On the Rational Choice Theory of Voter Turnout

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Abstract. I consider a two-candidate election in which there is aggregate uncertainty about the popularity of each candidate, where voting is costly, and where participants are instrumentally motivated. The unique equilibrium predicts substantial turnout under reasonable parameter configurations, and greater turnout for the apparent underdog offsets the expected advantage of the perceived leader. I also present clear predictions about the response of turnout and the election outcome to various parameters, including the importance of the election; the cost of voting; the perceived popularity of each candidate; and the accuracy of pre-election information sources, such as opinion polls.

THE TURNOUT PARADOX

Why do people vote? Across the supporters of different competing candidates, how is turnout likely to vary? Is the election result likely to reflect accurately the pattern of preferences throughout the electorate? These questions are central to the study of democratic systems; they are questions which have attracted the attention of the most insightful theorists and the most careful empirical researchers.

Nevertheless, the turnout question (why do people vote?) has proved problematic for theories based on instrumental actors. In an oft-quoted question based on a statement made by Fiorina (1989), Grofman (1993) provocatively asked: is turnout the paradox that ate rational choice theory? The paradox is this: people do vote and yet it is alleged that any "reasonable" rational-choice theory suggests that they should not.

Grofman (1993, p. 93) explained that the rational-choice-eating claim arises from two predictions: firstly, that "few if any voters will vote" and, secondly, that "turnout will be higher the closer the election." He claimed that these predictions "are contradicted, in the first case" and that, in the second case, "either contradicted or at least not strongly supported" by the empirical evidence. This led to his "heretical view" that the followers (himself included) of Downs (1957) were "fundamentally wrong" in their quest for empirically supported predictions from rational-choice models.

More recently, Blais (2000, p. 2) supported the wasted-vote argument: "however close the race, the probability of [an instrumentally motivated voter] being decisive is very small when the electorate is large." His view was this is true even in a moderately sized electorate: "with 70,000 voters, even in a close race the chance that both candidates will get exactly the same number of votes is extremely small." He concluded: "the rational citizen decides not to vote." While acknowledging that the cost of voting is small, he reasoned that "the expected benefit is bound to be smaller for just about everyone because of the tiny probability of casting a decisive vote." From all of this, he concluded that the-known calculus-of-voting model (Downs, 1957; Riker and Ordeshook, 1968) "does not seem to work." This is a well-established view; for instance, Barzel and Silberberg (1973) highlighted that Arrow (1969, p. 61) said that it is "somewhat hard to explain ... why an individual votes at all in a large election, since the probability that his vote will be decisive is so negligible." Many others, sometimes reluctantly, have accepted this conclusion; for instance, in their survey Dhillon and Peralta (2002) led by quoting Aldrich (1997), who said that "the rationality of voting is the Achilles' heel of rational choice theory in political science."

It seems that, with a few exceptions, it has been accepted that voters' voluntary and costly participation cannot be explained by conventional goal-oriented behavior; indeed, the turnout paradox has been used by some (Green and Shapiro, 1994, notably) to argue against the use of the rational-choice methods in political science.

I argue that a theory of turnout based upon instrumentally motivated actors can work well and so is not the Achilles' heel of the rational-choice approach. I model a two-candidate election, where voting is costly, and where participants are instrumentally motivated. Hence, a voter balances the individuals costs of participation against the possibility of determining the winner. The substantive and reasonable departure from most established theories is this: there is aggregate uncertainty about the popularity of each candidate. (Models without this feature have the unattractive property that, in a large electorate, voters are able to predict almost perfectly the outcome; such models have other problems too.)

The unique equilibrium arising from the simple model proposed here is consistent with "substantial turnout" under "reasonable parameter configurations." Of course, to make sense of this claim I need to say what I mean by "substantial" and "reasonable." The predictions are helpfully illustrated by the following vignette, which emerges from a particular numerical instance of the paper's results.

> *Numerical Example.* Consider a region with a population of 100,000 where 75% of the inhabitants are able to vote if they wish to do so. Suppose that a 95% confidence interval for the popularity of the leading candidate stretches from 56% to 61%. If each voter is willing to participate in exchange for a 1-in-2,500 chance of influencing the outcome of the election, then, under this specification, the equilibrium of the model presented here predicts that turnout will exceed 50% of eligible voters. Greater turnout for the underdog offsets her disadvantage.

This scenario specifies the voters' perceptions of the candidates' popularities; the confidence interval approximates that which would be obtained following a pre-election opinion poll with a sample size of 1,500 respondents.[2] The presence of a non-degenerate confidence interval reflects the existence of aggregate uncertainty over

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[2]The confidence interval concerns the popularity of the leading candidate (and so, implicitly, the popularity of the underdog) rather than the leader's anticipated vote share. This is because the actual vote shares depend, of course, on the turnout behavior of the two groups of voters.

the underlying popularity of the competing candidates. The remaining elements of the scenario concern the (unique) equilibrium prediction from the model presented later in this paper. A critical factor is each voter's willingness to participate; this is captured by the pivotal probability which induces a voter to show up. In the vignette, each voter "is willing to participate in exchange for a 1-in-2,500 chance of influencing the outcome." This implies that the instrumental benefit of changing the electoral outcome for 100,000 people is 2,500 times as big as the cost of voting. More generally, in equilibrium it turns out that the expected turnout rate satisfies

$$\text{Turnout Rate} \approx \frac{\text{Instrumental Benefit/Voting Cost}}{\text{Population} \times \text{Width of 95\% Confidence Interval}}, \qquad (\star)$$

A brief check confirms that $(\star)$ generates the vignette discussed above. This same rule implies that voters need to show up for a 1-in-25,000 influence if 50% turnout is to arise in a world with 1,000,000 inhabitants. This probability might be described as small. But how small is small? Some in the literature have described the likely pivotal probability as "miniscule" (Dowding, 2005, p. 442). A pivotal probability of 1-in-2,500 or 1-in-25,000 could hardly be described as such; with a 5% wide confidence interval the required pivotal probability of roughly $40/N$ (where N is the population size) is distinctly higher than the ball-park "$1/N$" arising from the Tullock's (1967) reasoning. Nevertheless, some have maintained that such odds remain too small to induce participation; Owen and Grofman (1984, p. 322) claimed that if a voter enjoys "only a one-in-21,000 chance of affecting the election" and his "only reason for voting was to influence the outcome" then he would be "best off staying home."

If the Owen and Grofman (1984) advice to the 1-in-21,000 voter is accepted, then of course the turnout-is-rational claim must fail. However, it seems that odds of this kind are entirely consistent with the motivations of an instrumental actor. Of course, a voter who cares only about his narrow material self-interest might find it difficult to turn out for the relatively moderate odds of 1-in-25,000; after all, if it costs $5 to vote then the identity of the winning candidate over a term of office must make a difference

of \$125,000 to the life of the voter. Viewed so narrowly (as the effect of a change in tax rates, public expenditure, and so forth, as it impacts a single individual) this instrumental benefit could seem large. However, nothing in rational-choice theory requires a voter to be so selfish. As soon as any element (even if it is a small element) of other-regarding preferences (perhaps a desire to elect "the best candidate for the people" rather than "the best candidate for my pocketbook") is incorporated then the odds of influence begin to look rather more attractive.

A worked example illustrates the impact of very mild other-regarding preferences. Consider a voter who believes (paternalistically) that the election of his preferred candidate will improve the life of each citizen by \$250 per annum over a five-year term. Suppose that his personal voting cost is \$5. It is easy to see that if his concern for others is only 0.01% (that is, 1-in-10,000) then, in a population of 1,000,000, he will be willing to participate in exchange for a 1-in-25,000 chance of influencing the outcome; this is enough to support a 50% turnout rate.

The idea that such other-regarding preferences may help to explain turnout in large electorates has been explored by recent contributors to the literature. For instance, Jankowski (2002) memorably depicted voting as "buying a lottery ticket to help the poor" in work which he developed more formally in a later paper (Jankowski, 2007). Similarly, in a persuasive article, Edlin, Gelman, and Kaplan (2007) argued (although their argument is only suggestive; their paper is insightful but nevertheless incompletely developed) that turnout is likely to be substantial if voters have social preferences. They noted (p. 293): "In a large election, the probability that a vote is decisive is small, but the social benefits at stake in the election are large, and so the expected utility benefit of voting to an individual with social preferences can be significant." Here I provide a complete analysis of a game-theoretic model which, when combined with this idea, provides a resolution to the turnout paradox.

RELATED LITERATURE

The vast turnout literature has been expertly surveyed by many authors, including Blais (2000), Dhillon and Peralta (2002), Feddersen (2004), Dowding (2005), Geys (2006a,b), and others. The literature moved from a view that the influence of an individual vote is too small to generate significant turnout, implying that other factors such as a direct benefit from political participation must be present (Riker and Ordeshook, 1968); there was a renaissance while some authors conjectured that game-theoretic reasoning could yield an equilibrium outcome with reasonable turnout levels (Palfrey and Rosenthal, 1983; Ledyard, 1981, 1984); and, finally, the renaissance view was overturned as it was recognized that the early game-theoretic models relied on a knife-edge properties (Palfrey and Rosenthal, 1985). The status quo is reflected in the conclusion (Feddersen and Sandroni, 2006b, p. 1271) that "there is not a canonical rational choice model of voting in elections with costs to vote."

