Read my Lips, Watch for Leaps: A Theory of Endogenous Political Instability*

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Abstract

As norms and tastes adapt to the social conditions shaped by policy variables, individuals' opinions about these policies may evolve as well. We show how this process can lead to political instability: Most voters may prefer the policy of the opposition over that of the current regime, but when the opposition party takes over and implements its policy, the new policy may influence the public's taste so that the old policy becomes more attractive. Alternatively, if a politician's platform is shaped opportunistically so as to gain the majority's support, once in office the politician *should not* implement the promised policy in order to remain popular and get re-elected. Rather, the effective policy in this respect is the one which will be deemed optimal by most voters *after* the norms and opinions will adjust to that very policy. We demonstrate these ideas when the policy variables are the level of public education and a subsidy for on-the-job education.

1 Introduction

Political instability is an inherent part of the democratic system. We are no longer surprised from a frequent change of elected leaders, neither from leaders not keeping up their campaign promises. At the same time we are also not surprised from frequent shifts in public opinion and the changing popularity of different policies or different politicians.

Adopting the paradigm of stable preferences and rational behavior, the question is what are the sources of such instability? One obvious possibility is that different exogenous factors affect voting patterns or electives' behavior. Often there are new events that require different policies or different skills. Contemporary commentators usually argue in

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such a case that it is the "daily reality" which forces leaders to change their recommended policies or forces the public to change its opinion. Another possible set of explanations relies on information revelation arguments. Sometimes while being in office, electives' actions reveal information about their personalities or abilities.[1] Voters may therefore reconsider their choices based on the leader's actions. Such a process, in turn, may affect electives' choice of policies or actions as they wish to signal their high competence[2].

In this article we present a theory of political instability in which changing voting patterns or public opinion occur *endogenously*. Our model is based on the view that policies and institutions, alongside their implications for incentives, may also influence the emergence of norms and the evolution of tastes and opinions. This point has been discussed already by Adam Smith, Alexis de Tocqueville, Karl Marx and John Stuart Mill (for recent surveys see Elster (1989), Aaron (1994) and Bowles (1998)). The interplay between policies and tastes may therefore have an important role in shaping the dynamics of the political system, via the candidates' choice of platforms and the actual policy implemented by the elected leader. In this study we show how this interaction may lead to endogenous political cycles or to deliberate discrepancies between promises and delivery of policy change.

The dynamics of opinions is the focus of many political studies, in which the standard approach is to conduct frequent polls that are designed to study changes in the public opinion and the popularity of different politicians or policies. Empirical research in political psychology suggests that voters' preferences regarding alternative courses of action are not fixed between (or within) election rounds (see Zaller 1991). Thus the speed of such changes makes them relevant for the discussion on political instability. Gerber and Jackson (1993) present empirical evidence that opinions may be altered in response to policies and positions taken by different political parties.[3]

The idea that life experience affects preferences via the accumulation of personal capital and social capital is already well established (see for example Becker (1976, 1996) and Akerlof (1983)). Preferences may evolve as a result of cultural transmission (Cavalli-Sforza and Feldman (1973, 1981), Boyd and Richerson (1985), Bisin and Verdier (1998, 2001)), by which parents socialize and transmit their preferences to their offspring, motivated by some form of paternalistic altruism. A different modelling approach is the use of an evolutionary setup in which individuals imitate or learn from the "successful"

[1] Or about expert analysis which is not directly available to the public or to competing candidates.

[2] For examples of such political instability see Alesina and Cukierman (1990), Canes- Wrone et al. (2001), Cukierman and Tomassi (1998), Harrington (1993), Roemer (1994), Rogoff (1990), Schultz (2002).

[3] Gerber and Jackson (1993) studied the relationship between voters' preferences and party platforms regarding civil rights between 1956-1964 and regarding the Vietnam War between 1968-1972. In their model, voters' preferences shift in direct response to the actions of the different parties. See also Carmines and Stimson (1989) for an illustration of how learders' actions regarding civil rights affect the public opinion on these issues.

individuals in their society and eventually adopt their preferences.[4] Such an evolutionary approach has been useful in studying and justifying different types of preferences (such as altruism, status seeking, overconfidence etc.) that emerge from the dynamic process.[5]

This suggests an important link between public policy and the evolution of values and norms. Public policy shapes the market conditions, which in turn may affect the prevailing norms. For example, different policies that encouraged women labor participation had an effect on feminist values and preferences with respect to family size. Aaron and Schwartz (1984) studied the physicians' attitude towards standards of care in the US and the UK. Their main finding is that the different resource constraints that physicians encounter in the two countries translate themselves into different views regarding standard of care. In particular, changes in the resources available to British physicians triggered not only a behavioral response in practice patterns, but also a change in their opinion and values. Wilson (1993) studied the problems of the urban underclass in the US. His main claim is that the "underclass problem" is not simply due to perverse incentives, but is rooted in the fact that members of this group were "habituated in ways that weaken their self control and their concern for others." Wilson defines habituation as "the process whereby people acquire a constant, often unconscious, way of doing something...". This process is therefore a form of the preference dynamics described above. Wilson emphasizes the distinction between "incentives" and "culture" and the immediate implications regarding the public policies that might be effective to counter the underclass problem.[6]

As tastes and opinions are responsive to policy, so are the choices of individuals at the ballots. In order to study the different possible interdependencies between the political system and endogenous formation of tastes, we examine two kinds of political processes, which differ by the type of the participating politicians. According to a famous statement by Margaret Thatcher there are two type of politicians; "*Conscious politicians*" and "*Consensus politicians*". Conscious politicians are politicians that have a strong, fixed political agenda, while consensus politicians are ones that are going to change their policy every time there is a shift in the public opinion.

We consider a model in which individuals interact in small groups (firms), and the outcome of this interaction depends on their respective abilities, on the level of public

[4] In this approach the meaning of a "successful" individual is exogenously given. However, most of the literature adopts the criterion of material payoffs for the definition of success. While such a choice is not necessarily realistic it serves as a dicipline device. In any case, the evolutionary approach endogenizes preferences and norms, but it requires an exogenous definition of a fitness function.

