8] This is where the assumption of no third party budget-breaking comes in. Without it, treating all 2's as perfect substitutes regardless of their liquidity would not be possible.

the problem

$$\max_{(\omega,t)} u_2(\omega,t)$$
$$u_1(\omega,0)\geq t$$
$$t\in[0,l_1].$$

In the contract (ω,t), the type 1 manager gets $u_1(\omega,t)+l_1$; the opportunity cost of the contract is to be unmatched and get l_1; hence the manager is willing to contract when $u_1(\omega,t)\geq 0$ which is equivalent to the condition stated since $u_1(\omega,t)=u_1(\omega,0)-t$. Simple computations show that the solution to this program is

$$\begin{aligned}\text{If } l_1 &\geq \frac{3}{8}A^2, (\omega,t)=(0,\frac{3}{8}A^2), \\ \text{If } l_1 &< \frac{3}{8}A^2, (\omega,t)=(\frac{3A^2-8l_1}{(2-A)^2},l_1).\end{aligned} \tag{5}$$

The willingness of a type 1 manager to pay for matching with a type 2 manager is then.

$$W(i)=\begin{cases}\frac{3}{4}A^2 & \text{if } l_1(i)\geq\frac{3}{8}A^2\\ \frac{3}{8}A^2+\left(\frac{3}{4}A^2-2l_1(i)\right)\frac{A}{2-A}+l_1(i) & \text{if } l_1(i)\leq\frac{3}{8}A^2\end{cases} \tag{6}$$

Since the frontier has slope magnitude less than unity above the 45^o-line, and since $l_1(i)$ is increasing in i, the willingness to pay of i is nondecreasing in i. If type 2 agents must get a payoff of v_2, the type 1 agents who are willing and able to pay this price is

$$D(v_2)=1-\min\{i\in[0,1]:W(i)\geq v_2\}.$$

The supply is vertical at n, the measure of 2's. Equilibrium is at the intersection of the two curves: this indicates that n of the 1's are matched, as claimed above, and that the marginal 1 is receiving zero surplus.

Proposition 2 *The equilibrium set of firms is $F=[1-n,1]$ and the equilibrium surplus of type 2 managers is*

$$v_2^*=\min\{\frac{3}{4}A^2,W\left(\bar{l}_1\right)\},$$

where $\bar{l}_1=l_1(1-n)$.

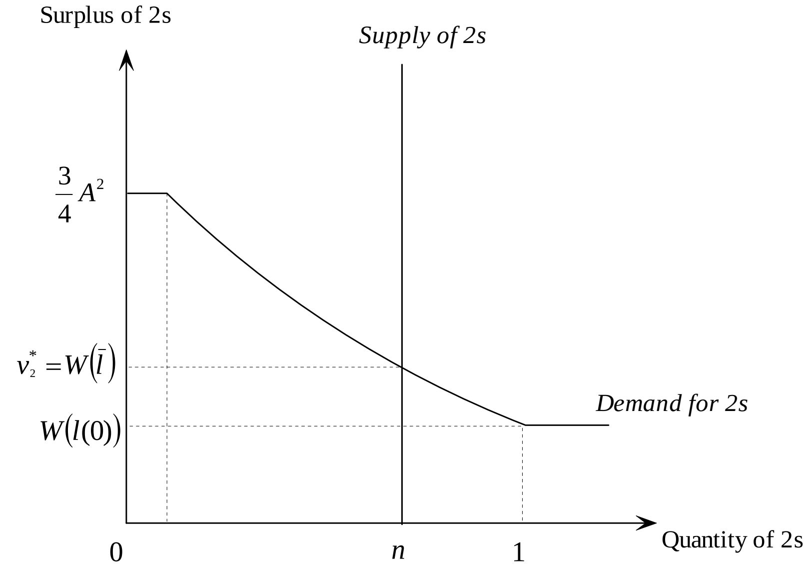

Figure 2: The Market for Ownership

If $\bar{l}_1 \geq 3A^2/8$, efficiency is obtained since each matched type 1 is able to pay $3A^2/8$ to the type 2 manager; note that in this case the equilibrium surplus of all type 1 managers is zero. We will consider below situations in which $\bar{l}_1 < 3A^2/8$.

In this case, the equilibrium surplus of type 2 managers is $v_2^* = W\left(\bar{l}_1\right) < \frac{3}{4}A^2$. The marginal type 1 manager $1-n$ has a surplus of 0, but the inframarginal type 1 managers with liquidity $l_1 > \bar{l}_1$ will be able to generate a positive surplus for themselves since they can transfer more liquidity than the marginal type 1. The surplus of an inframarginal type 1 with liquidity $l_1 \geq \bar{l}_1$ when the price is v_2^*, it the value of the problem

$$\max_{\omega} u_1\left(\omega, t\right) + l_1$$

$$u_2\left(\omega, 0\right) + t = v_2^*$$

$$t \leq l_1$$

The solution to this problem is $\omega\left(v_2^*, l_1\right), t\left(v_2^*, l_1\right)$ where

$$\begin{aligned}
\omega\left(v_2^*, l_1\right) &= 0, t\left(v_2^*, l_1\right) = v_2^* - \frac{3}{8}A^2, \text{ if } l_1 \geq v_2^* - \frac{3}{8}A^2 \\
\omega\left(v_2^*, l_1\right) &= 4\frac{v_2^* - \frac{3}{8}A^2 - l_1}{A\left(2-A\right)}, t\left(v_2^*, l_1\right) = l_1 \text{ if } l_1 \leq v_2^* - \frac{3}{8}A^2.
\end{aligned} \tag{7}$$

In this model, there is a piece-wise linear relationship between the liquidity, the degree of integration ω, the level of output and the managerial welfare. The fact that the degree of integration is a globally convex and decreasing function of liquidity and is an increasing function of the price v_2^* illustrates the internal and external effects we alluded to. The effect of a higher level of liquidity may be overcome by an increase in the price v_2^*. Of course, the price itself reflects the liquidity and the technology available in the economy. To study the effects of shocks systematically, we must take account of the fact that v_2^* itself is endogenous, which we do in the next section.