Many theories of voting (including models of strategic voting, as well as turnout-focused work) have suffered from a difficult feature: researchers have specified models in which voters' types (and so their voting decisions) are independent draws from a known distribution.[3] For instance, in a typical two-candidate model a voter prefers a right-wing candidate with probability p and her left-wing challenger with probability $1-p$, where p is commonly known by all, and types are independent draws. Under this specification there is no aggregate uncertainty: when the electorate is large the law of large numbers tells us that the support for each candidate (and the election outcome, if turnout is non-negligible) is known with high precision. The absence of any real uncertainty might set alarm bells ringing. Looking a little further, "independent type" models have other difficult features. For instance, if $p \neq 12$ (away from a knife edge) then the probability of tie vanishes to zero exponentially as the electorate

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[3]For examples, the classic turnout models of xxx, xxx, and xxxx all involve an independent-type specification; the models of strategic voting by xxx, xxxx, and xxxx do the same; and other work, including the comparison of the signaling and short-term instrumental motivations by xxxxx, and the evaluation of the welfare performance of voluntary-voting, also use independent-type specifications.

grows larger. The latter feature would seem (if an independent-type specification were to be accepted) to support the claim that the influence of an individual voter over an election's outcome is negligible.

However, Good and Mayer (1975) elegantly demonstrated that the absence of aggregate uncertainty is crucial. When every member of an n-strong electorate votes, but the probability p of support for the right-wing candidate is uncertain and drawn from a density $f(\cdot)$, then the probability of a pivotal event is $f(\frac{1}{2})/2n$; this is inversely proportional (so not exponentially related) to the electorate size.[4] Of course, this shrinks as n grows; however, it is difficult to conclude that it is negligibly small, and indeed the precise size (and presumably any predictions arising from a model with aggregate uncertainty) depend upon beliefs about the candidates' relative popularity. Sadly, the analysis of Good and Mayer (1975) was neglected; indeed, Fischer (1999) wrote that "Good and Mayer's method has largely been ignored, forgotten or unknown by most who have needed to calculate the value of [the probability of being decisive], although Chamberlain and Rothschild (1981) apparently independently discovered results which were in accord with those of Good and Mayer."

Fortunately, there are a few contributions which recognize the Good-Mayer analysis and its implications. A notable article here is by Edlin, Gelman, and Kaplan (2007), who recognized that a vote's influence is "roughly proportion to $1/n$." They developed ideas (to which I return later in the paper) suggesting that other-regarding concerns may increase with n, and so substantial turnout may be maintained even in large electorates. The ideas in their paper are good; however, their analysis is incomplete. Whereas they presented formally stated propositions, there described no fully specified model, and no proofs of their claims; their paper is suggestive rather than conclusive, although I argue that their suggestions are the conclusions that should be

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[4]When the electorate size n is even, the probability of an exact tie is approximately $f(\frac{1}{2})/n$. However, an individual vote can never change the election outcome for sure; it can only break or create a tie. For that reason, the additional factor $\frac{1}{2}$ is present when evaluating the influence of a vote.

reached. Indeed, it turns out that the results of Good and Mayer (1975) and Chamberlain and Rothschild (1981) cannot be directly used. The reason is that both of those early papers restricted to a world in which each voter has only two options; for instance, vote L or vote R. A model which allows for voluntary turnout must allow for a third option—namely, abstention—and yet the early results do not apply. One contribution of this paper, then, is to extend those earlier results to beyond the binary-option setting, so enabling the analysis of a game-theoretic model of voluntary turnout with aggregate uncertainty.[5]

Within the context of a generalized aggregate-uncertainty model, I show that the Good-Mayer result that a voter's influence is of order $1/n$ remains true. However, this influence is larger when the turnout rates of supporters of the different candidates offset any asymmetry in perceived support for them. For instance, if the right-wing candidate is expected to be twice as popular as the left-wing candidate (using then notation above, I mean that $\mathrm{E}[p] = 2/3$) then if the turnout rate of left-wing supporters is twice as high then the right-wing candidate's advantage is neutralized, and so the likelihood of a close race is maximized. A contribution of this paper is to show that this precisely what happens in the unique equilibrium.

A related point was made in a recent article by Taylor and Yildirim (2010). They considered an "independent types" model without aggregate uncertainty. They used the fact that supporters of different candidates are interested in slightly different close-call events. For instance, a left-wing voter is interested in situations in which the left-wing candidate is one vote behind her competitor (an extra vote will then create a tie, and so a possible left-wing win) while a right-wing voter considers outcomes in which the right-wing candidate is one vote behind. (Both types of voters are interested in situations with an exact tie.) In an equilibrium, both types of voters must

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[5]Although there is aggregate uncertainty (voters do not know for sure how popular the candidates are) each voter does know his own preference. So, this is a private-value model, unlike the common-value models applied to jury voting (for example, Feddersen and Pesendorfer, 1996, 1997, 1998) in which a voter performs a condition-on-being-pivotal calculation to ascertain his own preferred option.

perceive the same probability of being pivotal; the two events in which a favored candidate in one-vote-behind must be equally likely. This happens only when the turnout rates unwind any popularity-derived advantage for one of the candidates. The logic of Taylor and Yildirim (2010) suggesting that equilibrium considerations should enable an underdog to prosper is good; but nevertheless I show that their analysis does not work once a model with aggregate uncertainty is considered. In the presence of such aggregate uncertainty, the one-vote-behind outcomes are always equally likely in a large electorate.

So how do I generate the greater-turnout for the underdog effect? The answer is that different voter types (left-wing and right-wing) perceive the election differently because they are introspective. That is, a voter uses his type to update his beliefs about p. (Notice that this can only happen in a world with aggregate uncertainty; if $f(p)$ is degenerate then a voter has nothing new to learn.) His initial beliefs (before observing his type) are described by $f(p)$. Now, when evaluated at the mean $\bar{p} = \mathrm{E}[p]$ a straightforward application of Bayes' rule confirms that $f(\bar{p}\,|\,L) = f(\bar{p}\,|\,R) = f(\bar{p})$; that is, when thinking about the likelihood that the underlying division of support is equal the expected degree of support, a voter's beliefs do not shift when he conditions on his type. What this means is that voters' beliefs coincide when they worry about the likelihood that $p = \bar{p}$; and they are concerned about this only when $p = \bar{p}$ results in a close-run race. For this to be so, the turnout rates of the left-wing and right-wing voters must be inversely related to the corresponding candidates' expected popularities.

currently under construction — do feel free to jump ahead

A Simple Model of a Plurality Rule Election

The model is intentionally simple; it is a familiar two-candidate voting model but, crucially, with aggregate uncertainty over the candidates' relative support.

An electorate comprises $n+1$ voters. Each voter who is willing and able to participate casts a ballot for either candidate L (left) or candidate R (right). Throughout the paper, the pronoun "she" indicates a candidate, while the pronoun "he" indicates a voter. The simple plurality rule operates: the candidate with the most votes wins; if the vote totals are equal then a coin toss breaks the tie and decides the winner. Everything that I say is robust to the precise choice of tie-break rule.

Each voter is available to vote (that is, he is not precluded from doing so by some exogenous event) with probability a. Conditional on a, voters' availabilities are independent and identically distributed. However, there is aggregate uncertainty about a; it is drawn from a distribution with a continuous density $g(\cdot)$. I write $\bar{a} = \int_0^1 a g(a)\, da$ for the expectation of a. Hence, if everyone who was able to do so voted then the expected turnout would be $\bar{a}(n+1)$. Nothing in this paper relies on the fact that there is aggregate uncertainty about a. However, this specification allows for uncertainty about the precise size of the electorate, or the possibility that there may be an exogenous shock (inclement weather, perhaps) to the electorate's availability.

A randomly chosen voter prefers candidate R with probability p and candidate L with probability $1-p$. I refer to p as the true underlying popularity of R relative to L. Conditional on p, voters' types are independent and identically distributed. However, just as there is aggregate uncertainty about a, there is also aggregate uncertainty about p. Specifically, p is drawn from a distribution with a density $f(\cdot)$, where for ease of exposition I assume that there is full support on the unit interval. Thus, prior to their type realizations, voters share a common prior $p \sim f(p)$. I write $\bar{p} = \int_0^1 p f(p)\, dp$ for the prior expectation of p. However, each individual voter's belief about p can differ from $f(p)$, simply because his own type realization reveals extra information about p.