[5] See Bester and Guth (1998), Dekel et al. (1998), Dekel and Scotchmer (1999), Fershtman and Weiss (1997,1998), Guth and Yaari (1992), Heifetz and Spiegel (2000), Heifetz and Segev (2001) Huck and Oechssler (1999) Kockesen, Ok and Sethi (2000a, 2000b), Possajennikov (2000), Robson (1996), Rogers (1994), Rotemberg (1994), Vega-Redondo (1997).

[6] While most of the literature on the interdependence between policy and preferences is descriptive, for a formalization of this idea see Bar-Gill and Fershtman (2001) who considered the limit of the effectiveness of policies (like subsizing public goods) when preferences are endogenously determined. Guth and Ockenfels (2001) consider the co-evolution of morality and instituition in which the evolution of litigation or legal insurance is co-determined with the evolution of trust.

education they received as well as on their actions, which can be interpreted as effort or on-the-job training. Individuals, however, may also enjoy putting effort or being well trained *beyond* the direct monetary benefits that effort or training accrues. We assume that these tendencies are endogenously determined at equilibrium, so as to confer the maximal material reward upon each individual given the tastes of others. These tastes may therefore evolve when market conditions adapt to changes in public policy. Nevertheless, individuals cannot anticipate such an unconscious change of heart when they contemplate the implications of potential policies.[7]

There are two policy issues in our model – the level of public education and/or the subsidy for on-the-job training. Both types of public expenditures are financed by per capita taxation. First, we consider the political equilibrium that emerges when there are two conviction politicians, each of whom holds a firm view regarding her suggested public policy. We demonstrate a situation in which voters favor one policy and vote for one candidate, but the implementation of this policy triggers a change of tastes. The new dispositions affect the individuals' votes in the next round of elections, and we show the possibility of a sufficient change in the people's favorite policy so that in the next elections the other candidate wins. We further provide an example for a consecutive change of power, in which there is a permanent switch of regimes created not by exogenous shocks or new information but simply by the interplay between policies and tastes.

Assuming, next, that electives are consensus politicians, we trace the changes in their policies which follow the changes in the public opinion. Clearly, without endogenizing tastes and in the absence of shocks or new information, there is no reason for such politicians not to carry out a consistent policy, because there is no reason for changes in what the voters want. But endogenizing tastes may result in an interesting pattern of policy changes. When the elective implements the policy that the majority of people favor, she initiates a dynamics that may result in a different distribution of dispositions in the population, and hence in a change of the public opinion (i.e., the wish of the majority). As a consensus politician, she now follows the public opinion and adjusts her policy accordingly. This triggers a further change of opinions, and so on. We show, in particular, how this process may lead to a (popular) zig-zag policy.

Last, we ask what policy will make the leader most popular after the dispositions adapt to *that very policy*. With such a policy, the public may initially be dissatisfied with the leader, as that policy is typically different than the one promised in the pre-elections platform (which was devised to elicit the majority's support with the prevailing tastes at that time). However, the public will eventually appreciate the new policy when everybody's tastes adjust to the new regime. Consequently, the elective may end up

[7] The social discourse about intrinsic motives at work (e.g. about the appropriate balance between "materializing one's potential at work" versus "spending quality time with one's family") is ethical in nature and defies any relation to material rewards. That's why individuals cannot be aware that their intrinsic motives may, in fact, be adaptive to market conditions. See also Bewley (1999) for a similar modeling approach.

successful and popular at the end of her cadence and ensure her re-election: She will gain a reputation of being pragmatic – on account of deviating from her initial promises, but insisting on her correct vision – on account of implementing a successful policy, even though initially unpopular.

2 The Model

Consider a society with a large number of individuals. Individuals may have different innate abilities α_i which are uniformly distributed in the $[0,1]$ interval, and there is a large number of individuals of each given ability. Each individual, regardless of her ability, receives public education, denoted by e. Public education is supplied and financed by the government. The cost of supplying e is $\frac{1}{2}e^2$.

Work is done in this economy by small groups (which can be denoted as firms). To simplify our analysis we assume that each group consists only of two individuals and we further assume that such pairs are formed by a random matching process. In each firm individuals need to determine their action or activity level x_i. x_i can be viewed as on-the-job training. The government may choose to subsidize the activity level x_i with a monetary compensation of sx_i. Both public education and on-the-job training subsidy is financed by a per-capita tax.

2.1 Payoffs and Dispositions

The monetary payoff of individual i is derived from the firm she is working in and depends on her action x_i, the action of her co-worker x_j, her ability α_i, the level of public education e and the subsidy rate s. For simplicity, monetary payoff gross of tax is assumed to be given by

$$\Pi_i(x_i, x_j; \alpha_i, e, s) = (\alpha_i + e + s + \frac{1}{2}x_j - x_i)x_i. \tag{2.1}$$

Thus we assume that payoffs, as well as marginal payoffs, are increasing with the worker's ability with public education and with the action (on-the-job training) of her co-worker.[8]

While it looks from (2.1) that e and s may be two interchangeable policy variables, there is nevertheless an important difference between the two. Public education is given to everyone at the same level e and at the same cost. s, on the other hand, is a subsidy given in connection to the player's choice of on-the-job training, x_i, and therefore it is not equally distributed even though it is equally financed by all individuals.

[8] One can carry out a similar analysis assuming that payoffs as well as marginal payoffs are declining with the action of the co-worker.

Each individual i may derive also non-monetary utility, $\tau_i x_i$, from her work or on-the-job training. This benefit can simply arise from personal satisfaction of being trained, or from a competitive aspect of measuring oneself with respect to others.[9] The coefficient τ_i, which determines the relative importance of the non-monetary benefit, may differ across individuals and may depend on their abilities, the level of public education and the subsidy that is given to on-the-job training.

