Tailored Advertising and Consumer Privacy

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Abstract

The greater availability of personally identifiable data on the internet increasingly allows firms to tailor their advertising message to the preferences of particular consumers. In our model, firms selectively choose which of a product's attributes to communicate to a particular consumer. Consumers may be wary or naive about the potential for tailored advertising. We characterize how the impact of tailored advertising on efficiency and consumer surplus depends on consumer wariness, on firms' scope to price discriminate, and on competition. We also analyze the potential for policy intervention through stricter privacy laws that either limit the use of personally identifiable data or require consumers' explicit consent.

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1 Introduction

Technological progress in collecting and processing personally identifiable data has implications for firms, consumers, and policymakers. Firms are now able to better tailor their marketing effort to individual consumers by using data on consumers' past purchasing behavior, following the trails of consumers' browsing activity, and prying on the information shared on social networks. Even though personalized pricing (or, better, the scope to which this is feasible) will also play an important role in our analysis, this paper focuses on an hitherto relatively neglected aspect of this customization—the potential for firms to tailor their advertising to the perceived preferences of particular consumers in the presence of realistic bounds on the amount of information that can be effectively communicated.[1]

In the terminology of our model, consumers may receive marketing information about different attributes of a product or service depending on their individual preferences, provided that firms are able to gather this information. For example, the tailored advertising message may appeal more or less strongly to, say, style or comfort, depending on a consumer's recorded past purchases or the recently visited sites. A tailored advertising message may then devote more space or air time to display a product's stylish features. Alternatively, it may provide a more sombre checking list of a product's user-friendly features.

Our model builds on the presumption that the scope of firms' advertising is naturally restricted by time (e.g., airtime), space (e.g., on the screen, on packages, or on billboards), or simply consumers' limited attention. Given that there are some attributes on which firms are unable to provide information, firms' communication *must* remain selective. Unregulated profit-maximizing firms will then strategically select which attributes to communicate. Once firms gather information about consumers' particular preferences, they will want to tailor their communication and thus also the selection of revealed attributes accordingly, so as to raise the likelihood of a purchase. Such strategically selective communication (*tailored advertising*) relies, however, on firms' ability to identify consumers according to their preferences. This is where privacy law and regulation crucially comes into play.

Does tailored advertising benefit consumers? Should the scope of tailored advertising

[1] As discussed below in more detail, this is different from *targeting* a given advertising campaign to a narrow group of consumers, thereby increasing its effectivenss.

be regulated by restricting firms' ability to gather and store personally identifiable data? Or is it sufficient to require firms to obtain customers' consent to do so? What difference does it make if firms are able to additionally practice (first-degree) price discrimination? What is the impact of competition? How do the answers to these questions depend on whether consumers are sufficiently wary of firms' scope to tailor their advertising or, instead, remain naive about this? These questions should inform policy for consumer privacy. The European Union has increasingly raised the barriers and costs for firms to collect and use personally identifiable data about past purchases or recent browsing behavior; see the Data Protection Directive (95/46/EC) and the Privacy and Electronic Communications Directive (2002/58/EC). A change of privacy regulation is also presently discussed in the US; see Federal Trade Commission (2010).

Our answers are shaped by the following three key insights from our analysis. The first insight is that tailored advertising is more informative than non-selective communication. Intuitively, with tailored advertising consumers learn not only about the communicated attribute, but also about the attribute *not* communicated by the firm. Provided that consumers are wary that firms practice tailored advertising, we identify conditions for when their ex-ante expected valuation becomes more dispersed in the sense of a mean-preserving rotation. A priori this allows wary consumers to make a more informed decision. To what extent the resulting efficiency gains are shared between firms and consumers depends on competition and on whether firms can price discriminate according to the anticipated expected valuation of a particular consumer. Such price discrimination may only be feasible for either services or low-value products, for then there is little scope for arbitrage by customers or intermediaries.

Our second insight relates to the distinction between wary and naive consumers. Tailored advertising inflates the perceived valuation of naive consumers who fail to anticipate firms' incentives to collect and use personal data for such selective communication. While tailored advertising harms consumers of a price-discriminating firm that is not subject to competition, even naive consumers may benefit from tailored advertising when firms compete or when there is no scope for price discrimination. These results are driven by the comparison how much naive consumer lose due to biased purchases induced by their inability of properly account for the adverse selection associated with selective communication (*bias effect*) with how much consumers gain from the improved information contained

in the firms' choice of how to best use the limited communication channel (*value of information effect*). On the one hand, competition protects naive consumers because their inflated perceptions affect both the product that they purchase as well as the alternative, provided all firms engage in tailored advertising—thus the bias effect disappears with competition. This is not so when only a monopolistic firm practices tailored advertising. On the other hand, while consumers with inflated perceptions end up making purchases against their own interest, tailored advertising helps consumers avoid the opposite mistake of not purchasing even though they should do so—as we show, in same cases the value of information effect dominates the bias effect. Price discrimination, in turn, allows the firm to appropriate the value of information.

Our third theme relates to differentiation, which comes into play when there is competition and when firms can price discriminate. Then, tailored advertising dampens competition because it makes it more likely that firms are perceived differently by a given consumer from an ex-ante perspective. In our model, this effect is so strong that wary consumers, who strictly benefit from tailored advertising when there is no price discrimination, are strictly worse off under price discrimination and competition. Interestingly, consumers' ignorance about firms' practice of tailoring advertising is now a blessing: It reduces differentiation, thereby spurring competition, so that with price discrimination and tailored advertising naive consumers may be strictly better off than wary consumers!