The remaining elements of the model's specification are the players' payoffs. Voting is voluntary, but costly: a voter incurs a positive cost $c > 0$ if he goes to the polls; on the other hand, abstention is free. I assume that all voters share the same cost of voting, although in a later section I discuss what happens when I relax this part of the model's specification. A voter enjoys a positive benefit u if and only if his preferred candidate wins the election. Again, I assume that the benefit component u of a voter's payoff is common to everyone.[6]

With the model's basic ingredients in place, I describe one further piece of notation. The only decision available to a voter is one of participation; if he arrives at the voting booth then he (optimally) cast his ballot for his favorite candidate. I look at type-symmetric strategy profiles, and for most of the paper I focus on those profiles which involve incomplete turnout. By "type symmetric" I mean that voters of the same type (either L or R, according to their candidate preference) behave in the same way; by "incomplete turnout" I mean situations in which not everyone shows up to vote. A strategy profile that fits both of these criteria reduces to a pair of probabilities t_R and t_L which satisfy $0 < t_R < 1$ and $0 < t_L < 1$ respectively; these probabilities are the turnout rates amongst the two type-determined subsets of the electorate. (However, in a later section I also consider cases with complete turnout on at least one side, so that $t_L = 1$ or $t_R = 1$.) Given these parameters, the overall turnout rate is $t = a(pt_R + (1 - p)t_L)$, and the expected turnout rate is $\bar{t} = \bar{a}(\bar{p}t_R + (1 - \bar{p})t_L)$.

OPTIMAL VOTING

The pivotal-voter logic is very familiar: when a voter considers his turnout decision he balances the cost of participation against the likelihood of influencing the result. He changes the outcome, of course, only when he is pivotal.

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[6]Although I have not cluttered the notation to indicate this, I allow the benefit u and cost c terms to vary with electorate size n. In many existing papers, such parameters are fixed while the electorate expands. However, this is restrictive. A change in the electorate size is a change in the game played by voters, and so payoffs should change too. In particular, it seems reasonable to suppose that u is responsive (either positively or negatively) to n; later in the paper I consider this explicitly.

Here I consider the decision faced by someone who is available to vote, as he considers the likely outcome amongst the other n members of the electorate. I write b_L and b_R for the vote totals amongst these others. Note that $b_L + b_R \leq n$, and that the inequality can hold strictly (indeed, usually it does so) because some electors are exogenously unavailable (recall that a voter is unable to visit the polls with probability $1 - a$) and because others endogenously choose to abstain.

Consider the situation faced by a supporter of candidate R. If there is a tie amongst others (that is, if $b_R = b_L$) and if the tie-break coin toss goes against R, then his participation is pivotal to a win for R. Similar, if there is a near-tie, by which I mean that $b_R = b_L - 1$, and the tie-break coin toss is favorable, then his participation will again change the outcome from a win for L to a win for R. I have assumed that the returning officer employs a fair coin, and so the probability of each of the aforementioned coin-toss outcomes is simply $\frac{1}{2}$. In all other circumstances, the voter in question can have no influence on the election's outcome. Assembling these observations, and performing similar reasoning for a supporter of candidate L, I obtain

$$\Pr[\textbf{Pivotal} \,|\, R] = \frac{\Pr[b_R = b_L \,|\, R] + \Pr[b_R = b_L - 1 \,|\, R]}{2}, \quad \text{and} \tag{1}$$

$$\Pr[\textbf{Pivotal} \,|\, L] = \frac{\Pr[b_R = b_L \,|\, L] + \Pr[b_L = b_R - 1 \,|\, L]}{2}. \tag{2}$$

A supporter of R finds it strictly optimal to participate if and only if the expected benefit from voting exceeds the cost; that is, if and only if $u \Pr[\textbf{Pivotal} \,|\, R] > c$. Naturally, if this inequality (and the equivalent inequality for a supporter of L) holds then, given that the c and u payoff parameters are common to everyone, turnout will be complete. However, as turnout increases (that is, as the type-contingent turnout probabilities t_R and t_L rise) the pair of pivotal probabilities will typically fall. If the turnout strategies ensure that the expected costs and benefits of voting are equalized for both voter types then these strategies yield an equilibrium. (Formally, this is a type-symmetric Bayesian Nash equilibrium in mixed strategies.) For parameters in an appropriate

range, such an incomplete-turnout equilibrium (where $1 > t_R > 0$ and $1 > t_L > 0$) is characterized by a pair of equalities:

$$\Pr[\mathbf{Pivotal} \,|\, R] = \Pr[\mathbf{Pivotal} \,|\, L] = \frac{c}{v}. \tag{3}$$

Conceptually, an equilibrium characterization is quite straightforward: simply find a pair of turnout probabilities t_L and t_R such that these equalities are satisfied. However, in general the pivotal probabilities depend on t_L, t_R, $f(\cdot)$, $g(\cdot)$, and n in a complex way. Nevertheless, in the presence of aggregate-level uncertainty these probabilities become more tractable in larger electorates. I show this in the next section.

ELECTION OUTCOMES WITH AGGREGATE UNCERTAINTY

Taking a step back from the specific voting model, here I consider the properties of beliefs about the election outcome when there is aggregate uncertainty about the popular support for the various candidates. In the absence of such uncertainty, voting decisions may be modeled as independent draws from a known distribution, leading to a multinomial outcome for the vote totals. However, when these probabilities are unknown (this is the aggregate uncertainty) then the decisions are only conditionally independent; unconditionally there is correlation between voters' ballots.[7]

Consider, then, an election in which each of n participants casts a ballot for one of $m + 1$ options; so, there are m candidates, and the remaining option is abstention. Suppose that the electoral support for the options is described by $v \in \Delta$, where $\Delta = \left\{ v \in \mathcal{R}_+^{m+1} \,\middle|\, \sum_{i=0}^n v_i = 1 \right\}$ is the m-dimensional unit simplex. The interpretation here is that v is a vector of voting probabilities: a randomly selected elector votes for

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[7]When studying elections, a natural modeling approach is to think of the various voters as symmetric ex ante. One way of doing this is to suppose that their types are independent draws from the same distribution. This, however, is a particularly strong form of symmetry. The more general form of symmetry is that beliefs about a set of voters do not depend on the labeling of those voters. This does not imply that voting decisions are independent, but rather that they are exchangeable in the sense of de Finetti (see, for instance, Hewitt and Savage, 1955). Indeed, if a potentially infinite sequence of voters can be envisaged then (at least for binary type realizations) exchangeability ensures a conditionally independent representation, just as described in this section.

candidate i with probability v_i, and abstains with probability $v_0 = 1 - \sum_{i=1}^{n} v_i$. While v can be interpreted as the underlying electoral support for the different candidates, it does not necessarily represent the actual underlying popularity of them. The distinction is because v_i is the probability that an elector votes for i, and not the probability that he prefers that candidate. So, adapting the notation of the two-candidate model, if the supporters of candidate i turnout with probability t_i then $v_i = a p_i t_i$.

Even if the electoral support v for the candidates is known (with aggregate uncertainty, it is not) then the election outcome remains uncertain owing to the idiosyncrasies of individual vote realizations. That outcome is represented by $b \in \Delta^\dagger$, where $\Delta^\dagger = \left\{ b \in \mathcal{Z}_+^{m+1} \mid \sum_{i=0}^{n} b_i = n \right\}$; here, b_i is the number of votes cast for candidate i. Conditional on v, independent and identically distributed voting decisions ensure that b is the realization of a multinomial random variable. However, here I suppose that the underlying electoral support for the options (the m candidates and the abstention option) is unknown. Specifically, suppose that beliefs about v are represented by a continuous and bounded density function $h(v)$ ranging over Δ. Taking expectations over v, the probability of an election outcome b is

$$\Pr[b \mid h(\cdot)] = \int_\Delta \frac{\Gamma(n+1)}{\prod_{i=0}^{m} \Gamma(b_i + 1)} \left[\prod_{i=0}^{m} v_i^{b_i} \right] h(v)\, dv, \tag{4}$$

where the usual Gamma function $\Gamma(\cdot) \equiv \int_0^\infty y^{x-1} e^{-y}\, dy$ satisfies $\Gamma(x+1) = x!$ for $x \in \mathcal{N}$.

Obviously the expression for $\Pr[b \mid h(\cdot)]$ in (4) is somewhat complex, but for larger n all that really matters is the value of density $h(\cdot)$ evaluated at the peak of $v_i^{b_i}$. That peak occurs at $v = \frac{b}{n}$. An inspection of (4) confirms that $\prod_{i=0}^{n} v_i^{b_i}$ is sharply peaked around its maximum; as n grows $\prod_{i=0}^{n} v_i^{b_i}$ becomes unboundedly larger at its maximum then elsewhere. This is why only the density $h\left(\frac{b}{n}\right)$ really matters, and so

$$\Pr[b \mid h(\cdot)] \approx h\left(\frac{b}{n}\right) \int_\Delta \frac{\Gamma(n+1)}{\prod_{i=0}^{m} \Gamma(b_i + 1)} \left[\prod_{i=0}^{m} v_i^{b_i} \right] dv = \frac{xxx}{xxx} h\left(\frac{b}{n}\right),$$

where the final inequality follows from spotting that the integrand is the density of a Dirichlet distribution.