The overall utility of individual i (gross of tax) is therefore

$$U_i(x_i, x_j; \alpha_i, \tau_i, e, s) = \Pi_i(x_i, x_j; \alpha_i, e, s) + \tau_i x_i \tag{2.2}$$

We further assume that when paired to work together, the individuals get to know each other's ability α_i and type τ_i. In each such interaction they choose the (unique) Nash equilibrium activity levels[10]

$$\left(x_i(\alpha_i, \alpha_j; \tau_i, \tau_j; e, s),\ x_j(\alpha_j, \alpha_i; \tau_j, \tau_i; e, s)\right).$$

We denote the equilibrium monetary payoff of player i in this interaction by

$$f_i(\alpha_i, \alpha_j; \tau_i, \tau_j; e, s) = \Pi_i\left(x_i(\alpha_i, \alpha_j; \tau_i, \tau_j; e, s),\ x_j(\alpha_j, \alpha_i; \tau_j, \tau_i; e, s); \alpha_i, e, s\right).$$

2.2 Dispositions Dynamics

In the short run the disposition τ_i of an individual i with ability α_i is given, but in the long run it adjusts so as to maximize the monetary expected payoff of i (across her possible matches with a co-worker), given the adjustment of the dispositions of individuals with other abilities. This captures the idea that among the possible dispositions for those individuals with ability α_i, the one which will eventually become dominant, due to imitation or education, is the one which ultimately maximizes the material success.[11]

Definition 1 *For a given public education e, subsidy s, the dispositions $\tau_i^*(\alpha_i; e, s)$ across abilities constitute a "dispositions equilibrium" if it satisfies the following condition:*

$$\tau_i^*(\alpha_i; e, s) \in \arg\max_{\tau_i \in R} \int_0^1 f_i(\alpha_i, \alpha_j; \tau_i, \tau_j^*(\alpha_j; e, s); e, s) d\alpha_j. \tag{2.3}$$

[9] Since public education is common to all players the relative education is determined solely by on-the-job training.

[10] i.e. those satisfying

$$x_i(\alpha_i, \alpha_j; \tau_i, \tau_j; e, s) = \arg\max_{y_i \geq 0} U_i(y_i, x_j(\alpha_j, \alpha_i; \tau_j, \tau_i; e, s); \alpha_i, \tau_i, e, s).$$

[11] Note that while individuals may have utilities that depend on their "satisfaction" and not just monetary payoffs, we assume that the disposition adjustment is done such that monetary payoffs are maximized given the distribution of dispositions in the population.

Thus "dispositions equilibrium" is an assignment of dispositions for every α_i such that, given this assignment, no individual can get higher monetary payoffs with a different disposition.

Proposition 1 *(i) The model features a unique dispositions equilibrium.*

(ii) The equilibrium disposition, $\tau_i^*(\alpha_i; e, s)$, is increasing in all its arguments.

Proof. See the Appendix. ■

The equilibrium dispositions $\tau_i^*(\alpha_i; e, s)$ describe the non-monetary benefits individuals may have from their action x_i. If the interpretation of this action is on-the-job training, then the dispositions describes their enjoyment from being well trained or being better trained relative to other workers. Part (ii) of the proposition points out an interesting property. At equilibrium the disposition is increasing with the individual's own ability and with public education and subsidy for on-the-job training. Increasing e (public education), besides its direct effect on payoffs, increases also the equilibrium disposition of **all** individuals in this economy i.e., increases their non-monetary benefits from on-the-job training. Moreover, increasing e induces players to increase their x_i as it can be seen from the payoff function (2.1). But this is only the direct effect. An increase in public education causes an increase of the disposition – the non-monetary benefits from training – and thus induces the players to increase their x_i even further. Similarly, individuals with higher ability have greater incentives to choose a higher x_i but beside the direct incentives at equilibrium they are going to have a higher τ_i^* which induces them to increase x_i even further.

We denote the Nash equilibrium activity levels in a firm in which the individuals' abilities are (α_i, α_j) by

$$\begin{aligned} & \left(x_i^*(\alpha_i, \alpha_j; e, s),\ x_j^*(\alpha_j, \alpha_i; e, s)\right) \\ \equiv\ & \left(x_i(\alpha_i, \alpha_j; \tau_i^*(\alpha_i; e, s), \tau_j^*(\alpha_j; e, s); e, s),\ x_j(\alpha_j, \alpha_i; \tau_j^*(\alpha_j; e, s), \tau_i^*(\alpha_i; e, s); e, s)\right), \end{aligned}$$

and by $U_i^*(\alpha_i, \alpha_j; e, s)$ the individual i's equilibrium utility (gross of tax).

$$U_i^*(\alpha_i, \alpha_j; e, s) \equiv U_i(x_i^*(\alpha_i, \alpha_j; e, s),\ x_j^*(\alpha_j, \alpha_i; e, s); \alpha_i, \tau_i^*(\alpha_i; e, s), e, s). \tag{2.4}$$

Given our balanced-budget assumption, the government needs to finance its public education and its on-the-job training subsidy by imposing the appropriate per-capita tax. The level of such a tax will therefore be

$$T^*(e, s) = \frac{1}{2}e^2 + s \int_0^1 \int_0^1 \left(x_i^*(\alpha_i, \alpha_j; e, s) + \ x_j^*(\alpha_j, \alpha_i; e, s)\right) d\alpha_i d\alpha_j. \tag{2.5}$$

As explained in the introduction, we assume that when individuals make their choices they take their preferences as given, since they are unable to go through the mental exercise regarding the future formation of their dispositions. That is why actions and payoffs are evaluated given the individual's current disposition and given the current distribution of dispositions in the population.

It follows that when (e_0, s_0) are the current levels of public education and subsidy, and $\tau_i^*(\alpha_i; e_0, s_0)$ is the associated disposition equilibrium, individuals believe that if and when different levels of (e, s) are implemented, the corresponding Nash equilibrium actions will be

$$\hat{x}_i(\alpha_i, \alpha_j; e_0, s_0; e, s) \equiv x_i(\alpha_i, \alpha_j; \tau_i^*(\alpha_i; e_0, s_0), \tau_j^*(\alpha_j; e_0, s_0); e, s),$$

i.e.

$$\hat{x}_i(\alpha_i, \alpha_j; e_0, s_0; e, s) = \arg\max_{y_i \geq 0} U_i(y_i, \hat{x}_j(\alpha_j, \alpha_i; e_0, s_0; e, s); \alpha_i, \tau_i^*(\alpha_i; e_0, s_0), e, s),$$

where $\hat{x}_j(\alpha_j, \alpha_i; e_0, s_0; e, s)$ is similarly defined.