Lemma 3 *The degree of integration $\omega\left(v_2^*, l_1\right)$ is piece-wise linear: it is linear nondecreasing in v_2^*, nonincreasing in l_1 when $l_1 < v_2^* - \frac{3}{8}A^2$, it is equal to zero when $l_1 \geq v_2^* - \frac{3}{8}A^2$.*

3 Comparative Statics of Market Equilibrium

In equilibrium, there will typically be variation in organizational structure across firms, and this is accounted for by variation in their characteristics. In particular, "richer" firms are less integrated and generate greater surplus for the managers.[9]

But more liquidity overall can also lead to *more* integration: if the marginal firm's liquidity v_2^* rises, possibly by more than an inframarginal firm's gain in liquidity. As a result, the inframarginal firm may become more inte-

[9] Holmström and Milgrom (1994) emphasize a similar cross-sectional variation in organizational variables. In their model, the variation reflects differences in technology but not differences in efficiency, since all firms are surplus maximizing. Here by contrast, the variation stems from differences in liquidity and reflects differences in organizational efficiency.

grated, and indeed it is possible that the economy's average level of integration may increase via this external effect.

We shall consider three types of shocks that may lead to reorganizations in the economy: changes in the relative scarcity of the two types, changes in the distribution of liquidity and changes in the technological parameter A.

3.1 Relative Scarcity

In order to isolate the "external effect" our first comparative statics exercise involves changes in the tightness of the supplier market, i.e., in the relative scarcities of 1's and 2's.

Suppose that the measure of 2's increases, for instance from entry of downstream producers into the domestic market from overseas. Then just as in the standard textbook analysis, we represent this by a rightward shift of the supply schedule: the price of 2's decreases. Indeed, as n increases the liquidity of the marginal type 1 decreases since $l_1(1-n)$ is decreasing with n. What of course is different from the standard textbook analysis is that this change in price entails (widespread) refinancing or corporate restructuring.

Let $F(n)$ be the set of firms when there is a measure n of type 2 firms. As n increases to $\hat{n}$, there is an equilibrium set $F(\hat{n})$ where $F(n) \subset F(\hat{n})$; that is after the increase in supply, new firms are created but we can consider that previously matched managers stay together. The surplus of *all* type 1 managers in firms in $F(n)$ increases. Managers in a firm in $F(n)$ will restructure (decrease ω) in response to the reduction in the equilibrium value of v_2^*. The analysis is similar in the opposite direction: a decrease in the measure of 2's leads to an increase in v_2^*. Thus, we have

Proposition 4 *In response to an increase in the measure of 2's, the firms remaining in the market become less integrated.*

It is worth remarking that if the relative scarcity changes so drastically that the 2's become more numerous, then 1's get the preponderance of the surplus and tend to become the owners; the analysis is similar to what we have seen, with the role of 1's and 2's reversed. The point is that the owners of the integrated firm gain control because they are scarce, not because it is efficient for them to do so: in this sense, organizational power stems from market power.

As an application, if we interpret the assets as tasks or duties, the model can suggest a simple explanation for the "empowerment of talent" that has been noted by several authors (see Marin and Verdier, 2003; and references therein). Here empowerment means giving the highly skilled and professional workers decision rights over more of these tasks, i.e., more discretion. A large literature in labor economics has shown that in the last thirty years the demand for skilled workers in North America and Western Europe has outstripped the (nonetheless growing) supply. Interpreting the 1's in our model as the corporate demanders of talent and the 2's as the talented workforce, relative rightward shifts in demand mean more surplus to the type 2's, which will manifest itself variously as bigger cash payments and greater "empowerment," often in combination.[10] As long as firms' liquidity is restricted (relative to the scale of operations), tighter labor markets mean more control by these workers, not merely higher wages.

However, this story is heuristic: increases in demand for the talented workforce most likely emanate from entry of new firms (which in turn entails a change in the liquidity distribution among the active firms) and from increases in productivity (e.g., "skill-biased technical change"). Thus, a general analysis of the effects of changes in relative scarcity requires separate consideration of the effects of changes in liquidity and productivity; we provide this in the next two subsections.

3.2 Liquidity Shocks

Evaluating changes in the liquidity distribution is complicated by the interplay of the internal and external effects described above. The dependence of the ownership structure ω on the type-1 liquidity l_1 and the equilibrium sur-

[10] And greater shares of profit (use of bonus schemes or possibly stock options) in a model such as the one discussed in our previous working paper, in which profit shares are endogenous,

plus v_2^* was summarized in Proposition 2 and Lemma 3. Equipped with this result, we can derive simple comparative statics. We focus on the aggregate degree of integration in the market,

Suppose the initial liquidity endowment is $l_1(i)$ and that the economy receives a "shock" that transforms $l_1(i)$ into $\eta(l_1(i))$; the shock function $\eta(\cdot)$ is assumed continuous and increasing. We wish to compare the degree of integration before and after the shock. Let $\omega(v_2^*, l_1)$ be the degree of integration in a firm with a type 1 manager having liquidity l_1 when the equilibrium surplus to 2 is v_2^*.

The change in the average degree of integration is

$$\int_{1-n}^{1} \omega\left(v_2^*(\eta(\bar{l}_1)), \eta(l_1(i))\right) di - \int_{1-n}^{1} \omega\left(v_2^*\left(\bar{l}_1\right), l_1(i)\right) di$$

where $\eta(\bar{l}_1)$ and $\bar{l}_1$ are the respective marginal liquidity levels and the notation $v_2^*(\cdot)$ reflects the dependence of the 2's equilibrium surplus on the marginal liquidity as articulated in Proposition 2.

We study some special cases that place more structure on the problem.