Our analysis thus identifies various robust effects of tailored advertising on consumer surplus, firm profits, and efficiency. When there is limited scope to use personally identifiable data also for price discrimination (for example, because this would lead to arbitrage by consumers or intermediaries), our analysis suggests that the use of such data for tailored advertising alone tends to enhance efficiency and to make consumers better off. Surprisingly, this may also apply when consumers naively fail to anticipate that firms' communication is selective. Regulation, such as requiring firms to ask consumers' consent, then even risks harming consumers. Firms facing wary consumers would sometimes credibly abstain from practicing tailored advertising, and regulation could provide them with the respective commitment.

While the efficiency benefits of tailored advertising persist when firms can use personal data also to practice price discrimination, from a consumer policy perspective alone there is then a stronger rationale for intervention. When a firm has considerable pricing power,

naive consumers need protection against abusive pricing that exploits their inflated expectations under tailored advertising. For wary consumers, instead, tailored advertising becomes particularly detrimental under competition, as it induces greater differentiation and thus dampens price competition, again provided that firms can use personal data also to practice first-degree price discrimination.

The paper proceeds as follows. Section 2 reviews the literature. Section 3 develops the key differences between tailored and non-tailored advertising, which are then put to work in the rest of the paper: Section 4 analyzes the baseline case of a monopolistic firm; Section 5 introduces competition; and Section 6 introduces price discrimination. Section 7 offers some concluding remarks.

2 Literature Review

The literature on law and economics has much focused on the benefits of greater transparency to expand efficiency-enhancing trade (Stigler 1980, Posner 1981).[2] For instance, applied to our specific context, knowledge of consumer preferences may allow firms to customize their products. Our argument for tailored advertising is, however, less immediate, as there selective communication is intended not so much to reveal the value of a specific match but to inflate a consumer's overall perception of the product. Still, we find that tailoring advertising based on better knowledge about consumer preferences leads to more informed decision-making, at least for wary consumers. That the distribution of the resulting efficiency gains depends on firms' scope to practice first- order price discrimination is immediate. However, core insights with respect to the impact of price discrimination, together with tailored advertising, seem less obvious. In particular, we find that there is scope to protect naive consumers when a firm has monopolistic pricing power, but that discriminatory pricing together with tailored advertising harms wary consumers and tends to benefit naive consumers when there is competition.

The literature has further recognized that incentives to collect information may be too high when its prime purpose is to affect the distribution of surplus, as is possibly the case when it serves firms to better price discriminate (Hirshleifer 1971). In our model, we fully abstract from the costs of information acquisition. To better trade-off the social costs and

[2]Hermalin and Katz (2006) show, however, that trade efficiency may not monotonically increase with information.

benefits of collecting and using personally identifiable data, instead of prohibiting these practices, it has been proposed to essentially grant the respective agents property rights over such information (e.g., Shapiro and Varian 1997). Our analysis reveals a particular twist to this policy. We show that, when the scope for price discrimination is limited, wary consumers always benefit if firms can use information to tailor their advertising, but that this practice may not be in firms' interest as long as it is observed by consumers. A policy that requires consumers' consent may then give firms sufficient commitment to abstain from tailored advertising, thereby harming consumers.

The industrial organization literature has focused much on price discrimination based, in particular, on information obtained from consumers' past purchasing history (e.g., Villas-Boas 1999, Acquisti and Varian 2005). In this literature, Taylor (2004) considers explicitly the case when naive consumers fail to anticipate that firms base future pricing decisions on consumers' past purchase decisions. In our model, naive consumers fail to anticipate firms' incentives to collect and use personal information so as to tailor advertising. Our case of *tailored* advertising, where a firm reveals different attributes to different consumers, is also different to that of *targeted* advertising, which allows firms to better restrict the scope of its marketing to those consumers who are likely to purchase in the first place (cf. Athey and Gans 2010 for its impact on media competition). Several recent papers in marketing (e.g., Goldfarb and Tucker 2011; Campbell, Goldfarb, and Tucker 2011) analyze both theoretically and empirically, how more restrictive privacy rights affect competition and welfare by potentially making advertising campaigns less (cost-)effective.

In our model, firms cannot misrepresent information. Our key departure from much of the literature on strategic disclosure, as initiated by Grossman (1981) and Milgrom (1981), is that constrained by time, space, or consumers' limited attention, firms cannot communicate all information (attributes).[3] Fishman and Hagerty (1990) and Glazer and Rubinstein (2004) also consider models with constraints to the disclosure of verifiable information, albeit their focus is different. In Glazer and Rubinstein (2004) it is the receiver who can choose which information is revealed. In contrast to the disclosure literature, including Fishman and Hagerty (1990), ours is essentially a model of horizontal differentiation: Consumers differ in their preferences, which makes it optimal for firms to

[3]When the information partition of the sender is commonly known and there are no such constraints, there is complete unravelling so that all information is disclosed (Milgrom and Roberts 1986).