In such a case individual i's utility (gross of tax) will be

$$\hat{U}_i(\alpha_i, \alpha_j; e_0, s_0; e, s) \equiv U_i\left(\hat{x}_i(\alpha_i, \alpha_j; e_0, s_0; e, s),\ \hat{x}_j(\alpha_j, \alpha_i; e_0, s_0; e, s)\right),$$

while the per-capita tax will be

$$\hat{T}\left(e_0, s_0; e, s\right) = \frac{1}{2}e^2 + s\int_0^1 \int_0^1 \left(\hat{x}_i(\alpha_i, \alpha_j; e_0, s_0; e, s) + \ \hat{x}_j(\alpha_j, \alpha_i; e_0, s_0; e, s)\right) d\alpha_i d\alpha_j.$$

However, this will be the case only in the short run. In the long run the new policies (e, s) will affect the dispositions, which will adjust to their new equilibrium level $\tau_i^*(\alpha_i; e, s)$.

2.3 Policy Issues and Political Equilibrium

The role of the government in our model is to determine the level of public education e, direct subsidy for intra-firm activity, s, and financing these expenses by levying a per-capita tax. We assume a democratic regime with two parties. The levels of public education and subsidy are the only issues of public debate. Moreover, we assume that in each round of elections only one of these issues is of central concern, so that the elected candidate is the one whose platform regarding the issue at stake is favored by most voters. We denote by (e_0, s_0) the pre-elections policy, and assume that the dispositions are at the equilibrium levels $\tau_i^*(\alpha_i; e_0, s_0)$ which correspond to this initial policy.

Given any policy (e_0, s_0) and the equilibrium dispositions $\tau_i^*(\alpha_i; e_0, s_0)$ it induces, individuals evaluate alternative policies (e, s) according to their expected utility (net of tax)

$$\hat{H}_i(\alpha_i; e_0, s_0; e, s) = \int_0^1 \hat{U}_i(\alpha_i, \alpha_j; e_0, s_0; e, s) d\alpha_j - \hat{T}\left(e_0, s_0; e, s\right). \tag{2.6}$$

Denote

$$E\left(\alpha_i; e_0, s_0; s\right) = \arg\max_{e\geq 0} \hat{H}_i(\alpha_i; e_0, s_0; e, s).$$

$$S\left(\alpha_i; e_0, s_0; e\right) = \arg\max_{s\in R} \hat{H}_i(\alpha_i; e_0, s_0; e, s).$$

$E\left(\alpha_i; e_0, s_0; s\right)$ is the optimal public education level for an individual with ability α_i given the equilibrium disposition $\tau_i^*(\alpha_i; e_0, s_0)$ and given the subsidy level s. Similarly $S\left(\alpha_i; e_0, s_0; e\right)$ is the optimal subsidy for the same individual given the public education level e.

Proposition 2 *(i) $\hat{H}_i(\alpha_i; e_0, s_0; e, s)$ are single-peaked in each of the policy variables e, s.*

(ii) The preferred level of public education $E\left(\alpha_i; e_0, s_0; s_0\right)$ by an individual with ability α_i when the subsidy level is s_0, and her preferred level of subsidy $S\left(\alpha_i; e_0, s_0; e_0\right)$ when the public education level is e_0 are both monotonic in the ability α_i.

Proof. See the Appendix. ■

Proposition 2 has two parts. The first asserts that preferences over policy variables are single-peaked, which simplifies our political equilibrium analysis. Second, we have the interesting property that people with higher ability wish to have higher levels of public education and subsidy for on-the-job training.[12]

3 Conviction Politicians

We start by analyzing the model under the assumption that politicians' platforms represent their convictions, and that they are committed to implement these convictions if and when they are elected. We assume that there are only two candidates each is a conviction politician and we explore the implications of the dispositions dynamics on the popularity of candidates and leaders and on the pattern of public voting.

We demonstrate in this section two possible effects. The first is reinforced political stability in which once the preferences adjust in response to the policy implemented by the chosen candidate, the elective in office becomes more popular relative to the opposition leader. The second is political instability in which the implementation of the policy advocated by the elective triggers a disposition dynamics that makes this policy unpopular relative to the policy suggested by the opposition. Moreover, as we demonstrate below, the latter possibility may entail a political cycle, under which the government changes from "right" to "left" and vice versa in each round of elections. This instability is not due to any external shock which influences the objective features of economic interaction, and neither is it due to any information which is revealed to the

[12] Note that in the payoff function the effects of public education and ability are additive, so ability does not affect the direct benefits from public education or from on-the-job training.

public or to the politicians over time. Rather, it is implied by the very nature of the economic activity and the intervening policy, via their *endogenous* implications on the individuals' intrinsic motives at work.

3.1 Reinforcing platforms: Political Stability

Consider a society in which there is no direct subsidy for within-firm activities; i.e., $s_0 = 0$. The policy variable which is under public debate is public education. The initial level of public education is assumed to be e_0. There are two possible parties, headed by two conviction politicians. One party is committed to the existing level of public education; e_0, while the second party supports changing the level to e_1.

Proposition 3 ***(Political Stability):*** *Assume a zero subsidy rate $s_0 = 0$, an initial level of public education e_0, and an alternative suggested policy $e = e_1$. If, with the dispositions $\tau_i^*(\alpha_i; e_0, s_0)$ induced by the current policy, most people prefer e_1 over e_0, then most people will also prefer e_1 over e_0 once e_1 is implemented and the dispositions adjust to their new equilibrium levels $\tau_i^*(\alpha_i; e_1, s_0)$.*

Proof. Since, by proposition 2, $\hat{H}_i(\alpha_i; \tilde{e}, s_0; e, s)$ is single-peaked as a function of e and the maximizer is monotonic in α_i, it follows that a majority of people prefer e_1 over e_0 when the dispositions are $\tau_i^*(\alpha_i; \tilde{e}, s_0)$ if and only if the median voter, with ability $\alpha_i = \frac{1}{2}$, prefers e_1 over e_0. This is the case when

$$\left| e_1 - E\left(\frac{1}{2}; \tilde{e}, s_0; s\right)\right| < \left| e_0 - E\left(\frac{1}{2}; \tilde{e}, s_0; s\right)\right| \tag{3.1}$$

as $\hat{H}_i(\frac{1}{2}; \tilde{e}, s_0; e, s)$ is quadratic in e.