3.2.1 Positive Shocks to Liquidity

Suppose that the shocks $\eta(l_1) - l_1$ are both positive and nondecreasing in l_1. Note that a *uniform* shock in which every type 1 receives the same increase to his endowment is a special case. So is a multiplicative shock in which the percentage increase to the endowment is the same for all 1's. The shock will increase both the willingness to pay of the type 1's, which, via the internal effect, reduces the degree of integration, but also will increase the equilibrium surplus to 2, which, via the external effect, has the opposite impact. However, it is a simple matter to demonstrate that in this case, the internal effect dominates: more liquidity implies less integration.

The change in v_2^* is the change in the willingness to pay $W\left(\bar{l}_1\right)$ times the change in $1-n$'s liquidity. From (6), $W'(l_1) = 1 - \alpha < 1$: this is smaller than the liquidity increase and thus $1-n$ can cover the new price and still buy back some assets; all $i > 1-n$ have at least as large an increase in their endowments and can therefore do the same. Of course, negative, nonincreasing shocks yield the opposite changes in surplus and organization.

Proposition 5 *Under positive, nondecreasing, shocks to the liquidity distribution of type 1 the aggregate degree of integration decreases.*

To maintain this conclusion, the proviso that the shocks are monotonic can be relaxed, but not arbitrarily. Positive shocks alone are not enough, and having more liquidity in the economy may actually imply that there is higher overall degree of integration. Intuitively, if the positive shock hits only a small neighborhood of the marginal type 1, the price v_2^* will increase and the inframarginal unshocked firms will choose to integrate more in response to the increase in v_2^*. This is formally stated below. (Missing proofs in the text are available in Appendix II.)

Proposition 6 *There exist first order stochastic dominant shifts in the distribution of type-1 liquidity that lead to more integration.*

We turn now to consider other types of distributional changes.

3.2.2 Liquidity Heterogeneity and Degree of Integration

Suppose first that l_1 and $\eta \circ l_1$ have a single crossing property at $1-n$: $\eta(l_1(i)) < l_1(i)$ for $i < 1-n$ and $\eta(l_1(i)) > l_1(i)$ for $i > 1-n$. Since all *matched* 1's have greater liquidity and the equilibrium surplus v_2^* is by construction fixed, the surplus accruing to 2 falls in every firm, and the economy becomes less integrated. If one supposes further that $\int_0^1 l_1(i)di = \int_0^1 \eta(l_1(i))di$, then in fact the new liquidity distribution (which is essentially the inverse of the liquidity endowment function) is riskier then the old one in the sense of second order stochastic dominance (equivalently, it is more unequal in the sense of Lorenz dominance). This is an instance in which *increasing inequality may lower integration and raise efficiency.*

Now maintain the common marginal liquidity assumption, and denote the inverses of the restrictions of $l_1(\cdot)$ and $\eta(l_1(\cdot))$ to $[1-n, 1]$ as l_1^* and $(l_1 \circ \eta)^*$: these are just the conditional distributions of liquidity above $\bar{l}_1$. Suppose that l_1^* is more unequal than $(l_1 \circ \eta)^*$. Then because ω is linear in

l_1 and v_2^* is the same for both distributions, there is less integration under the new distribution.

This suggests the opposite of the previous conclusion: *increasing inequality may raise integration and lower efficiency.* These two results are easily reconciled: while the single-crossing result refers to the distribution for the economy as a whole, the second result refers to the distribution *only among the existing firms.* From the empirical point of view the important distinction is between overall inequality and inequality among the selected sample of matched firms, which in this model at least, can work in opposite directions.

If one is interested in minimizing the degree of integration in the economy (this maximizes the surplus), it is clear that one wants the marginal liquidity as low as possible, so as to minimize the equilibrium price, and one wants to maximize the liquidity of the inframarginal firms. If $\bar{l}_1 = 0$, $v_2^* = (1+\alpha)\,3A^2/8$, and the degree of integration $\omega\left(v_2^*, l_1\right)$ for an inframarginal firm with liquidity l_1 is as given in (7). If $G\left(l_1\right)$ is the distribution of liquidity among the inframarginal type 1's, the average degree of integration is

$$E\omega = \int \omega\left(v_2^*, l_1\right) dG\left(l_1\right).$$

Because the function $\omega\left(v_2^*, l_1\right)$ is globally convex in l_1, $E\omega$ is minimal when all firms have the same level of liquidity; more generally, there is a simple characterization of the set of distributions that minimize average integration in the economy.

Proposition 7 *Let L be the average liquidity in the economy. The degree of integration is minimized when the marginal type 1 has no liquidity and when the distribution of liquidity among the inframarginal type 1 agents is such that G has a support in $[0, \alpha 3A^2/8]$ when $L < \alpha 3A^2/8$ and G has support in $[\alpha 3A^2/8, \infty)$ when $L > \alpha 3A^2/8$.*

This leaves considerable freedom to vary the distribution without changing the average level of integration. However, the degree of heterogeneity of ownership structures will be more sensitive to the liquidity distribution. The linearity of the degree of integration in l_1 implies a monotonic relationship between the first two moments of the distribution of liquidity and those of the distribution of ownership when all firms choose integration.

Proposition 8 *Consider two distributions of liquidity G and H such that $v_2^{*G} = v_2^{*H} = v_2^*$ and that the support of inframarginal firms is $\left[\bar{l}_1, v_2^* - \frac{3}{8}A^2\right]$. Then the variance of the degree of integration is lower with G than with H if and only if the variance of liquidity is lower with G than with H. Similarly the mean degree of integration is lower with G than with H if and only if the mean liquidity is lower at G than at H.*

Until now we have considered distributions having the same marginal level of liquidity, and differences in the distribution of ownership reflect the internal effect of liquidity. Consider now two distributions of liquidity G and H and suppose that the maginal level of liquidity is larger at H than at G : $G(\bar{l}_1^G) = H(\bar{l}_1^H) = 1 - n$ implies $\bar{l}_1^G < \bar{l}_1^H$. It follows that the price of type 2 is greater with H than with G; in fact from (6), $v_2^{*H} = v_2^{*G} + (1-\alpha)(\bar{l}_1^H - \bar{l}_1^G)$. Hence each type 1 who is inframarginal with H uses a greater degree of integration than with G. However this is not incompatible with a decrease in the average degree of integration if the average liquidity increases enough: the internal effect must compensate for the external effect.