communicate different attributes to different consumers. This focus on horizontal differentiation and thus on information about individual suitability is, instead, shared with Lewis and Sappington (1995) and Johnson and Myatt (2006). Lewis and Sappington (1995) consider a firm's incentives to provide consumers with more precise or more noisy information, restricting attention to a "truth-or-noise" information structure. Johnson and Myatt (2006) define more precise information through a rotation of the posterior distribution of receivers' beliefs. We identify conditions for when the use of tailored advertising represents a rotation of consumers' expected valuation. The rest of our analysis is then based on the use of this rotation principle, which allows us to obtain often clear-cut results on how such selective communication affects consumer surplus and efficiency.[4]

3 Tailored vs. Non-Tailored Advertising

At the heart of our analysis is a game of communication between firms and consumers. We focus on the case where firms' communication must be truthful. This may be due to the verifiability of the transmitted information or as the respective firm may, otherwise, fear for its reputation or to be held liable. Communication affects a consumer's perception of her valuation for the firm's product relative to an alternative option. In the monopolistic (baseline) case this is an outside option of known value, while otherwise it is the consumer's option to purchase from a different firm.

Firms may choose different communication strategies, which are at the heart of our analysis. In this Section, we first analyze how consumers update their beliefs when faced with the communication of a given firm. This analysis is subsequently embedded in different market environments.

Information and Preferences. As discussed in the introduction, we impose the key restriction that not all information that a firm possesses can be communicated to a given consumer. More precisely, in the specific context that we consider, we talk about different *attributes* that can be communicated. We suppose that there are exactly two such attributes for a given firm's product or service, $n = 1, 2$, and that only one can be com-

[4]In contrast, Rayo and Segal (2010) and Kamenica an Gentzkow (2011) ask more generally whether and how a sender can choose his information policy to optimally tweak a receiver's beliefs so as to induce certain actions

municated. If a consumer knew both attributes n, we suppose that she would, thereby, learn two values u_n, resulting in her true valuation

$$u = \sum_{n=1,2} u_n. \tag{1}$$

Ex-ante, for a given consumer each value u_n is independently distributed according to $F(u_n)$, which is atomless and has everywhere a strictly positive density $f(u_n)$. This captures the notion that the characteristics of a given attribute may represent a good match for some but not all consumers. For now we suppose that the support of u_n is bounded and given by $[\underline{u}, \overline{u}]$; later we extend our results to allow for unbounded support. Denote $\underline{U} = 2\underline{u}$ and $\overline{U} = E[u] + \overline{u}$.[5]

We next consider two communication or advertising strategies for firms, where the single communicated attribute is either selectively chosen or non-selectively. Note that this entails the restriction that the firm must communicate one of the two attributes, $d \in \{1, 2\}$. Hence, we presently do not consider the strategy of a firm not to communicate any of the two attributes. We show later that when consumers are wary of firms' choices, such a strategy would not arise in equilibrium due to a standard unraveling argument. Also, when consumers naively fail to anticipate firms' strategies, in a sense that will be made more precise below, all our insights will hold qualitatively, though such a strategy may then sometimes be chosen in equilibrium. We make this assertion more formal in what follows.

Non-Tailored Advertising. Advertising is *non-tailored* (or, more generally, communication is *non-selective)* if it is a priori known that a firm always discloses the same attribute d to all consumers. In our subsequent analysis this will be the case when a firm cannot learn about a consumer's preferences, so that each consumer looks identical, with the respective match qualities drawn from $F(u_n)$. Given symmetry over the two attributes, communicating $d \in \{1, 2\}$ to all consumers will then be optimal for a firm.

When advertising is non-tailored, a consumer rationally continues to hold her prior

[5] An alternative interpretation is that through a firm's communication, which is specified below, a consumer may learn the distances between the product's true characteristics and her own preferred characteristics. If we took the distributions of these differences as primitives, while assuming that consumer surplus is reduced by the respective differences, we can show that our key results still hold. This holds despite the fact that the distribution of utility does then not necessarily inherit the properties of the distribution of distances, e.g., once we write $u = u_0 - d_1 - d_2$, for instance.

beliefs about the non-disclosed attribute. Consequently, when attribute d is known and, thereby, the respective realization u_d, the consumer's expected valuation for the product is $U = u_d + E[u]$. From an ex-ante perspective, a consumer's expected valuation under non-tailored advertising is then distributed according to

$$H_N(U) = F(U - E[u]). \tag{2}$$

Tailored Advertising. For a firm to employ tailored advertising vis-à-vis a particular consumer, it must have learnt the consumer's preferences, as represented by the respective values u_n. The firm then strategically chooses which attribute d to disclose. Suppose now that it is known to a consumer that the firm always chooses the attribute d with the highest realization u_d. In our subsequent applications this will indeed be optimal. When $u_1 = u_2$, which it is a zero-probability event, it is likewise known that the firm randomizes.