This logic was used originally by Good and Mayer (1975) and subsequently by Chamberlain and Rothschild (1981). In a two-option election, they considered the probability of a tie (here, this corresponds to $m = 1$ and the event $b_0 = b_1 = \frac{n}{2}$, where n is even) and demonstrated that (in an obvious notation) it converges (in an appropriate sense) to $\frac{1}{n} h\left(\frac{1}{2}, \frac{1}{2}\right)$ as n grows. The same logic holds, of course, for larger m and for more general electoral outcomes. This is confirmed by Lemma 1, which provides the first step in a generalization of the Good and Mayer (1975) and Chamberlain and Rothschild (1981) results.[8]

Lemma 1. *Consider a sequence of elections indexed by $n \in \mathcal{N}$, but where the voting probabilities are described by $h(\cdot)$. Then $\lim_{n \to \infty} \max_{b \in \Delta^\dagger} |n^m \Pr[b] - h(b/n)| = 0$.*

An implication of this lemma is that what really matters when thinking about the likelihood of electoral outcomes is not the idiosyncratic type realizations which determine the probability of an outcome b conditional on the underlying v, but rather than density $h(\cdot)$ which captures the aggregate uncertainty about candidates' popular support. The law of large numbers ensures that any idiosyncratic noise is averaged out. In an election with few voters (a committee, perhaps) idiosyncratic noise remains. In the committee context, then, a theoretical model which specifies only idiosyncratic uncertainty (so that players' types are independent draws from a known distribution) can be useful. When there are more voters, however, the use of "independent type" models is discomforting. When there is no aggregate uncertainty, the modeler is forcing voters' beliefs to be entirely driven by factors (idiosyncratic type realizations) which are simply eliminated when there is mild aggregate uncertainty.

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[8]Good and Mayer (1975) and Chamberlain and Rothschild (1981) considered elections with two options (so $m = 1$ in the notation of this section), where n is even. They considered the probability of a tied outcome; this corresponds to a sequence of election outcomes of the form $b^n = \left(\frac{n}{2}, \frac{n}{2}\right)$, which obviously satisfies $\lim_{n \to \infty} \left(\frac{b^n}{n}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$. Equation (2) from Good and Mayer (1975) corresponds to Lemma 1 for this special case; Proposition 1 from Chamberlain and Rothschild (1981) reports the same result.

My attention now turns to the probability of pivotal events in elections with only two competing candidates. Here I return from the general m-candidate notation used in the previous section and move back to the (L, R)-notation used throughout the remainder of the paper. For this case, the probability of an exact tie (a near tie, where the candidates are separated by a single vote, takes a similar form) is

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\Pr[b_L = b_R] = \sum\nolimits_{z=0}^{\lfloor n/2 \rfloor} \Pr[b_L = b_R = z]
$$
$$
= \sum\nolimits_{z=0}^{\lfloor n/2 \rfloor} \frac{\Gamma(n+1)}{[\Gamma(z+1)]^2 \Gamma(n-2z+1)} \int_\Delta (v_L v_R)^z v_0^{n-2z} h(v)\, dv, \quad (5)
$$

where $v \in \Delta$ combines the voting probabilities v_L, v_R, and $v_0 = 1 - v_L - v_R$, and where $\lfloor n/2 \rfloor$ indicates the integer part of $\frac{n}{2}$. The probability in (5) is, of course, relatively complex. However, for larger n Lemma 1 can be exploited and this probability simplifies dramatically. A rigorous analysis is contained within the proof of Lemma 2, and so here I use a rough heuristic argument. Applying Lemma 1,

$$
\Pr[b_L = b_R = z] \approx \frac{h\left(1 - \frac{2z}{n}, \frac{z}{n}, \frac{z}{n}\right)}{n^2}. \quad (6)
$$

This expression can be substituted into the summation of (5), so that

$$
\Pr[b_L = b_R] \approx \frac{1}{n} \sum\nolimits_{z=0}^{\lfloor n/2 \rfloor} \frac{h\left(1 - \frac{2z}{n}, \frac{z}{n}, \frac{z}{n}\right)}{n}. \quad (7)
$$

Allowing n to grow large, notice that summation defines a familiar Riemann integral of the integrand $h(1-2x, x, x)$ over the range $[0, 1/2]$. Dealing with all of these heuristic steps rather more carefully generates the following lemma.

Lemma 2. *In a two-candidate election, with abstention allowed, and support for the two candidates and abstention described by the density $h(v_0, v_L, v_R)$, the probability of a tie between the two candidates satisfies $\lim_{n \to \infty} n \Pr[b_L = b_R] = \int_0^{1/2} h(1 - 2x, x, x)\, dx$.*

Notice once again that (in the limit) the probability of a tie depends solely on the properties of aggregate uncertainty via the density $h(\cdot)$.

An instrumental voter has a particular interest in the probability of a tied outcome: his participation in the election can serve to break the tie. However, such an instrumental voter is also interested in situations which are one away from a tie.

When there is no aggregate uncertainty, the probability of different near-tie events can be very different. This easiest to see by considering a world in which n is odd and there is no abstention, so that $v_0 = 0$. In this world, if v is known, then

$$\Pr[b_L = b_R \pm 1] = \frac{\Gamma(n+1)}{\Gamma\left(\frac{n+1}{2} + 1\right)\Gamma\left(\frac{n-1}{2} + 1\right)} v_L^{(n\pm 1)/2} v_R^{(n\mp 1)/2}, \qquad (8)$$

where the "$\pm$" and "$\mp$" notation should have an obvious interpretation. The relative likelihood of the two different kinds of close ties is

$$\frac{\Pr[b_L = b_R + 1]}{\Pr[b_L = b_R - 1]} = \frac{v_L}{v_R} \neq 1, \qquad (9)$$

where of course the "$\neq 1$" claim holds if and only if $v_L \neq v_R$. Equation (9) holds for any n, and once again illustrates the worrying properties of IID-specified models; surely two such close events should not have radically different probabilities in a large electorate?[9] In fact, once aggregate uncertainty is introduced they do not have different (limiting) probabilities. This is confirmed in the next lemma.

Lemma 3. *In a two-candidate election with abstention, where the beliefs of a type $i \in \{L, R\}$ about the electoral support for the candidates are described by the density $h(v_0, v_L, v_R \,|\, i)$, the probabilities of a tie and a near tie are asymptotically equivalent:*

---

[9]Some contributors to the literature have relied on the property reported in equation (9). For example, Taylor and Yildirim (2010) employed an IID specification, and in their model (as in this one, as I shall show) an equilibrium requires different types to perceive the same probability of pivotality, and so $\Pr[b_L = b_R + 1] = \Pr[b_L = b_R - 1]$. Given (9), this can only be true if $v_L = v_R$, and so turnout must be inversely proportional to the popularity of a candidate, so that $p_L t_L = p_R t_R$. Once aggregate uncertainty is introduced, their argument no longer applies; it relies on the knife-edge properties of a IID-specified model. Fortunately, however, their conclusion (that turnout should be inversely related to the perceived relative popularity of the competing candidates) remains, as I confirm in this paper.

$\lim_{n\to\infty} n \Pr[b_L = b_R \pm 1 \mid i] = \lim_{n\to\infty} n \Pr[b_L = b_R \mid i]$. **Furthermore,**

$$\lim_{n\to\infty} n \Pr[\textbf{\textit{Pivotal}} \mid i] = \int_0^{1/2} h(1 - 2x, x, x \mid i) \, dx \quad \textit{for} \quad i \in \{L, R\}. \tag{10}$$

From Lemma 3, a voter's beliefs about pivotal events are determined by $h(\cdot \mid i)$; this is conditional on the voter's type $i \in \{L, R\}$, simply because his own type teaches him, via introspection, about the popularity of the candidates. But of course, the density which appears in (10) concerns the electoral support for the candidates, rather than their popularities. To form $h(\cdot \mid i)$, a voter takes into account the turnout probabilities on the two sides, as well as possible variation in the size of the available electorate.