For a distribution K, let $\pi^K = K(v_2^{*K} - \frac{3}{8}A^2) - K\left(\bar{l}_1^K\right)$ be the measure of firms with a positive degree of integration and let $\mu^K = \frac{1}{\pi^K}\int_{\left\{l \geq \bar{l}_1^H\right\}} l dK(l)$ be the average liquidity levels among the type 1 in this group.

Proposition 9 *Consider two distributions of liquidity G and H. If the measure of firms having a positive degree of integration is the same, that is if $\pi^G = \pi^H$, then the average degree of ownership is lower with H than with G if and only if* $\underbrace{(1-\alpha)(\bar{l}_1^H - \bar{l}_1^G)}_{\text{change in price}} < \underbrace{\mu^H - \mu^G}_{\text{change in average liquidity}}$.

3.3 Productivity Shocks

The external effect outlined in the previous section offers a propagation mechanism whereby local shocks that affect only a few firms initially may nevertheless entail widespread reorganization. Empirically this implies that to explain why a particular reorganization happens, there is no need to find a smoking gun in the form of a change *within* that organization: instead the impetus for such change may originate elsewhere in the economy. The same logic applies to other types of shocks, most prominently among them innovating productivity shocks. These are often thought to be the basis of large-scale reorganizations such as merger waves (Jovanovic and Rousseau, 2002).

We model a (positive) productivity shock or technological innovation as an increase in A. We suppose the shock inheres in the type 1's. Suppose that in the initial economy, all firms have the same technology; after a shock, a subset of them, an interval $[i_0, i_1]$, have access to a better technology (for them, $\hat{A} > A$). We restrict ourselves to considering "small" shocks in the sense that $\hat{A} < 2/3$.

Raising A modifies the game that managers play given a contract ω : it is clear from (1) and (2) that both managers obtain a larger surplus from a given contract. Hence the feasible set expands and the type-1's willingness to pay also increases. What is perhaps less immediate is that there is also more transferability within the firm.

Lemma 10 *Let A be the initial productivity. After a positive productivity shock,*
(i) the feasible set expands.
(ii) For any $t < 3A^2/8$, the degree of integration solving $u_1(\omega, t) = 0$ increases.
(iii) there is more transferability in the sense that the slope of the frontier is steeper in the region $v_2 \geq v_1$ when A increases.

Proof. (i) From (3), differentiating (1) and (2) with respect to A, shows that for any contract (ω, t), both $u_1(\omega, t)$ and $u_2(\omega, t)$ are increasing in A. (ii) Use (5). (iii) The absolute value of the slope of the frontier in the region $v_2 \geq v_1$is $\alpha = 2A/(2-A)$ which is also increasing in A. ■

The willingness to pay (6) depends on the technology available to the firm; since we assume that some firms have a different technology, we can make explicit the relationship between technology and willingness to pay:

$$W(i;A_i) = \min\left\{\frac{3}{4}A_i^2, \frac{3}{8}A_i^2 + \left(\frac{3}{4}A_i^2 - 2l_1(i)\right)\frac{A_i}{2-A_i} + l_1(i)\right\}, \quad (8)$$
$$\text{with} \begin{cases} A_i = A & \text{if } i \notin [i_0, i_1] \\ A_i = \hat{A} & \text{if } i \in [i_0, i_1]. \end{cases}$$

Lemma 10(iii) implies that – for a *fixed* equilibrium surplus for 2 – a shocked firm integrates less since it is able to transfer surplus via ω in a more efficient way. Hence when the 2s' equilibrium surplus is fixed, positive technological shocks lead to less integration in the economy.

However, Lemma 10(ii) implies that when the marginal firm is shocked, the price will increase. Since by (iii) there is more transferability with ω, liquidity has less value: the inefficiency linked to the use of integration is lower and integration is a better substitute to liquidity transfers. This implies that type 1 agents find it more expensive, in terms of liquidity, to "buy" decision rights or reduce the degree of integration. Therefore, if the 2s' equilibrium surplus increases, there is a force toward more integration. Unshocked firms certainly integrate more; for shocked firms, we show below that while they benefit internally from the technological shock, the countervailing effect of an increase in the 2s' equilibrium surplus dominates. The net effect is towards more integration *for all firms in the economy* if the marginal firm is a shocked firm. Other results are contained in the following proposition:

Proposition 11 *(i) (Inframarginal shocks) If $i_0 > 1-n$ the shocked firms become less integrated and the unshocked firms remain unaffected*
(ii) (Marginal shocks) If $1-n \in (i_0, i_1)$ and $1-n$ is still the marginal type 1 agent, the equilibrium price increases and all firms, shocked and unshocked, integrate more.
(iii) If there is a uniform shock to the technology ($i_0 = 0, i_1 = 1$) each firm integrates more.

Thus the effect of small positive productivity shocks depends on what part of the economy they affect. If they occur in "rich" firms (case (i)), only the innovating firms are affected, and they become less integrated. But innovations that occur in "poor" firms (case (ii)) may affect the whole economy: even firms that don't possess the new technology become more integrated.

The first result suggests that "local" reorganizations involving established firms originate within those firms, and is consistent with the view (e.g., Rajan and Wulf, 2003) that recent technological advances may be responsible for a palpable "flattening" of corporate hierarchies, at least if we interpret this as reduced integration. On the other hand, since new technologies are often introduced by new, small firms – the very ones that are likely to be liquidity poor – the second result suggests that widespread reorganizations (such as merger waves) are more likely to be set off by the entry of new firms embodying these new technologies. This interpretation supports the view (Jovanovic and Rousseau, 2002) that technological advances are responsible for the recent wave of mergers, i.e., increased integration! The point is that technological advances can have both effects: the origins of the innovations are crucial to determining how reorganization plays out in the economy.

Proposition 11 (iii) emphasizes that, in contrast to reduced integration after a positive uniform liquidity shock, a uniform positive productivity shock will have the *opposite* effect. In this sense the external effect of productivity shocks is more powerful than that for liquidity shocks.