The consumer should thus rationally update that $u_n \leq u_d$ holds for the attribute $n \neq d$ that has not been disclosed. Consequently, her expected valuation should be

$$U = u_d + E[u \mid u \leq u_d]. \tag{3}$$

This can be solved implicitly for a unique and monotone function $u_D(U)$, which obtains the value u_d that the consumer must have learnt so that her expected valuation is U. Under tailored advertising, the ex-ante distribution of the expected valuation is then

$$H_T(U) = F^2(u_D(U)). \tag{4}$$

The conditional expected valuation U in (3) and its distribution (4) were derived under the assumption that the consumer is wary of the fact that the firm selectively communicates one of the two attributes. One reason why this may not be the case is that the consumer wrongly anticipates that advertising is not tailored to particular consumers. This may be either off-equilibrium or as a consumer remains naive about a firm's (technological) capabilities. Then, when faced with u_d, the consumer wrongly *believes* that her valuation is $\widehat{U} = u_d + E[u]$, i.e., she neglects to discount her valuation as in expression (3). (In what follows, we use the notation $\widehat{U}$ more generally to denote consumers' perceived expected valuation, which may be different from their true expected valuation.) As the firm, however, selectively communicates u_d in this case, the revealed attribute is no longer

distributed according to $F(u_d)$, but according to $F^2(u_d)$. A non-knowledgeable or naive consumer's perceived expected valuation is then distributed according to

$$H_{Tn}(\widehat{U}) = F^2(\widehat{U} - E[u]).$$

Comparison. In the rest of this Section we take the derived distributions of a consumer's (true or perceived) expected valuation and compare the cases of tailored and non-tailored advertising. The case with a naive consumer is most immediate. Then, the perceived expected valuation under tailored advertising and the true expected valuation under non-tailored advertising have the same support, ranging from $\underline{u} + E[u]$ to $\overline{u} + E[u]$. (Recall that we presently suppose that the distribution of u_n is bounded.) However, the respective distribution of the perceived expected valuation $\widehat{U}$ under tailored advertising dominates the distribution of the true expected valuation U under non-tailored advertising in the sense of strict first-order stochastic dominance (FOSD). (As an immediate consequence, also the respective expected value $E[\widehat{U}]$ strictly exceeds the ex-ante expected value under non-tailored advertising $E[U] = 2E[u]$.)

The comparison with rational updating under tailored advertising, when consumers are wary, is more intricate. What is immediate there, however, is that compared to the distribution with non-tailored advertising, $H_N(U)$, the distribution with tailored advertising, $H_T(U)$, puts more mass on lower values of U. In fact, the lower bound of the support is then no longer $\underline{u} + E[u]$, but $\underline{U} = 2\underline{u}$, which is strictly smaller. We next compare the two distributions, $H_T(U)$ and $H_N(U)$, at the upper end of their support, which is $\overline{U} = \overline{u} + E[u]$ in either case. For a *given* value of the consumer's expected valuation U, let us presently denote the respective disclosed values by $u_N = U - E[u]$ and $u_D = u_D(U)$ (as defined previously). We then obtain for the densities

$$\begin{aligned} H_N'(U) &= \frac{dH_N(U)}{dU} = f(u_N), \\ H_T'(U) &= \frac{dH_T(U)}{dU} = f(u_D)\frac{2F(u_D)}{1 + \frac{dE[u|u \leq u_D]}{du_D}}. \end{aligned}$$

Suppose now that f is logconcave. As is well known, this implies that

$$\frac{dE[u \mid u \leq u_D]}{du_D} \leq 1.$$

Together with $u_D = u_N$ at the highest realization of the expected valuation $U = \overline{U}$, next to $F(u_D) = F(u_N) = 1$, this yields $H_N'(\overline{U}) \leq H_T'(\overline{U})$. Thus, when $f(u_n)$ is logconcave,

with tailored advertising the distribution of a wary consumer's posterior valuation has also more mass in the upper tail, compared to the distribution when communication is non-selective: $H_N(U) > H_T(U)$ for all sufficiently large values of U. Taken together, when the distribution $F(u_n)$ is logconcave, then tailored advertising to a wary consumer results in a distribution of the expected valuation U that has more mass in the tails (while clearly not affecting the expected value $E[U] = 2E[u]$).

To go beyond this tail result, let us take first a specific, tractable example: that of a uniform distribution on $[\underline{u}, \overline{u}]$. As is easily confirmed, we then obtain the distributions

$$H_N(U) = \frac{2U - 3\underline{u} - \overline{u}}{2(\overline{u} - \underline{u})} \text{ and } H_T(U) = \left(\frac{2(U - 2\underline{u})}{3(\overline{u} - \underline{u})}\right)^2.$$

In this case, there is a single point of intersection in the interior of both supports, namely at $\widetilde{U} = (3\overline{u} + 5\underline{u})/4$. In other words, starting from the case of non-tailored advertising with $H_N(U)$, the distribution with tailored advertising, $H_T(U)$, represents a *rotation* around $\widetilde{U}$: $H_T(U) > H_N(U)$ for $U < \widetilde{U}$ and $H_T(U) < H_N(U)$ for $U > \widetilde{U}$.

Another specific case for which we obtain a general result is that where u_n has an exponential distribution. Here, note first that in this case the support is no longer bounded from above. As we also show in the proof of Proposition 1, when $f(u_n)$ is logconcave while now $\overline{u} = \infty$, then generally there is still more mass in the tail when advertising is tailored and communication thus selective (i.e., $H_T(U) > H_N(U)$ for all sufficiently large U). With an exponential distribution, we can also show more: Again, $H_T(U)$ results from $H_N(U)$ through a rotation around some interior value $\widetilde{U}$.