Recall that each elector is available on polling day with probability a, and prefers candidate R rather than L with probability p. Beliefs about these two aspects of the electoral environment arise from $g(a)$ and $f(p)$ respectively, and so the underlying electoral situation is described by $(a, p) \in [0, 1]^2$ with density $f(p)g(a)$. However, a voter updates his beliefs based on his own availability and his own preference. Abusing notation a little, I write $g(a \mid i)$ for the conditional belief of an available voter about the availability of others. From Bayes' rule, it is easy to see that

$$g(a \mid i) = \frac{g(a)a}{\bar{a}}, \quad f(p \mid L) = \frac{f(p)(1 - p)}{1 - \bar{p}} \quad \text{and} \quad f(p \mid R) = \frac{f(p)p}{\bar{p}}, \tag{11}$$

where $\bar{a}$ is the ex ante expected availability of voters, and $\bar{p}$ is the ex ante expected popularity of candidate R. Of course, a voter's conditional beliefs about a and p must be transformed into beliefs about v_L and v_R (and, of course, the abstention probability via $v_0 = 1 - v_L - v_R$). If supporters turn out with probabilities t_R and t_L then the corresponding voting probabilities for the candidates are $v_R = apt_R$ and $v_L = a(1-p)t_L$. The Jacobian of this transformation is readily obtained:

$$\frac{\partial(v_L, v_R)]}{\partial(p, a)} = \begin{bmatrix} at_R & pt_R \\ -at_L & (1-p)t_L \end{bmatrix} \quad \Rightarrow \quad \left| \frac{\partial(v_L, v_R)]}{\partial(p, a)} \right| = at_L t_R. \tag{12}$$

Looking back to Lemma 3, the density $h(\cdot \,|\, i)$ is only evaluated where $v_L = v_R = x$. Using these inequalities it is straightforward to solve for p and a, and so

$$h(x, x, 1 - 2x) = \frac{f(p \,|\, i) g(a \,|\, i)}{a t_L t_R} \quad \text{where} \quad p = \frac{t_L}{t_L + t_R} \quad \text{and} \quad a = \frac{x(t_R + t_L)}{t_R t_L}. \tag{13}$$

Using (11) and (13) together with Lemma 3, yields the following result.

Lemma 4. *Consider an election in which the supporters of candidates L and R partic- ipate with probabilities t_L and t_R respectively. From the perspective of a voter thinking about the outcome amongst the other n voters:*

$$\lim_{n \to \infty} n \Pr[\textbf{\textit{Pivotal}} \,|\, L] = \frac{f(p^\star)}{\bar{a}(t_L + t_R)} \frac{1 - p^\star}{1 - \bar{p}} \quad and \tag{14}$$

$$\lim_{n \to \infty} n \Pr[\textbf{\textit{Pivotal}} \,|\, R] = \frac{f(p^\star)}{\bar{a}(t_L + t_R)} \frac{p^\star}{\bar{p}}, \quad where \quad p^\star = \frac{t_L}{t_L + t_R}, \tag{15}$$

and where $\bar{p}$ is the expected popularity of R and $\bar{a}$ is the expected availability of voters.

This lemma uses the notation "$p^\star$" for a critical threshold level of the underlying pop- ularity of R relative to L. Notice that $v_L = v_R$ if and only if $p = p^\star$. Hence, if candidate R is to win then her popularity p must exceed $p^\star$. A tied outcome is only really feasible when p is close to $p^\star$, and so when contemplating the likelihood of a pivotal event a voter asks how likely this is by evaluating the density $f(p^\star)$.

Several other aspects of Lemma 4 are worthy of note. Firstly, the likelihood of a pivotal outcome is, of course, inversely proportional to the electorate size n.[10] A con- sequence of this is that the relative size of benefits and costs, captured by the ratio $\frac{v}{c}$, needs to be larger in a larger electorate if the same turnout behavior is to be sup- ported. Secondly, and relatedly, the pivotal probability is inversely proportional to the overall level of turnout (doubling both t_L and t_R, for instance, halves the probability of

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[10]As observed by Good and Mayer (1975), this is not the case when the electoral support for the can- didates is known, so that voting decisions are independent draws from a known distribution. Under an IID specification, the probability of a pivotal event is inversely proportional to the square root of the electorate size in the knife-edge case where the underlying support of the candidates is precisely balanced; otherwise, the probability disappears exponentially with the electorate size.

a pivotal outcome) and is inversely proportional to the expected availability of voters at the polls. Thirdly, and perhaps most interestingly, the probability of a tied outcome depends on the nature of the voters' ex ante beliefs $f(p)$ about the relative support of the candidates. Indeed, the probability is higher as $p^\star = t_L/(t_L + t_R)$ moves closer to the mode of $f(\cdot)$.

EQUILIBRIUM TURNOUT

Fixing the conjectured turnout behavior (that is, t_L and t_R) of others, Lemma 4 characterizes the impact of an additional vote for both types of elector in a large electorate. Here I turn my attention toward equilibrium considerations.

Recall, from equation (3), that an equilibrium with incomplete turnout (that is, where $1 > t_L > 0$ and $1 > t_R > 0$) is characterized by the pair of equalities

$$\Pr[\text{Pivotal} \,|\, R] = \Pr[\text{Pivotal} \,|\, L] = \frac{c}{u}. \tag{16}$$

In general, these probabilities take a relatively complex form and so this characterization is obtuse. However, Lemma 4 demonstrates that the approximations

$$\Pr[\text{Pivotal} \,|\, L] \approx \frac{f(p^\star)}{\bar{a}(t_L + t_R)n} \frac{1 - p^\star}{1 - \bar{p}} \quad \text{and} \quad \Pr[\text{Pivotal} \,|\, R] \approx \frac{f(p^\star)}{\bar{a}(t_L + t_R)n} \frac{p^\star}{\bar{p}}$$

$$\text{where} \quad p^\star \equiv \frac{t_L}{t_L + t_R} \tag{17}$$

work well when the electorate is large. I proceed, then, in a pragmatic way by assuming that voters employ the approximations in (17) when they evaluate their decisions.

Definition. *A* voting equilibrium *is a pair of voting probabilities t_L and t_R such that voters act optimally given that they use the asymptotic approximations of* (17).

This is a kind of "ε equilibrium" in the sense that, for finite n, voters are only approximately optimizing. Nevertheless, for even moderately large elections the approximations in (17) are extremely good. Furthermore, in a later section of the paper (missing

from this draft manuscript) I consider other justifications for the solution concept employed here. I also evaluate (in a section which is also missing from this preliminary draft) the extent of the error in the approximations via a computational study, and demonstrate that the approach works well even in a relatively small electorate.[11]

I now characterize the unique voting equilibrium. I write $\Pr^\dagger[\text{Pivotal}\,|\,i]$ for $i \in \{L, R\}$ for the approximations of (17). If turnout is incomplete then a voting equilibrium must satisfy $\Pr^\dagger[\text{Pivotal}\,|\,L] = \Pr^\dagger[\text{Pivotal}\,|\,R] = \frac{c}{u}$. Inspecting (17), notice that the equality of the pivotal probabilities holds if and only if $p^\star = \bar{p}$.

Lemma 5. *A voting equilibrium with incomplete turnout must satisfy* $\bar{p} = t_L/(t_L + t_R)$.

This lemma is of independent interest. It says that the turnout rates amongst the two factions of the electorate must exactly offset the prior expected asymmetry between their popularity. Recall that $p^\star \equiv t_L/(t_L + t_R)$ is a critical threshold in the sense that the true underlying popularity of candidate R needs to exceed $p^\star$ if candidate R is to win, at least in expectation. By showing that $p^\star = \bar{p}$, Lemma 5 reveals that a candidate's true popularity (that is, either p or $1 - p$) must exceed her perceived popularity (either $\bar{p}$ of $1 - \bar{p}$) if she is going to carry the election.

Lemma 5 characterizes the relative size of the turnout rates t_L and t_R by solving the equation $\Pr^\dagger[\text{Pivotal}\,|\,L] = \Pr^\dagger[\text{Pivotal}\,|\,R]$. However, it does not tie down the level of these rates. This second step may be performed via the equation $\Pr^\dagger[\text{Pivotal}\,|\,i] = \frac{c}{v}$. Before doing this, it is useful to introduce one additional item of notation. Define:

$$\bar{t} = \bar{a}\left[\bar{p}t_R + (1 - \bar{p})t_L\right] \tag{18}$$

---

[11]The reason is that the approximations reported in (17) are obtained by averaging out the idiosyncratic noise. The law of large numbers bites relatively quickly as n increases, and so aggregate-level uncertainty dominates (and the approximations work well) even for moderate electorate sizes.

for the expected turnout rate from an ex ante perspective. Dividing this by $t_L + t_R$, applying Lemma 5, and using the approximations of (17), it is easy to see that

$$\frac{\bar{t}}{\bar{a}(t_L + t_R)} = \frac{\bar{p}t_R + (1 - \bar{p})t_L}{t_L + t_R} = 2\bar{p}(1 - \bar{p}) \quad \Rightarrow$$

$$\Pr^\dagger[\mathbf{Pivotal} \,|\, L] = \Pr^\dagger[\mathbf{Pivotal} \,|\, R] = \frac{f(\bar{p})}{\bar{a}(t_L + t_R)} = \frac{2\bar{p}(1 - \bar{p})f(\bar{p})}{\bar{t}}. \quad (19)$$

Equating this final expression to the cost-benefit ratio $\frac{c}{u}$ pins down the equilibrium.