This result also helps confirm the conjecture at the end of Section 3.1 that increased demand for skilled workers would lead to their empowerment. The growth in demand for skilled workers alluded to in Section 3.1 is often attributed to "skill-biased technical change." In terms of our model, with the 2's interpreted as the skilled labor force, this corresponds to a widespread increase in A, and the result implies that the skilled will get more control.

Similarly, the introduction of on-board computers and similar technologies in the trucking industry in the 1980s has been associated with a widespread restructuring toward the employee driver and away from the independent trucker (Baker and Hubbard, 2004). Taking the drivers as the long side of the market, one can make sense of this finding by noting that if the new technology is represented by an increase in the level of A available to all firms, the model predicts an increase in integration.

4 Discussion

If one asks the question "who gets organizational power in a market economy?," one is tempted to answer "to the scarce goes the power." There is a tradition in the business sociology literature (reviewed in Rajan and Zingales 2001) which ascribes power or authority to control of a resource that is scarce within the organization. Similar claims can be found in the economic literature (Hart and Moore, 1990; Stole and Zweibel, 1996). Our results suggest that organizational power may emanate from scarcity *outside* the organization, i.e., from market power. But this result has to be qualified somewhat: Proposition 6 suggests that having more liquidity may actually cause one to *lose* power, via what we have called the external effect of shocks to fundamentals. Similarly, the possessors of a new technology, if they are inframarginal, will gain ownership (Proposition 11 (i)), but if they are marginal may lose it. This is evidence of the importance of market effects for the allocation of power inside firms and more generally of their importance for the study of organizations.

We now discuss some other implications of the model.

4.1 Interest Rate

We have assumed that the interest rate (the rate of return on liquidity) is exogenous and is not affected by changes in the liquidity distribution or the technology available to firms. One can easily extend the model to allow for liquidity that yields a positive return though the period of production. Because liquidity in this model is used only as a means of surplus transfer, and not as a means to purchase new assets, the effects of this can be somewhat surprising. Raising this interest rate means that liquidity transferred at the beginning of the period has a higher value to the recipient than before: formally, the effect is equivalent to a multiplicative positive shock on the distribution of liquidity, and by Proposition 5, firms will integrate less if the interest rate increases, and will integrate more if the interest rate decreases. If liquidity transfers made in the economy affect the interest rate, then increases in the aggregate level of liquidity, by lowering interest rates, may constitute a force for integration above and beyond that suggested by the example in Proposition 6. These observations suggest that the relationship between aggregate liquidity and aggregate performance is unlikely to be straightforward; whether the potentially harmful organizational consequences would counter or even outweigh the traditional real investment responses is a question for future research.

4.2 Product Market

If we imagine all the firms sell to a competitive product market, then the selling price inheres in R, which we have thus far viewed as exogenous (for instance the supplier market is contained in a small open economy, with prices determined in the world market). But if instead price is determined endogenously in the product market, then shocks to product demand will change the price, which has the effect of changing A for all firms. Suppose the price increases. Then from Proposition 11(iii) in the analysis of productivity shocks, all firms become more integrated.

Next, notice that expected output is proportional to A for nonintegrated firms and proportional to $A + \omega(1 - A/2)$ for integrated ones. Integrated firms produce more than nonintegrated ones, and since from Proposition 11 the aggregate ω increases with A, aggregate output rises in response to an increase in A. Thus, if product price rises, so does output, and we conclude that the product supply curve is upward sloping. An increase in consumer demand therefore raises equilibrium price: increasing demand results in greater integration.

What is more, the product market price effect now means that more local shocks will result in widespread reorganization: more than just the very poorest firms in the economy may be "marginal." To see this, suppose a number of perfectly nonintegrated firms innovate. With fixed prices, these firms produce more output, but nothing further happens. With endogenous prices, the increased output in the first instance lowers product price; all other firms in the economy treat this exactly like a (uniform) negative productivity shock: they all become less integrated. Thus product market price adjustment has a kind of "amplification" effect on organizational restructuring.[11]

Previous work has analyzed how the intensity of product market competition may act as an incentive tool for managers.[12] In this literature the set of firms and their internal organization are exogenous. Here we wish to emphasize a causal relation in the opposite direction that becomes apparent once organization is allowed to be endogenous: organizations may affect product market prices, even when there is perfect competition. As discussed in Legros and Newman (2004), the fact that the product market – even a competitive one – can be affected by the internal organization decisions of firms has implications for consumer welfare, the regulation of corporate governance, and competition policy.

5 Appendix I: Contracting

We have defined contracts by (ω, t) and equal sharing of the output ex-post. This definition could be restrictive because it ignores the following four potential extensions.

- *Contingent shares.* A contract could specify state contingent revenues $x_i(R), x_i(0)$ to $i = 1, 2$.
- *Debt contract.* Type 1 borrows B from a financial institution in exchange for a repayment of D after output is realized.
- *Ex-post transfers of liquidity.* The total liquidity available in the firm is $L = l_1 + l_2$. This liquidity can be transferred either ex-ante or added to the revenue of the firm ex-post.
- *Asset swapping.* This is a means of effectively committing the managers to high levels of q. This commitment is only worthwhile if productivity

[11] Of course the effect is self-limiting because as they become less integrated, they lower their output, causing the price to go up again. As shown in Legros-Newman (2004), these product market effects can be more pronounced in models that rely on somewhat different trade-offs in their basic organizational model than the one considered here.

[12] See Hart (1983), Scharfstein (1988) and Schmidt (1997).

is sufficiently high relative to costs, which will nto be the case given our parametric case. If assets are to be swapped, we can characterize the situation via two ownership parameters ψ and ω : manager 1 owns $k \in [0, 1-\omega)$ and $k \in [2-\psi, 2)$, and 2 owns the other assets.

We show that our definition of contracting is without loss of generality by introducing into the contracting model described in the text a moral hazard element. The incentive compatibility condition associated to this moral hazard problem will restrict the marginal revenue $x_i(R) - x_i(0)$ to be equal to $R/2$ for each agent. The result will then follows.