We have thus established the *rotation property* generally for the tractable uniform case and, in the family of distributions with unbounded support, for the case with an exponential distribution. We have also made numerous numerical calculations with all typically used logconcave distributions and always found such a rotation.[6] In what follows, we restrict the analysis to distribution functions $F(u_n)$ that indeed imply such a rotation.

Assumption A1. *Given the distribution $F(u_n)$, $H_T(U)$ results from $H_N(U)$ through a rotation around some interior value $\widetilde{U}$:*

$$H_T(U) \gtreqless H_N(U) \text{ for } U \lesseqgtr \widetilde{U}. \tag{A1}$$

We also summarize the preceding observations as follows.

[6] Precisely, we have experimented with all distributions with logconcave density functions listed in Table 1 of Bagnoli and Bergstrom (2005).

Proposition 1 *Comparing the cases with tailored and non-tailored advertising, where in the former case a firm always communicates the attribute with the highest realization u_n, we have the following results:*
i) The distribution of a naive consumer's perceived valuation $\widehat{U}$ under tailored advertising, $H_{Tn}(\widehat{U})$, dominates the distribution of his true valuation U under non-tailored advertising, $H_N(U)$, in the sense of FOSD.
ii) The distribution of a wary consumer's valuation under tailored advertising, $H_T(U)$, compares with $H_N(U)$ as follows: When $f(u_n)$ is logconcave, $H_T(U)$ always has more mass in both tails, and for the cases of a uniform or exponential distribution it generally holds that $H_T(U)$ results from $H_N(U)$ through a rotation (as described in A1).

Proof. See Appendix.

4 Baseline Case

Our baseline analysis makes the following two specifications. First, there is only a single monopolistic firm. Apart from abstracting from competition, this also entails for our analysis the following specification: A consumer's value of his alternative option, i.e., that of not buying from the monopolistic firm, is then not affected by advertising and thus by whether personal data is collected and used by firms or not. Second, we stipulate that the firm cannot use such information for price discrimination. That is, the same price applies to all consumers. Note here that, indeed, all consumers are sold the same product or service, even though the tailored information that they receive may be different. This should, indeed, seriously restrict a firm's ability to practice (first-degree) price discrimination, in particular when we consider physical goods rather than the sale of service contracts. With physical products of non-negligible value, price discrimination would create scope for arbitrage, either through a grey (or parallel) market between consumers or through the activity of intermediaries. Also, price discrimination may be limited due to consumers' concern about fairness.[7]

In order to economize on notation for the present analysis, we simply stipulate that the firm's margin from a sale is strictly positive and that a consumer purchases whenever

[7]Price (or rate) parity has become a major objective for firms, e.g., hotels, given the increasing transparency over online channels.

his perceived valuation does not fall below a (reservation) threshold R:[8] $U \geq R$ (or $\widehat{U} \geq R$ when a (naive) consumer's true and perceived expected valuations differ). We stipulate that $\underline{u} + E[u] < R < \overline{u} + E[u]$, which ensures that in equilibrium both the option to purchase as well as the alternative not to purchase are taken up with positive probability. To be precise, in the baseline analysis we thus consider the following game. In $t = 1$ a monopolistic firm may or may not learn about the preferences of an individual consumer, depending on whether the firm is allowed to acquire information and actually does so. In $t = 2$ the firm then discloses a particular attribute, d, thereby revealing u_d to the respective consumer. In $t = 3$ the consumer then decides whether to purchase or to take up the alternative option.

Before applying the results from Section 3, we must consider one caveat. There, we stipulated that under tailored advertising the firm always chooses to disclose the highest value u_n. Take now, for instance, the case with a wary consumer, who rationally updates her beliefs about the non-disclosed attribute. If both $u_1 + E[u \mid u \leq u_1] \geq R$ and $u_2 + E[u \mid u \leq u_2] \geq R$, then the consumer would purchase irrespective of whether u_1 or u_2 was disclosed. The firm would then be indifferent over the choice of attribute that is advertised to this particular consumer. The same holds when the consumer would not want to purchase regardless of which $d \in \{1, 2\}$ was chosen, as then $u_1 + E[u \mid u \leq u_1] < R$ and $u_2 + E[u \mid u \leq u_2] < R$. Clearly, this indifference would no longer prevail if with arbitrary small probability $\varepsilon > 0$ the consumer were to privately observe an additional component (with sufficiently large support) to her utility from either the product or the outside option. In what follows, we thus still focus on the equilibrium where the firm follows the strategy to disclose the attribute associated with the highest value u_n.

4.1 Firm Preferences over Tailored versus Non-tailored Advertising

Ex-ante, when the firm does not gather information about consumer preferences and thus ends up with non-tailored advertising, there will be a purchase with probability $1 - H_N(R)$. If a consumer (naively) always holds the view that, in the present application, advertising remains non-tailored, recall that her expectations about the non-advertised attribute re-

[8] Thereby, we economize on a separate notation for the price and cost of the product, as well as the consumer's reservation value.

main unchanged. When the firm then *does* gather information, the likelihood of a purchase becomes $1 - H_{Tn}(R)$. From Proposition 1 this is strictly larger, so that firm profits are strictly higher.