Assume $s_0 = s = 0$. Then

$$E\left(\frac{1}{2}; \tilde{e}, s_0; s\right) = E\left(\frac{1}{2}; \tilde{e}, 0; 0\right) = \frac{8}{11}\tilde{e} + \frac{48}{11}$$

which is increasing in $\tilde{e}$. Therefore $e_1 - e_0$ has the same sign as $E\left(\frac{1}{2}; e_1, 0; 0\right) - E\left(\frac{1}{2}; e_0, 0; 0\right)$. Hence

$$\left| e_1 - E\left(\frac{1}{2}; e_0, 0; 0\right)\right| < \left| e_0 - E\left(\frac{1}{2}; e_0, 0; 0\right)\right| \tag{3.2}$$

implies

$$\left| e_1 - E\left(\frac{1}{2}; e_1, 0; 0\right)\right| < \left| e_0 - E\left(\frac{1}{2}; e_1, 0; 0\right)\right|. \tag{3.3}$$

This means that if the median voter (and hence most people) prefers e_1 over e_0 with the dispositions $\tau_j^*(\alpha_j; e_0, 0)$, she continues to prefer e_1 over e_0 with the adjusted dispositions $\tau_j^*(\alpha_j; e_1, 0)$. ■

Proposition 3 gives an example of situations with endogenous political stability. Changing public education to the level of e_1 indeed induces an endogenous change of preferences (or disposition dynamics) but under the new disposition equilibrium the majority of people still prefer the policy e_1 over the old policy e_0. This does not mean that there will not be any further changes in the regime. However in order to beat the incumbent party one needs to come up with new public education policy recommendations.

3.2 Self-defeating Platforms: Political Instability

We will now try to illustrate the reverse case of political instability. In such a case the implementation of a new policy entails longing for the old regime. For this example consider a society in which public education is fixed at the level of $e_0 = 1$ and is not subject to political debate. The two political parties support two different levels of subsidies $s_0$and s_1.

Proposition 4 ***(Political Instability)****: Assume a unit level of public education $e_0 = 1$. There is a continuum of policy pairs $s_1 < 0 < s_0$ such that if the parties' platforms promise subsidy rates s_0, s_1, respectively, then most people prefer s_1 (i.e. a tax) over s_0 with their initial dispositions $\tau_i^*(\alpha_i; e_0, s_0)$, but most people prefer s_0 over s_1 once their dispositions adjust to their new equilibrium levels $\tau_i^*(\alpha_i; e_0, s_1)$.*

Proof. Since, by proposition 2, $\hat{H}_i(\alpha_i; e_0, \tilde{s}; e, s)$ is single-peaked as a function of s and the maximizer is monotonic in α_i, it follows that a majority of people prefer s'' over s' when the dispositions are $\tau_i^*(\alpha_i; e_0, \tilde{s})$ if and only if the median voter, with ability $\alpha_i = \frac{1}{2}$, prefers s'' over s'. This is the case when

$$\left| s'' - S\left(\frac{1}{2}; e_0, \tilde{s}; e\right)\right| < \left| s' - S\left(\frac{1}{2}; e_0, \tilde{s}; e\right)\right| \tag{3.4}$$

as $\hat{H}_i(\frac{1}{2}; e_0, \tilde{s}; e, s)$ is quadratic in s.

Assume $e_0 = e = 1$. Then

$$S\left(\frac{1}{2}; e_0, \tilde{s}; e\right) = S\left(\frac{1}{2}; 1, \tilde{s}; 1\right) = -\frac{1}{44}\tilde{s} - \frac{9}{22}.$$

Consider the inequalities

$$S\left(\frac{1}{2}; 1, s_0; 1\right) - s_1 \;<\; s_0 - S\left(\frac{1}{2}; 1, s_0; 1\right) \tag{3.5}$$

$$s_0 - S\left(\frac{1}{2}; 1, s_1; 1\right) \;<\; S\left(\frac{1}{2}; 1, s_1; 1\right) - s_1 \tag{3.6}$$

or, explicitly,

$$\begin{aligned}\left(-\frac{1}{44}s_0-\frac{9}{22}\right)-s_1 &< s_0-\left(-\frac{1}{44}s_0-\frac{9}{22}\right)\\ s_0-\left(-\frac{1}{44}s_1-\frac{9}{22}\right) &< \left(-\frac{1}{44}s_1-\frac{9}{22}\right)-s_1\end{aligned}$$

amounting to

$$-\frac{23}{22}s_0-\frac{9}{11}<s_1<-\frac{22}{23}s_0-\frac{18}{23}. \tag{3.7}$$

For $s_0>-\frac{2}{5}$ there is a range of values s_1 satisfying these inequalities, and for such s_1 it is also the case that

$$\begin{aligned}s_1 &< S\left(\frac{1}{2};1,s_0;1\right)=-\frac{1}{44}s_0-\frac{9}{22}<-\frac{1}{44}\left(-\frac{22}{23}s_0-\frac{18}{23}\right)-\frac{9}{22}\le S\left(\frac{1}{2};1,s_1;1\right)\\ &\le -\frac{1}{44}\left(-\frac{23}{22}s_0-\frac{9}{11}\right)-\frac{9}{22}<s_0.\end{aligned}$$

Hence for $s_0>-\frac{2}{5}$ and s_1 in the range (3.7), inequality (3.5) implies that (3.4) obtains with $s''=s_1$, $s'=s_0$ and $\tilde{s}=s_0$, while inequality (3.6) implies that (3.4) obtains with $s''=s_0$, $s'=s_1$ and $\tilde{s}=s_1$. In other words, the median voter (and hence the majority of voters) prefers s_1 over s_0 with the disposition $\tau_i^*(\frac{1}{2};1,s_0)$, while she has the reverse preference with the disposition $\tau_i^*(\frac{1}{2};1,s_1)$. ■

Proposition 4 demonstrates a scenario in which an elected politician, by implementing the policy in which she believes and advocates, actually initiates a process of preference change that ultimately leads to her failure in the next elections. In such a case there is an inherent political instability which is due to the preference dynamics induced by the different policies.