A manager has the opportunity to divert revenue R in the high state by choosing an effort $e \in [0,1]$: if the state is high, with probability e the perceived output in the firm will be R while with probability $1-e$ the perceived output in the firm is 0, in which case the manager diverts a share cR and $(1-c)R$ is lost; if the state is low, the perceived output in the firm will be 0 independently of e. Only one manager has the opportunity to divert (the identity of that manager being chosen by nature).

The ex-post revenue of the firm consists of two components: the risky component with realizations 0 and R and a non-risky component denoted by T, typically the amount of ex-ante liquidity than is pledged (in an escrow account) to the firm. By choosing e, the manager can "hide" R but not T.

Let $x_i(R)$ and $x_i(0)$ be the revenues to the manager if the perceived realization of the risky component is R and 0 respectively. Then, with $e = 1$, the expected revenue to the manager is $px_i(R) + (1-p)x_i(0)$. With $e = 0$, the expected revenue is $p(cR + x_i(0)) + (1-p)x_i(0)$. Hence $e = 1$ is optimal when $x_i(R) \geq cR + x_i(0)$, or $x_i(R) - x_i(0) \geq cR$. Clearly if $c > 1/2$, both incentive compatibility constraints cannot hold. By choosing $c = 1/2$, we have

$$x_i(R) - x_i(0) = R/2 \tag{9}$$

as claimed. If $c < 1/2$, there is scope for unequal marginal revenues for the two agents, but it still remains true that there is no loss in assuming that $T = 0$ and that debt contracts are weakly dominated by non-debt contracts.

Suppose that (9) holds. A contract is $((\omega,\psi),(B,D),(x_1,x_2)(t_1,t_2))$, where we assume without loss of generality that only agent 1 engages in a debt contract. Let x_i^* be the state contingent revenue equal to $R/2$ in state R and 0 in state 0. We want to show that there exists a contract $\left((\hat{\omega},0),(0,0),(x_1^*,x_2^*),\left(\hat{t}_1,L-\hat{t}_1\right)\right)$ that leads to payoffs that are weakly greater for both managers. We establish this result sequentially: first by showing that $((\omega,\psi),(B,D),(x_1,x_2),(t_1,t_2))$ is weakly dominated by the contract $((\omega,\psi),(0,0),(x_1^*,x_2^*),(t_1+x_1(0),t_2+x_2(0)))$, where neither debt nor ex-post transfers of liquidity are used, second by showing that this contract is dominated by a contract in which only part of the assets of type 1 are reassigned to type 2 $\left((\hat{\omega},0),(0,0),(x_1^*,x_2^*),\left(\tilde{t},L-\tilde{t}\right)\right)$.

Step 1. In a contract $((\omega,\psi),(B,D),(x_1,x_2),(t_1,t_2))$, feasibility requires that $t_1+t_2\leq L+B$ and $t_i\geq 0$, $i=1,2$. We write $t=t_1+t_2$ the total liquidity ex-ante and $T=L+B-t$ the liquidity that is pledged to the firm. Ex-post total revenues are then T and $T+R$. Managers get state contingent revenues $x_i(0),x_i(R)$ satisfying budget balancing and limited liability: $x_1(0)+x_2(0)=T$, $x_1(R)+x_2(R)=T+R$, $x_i(0)\geq 0$, $x_i(R)\geq 0$.

If there is a debt contract, manager 1 has to repay $\min\{D,x_1(0)\}$ in state 0 and $\min\{D,x_1(R)\}$ in state R. Since by (9),we need $x_2(R)-x_2(0)=R/2$, we have $x_1(R)-x_1(0)=R/2$, however since manager 1 has to repay the debt, his effective marginal compensation is $x_1(R)-x_1(0)-[\min\{D,x_1(R)\}-\min\{D,x_1(0)\}]$. This is consistent with (9) only if $\min\{D,x_1(R)\}=\min\{D,x_1(0)\}$, or if $D\leq x_1(0)$. In this case, debt is not risky; the creditor makes a non-negative profit only if $D\geq B$, but then we need $x_1(0)\geq B$ and therefore $x_2(0)\leq L+B-t-B=L-t$. It follows that the initial contract $((\omega,\psi),(B,D),(x_1,x_2),(t_1,t_2))$ is weakly dominated by the contract $((\omega,\psi),(0,0),(x_1^*,x_2^*)(t_1+x_1(0),t_2+x_2(0)))$. Since $\sum_{i=1,2}(t_i+x_i(0))=L$, there is no liquidity transfered ex-post.

Step 2. Finally we show that swapping of assets is dominated by no swapping of assets

Consider a contract $((\omega,\psi),(0,0),(x_1^*,x_2^*),(t,L-t))$ consisting of a swap of assets and ex-ante transfers; we denote such contracts by $((\omega,\psi),t)$. We have the following Nash equilibrium payoffs:

$$
\begin{aligned}
u_1(\omega,\psi,t) &= \frac{A}{2}\left((2-\omega-\psi)\frac{A}{2}+\omega+\psi\right)-\frac{1}{2}\left(\omega+(1-\omega)\frac{A^2}{4}\right)-t \\
u_2(\omega,\psi,t) &= \frac{A}{2}\left((2-\omega-\psi)\frac{A}{2}+\omega+\psi\right)-\frac{1}{2}\left(\psi+(1-\psi)\frac{A^2}{4}\right)+t
\end{aligned}
$$

Suppose without loss of generality that $t > 0$ and that $u_2(\omega,\psi,t) - t > u_1(\omega,\psi,t) + t$; then we must have $\omega > \psi$.

Let $\omega^0 = \omega - \psi A/(1-A/2)$; since $A/(1-A/2) < 1$ and $\omega > \psi$, $\omega^0 > 0$. Then, $u_1(\omega^0,0,t) = u_1(\omega,\psi,t)$ while $u_2(\omega^0,0,t) - u_2(\omega,\psi,t) = \psi(2-A-A^2)/4 > 0$ since $A < 1$. By continuity there exists $\hat{\omega} < \omega^0$ such that the contract $((\hat{\omega},0),t)$ strictly pareto dominates the contract $((\omega,\psi),t)$. If $u_2(\omega,\psi,t) - t < u_1(\omega,\psi,t) + t$, a similar argument applies by decreasing the value of ψ appropriately.