Take next the case with wary consumers. When they expect the firm to gather information, which the firm will then optimally use to tailor advertising, a purchase takes place with probability $1 - H_T(R)$. Given Assumption (A1), this is higher than the likelihood of a purchase with non-tailored advertising if R lies to the right of the rotation point $\widetilde{U}$, while otherwise it is strictly lower. As tailored advertising puts more mass into the tails of the distribution of wary consumers' expected utility, firm profits increase only if *a priori* a purchase is not too likely. Otherwise, the firm would want to keep advertising non-tailored, if it could commit to do so. However, if the firm's activities to gather information are non-observable, such commitment is not feasible. This result follows immediately from our preceding observation that the firm strictly benefits to gather information and tailor advertising as long as this is not anticipated by consumers.[9] We have thus arrived at the following conclusions.

Proposition 2 *When consumers are naive, a firm always strictly benefits from gathering information about consumer preferences so as to then tailor its advertising. When consumers are wary, under the rotation property (A1), a firm would, instead, like to commit not to gather such information when a priori a purchase was already sufficiently likely (sufficiently low R). If gathering information is, however, not observable, then even with wary consumers a firm would always end up gathering information.*

4.2 Regulation and Consumer Preferences

We next turn to the question how consumers are affected by tailored advertising. We do so while asking whether consumers benefit when firms are prohibited from gathering and using customer-specific information or whether they benefit when they must consent to

[9] In addition, note that if a firm was thought of gathering information so as to tailor advertising, while it effectively did *not* do so, a consumer would value the product at $u_d + E[u \mid u \leq u_d]$ while the true value was $u_d + E[u]$. Consumers' suspicion of tailored advertising thus unfavorably shifts the distribution of their expected perceived utility in the sense of strict FOSD, making it ex-ante strictly less likely that a given consumer will purchase. If the rotation property (A1) prevails, consumers' suspicion of tailored advertising thus even makes it more likely that the firm indeed benefits from tailoring advertising. However, by the preceding remarks there is no scope for multiple equilibria as, when information gathering is not observed, the firm will always strictly prefer targeted advertising.

the collection and usage of such information.

Wary Consumers. When consumers are wary, recall that they purchase whenever $U \geq R$. After integration by parts, it is immediate that their ex-ante expected utility is higher under tailored advertising when

$$\int_{R}^{\overline{U}} \left[H_N(U) - H_T(U)\right] dU > 0. \tag{5}$$

To see that this holds when H_T is obtained from H_N through a rotation (A1), note that, first, $E[U]$ remains unchanged, so that

$$\int_{\underline{U}}^{\overline{U}} \left[H_N(U) - H_T(U)\right] dU = 0, \tag{6}$$

and that, second, we have from (A1) for an interior rotation point $\widetilde{U}$ that $H_T(U) > H_N(U)$ for $U < \widetilde{U}$ and $H_T(U) < H_N(U)$ for $U > \widetilde{U}$.

We have thus shown that wary consumers strictly benefit when the firm learns their preferences and tailors its advertising, thereby revealing the attribute that the consumer likes most and hiding the attribute that they like least. It is in this sense that we can unambiguously say that tailored advertising is thus more informative for wary consumers. Regulation that would prohibit the use of such information would thus hurt wary consumers. This would also be the case with a less restrictive regulation that would only require firms to obtain consumers' consent before gathering information. Wary consumers would always consent to this, but we know from Proposition 2 that a firm would only ask for it when it was a priori sufficiently unlikely that the consumer would buy (high R). When R is low, however, with wary consumers the requirement to ask consumers' permission would act as a commitment device in the interest of firms, but not in the interest of consumers. We have obtained the following results regarding regulation when consumers are wary.

Proposition 3 *As a result of regulation that prohibits firms from gathering customer-specific information or requires firms to obtain consumers' consent, wary consumers are made worse off under the rotation property (A1).*

Naive Consumers. We know that with naive consumers, a firm would always want to gather customer-specific information, as from an ex-ante perspective this inflates consumers' perceived valuations. Somewhat surprisingly, however, even naive consumers may be better off under tailored advertising.

To see this, take the two types of errors that consumers can make. The mistake of erroneously making a purchase even though $u_1 + u_2 < R$ evidently becomes larger when a naive consumer faces tailored advertising. This is an immediate implication of the observation that with tailored advertising, a naive consumer always ends up purchasing whenever he would do so with non-tailored advertising. On the other hand, however, it is also less likely that the consumer does not purchase even when $u_1 + u_2 > R$. How these two errors (weighted by the respective relative loss $u - R$) trade off should generally depend on the distribution $F(u_n)$. In fact, we find for the case of a uniform distribution, where we obtain explicit solutions, that the naive consumer's true expected utility is exactly the same under the two regimes. And for the second tractable case with an exponential distribution we can prove that naive consumers are in fact strictly better off with tailored advertising.

Proposition 4 *A regulation that prohibits firms from gathering customer-specific information does not necessarily benefit even naive consumers. More precisely, as a result of this regulation naive consumers' true expected utility remains unchanged if u_n is uniformly distributed and is strictly reduced if u_n is exponentially distributed.*

Proof. See Appendix.

4.3 No Advertising

So far we have specified that exactly one attribute is disclosed to consumers. As discussed in the introduction, in many applications it seems indeed reasonable to assume that there are limits to consumers' attention or other limitations, say in air time or space on a package or billboard, so that not all attributes can be equally well (or at all) disclosed. We now expand the firm's strategy space by allowing not to reveal *any* attribute at all.