3.3 Permanent Political Instability

Consider a society in which public education is constant and given at the level $e_0=1$. Further assume that there are two parties in this society; "Left" and "Right" which are committed to the subsidy levels s_L and s_R respectively. If the positions of the two parties are "opposed in a balanced way" – one advocating a subsidy and the other advocating tax, a political cycle will emerge, in which "Right" subsumes "Left" and vice versa in each round of elections:

Proposition 5 ***(Political Cycle)****: Assume a unit level of public education $e_0=1$. There is a continuum of policy pairs $s_1<0<s_0$ such that if the parties' platforms promise subsidy rates $s_L=s_0$ and $s_R=s_1$,, the parties will switch power in each round of elections, assuming that by the end of each cadence the dispositions have adjusted to the equilibrium levels $\tau_i^*(\alpha_i;e_0,\tilde{s})$ pertaining to the prevailing subsidy policy $\tilde{s}$.*

Proof. This follows immediately from the proof of proposition 4. ■

The situation described in Proposition 5 of permanent instability and constant switching of political regimes illustrates an interesting problem that may arise in settings with endogenous preferences. It is a situation in which individuals are never satisfied with their current regime and permanently wish to change it.[13] The implementation of s_L triggers a disposition dynamics that makes the majority of individuals wish to have s_R and visa versa.

4 Consensus Politicians

In the previous section we assumed that the platforms of the two parties are fixed and that, by conviction, once a candidate is elected she indeed implements the policy promised in her ticket. We now turn to explore the implementation of policies when leaders are consensus politicians that adapt their platforms in an attempt to be popular and get elected. Moreover, we assume that once in office, the leader may implement a different policy than she initially promised, or change the policy in the course of her cadence following the popularity of different policies among their constituencies.

By the "median voter theorem" and Proposition 2, a candidate can guarantee her election by promising the policy most preferred by the median voter; i.e., the player with ability $\alpha_i = \frac{1}{2}$. If the pre-elections prevailing policy is (e_0, s_0), then given the monotonicity of $E(\alpha_i; e_0, s_0; s_0)$ and $S(\alpha_i; e_0, s_0; e_0)$ in α_i (see Proposition 2) the policy that will guarantee winning the elections is $E\left(\frac{1}{2}; e_0, s_0; s_0\right)$ when the level of public education is of major concern, or $S\left(\frac{1}{2}; e_0, s_0; e_0\right)$ when the subsidy level is at stake.

However, if this promised policy is different than the prevailing one, its implementation will trigger a change of dispositions. This means that in order to please her people, the politician in office may need to implement a different policy than the one she promised before the elections.

In this section we thus do not consider the political instability that is derived from switches of ruling parties or leaders but the instability associated with changes of policies by leaders that seek to be popular or strive to get re-elected. Since the incumbent leader implements the popular policy she will always be re-elected, and therefore our focus is on the pattern of the endogenous change of the popular policy.

[13] Had we considered a setup in which individuals are aware of the evolution of their own preferences, then under the circumstances described in this proposition they would have been unable to decide which regime they prefer.

4.1 Following the Public Opinion

One type of a consensus leader is a leader that chooses a policy according to the current public opinion. That is, when people's dispositions adjust the leader implements a new policy pleasing the majority of the public (or equivalently pleasing the median voter). However, every policy change will lead to yet another change of dispositions, hence the leader is bound to change her policy again in order to maintain her popularity. This, however, is not an example of a politician that does not keep her campaign promises. On the contrary, after the elections the campaign promises are kept and the politician implements the strategy she advocated, but then the policy is "modified" as the public opinion changes. What will be the nature of these policy adjustments? In our model, this depends on the type of the policy variable.

4.1.1 Monotonic Policy Adjustments

When the policy variable is the level of pubic education, the resulting sequence of policy changes will be monotonic, resulting in a sequence of successive augmentations or successive reductions. Specifically,

Proposition 6 *A popular raise of the public education level following the public opinion will result in a change of dispositions and public cry for another raise, and vice versa. That is, if*

$$e_1 \equiv E\left(\frac{1}{2}; e_0, s_0; s_0\right) > e_0, \tag{4.1}$$

then

$$e_2 \equiv E\left(\frac{1}{2}; e_1, s_0; s_0\right) > e_1, \tag{4.2}$$

and vice versa.

Proof. It follows from equation[14] (A.2) that the desired public education level, in particular by the median voter $\alpha_i = \frac{1}{2}$, is increasing in the prevailing policy e_0. Hence,

$e_1 = E\left(\frac{1}{2}; e_0, s_0; s\right) \gtrless e_0$ implies $e_2 = E\left(\frac{1}{2}; e_1, s_0; s\right) \gtrless e_1$. ■

The outcome of the above popular policy adjustments is a subsequent increase of public education level.

4.1.2 Zig-Zag Policy Adjustments

When the policy variable is the subsidy for on-the-job training, the resulting sequence of policy changes will be non-monotonic. Specifically,

[14] In the appendix.

Proposition 7 *A popular raise of the subsidy rate will result in a change of dispositions and public cry for its reduction, and vice versa. That is, if*

$$s_1 \equiv S\left(\frac{1}{2}; e_0, s_0; e_0\right) > s_0, \tag{4.3}$$

then

$$s_2 \equiv S\left(\frac{1}{2}; e_0, s_1; e_0\right) < s_1, \tag{4.4}$$

and vice versa.

Proof. It follows from equation[15] (A.3) that the desired subsidy level, in particular by the median voter $\alpha_i = \frac{1}{2}$, is decreasing in the prevailing policy s_0. Hence

$$s_1 = S\left(\tfrac{1}{2}; e_0, s_0; e\right) > s_0 \quad \text{implies} \quad s_2 = S\left(\tfrac{1}{2}; e_0, s_1; e\right) < s_1,$$

and vice versa. ■

Proposition 7 provides an example for a zig-zag policy which arises endogenously from the elective's concern about her popularity. Interestingly, this zig zag public policy is not necessarily unpopular although the elective does not necessarily follow her campaign promise. She can always justify her zig-zag policy by a zig-zag in the public opinion.