6 Appendix II: Proofs

6.1 Proof of Proposition 6

It is enough to provide an example. Consider the liquidity distribution $l_1(i) = \varepsilon i$ where $i \in [0,1]$ and $\varepsilon < \frac{3}{8}A^2$. Suppose that $n = 1$, that is that the marginal liquidity is 0 and the maximal liquidity is ε. The frontier when there is no liquidity is linear and can be written $v_2 = -\alpha v_1 + v^0$, where $\alpha = \frac{2A}{2-A} \in (0,1)$ and $v^0 = (\alpha+1)\frac{3}{8}A^2$. The equilibrium surplus is $v_2^*(0) = v^0$ and if type 1 has liquidity l_1, the degree of integration is given by (7).

Define $\eta(l_1)$ by

$$
\eta(l_1) = \begin{cases} \delta, \text{if } l_1 \leq \delta \\ l_1 \text{ if } l_1 \geq \delta. \end{cases}
$$

where we choose $\delta < \varepsilon$. $\eta(l_1)$ is increasing and continuous. The marginal liquidity is now $\eta(0) = \delta$ and the new equilibrium surplus is $v_2^*(\delta) = v^0 + (1-\alpha)\delta$.

Firms with $i \geq \delta$ have the same liquidity as before but a higher equilibrium surplus accrues to type 2, and therefore a lower equilibrium surplus accrues to type 1, that is they choose a higher level of integration. From (7), if $\varepsilon i > \delta$,

$$\omega\left(v_2^*\left(\delta\right), \varepsilon i\right) - \omega\left(v^0, \varepsilon i\right) = \frac{\left(1-\alpha\right)\delta}{A\left(2-A\right)}$$

By contrast, all firms with $\varepsilon i < \delta$ are marginal after the shock and have the same degree of integration

$$\omega\left(v_2^*\left(\delta\right), \delta\right) = \frac{3A^2 - 8\delta}{\left(2-A\right)^2}$$

while before the shock

$$\omega\left(v^0, \varepsilon i\right) = 4\frac{v^0 - \frac{3}{8}A^2 - \varepsilon i}{A\left(2-A\right)}.$$

There exists $i\left(\delta\right)$ such that $\omega\left(v_2^*\left(\delta\right), \delta\right) - \omega\left(v^0, \varepsilon i\right) > 0$ when $i < i\left(\delta\right)$ and $\omega\left(v_2^*\left(\delta\right), \delta\right) - \omega\left(v^0, \varepsilon i\right) < 0$ when $i < i\left(\delta\right)$. It is immediate that $i\left(\delta\right)$ is a strictly increasing function of δ.

Let

$$\Delta\left(i\right) = \begin{cases} \omega\left(v_2^*\left(\delta\right), \delta\right) - \omega\left(v^0, \varepsilon i\right) & \text{if } i \leq \delta/\varepsilon \\ \frac{(1-\alpha)\delta}{A(2-A)} & \text{if } i > \delta/\varepsilon. \end{cases}$$

In the aggregate, the change in the average degree of integration is

$$\Delta\left(\delta\right) = \int_0^{i(\delta)} \underbrace{\Delta\left(i\right)}_{<0} + \int_{i(\delta)}^{\delta/\varepsilon} \underbrace{\Delta\left(i\right)}_{>0} + \left(1 - \frac{\delta}{\varepsilon}\right)\frac{\left(1-\alpha\right)\delta}{A\left(2-A\right)}.$$

As $\delta \to 0$, $i\left(\delta\right) \to 0$; therefore $\Delta\left(\delta\right) > 0$ for δ small enough, proving that integration increases on average.

6.2 Proof of Proposition 7

If $\bar{l}_1 = 0$, note that $v_2^* = W\left(0\right) = \left(1+\alpha\right)\frac{3}{8}A^2$ and from (7), $\omega\left(v_2^*, l_1\right)$ has a kink at $l_1 = \alpha\frac{3}{8}A^2$: for lower values the degree of integration is linear and for larger values it is zero; hence $\omega\left(v_2^*, l_1\right)$ is indeed globally convex in l_1. Suppose that $L < \alpha 3A^2/8$. Let $L_0 = \int_{l_1 < \alpha 3A^2/8} l_1 dG\left(l_1\right)$ and $L_1 = \int_{l_1 > \alpha 3A^2/8} l_1 dG\left(l_1\right)$. Note that by (7), $\int_{l_1 < \alpha 3A^2/8} \omega\left(v_2^*, l_1\right) dG\left(l_1\right) = \omega\left(v_2^*, L_0\right)$ and that $\int_{l_1 > 3A^2/8} \omega\left(v_2^*, l_1\right) dG\left(l_1\right) = \omega\left(v_2^*, L_1\right)$. Hence, $E\omega = G\left(\alpha 3A^2/8\right)\omega\left(v_2^*, L_0\right) + \left(1 - G\left(\alpha 3A^2/8\right)\right)\omega\left(v_2^*, L_1\right)$. However since $\omega\left(v_2^*, L_1\right) =$

0, and since ω is globally convex, $L = G(\alpha 3A^2/8)\, L_0 + (1 - G(\alpha 3A^2/8))\, L_1$ implies that $E\omega > \omega(v_2^*, L)$. This shows that $L_1 = 0$ and that the support of G is contained in $[0, \alpha 3A^2/8]$. The same argument applies when $L > \alpha 3A^2/8$.