Take first the case where the firm did not gather information and where this was also not expected by consumers. Then, the firm also does not know consumers' preferences. As

is immediate, the firm will choose not to disclose any of the two attributes if consumers' a priori valuation is sufficiently large so that they are willing to make a purchase even then: $2E[u] \geq R$. Otherwise, when $2E[u] < R$, the firm will disclose one of the attributes, as without disclosure it would not make a sale with positive probability.

Suppose next that the firm gathers information but that consumers are naive about this. When $2E[u] \geq R$ there will always be trade, as the firm decides not to reveal any attribute when the respective utilities both fall short of $E[u]$, i.e., when $\min\{u_n\} < E[u]$. In this case, i.e., when $2E[u] \geq R$, it thus does not make any difference now whether the firm gathers information about naive consumers' preferences. When $2E[u] < R$, the strategy not to reveal any attribute always results in no purchase and is thus not optimal. Summing up and focusing on the impact of regulation, when $2E[u] \geq R$ regulation has now no impact as naive consumers will always purchase in equilibrium, while when $2E[u] < R$ the previous analysis applies.

Finally, turn to the case with wary consumers. Suppose that the firm gathers information and that this is expected by wary consumers. While a priori a given set of attributes represents a better match for some but a worse match for other consumers (horizontal differentiation), once the firm knows a *given* consumer's preferences the setting becomes one of vertical differentiation: For *this* consumer, the firm knows the value u_n that the respective attribute will generate. With wary consumers we can now appeal to standard unraveling arguments à la Grossman (1981), by which the strategy of not disclosing anything would be interpreted as a bad signal. As a consequence, when information is gathered and consumers are wary, the additional strategy of not disclosing any attribute will not become relevant. Note further that when information gathering is non-observable, the firm still faces a commitment problem unless there is regulation. When $2E[u] \geq R$, regulation now allows the firm to commit to reveal none of the attributes, thereby inducing a purchase with probability one. When $2E[u] < R$, the previous analysis again fully applies, and regulation allows the firm to commit not to tailor its advertising when $R < \widetilde{U}$, even though this would be in consumers' interest.

We have thus arrived at the following extension to our previous results.

Proposition 5 *Suppose that a firm can also choose not to reveal any attribute. Then, under (A1) the results of Propositions 2 to 4 still apply qualitatively. In particular, regulation that prohibits information gathering or requires consumer consent hurts wary con-*

sumers. For naive consumers the impact on consumer surplus is generally ambiguous if $2E[u] < R$*: consumer surplus is unaffected when* u_n *is uniformly distributed and strictly reduced when* u_n *is exponentially distributed; if, instead,* $2E[u] \geq R$ *consumer surplus is unaffected. Firms facing wary consumers benefit from regulation whenever* R *is sufficiently low. Firms facing naive consumers are strictly worse off with regulation if* $2E[u] < R$*, but they are unaffected if* $2E[u] \geq R$*.*

5 Competition

So far a consumer's alternative option was that of not purchasing or of purchasing an alternative "outside good" of given value. That is, the value of a consumer's alternative option was not affected by advertising and thus the potential use of personal data. This is now the key point of departure in this section. Once we allow for competition and thus for tailored advertising to affect also a consumer's alternative option, we will see that our key insight on the benefits of tailored advertising is even strengthened, as also naive consumers unambiguously benefit.

For this we consider two a priori symmetric firms $m = a, b$. For each firm, the respective draws of match values are independent and denoted by u_n^m (for firm m and attribute n). With respect to the game form, we require that firms simultaneously decide in $t = 1$ whether to acquire information, provided that they are allowed to do so, and then simultaneously disclose in $t = 2$ the respective attribute $d^m \in \{1, 2\}$.

Depending on the disclosed information, the respective expected valuations are denoted by U^m.[10] A wary consumer, who rationally anticipates the communication regime, compares these utilities and realizes the maximum: $U^* = \max\left\{U^a, U^b\right\}$. With non-tailored advertising for both firms, U^* is distributed according to

$$H_N^*(U^*) = F^2(U^* - E[u])$$

and with tailored advertising it is distributed according to

$$H_T^*(U^*) = F^4(u_D^*(U^*)),$$

where now $u_D^*(U^*)$ is determined implicitly from $U^* = u_D^* + E[u|u \leq u_D^*]$.

[10] As we consider the case of symmetry, we suppose that firms charge the same price, so that it is indeed sufficient for what follows to focus on a comparison of the values U^m.

As we show in the proof of Proposition 6, when (A1) is satisfied, then also H_T^* is obtained from H_N^* through a rotation. From this observation we can immediately apply the insight from Proposition 3 that also with competition, wary consumers strictly prefer that firms gather information to tailor their advertising.

Turn now to naive consumers. What is key to note is that now a naive consumer no longer overestimates the value of an advertised product relative to the fixed value of an alternative: She also overestimates the value of her alternative option of purchasing from a rival firm. Though a naive consumer, when faced with tailored advertising, wrongly uses the perceived valuations $\widehat{U}^m = u_d^m + E[u]$, she now ends up with the same decision as a wary consumer who compares the true valuations, $U^m = u_d^m + E[u|u \leq u_d^m]$. Competition thus not only protects naive consumers from exploitation through tailored advertising, but it also ensures that they are now always strictly better off with tailored advertising, just as wary consumers.