4.2 The Unpopular Way to Populism

Alternatively, if the politician in office is able to forecast the dispositions dynamics, she may try to implement immediately the policy which will *eventually* please the median voter. If the public education level is the main issue, this is the "fixed point" education level satisfying

$$e^* = E\left(\frac{1}{2}; e^*, s_0; s_0\right). \tag{4.5}$$

Similarly, if the subsidy level is the central issue, it will be the subsidy rate satisfying

$$s^* = S\left(\frac{1}{2}; e_0, s^*; e_0\right). \tag{4.6}$$

With such a policy, the public will initially be dissatisfied with the leader, as the policy is different than the one promised in the platform (which solicited the median voter and ensured popularity). However, the public will eventually appreciate the new policy as everybody's dispositions adjust to the new regime. Consequently, the leader may end up successful and popular at the end of her cadence and ensure her re-election.

The policy change needed in the latter strategy may be more or less extreme than in the former, depending on the policy variable at stake:

[15] In the appendix.

Proposition 8 *(i) When changing the level of public education, following the public opinion requires a less extreme policy change than implementing the eventually-popular policy:*

$$\left| E\left(\frac{1}{2}; e_0, s_0; s_0\right) - e_0 \right| \leq |e^* - e_0| \tag{4.7}$$

(ii) When changing the subsidy rate, following the public opinion requires a more extreme policy change than implementing the eventually-popular policy:

$$\left| S\left(\frac{1}{2}; e_0, s_0; e_0\right) - s_0 \right| \geq |s^* - s_0| \tag{4.8}$$

Proof. See Appendix. ■

5 Concluding Remarks

One of the difficulties facing politicians is that public opinion keeps changing. A policy that is popular today becomes unpopular tomorrow. The changing public opinion reflects the dynamics of our life. Public opinion may change as a result of new events, realizations of uncertainties or changes in the population. Beside these well documented changes of public opinion, the view taken in this paper is that there are also some inherent endogenous concerns which are the outcome of the interplay between the economic implications of different policies and the formation of tastes. Leaders that were elected by promising one policy may become unpopular not because of a change of circumstances or new information regarding the policy or the leader, but rather because the policy induces an endogenous change of heart which affects the popularity of different policies.

One of the concerns in modelling such an effect is the time frame needed for such changes. The dynamics involving preference change might be slower than the political cycle of elections. This, however, does not imply that it is not relevant. It is possible that a policy implemented by one leader induces a disposition dynamics, and that before this dynamics settles down completely the society is going to face a new election round. But even in such a case opinions were affected by policy implementation and thus may affect, in turn, the outcome of the upcoming elections. As a modelling approach we chose to assume that the preferences dynamics is settled before the elections, as we need a form of analytical discipline in order to evaluate the popularity of different policies. However, the validity of our main message stands out independently of this modeling choice.

6 References

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7 Appendix: Proofs

Proof of Proposition 1. Given the action x_j of player j, player i's utility (gross of the tax, which she cannot influence with her actions)

$$U_i(x_i, x_j; \alpha_i, \tau_i, e, s) = (\alpha_i + \tau_i + e + s + \frac{1}{2}x_j - x_i)x_i$$

is maximized by $x_i = (2(e + s + \kappa_i) + x_j)/4$ where we denote $\kappa_i \equiv \alpha_i + \tau_i$.

When matched with an individual α_j with disposition τ_j, the Nash equilibrium actions are

$$\begin{aligned} x_i(\alpha_i, \alpha_j; \tau_i, \tau_j; e, s) &= \frac{10(e+s) + 8\kappa_i + 2\kappa_j}{15}, \\ x_j(\alpha_i, \alpha_j; \tau_i, \tau_j; e, s) &= \frac{10(e+s) + 8\kappa_j + 2\kappa_i}{15}. \end{aligned}$$

The material payoff to α_i in this interaction (gross of the tax) is

$$\begin{aligned} & f_i(\alpha_i, \alpha_j; \tau_i, \tau_j; e, s) = \left(e + s + \alpha_i + \frac{1}{2}x_j - x_i\right) x_i \\ = \ & \frac{2}{225}\left(2\kappa_j^2 + (20(e+s) + 16\alpha_i + \tau_i)\kappa_j + (10(e+s) + 8\alpha_i - 7\tau_i)(5(e+s) + 4\alpha_i + 4\tau_i)\right). \end{aligned}$$

So if K is the current average of κ_j in the population and V is its variance, the average monetary payoff to α_i when she has the disposition τ_i is

$$\begin{aligned} f_i(\alpha_i; \tau_i; e, s) &\equiv \int f_i(\alpha_i, \alpha_j; \tau_i, \tau_j; e, s)dj \\ &= \frac{2}{225}2\left(K^2 + V\right) + (20(e+s) + 16\alpha_i + \tau_i)K + \\ &\quad + (10(e+s) + 8\alpha_i - 7\tau_i)(5(e+s) + 4\alpha_i + 4\tau_i). \end{aligned}$$

The best (monetary-wise on average) disposition τ_i for α_i to have is therefore

$$\tau_i^*(\alpha_i; e, s) \equiv \arg\max_{\tau_i} f_i(\alpha_i; \tau_i; e, s) = \frac{1}{56}\left(K + 5\left(e + s\right) + 4\alpha_i\right). \tag{A.1}$$

Adding α_i on both sides gives

$$\tau_i^*(\alpha_i, e, s) + \alpha_i = \frac{1}{56}\left(K + 5\left(e + s\right) + 60\alpha_i\right).$$

Since the average α_i is $\frac{1}{2}$, taking the average on both sides at a dispositions equilibrium (in which an individual with ability α_j has the disposition $\tau_j^*(\alpha_j, e, s)$) gives

$$K = \frac{1}{56}\left(K + 5\left(e + s\right) + 30\right).$$

Solving for K gives

$$K = \frac{1}{11}\left(e + s + 6\right).$$

Substituting in $(A.1)$ and simplifying gives

$$\tau_i^*(\alpha_i; e, s) = \frac{1}{11}\left(e + s\right) + \frac{1}{14}\alpha_i + \frac{3}{308}$$

which yields the unique dispositions equilibrium and proves part (i) of the proposition. This disposition is increasing with e, s and $\alpha_i$which proves part (ii) of the proposition. ■