6.3 Proof of Proposition 9

We know from (7) and Proposition 2 that for a given distribution K the degree of integration is positive when l belongs to $\left[\bar{l}_1^K, v_2^{*K} - \frac{3}{8}A^2\right)$. In this case we can write $\omega\left(v_2^{*K}, l_1\right) = \omega_0 + a\bar{l}_1^K - bl_1$ where $\omega_0 = \frac{3A^2}{(2-A)^2}$, $a = 4\frac{2-3A}{A(2-A)^2}$, $b = \frac{4}{A(2-A)}$, note that $a/b = 1 - \alpha$. It follows that

$$
\begin{aligned}
\int \omega\left(v_2^{*K}, l_1\right) dK(l_1) &= \int_{\bar{l}_1^K}^{v_2^{*K} - \frac{3}{8}A^2} \left(\omega_0 + a\bar{l}_1^K - bl_1\right) dK(l_1) \\
&= \pi^K\left(\omega_0 + a\bar{l}_1^K\right) - b\int_{\bar{l}_1^K}^{v_2^{*K} - \frac{3}{8}A^2} l_1 dK(l_1) \\
&= \pi^K\left(\omega_0 + a\bar{l}_1^K - b\mu^K\right).
\end{aligned}
$$

If $\pi^G = \pi^H$, $\int \omega\left(v_2^{*H}, l_1\right) dH(l_1) < \int \omega\left(v_2^{*G}, l_1\right) dG(l_1)$ if and only if $a\bar{l}_1^H - b\mu^H < a\bar{l}_1^G - b\mu^G$ or if $(1-\alpha)\left(\bar{l}_1^H - \bar{l}_1^G\right) < \mu^H - \mu$ since $\frac{a}{b} = 1 - \alpha$.

6.4 Proof of Proposition 11

Let

$$
\begin{aligned}
\pi &: [0,1] \rightarrow [0,1] \\
\pi(i) &\geq \pi(\hat{\imath}) \Leftrightarrow W(i) \geq W(\hat{\imath}).
\end{aligned}
$$

be a reordering of the indexes of type 1 managers that is consistent with the reordering on willingness to pay induced by the shock. The marginal type 1 agent is i_π such that the Lebesgue measure of the set $\{i : W(i) \geq W(i_\pi)\}$ is n and the set of equilibrium firms is $F = \{i : \pi(i) \geq \pi(i_\pi)\}$.

Let $v_2^*(A)$ be the equilibrium price in the initial situation and $v_2^*\left(\hat{A}\right)$ the equilibrium price after the shock to the technology available to agents in $[i_0, i_1]$.

Remark 1 *Proposition 11 is concerned with situations where* $i_\mu = 1-n$. *However, note that the marginal type may not be* $1-n$. *This can happen in two cases.*

Case 1: A first possibility is $i_1 < 1-n$, *that is, shocked firms were not matched in the initial economy but because* $W(i_1) > v_2^*(A)$, *some of these firms will be matched. In this case, the set of "new entrants" are firms with* $i \in [i_\pi, i_1]$ *while the set of "old firms" are those with index* $i \geq k$, *where* $k \geq 1-n$ *satisfies* $i_1 - i_\pi = k - (1-n)$ *(hence firms* $i \in [i_\pi, i_1]$ *"replace" firms* $i \in [1-n, k]$*). Since* $W(i_\pi) > v_2^*(A)$, *the degree of integration in old firms increases.. For new firms, the question is whether the increase in price* $W(i_\pi) - W(1-n)$ *is large enough to overcome the internal effect of technology shock pushing towards less integration.*

Case 2: Another possibility is $1-n \in (i_0, i_1)$ *and* $W(1-n) > \lim_{\varepsilon\downarrow 0} W(i_1+\varepsilon)$. *Then there exists* $k > i_1$ *such that* $W(k) = W(1-n)$, *and either* $i_\pi \in (i_1, k]$ *or* $i_\pi \in [i_0, 1-n)$. *In either case, if* $l_1(i_\pi)$ *is low enough, the increase in equilibrium surplus to the 2 may be small enough that the internal effect dominates and shocked firms integrate less.*

(i) (Inframarginal shocks) If $i_0 > 1-n$, then $i_\pi = 1-n$ *and* $W(i_\pi) = v_2^*(A)$, *then the shocked firms become less integrated while the unshocked firms remain unaffected*

This is a direct consequence of Lemma 10

(ii) (Marginal shocks) If $1-n \in (i_0, i_1)$ *is still the marginal type 1, the equilibrium price increases and all firms, shocked and unshocked, integrate more.*

Note that $1-n$ is still the marginal type if and only if $W(1-n) \leq \lim_{\varepsilon\downarrow 0} W(i_1+\varepsilon)$, for in this case, all agents $i > 1-n$ have higher willingness to pay than $1-n$.

>From (8), $v_2^*(A) = W(1-n)$ is increasing in A, hence $v_2^*\left(\hat{A}\right) > v_2^*(A)$ and it follows that all unshocked firms $[i_1, 1]$ integrate more.

If the firm $1-n$ did not integrate before the shock (that is chose $\omega = 0$),then all $i > 1-n$ firms also chose not to integrate since ω is decreasing in the liquidity of type 1. Hence, it is immediate that an increase in A can only lead to more integration.

Consider now the case where firm $1-n$ integrated before, that is chose a contract with $\omega > 0$. If i_1 chose initially a contract $\omega = 0$, there exists $k \in (1-n, i_1)$ such that all firms with $i < k$ integrate ($\omega > 0$) and all firms with $i \geq k$ do not integrate; firms with $i \geq k$ will necessarily integrate more after the shock. We have $v_2^*(A) = W(1-n; A)$, $v_2^*\left(\hat{A}\right) = W\left(1-n; \hat{A}\right)$, and from (7), (8), for all shoked firms $i \in [1-n, k)$, the difference in the degree of integration after and before the shock is

$$\frac{3\hat{A}^2 - 4\bar{l}_1}{\left(2-\hat{A}\right)^2} - \frac{3A^2 - 4\bar{l}_1}{(2-A)^2} > 0.$$

(here $\bar{l}_1 = l_1(1-n)$) and all firms integrate more as claimed.

(iii) If $i_0 = 0$ and $i_1 = 1$, the arguments for (ii) apply since $1-n$ is still the marginal type 1 manager.

7 References

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