Proposition 1. *If $\frac{u}{c}$ is not too large then there is a unique voting equilibrium with incomplete turnout. Ex ante, the expected turnout rate $\bar{t}$ satisfies*

$$\bar{t} = \frac{2\bar{p}(1 - \bar{p})f(\bar{p})u}{cn}, \quad (20)$$

while the individual turnout rates amongst the two groups of supporters are

$$t_L = \frac{\bar{p}f(\bar{p})u}{\bar{a}cn} \quad and \quad t_R = \frac{(1 - \bar{p})f(\bar{p})u}{\bar{a}cn}. \quad (21)$$

The asymmetry in these turnout rates offset any prior asymmetry in the perceived popularity of the candidates: the less popular candidate enjoys greater turnout, and so $\mathrm{E}[v_L] = \mathrm{E}[v_R] = \frac{1}{2}$. The overall expected turnout $\bar{t}n$ is increasing in the importance of the election and decreasing in the cost of voting. Fixing $f(\bar{p})$, turnout increases as the margin between the expected popularity of the two candidates falls.

The final comparative-static prediction holds because the quadratic term $\bar{p}(1 - \bar{p})$ in $\bar{t}$ peaks at $\bar{p} = \frac{1}{2}$. This is a familiar prediction: turnout is higher in marginal elections. Note, however, that the effect of increased asymmetry is weak when the candidates are evenly matched. That is, beginning from $\bar{p} = \frac{1}{2}$, a local change in $\bar{p}$ has only a second-order effect on turnout. In the introductory remarks to this paper, I commented on the claim, from Grofman (1993), that "turnout will be higher the closer the election" and that this is "not strongly supported" by the empirical evidence. The

claim is weakly supported here, but it should not necessarily be "strongly supported" owing to the second-order effect of candidate asymmetry close to $\bar{p} = \frac{1}{2}$.

The other properties of a voting equilibrium are unsurprising. In particular, turnout $\bar{t}$ is, other things equal, inversely proportional to electorate size. However, the "other things equal" is critical: as the electorate size changes, then so may the other parameters, in particular the payoff u which an instrumental voter enjoys from changing the identity of the winner. Also, turnout depends on the density $f(\bar{p})$ of beliefs about p. I consider this in more detail in the next section; however, it is worth noting that pre-election information and so the nature of $f(\cdot)$ may be different in larger electorates.

A further observation is that Proposition 1 imposes the condition that "u/c is not too large." The reason is that an equilibrium exhibits incomplete turnout if and only if $\max\{t_L, t_R\} < 1$. Applying the solutions from (21), this holds if and only if

$$\frac{u}{c} < \frac{\bar{a}n}{\max\{\bar{p}, (1-\bar{p})\}f(\bar{p})}. \tag{22}$$

If this inequality fails, then the voting equilibrium instead involves complete turnout for the candidate with the lower expected popularity; I deal with this in a later section of the paper (again, omitted from this preliminary draft manuscript).

A final observation I make here is that the equilibrium solution for the expected turnout $\bar{t}$ given by Proposition 1 does not depend on $\bar{a}$ or any other aspect of voters' aggregate availability. Hence, if in aggregate fewer electors are available or willing to vote, then aggregate turnout is unaffected. Inspecting the solutions for t_L and t_R given in (21), this is because the turnout rates of those who are "playing the turnout game" rise as $\bar{a}$ falls. This implies that the solution for turnout is robust to the supposition that some voters (a fraction $1 - \bar{a}$ in expectation) have (erroneously, I claim) reached the conclusion that their voter cannot count; the behavior of the "real players" endogenously adjusts. Of course, this is all true only so long as there is a voting equilibrium

with incomplete turnout. As the discussion just above has established, such an equilibrium exists only if the inequality (22) holds. By inspection, this inequality holds only if $\bar{a}$ is large enough. If $\bar{a}$ is sufficiently small (so that the voting-is-worthless message has really taken hold) then the inequality fails. If this happens, then a voting equilibrium involves incomplete turnout only on one side (the side with the perceived advantage) and complete turnout (amongst those voters who are willing and able to show up) on the other side. I study this case in a later section of the paper.

POPULARITY AND BELIEFS

From Proposition 1 it is clear that the properties of beliefs about the candidates' popularity, determined by the density $f(\cdot)$, are critical to the level of expected turnout. Note in particular that the density which enters the solution for $\bar{t}$ is evaluated at the expectation $\bar{p}$. For a well-behaved density this expectation will be close to the mode, which clearly maximizes the expected turnout rate. To move further, here I place a little more structure on the density $f(\cdot)$.

A natural specification for voters' beliefs is to assume that p follows a Beta distribution. Recall that p follows such a distribution with parameters β_R and β_L if

$$f(p) = \frac{\Gamma(\beta_R + \beta_L)}{\Gamma(\beta_R)\Gamma(\beta_L)} p^{\beta_R - 1} p^{\beta_L - 1}, \tag{23}$$

where $\Gamma(\cdot)$ is the Gamma function. A special case of this is obtained, for instance, when $f(p)$ is uniform: $\beta_R = \beta_L = 1$. The Beta also has the convenient property that it is conjugate with the binomial distribution. Hence, beginning from a Beta prior and updating via Bayes' rule following the observation of an opinion poll, beliefs are Beta distributed. Specifically, if a voter begins with a uniform prior over p (a special case of the Beta) and observes a random sample of potential voters containing $\beta_R - 1$ supporters of R and $\beta_L - 1$ supporters of L, then the voter's posterior belief about p follow the Beta with parameters β_R and β_L. Thus $s = \beta_R + \beta_L$ indexes the size of

the sample (allowing for information contained in the prior, together with the actual sample of size $s - 2$) used by a voter to form beliefs.

Other properties of the Beta concern its mean. In particular $\bar{p} = \mathrm{E}[p] = \beta_R/(\beta_R + \beta_L)$. This means that a Beta may be conveniently rewritten in terms of its mean $\bar{p}$ and a parameter s which corresponds to the information available to a voter; as explained above, $s - 2$ would corresponds to the sample size of an opinion poll (or set of opinion polls), yielding an effective precision proportional to s once the prior is taken into account. Using this formulation, Beta distributed beliefs satisfy

$$f(p) = \frac{\Gamma(s)}{\Gamma(\bar{p}s)\Gamma((1-\bar{p})s)} p^{\bar{p}s-1}(1-p)^{(1-\bar{p})s-1}.$$

This can be substituted into the turnout solution from Proposition 1. Doing so:

$$\bar{t} = \frac{2u}{cn} \frac{\Gamma(s)}{\Gamma(\bar{p}s)\Gamma((1-\bar{p})s)} \frac{2v[\bar{p}^{\bar{p}}(1-\bar{p})^{(1-\bar{p})}]^s}{cn}. \tag{24}$$

To see things a little more clearly, and when s is large enough, we can approximate the Beta density with a normal distribution. The variance of the Beta, in terms of s and the mean $\bar{p}$, satisfies $\mathrm{var}[p] = \bar{p}(1-\bar{p})/(s+1)$. So, using a normal approximation

$$f(p) \approx \sqrt{\frac{s+1}{2\pi\bar{p}(1-\bar{p})}} \exp\left(-\frac{(s+1)(p-\bar{p})^2}{2\bar{p}(1-\bar{p})}\right), \tag{25}$$

where here π indicates the mathematical constant and not a model parameter. When evaluated at $\bar{p}$ the exponential term disappears, so generating the next result.

Proposition 2. *Using a normal specification for voters' beliefs, turnout satisfies*

$$\bar{t} = \frac{u\bar{p}(1-\bar{p})}{cn}\sqrt{\frac{2}{\pi\,\mathrm{var}[p]}}. \tag{26}$$

This is increasing in the precision of voters' beliefs about the candidates' popularity.

Equation (26) offers a simple closed-form solution to the equilibrium turnout rate. All of the existing predictions are maintained: the turnout rate is greatest when the election is viewed as important; when voting costs are low; when candidates are perceived to be evenly matched; when (other things equal) the electorate is smaller; and, finally, when there is good pre-election information about the popularity of the candidates.

RESOLVING THE TURNOUT PARADOX

It is often claimed that rational-choice theory predicts extremely low turnout. The classic reasoning is that if the turnout rate were high then the probability of a tie in a large electorate is far too low to justify the cost of voting. Indeed, in a critical assessment of rational-choice methods, Green and Shapiro (1994, Chapter 4) claimed:

> "Although rational citizens may care a great deal about which person or group wins the election, an analysis of the instrumental value of voting suggests that they will nevertheless balk at the prospect of contributing to a collective cause since it is readily apparent that any one vote has an infinitesimal probability of altering the election outcome."

It is certainly true that, other things equal, the probability of a tie declines as the electorate size grows. This, however, does not automatically justify the "too low to vote" conclusion. Indeed, it is not clear that the probability of a pivotal event is "infinitesimal" and (at least to me) it is not "readily apparent" that there is no hope for an instrumental explanation for the turnout decision. What is needed is an assessment of precisely how big or small the pivotal probability is. There have been attempts at this in the literature, but each of them have serious methodological problems.