Proof of proposition 2. Suppose the initial policy variables are set at (e_0, s_0), and that individual i with ability α_i has the equilibrium disposition $\tau_i^*(\alpha_i; e_0, s_0)$. If a different policy (e, s) is considered ,individual i ,being unaware of the forthcoming adjustment of dispositions in response to the change in policy, believes that the Nash equilibrium actions when working with α_j will be

$$\begin{aligned}
&\hat{x}_i(\alpha_i, \alpha_j; e_0, s_0; e, s) = x_i(\alpha_i, \alpha_j; \tau_i^*(\alpha_i; e_0, s_0), \tau_j^*(\alpha_j; e_0, s_0); e, s) = \\
= \;& \frac{10\left(e + s\right) + 8\left(\frac{1}{11}\left(e_0 + s_0\right) + \frac{15}{14}\alpha_i + \frac{3}{308}\right) + 2\left(\frac{1}{11}\left(e_0 + s_0\right) + \frac{15}{14}\alpha_j + \frac{3}{308}\right)}{15} \\
= \;& \frac{2\left(e + s\right)}{3} + \frac{2\left(e_0 + s_0\right)}{33} + \frac{4\alpha_i + \alpha_j}{7} + \frac{1}{154}
\end{aligned}$$

Consequently, i believes that her utility (gross of tax) when working with a partner with ability α_j will be

$$\begin{aligned}
&\hat{U}_i(\alpha_i, \alpha_j; e_0, s_0; e, s) = \\
(\;& \left(\frac{\left(e_0 + s_0\right)}{11} + \frac{15\alpha_i}{14} + \frac{3}{308}\right) + e + s + \frac{1}{2}\left(\frac{2\left(e + s\right)}{3} + \frac{2\left(e_0 + s_0\right)}{33} + \frac{4\alpha_j + \alpha_i}{7} + \frac{1}{154}\right) \\
& - \left(\frac{2\left(e + s\right)}{3} + \frac{2\left(e_0 + s_0\right)}{33} + \frac{4\alpha_i + \alpha_j}{7} + \frac{1}{154}\right)) \left(\frac{2\left(e + s\right)}{3} + \frac{2\left(e_0 + s_0\right)}{33} + \frac{4\alpha_i + \alpha_j}{7} + \frac{1}{154}\right)
\end{aligned}$$

and that the per-capita tax will be

$$\hat{T}\left(e_0, s_0; e, s\right) = \frac{1}{2}e^2 + s\int_0^1\int_0^1 \left(\frac{4\left(e+s\right)}{3} + \frac{4\left(e_0+s_0\right)}{33} + \frac{5\alpha_i + 5\alpha_j}{7} + \frac{2}{154}\right) d\alpha_i d\alpha_j.$$

Thus, i believes that her average net utility will be

$$\begin{aligned}
&\hat{H}_i(\alpha_i; e_0, s_0; e, s) = \int_0^1 \hat{U}_i(\alpha_i, \alpha_j; e_0, s_0; e, s) d\alpha_j - \hat{T}\left(e_0, s_0; e, s\right) \\
= \; & -\frac{1}{18}e^2 - \frac{8}{9}s^2 + \left(\frac{8}{77} + \frac{16}{21}\alpha_i + \frac{8}{99}\left(e_0+s_0\right) - \frac{4}{9}s\right)e + \left(-\frac{4}{99}\left(e_0+s_0\right) - \frac{48}{77} + \frac{16}{21}\alpha_i\right)s \\
& + \frac{4}{1089}\left(e_0+s_0\right)^2 + \left(\frac{16}{231}\alpha_i + \frac{8}{847}\right)\left(e_0+s_0\right) + \frac{16}{49}\alpha_i^2 + \frac{48}{539}\alpha_i + \frac{79}{10\,164}
\end{aligned}$$

This average net utility is therefore quadratic and single-peaked as a function of either e or s, with maxima at

$$E\left(\alpha_i; e_0, s_0; s\right) \equiv \arg\max_{e \geq 0} \hat{H}_i(\alpha_i; e_0, s_0; e, s) = \frac{48}{7}\alpha_i + \frac{8}{11}\left(e_0+s_0\right) + \frac{72}{77} - 4s \qquad \text{(A.2)}$$

(we will only consider s_0, s with which this maximizer is indeed non-negative) and

$$S\left(\alpha_i; e_0, s_0; e\right) \equiv \arg\max_{s \in R} \hat{H}_i(\alpha_i; e_0, s_0; e, s) = \frac{3}{7}\alpha_i - \frac{1}{44}\left(e_0+s_0\right) - \frac{27}{77} - \frac{1}{4}e \qquad \text{(A.3)}$$

These maxima are both monotonic in the ability α_i. ■

Proof of Proposition 8. (A.2) implies that the unique "fixed point" education level is

$$e^* = E\left(\frac{1}{2}; e^*, s_0; s_0\right) = 16 - 12s_0$$

while

$$E\left(\frac{1}{2}; e_0, s_0; s_0\right) = \frac{48}{11} + \frac{8}{11}e_0 - \frac{36}{11}s_0.$$

As the latter is an increasing linear function of e_0 with slope smaller than 1, $e_0 \gtrless e^*$ implies that $e_0 \gtrless E\left(\frac{1}{2}; e_0, s_0; s_0\right) \gtrless e^*$, and hence

$$\left|E\left(\frac{1}{2}; e_0, s_0; s_0\right) - e_0\right| \leq \left|e^* - e_0\right|.$$

Similarly, it follows (A.3) that the "fixed-point" subsidy rate is

$$s^* = S\left(\frac{1}{2}; e_0, s^*; e_0\right) = -\frac{2}{15} - \frac{4}{15}e_0.$$

The function

$$S\left(\frac{1}{2}; e_0, s_0; e_0\right) = -\frac{3}{22} - \frac{3}{11}e_0 - \frac{1}{44}s_0.$$

is a decreasing linear function of s_0. Therefore $s_0 \gtrless s^*$ if and only if $s_0 \gtrless s^* \gtrless S\left(\frac{1}{2}; e_0, s_0; e_0\right)$, and we conclude that

$$\left|S\left(\frac{1}{2}; e_0, s_0; e_0\right) - s_0\right| \geq |s^* - s_0|$$

■