To move forward, then, I proceed with a calibration exercise. Specifically, I choose reasonable parameter choices and then ask whether plausible levels of turnout emerge.

I begin with the precision of voters' beliefs. In the context of Proposition 2, the variance of the normal beliefs $\text{var}[p] = \bar{p}(1 - \bar{p})/(s + 1)$ can be used to construct the size

of a confidence interval regarding the popularity of candidate R. I write Δ for the width of such a confidence interval. Familiar calculations from classical statistics yield $\Delta \approx 3.92 \times \sqrt{\mathrm{var}[p]}$ for a confidence interval at the usual 95% level. Now

$$\Delta \approx 3.92 \times \sqrt{\mathrm{var}[p]}. \quad \Leftrightarrow \quad s + 1 \approx 15.37 \times \frac{\bar{p}(1 - \bar{p})}{\Delta^2} \tag{27}$$

Substituting this into the turnout solution (26) from Proposition 2,

$$\bar{t} \approx 3.13 \times \frac{u\bar{p}(1 - \bar{p})}{cn\Delta}. \tag{28}$$

Rather than fix the payoff parameters c and u, a rather more informative perspective is to pick a particular turnout rate and ask what is required of the payoff parameters to justify that rate. That is, the simple inversion of (28) yields

$$\frac{c}{u} \approx 3.13 \times \frac{\bar{p}(1 - \bar{p})}{\bar{t}n\Delta}. \tag{29}$$

Of course, the ratio $\frac{c}{u}$ is the probabilistic influence that an instrumental voter needs to have on the outcome before she is willing to show up to the polls. In equilibrium, it is equal to the probability of a pivotal event in the eyes of each type of voter.

So, let me work through these calculations. Consider a region with a population of 100,000; this approximates the population of a city such as Cambridge, Massachusetts. In such a democracy it is reasonable to suppose that 75% of the population are eligible to vote, and so $n \approx 75,000$. Next, suppose that the width of a 95% confidence interval for the support of the leading candidate is 5%, or equivalently $\Delta = 0.05$. (An opinion poll with a sample size of around $1,500$ would generate such confidence.) Suppose also that candidate R is believed to be more popular, and that 54.5% of the electorate are expected to prefer her; thus $\bar{p} = 0.545$. (Equivalently, there is 95% confidence that her popularity lies between 52% and 57%.) I now ask: how small does the cost-benefit ratio $\frac{c}{u}$ have to be in order to generate turnout of 50%?

$$\frac{c}{u} = \frac{3.13 \times 0.545 \times 0.455}{0.5 \times 75,000 \times 0.05} \approx 0.0004 \quad \Leftrightarrow \quad \frac{u}{c} \approx 2,500. \tag{30}$$

In other words, if the cost-to-benefit ratio is less than 1-in-2,500, or equivalently a voter would be willing to participate in a 1-in-2,500 lottery which enabled her to change the outcome from the disliked to the preferred candidate, then turnout of 50% (slightly greater, in fact) can be supported as part of the unique voting equilibrium.

Proposition 3. *Consider a region with a population of 100,000 where 75% of the inhabitants are eligible to vote. Suppose that a 95% confidence interval for the popularity of the leading candidate stretches from 52% to 57%. If each voter is willing to participate in exchange for a 1-in-2,500 chance of influencing the outcome of the election, then in the unique voting equilibrium turnout will exceed 50%.*

Is this consistent with instrumental explanations for voting? Would a voter show up for a 1-in-2,500 of instrumentally changing the lives of 100,000 people?

Other-regarding preferences section to go here.

<div align="center">COMPLETE TURNOUT?</div>

So far, my equilibrium analysis restricts to a context with incomplete turnout; that is, where the turnout rates within each wing of the political spectrum satisfy $0 < t_L < 1$ and $0 < t_R < 1$. This, of course, requires the inequality (22) to hold. If voters beliefs are normally approximated, as in (25), then this inequality becomes

$$\frac{u}{c} < \frac{\bar{a}n\sqrt{2\pi \operatorname{var}[p]}}{\max\{\bar{p}, (1-\bar{p})\}}. \tag{31}$$

This fails whenever the election is viewed as important (so that u is large); when the cost of voting is small; when the electorate is relatively small; when relatively few are willing to contemplate participation (that is, when $\bar{a}$ is low); when one candidate is perceived to enjoy a strong advantage; and, finally, whenever the voters beliefs about the candidates' positions are relatively precise (so that $\operatorname{var}[p]$ is small).

In the context of a voting equilibrium, what happens if (31) fails? One possibility is that there is no turnout on either side. However, I can rule out this uninteresing case

by simple assuming (quite reasonably) that $c < \frac{v}{2}$. This inequality means that if a voter were given dictatorial powers then he would choose to exercise them. Another possibility is that there is complete turnout on at least one side, so that $\max\{t_L, t_R\} = 1$. If this is the case, then the side with the least popular candidate (in expectation) will be the one that maximizes turnout. This extends a property of the incomplete-turnout equilibrium: the underdog enjoys greater turnout than the perceived leader.

Lemma 6. *Suppose that $\bar{p} > \frac{1}{2}$ so that candidate R is perceived to enjoy greater popularity. A voting equilibrium satisfies $t_R \leq t_L$. This holds strictly if $\min\{t_L, t_R\} < 1$.*

Thus it must be the case that $p^* \geq \frac{1}{2}$, where p^* is the the critical threshold which the true popularity of candidate R needs to exceed if she is to win (at least in expectation). Inspecting Lemma 4, this means that the supporters of the stronger candidate have a weaker incentive to participate, and so the probability $\Pr^\dagger[\text{Pivotal} \mid R]$ is the critical factor in any equilibrium. One possibiblity, of course, is that there is complete turnout from both sides, so that $t_L = t_R = 1$. This can be voting equilibrium if and only if $\Pr^\dagger[\text{Pivotal} \mid R] \geq \frac{c}{v}$. Substituting $t_L + t_R = 1$ and $p^\star = \frac{1}{2}$ into the relevant pivotal probability from equation (17), and using the normal approximation to voter beliefs, the condition for this inequality, and hence for a complete-turnout equilibrium, is

$$\Pr^\dagger[\text{Pivotal} \mid R] = \frac{f\left(\frac{1}{2}\right)}{4\bar{a}n} \frac{1}{\bar{p}} = \frac{1}{4\bar{a}n\bar{p}\sqrt{2\pi \operatorname{var}[p]}} \exp\left(-\frac{\left(\frac{1}{2} - \bar{p}\right)^2}{2\operatorname{var}[p]}\right) \geq \frac{c}{u}. \tag{32}$$

Fixing all of the other parameters of the model, this inequality must fail whenever $\operatorname{var}[p]$ is sufficiently small. In other words, if the voters' collective prior beliefs about the underlying popularities of the candidates are sufficiently precise, then there cannot be a voting equilibrium with complete turnout.

The remaining case is an equilibrium with complete turnout on one side of the political spectrum (so that $t_L = 1$ for supporters of the underdog candidate L) and incomplete supporter on the other side. Given that $t_L = 1$, such an equilibrium is pinned

down by the equation $\mathrm{Pr}^{\dagger}[\text{Pivotal} \,|\, R] = \frac{c}{v}$. This equation reduces to

$$f(p^{\star}) = \frac{c\bar{a}\bar{p}n}{u(p^{\star})^2} \quad \text{where} \quad p^{\star} = \frac{1}{1 + t_R}. \tag{33}$$

Looking for a solution $t_R \in [0, 1]$ is equivalent to seeking a solution $p^{\star} \geq \frac{1}{2}$. If $f(\cdot)$ is a unimodal distribution with its mode at $\bar{p}$, then $f(p)$ is increasing for all $p < \bar{p}$. This means that there is a unique solution to (33) satisfying $p^{\star} < \bar{p}$. Of course, parameter restrictions need to be checked to ensure that this solution satisfies $p^{\star} > \frac{1}{2}$. Once this has been done, the following result emerges.

Proposition 4. *Suppose that voters' beliefs about the relative popularity of the candidates are approximated by a normal distribution. If u/c is small enough, then there is a unique voting equilibrium with incomplete turnout on both sides. If u/c is large enough, then there is a unique voting equilibrium with complete turnout. For intermediate values of u/c, however, there is a unique voting equilibrium with complete turnout by supporters of the underdog, but only partial turnout of the leaders' supporters.*

— in progress —

OMITTED PROOFS

— this section under construction —

Proof of Lemma 1. For each $\gamma \in \Delta$ and each n define $P_n(\gamma) \equiv \int_{\Delta} \left[\prod_{i=0}^{m} v_i^{\gamma_i}\right]^n h(v)\, dv$. In terms of this function, the probability of a ballot outcome $b \in \Delta_n^{\dagger}$ is

$$\mathrm{Pr}[b \,|\, h(\cdot)] = \frac{\Gamma(n+1)}{\prod_{i=0}^{n} \Gamma(b_i + 1)} P_n\left(\frac{b}{n}\right).$$

$P_n(\gamma)$ is a continuous function of γ over the compact set Δ. The first step of the proof is to show that $P_n(\gamma)$ can be replaced with $h(\gamma) \int_{\Delta} \left[\prod_{i=0}^{m} v_i^{\gamma_i}\right]^n dv$ as n grows large. To take this step, first note that the quasi-concave function $\prod_{i=0}^{m} v_i^{\gamma_i}$ is maximized by $v = \gamma$. For small $\varepsilon > 0$ define $\Delta_{\varepsilon} = \{v \in \Delta \,|\, \prod_{i=0}^{m} v_i^{\gamma_i} \geq \prod_{i=0}^{m} \gamma_i^{\gamma_i} - \varepsilon\}$. This is a compact and

convex neighborhood of γ. A lower bound to an expression of interest can be obtained:

$$\frac{P_n(\gamma)}{\int_\Delta [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv} \geq \frac{\int_{\Delta_\varepsilon} [\prod_{i=0}^m v_i^{\gamma_i}]^n \, h(v) \, dv}{\int_\Delta [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv} \geq \frac{\min_{v \in \Delta_\varepsilon}\{h(v)\}}{1 + R_n}$$

$$\text{where} \quad R_n = \frac{\int_{\Delta/\Delta_\varepsilon} [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv}{\int_{\Delta_\varepsilon} [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv}.$$

and where $\Delta/\Delta_\varepsilon = \{v \in \Delta \mid \prod_{i=0}^m v_i^{\gamma_i} < \prod_{i=0}^m \gamma_i^{\gamma_i} - \varepsilon\}$. Similarly,

$$\frac{P_n(\gamma)}{\int_\Delta [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv} \leq \frac{\int_{\Delta/\Delta_\varepsilon} [\prod_{i=0}^m v_i^{\gamma_i}]^n \, h(v) \, dv + \int_{\Delta_\varepsilon} [\prod_{i=0}^m v_i^{\gamma_i}]^n \, h(v) \, dv}{\int_\Delta [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv}$$

$$\leq \frac{\max_{v \in \Delta}\{h(v)\} R_n + \max_{v \in \Delta_\varepsilon}\{h(v)\}}{1 + R_n},$$

where R_n is defined as before, and so

$$\frac{\max_{v \in \Delta}\{h(v)\} R_n + \max_{v \in \Delta_\varepsilon}\{h(v)\}}{1 + R_n} \geq \frac{P_n(\gamma)}{\int_\Delta [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv} \geq \frac{\min_{v \in \Delta_\varepsilon}\{h(v)\}}{1 + R_n}.$$

The maxima are well-defined: $h(v)$ is continuous and Δ and Δ_ε are compact. Examining R_n,

$$R_n \leq \frac{\int_{\Delta/\Delta_\varepsilon} [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv}{\int_{\Delta_\varepsilon} [\prod_{i=0}^m \gamma_i^{\gamma_i} - \varepsilon]^n \, dv} = \frac{1}{\int_{\Delta_\varepsilon} dv} \int_{\Delta/\Delta_\varepsilon} \left[\frac{\prod_{i=0}^m v_i^{\gamma_i}}{\prod_{i=0}^m \gamma_i^{\gamma_i} - \varepsilon}\right]^n \, dv \longrightarrow 0 \quad \text{as} \quad n \to \infty,$$

where the final claim follows because $\prod_{i=0}^m v_i^{\gamma_i} < \prod_{i=0}^m \gamma_i^{\gamma_i} - \varepsilon$ for all $v \in \Delta/\Delta_\varepsilon$. Hence:

$$\max_{v \in \Delta_\varepsilon}\{h(v)\} \geq \lim_{n \to \infty} \left[\frac{P_n(\gamma)}{\int_\Delta [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv}\right] \geq \min_{v \in \Delta_\varepsilon}\{h(v)\}.$$

By choosing ε small enough I can ensure that both $\min_{v \in \Delta_\varepsilon}\{h(v)\}$ and $\max_{v \in \Delta_\varepsilon}\{h(v)\}$ are arbitrarily close to $h(\gamma)$, since $h(\cdot)$ is continuous, and so I conclude that

$$\lim_{n \to \infty} \left[\frac{P_n(\gamma)}{\int_\Delta [\prod_{i=0}^m v_i^{\gamma_i}]^n \, dv}\right] = h(\gamma).$$

Notice also that each member of the sequence of functions (indexed by n) is continuous in γ, and so is the limiting function $h(\gamma)$. Thus, looking across the compact set Δ, the

convergence reported above is uniform. That is,

$$\lim_{n \to \infty} \sup_{\gamma \in \Delta} \left| \frac{P_n(\gamma)}{\int_\Delta \left[\prod_{i=0}^m v_i^{\gamma_i} \right]^n \, dv} - h(\gamma) \right| = 0.$$

With the first step of the proof in hand in hand, I turn to the probability of $b \in \Delta_n^\dagger$.

$$n^m \Pr[b \,|\, h(\cdot)] = n^m \frac{\Gamma(n+1)}{\prod_{i=0}^m \Gamma(b_i+1)} P_n\left(\tfrac{b}{n}\right)$$

$$= n^m \frac{\Gamma(n+1)}{\Gamma(n+m)} \frac{P_n\left(\tfrac{b}{n}\right)}{\int_\Delta \left[\prod_{i=0}^m v_i^{(b_i/n)} \right]^n \, dv} \times \int_\Delta \underbrace{\frac{\Gamma(n+m)}{\prod_{i=0}^m \Gamma(b_i+1)} \left[\prod_{i=0}^m v_i^{(b_i/n)} \right]^n}_{\text{density of Dirichlet}} dv$$

$$= \frac{n^m}{n \times (n+1) \times \cdots \times (n+m-1)} \times \frac{P_n\left(\tfrac{b}{n}\right)}{\int_\Delta \left[\prod_{i=0}^m v_i^{(b_i/n)} \right]^n \, dv}.$$

The final equality follows from noting that the third term in the previous expression is the integral of the density of a Dirichlet distribution. Looking at the final line, the first ratio converges to one as $n \to \infty$. The second ratio is the term considered earlier in the proof, evaluated at $\gamma = b/n$. The convergence established previously was uniform. I conclude that $\lim_{n\to\infty} \max_{b \in \Delta_n^\dagger} |n^m \Pr[b \,|\, h(\cdot)] - h(\gamma)| = 0$. $\qquad\square$

Proof of Lemma 2. From (5) in the text, $n \Pr[b_L = b_R] = \frac{1}{n} \sum_{z=0}^{\lfloor n/2 \rfloor} n^2 \Pr[b_L = b_R = z]$. Using Lemma 1, $n \Pr[b_L = b_R = z]$ converges uniformly. That is,

$$\lim_{n \to \infty} \max_{z \in \{0,1,\ldots,\lfloor n/2 \rfloor\}} \left| n^2 \Pr[b_L = b_R = z] - h\left(\frac{n-2z}{n}, \frac{z}{n}, \frac{z}{n} \right) \right| = 0$$

$$\Rightarrow \quad \lim_{n \to \infty} n \Pr[b_L = b_R] = \lim_{n \to \infty} \frac{1}{n} \sum_{z=0}^{\lfloor n/2 \rfloor} h\left(\frac{n-2z}{n}, \frac{z}{n}, \frac{z}{n} \right),$$

where this equality holds so long as the right-hand side limit is well-defined. As noted in the text, the right-hand side defines a Riemann integral of $h(1 - 2x, x, x)$ over the range $[0, 1/2]$, and so converges to $\int_0^{1/2} h(1 - 2x, x, x) \, dx$ as $n \to \infty$. $\qquad\square$

Proof of Lemma 5. As noted in the main text, an inspection of (17) shows that $\Pr^\dagger[\textbf{Pivotal} \,|\, L] = \Pr^\dagger[\textbf{Pivotal} \,|\, R]$ if and only if $\bar{p} = p^\star = t_L/(t_L + t_R)$. $\qquad\square$

Proof of Lemma 6. I have assumed that $c < \frac{v}{2}$ and so there must be positive probability turnout from at least one voter type. This implies that $\min\{t_L, t_R\} > 0$, and so $p^\star$ (from Lemma 4) is defined. If $t_R \geq t_L$ then there is positive turnout from the supporters of R and so $\Pr^\dagger[\text{Pivotal} \,|\, R] > 0$. $t_R \geq t_L$ implies that $p^\star \leq \frac{1}{2}$ and so, given that $\bar{p} > \frac{1}{2}$, the expressions in (17) ensure that $\Pr^\dagger[\text{Pivotal} \,|\, R] < \Pr^\dagger[\text{Pivotal} \,|\, L]$. This, however, means there is a strict incentive for supporters of L to participate, and so $t_L = 1$. Hence, $t_R \geq t_L$ can only hold if $t_L = t_R = 1$. I conclude that it cannot be the case that $t_R > t_L$. The final claim of the lemma follows directly. $\square$

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