Proposition 6 *Take the case with competition, where now consumers may purchase from one of two a priori symmetric firms $m = a, b$. Then, under (A1) both wary and naive customers strictly benefit when firms gather information and choose tailored advertising.*

Proof. See Appendix.

Under competition, Proposition 6 strengthens our previous key insight that tailored advertising, based on firms' recognition of different preferences, benefits consumers. In fact, in the presence of competition, under the rotation property (A1) this now holds unambiguously for both naive and wary consumers.

6 Tailored Advertising with Price Discrimination

As discussed previously, even when a firm learns about the preferences of a particular consumer and may then tailor its advertising accordingly, it may not be able to price discriminate. Such limitations may arise from arbitrage by customers or intermediaries, given that all consumers are sold the same product, or from a firm's fear of negative reputational repercussions, as a customer may be irritated when finding out that other customers had purchased the same product cheaper. When firms sell a service, however, price discrimination may be more easily accomplished and less transparent to consumers.

In what follows, we now allow firms that tailor their advertising to also set an individualized price for each consumer.

While previously we could restrict our analysis to a comparison of a consumer's expected valuation U^m with some threshold, R, we must now be more explicit. We refer to the price of firm m as p^m, which the firm chooses jointly with the specific communication in $t = 2$.[11] Firms have homogeneous costs c. In the baseline case of a monopoly, we again drop the superscript m, while there R refers to consumers' utility when not purchasing, which they compare to $U - p$ or, if this differs, to the perceived expected net utility $\widehat{U} - p$.

6.1 The Baseline (Monopolistic) Case

The monopolistic firm optimally sets its price to extract all of a consumer's perceived net utility:

$$p(\widehat{U}) = \widehat{U} - R, \tag{7}$$

provided that $p(\widehat{U}) \geq c$, as otherwise there would be no trade. Note also that as $\widehat{U}$ is strictly increasing in the disclosed attribute, the firm now *strictly* prefers to *always* disclose the highest attribute, provided that it is not restricted by regulation.

The case with wary consumers is now rather immediate, given our previous results. As the monopolistic firm can now, through adjusting the price, extract the full (incremental) surplus, a consumer is indifferent to the disclosure regime (realizing always R). Instead, the full benefits arising from the more informative selective disclosure policy under tailored advertising accrue to the firm, which thus now always strictly prefers tailored advertising.

When consumers are naive, the fact that the firm now adjusts its price conditional on the (inflated) perceived valuation of all consumers implies that consumers not only do not reap benefits from selective disclosure, but they are also harmed unambiguously: Under selective disclosure, naive consumers (truly) realize strictly less than their reservation value R. Firms strictly benefit from selective disclosure, and the implications for total welfare are generally ambiguous. However, we obtain clear-cut results for two tractable cases, in analogy to Proposition 4.

Proposition 7 *Suppose that a monopolistic firm that practices tailored advertising can also price discriminate. Consider regulation that would prevent this through prohibiting*

[11] Note that there is no scope for signaling the value of the non-disclosed attribute through the choice of p^m.

the collection and use of personal data. This has the following implications:
i) When consumers are wary, such regulation leaves consumer surplus unaffected but reduces firm profits and welfare.
ii) When consumers are naive, such regulation protects consumers and increases their true expected surplus, while reducing firm profits. The impact on welfare is generally ambiguous (though it is unaffected when u_n is uniformly distributed and strictly lower under regulation when it is exponentially distributed).

Proof. See Appendix.

Tailored advertising generates efficiency gains. In this respect, our previous insights are robust to the introduction of price discrimination. Albeit, price discrimination generates an important difference, as now - from an ex-ante perspective - price discrimination ensures that the efficiency gains are appropriated by the monopolistic firm. While wary consumers thereby realize just their reservation value, irrespective of the advertising regime, naive consumers are now strictly worse off under tailored advertising. In the latter case, the firm can charge an excessively high price due to consumers' inflated expectations. Thus, when the monopolistic firm can adjust its price along with consumers' expectations, there is scope for consumer protection. As we show next, however, our conclusions are markedly different when we allow for competition, which once again protects naive consumers.

6.2 Competition

Recall that with competition, we consider two ex-ante symmetric firms $m = a, b$. Also recall that presently, in contrast to Section 5, both firms can adjust their prices p^m jointly with disclosure. We assume that both values U^m are commonly known, i.e., based on its own knowledge about a consumer's preferences, each firm knows or, at least, can correctly anticipate what attribute the other firm communicates. When no firm chooses weakly dominated prices, the customer's surplus (net of the respective price) equals

$$\breve{U}^* = \min\left\{\widehat{U}^a, \widehat{U}^b\right\}, \tag{8}$$

i.e., the minimum of the two perceived expected valuations. (In equilibrium, when the consumer is wary, we can also use $U^m = \widehat{U}^m$). Thus, with competition a firm no longer extracts the full perceived (incremental) surplus, but only the difference to the perceived

surplus that its competitor offers. It is immediate from (8) that, by putting more mass in the tails of the ex-ante distribution of U^m (cf. (A1)), tailored advertising now harms wary customers compared to non-tailored advertising. Intuitively, from an ex-ante perspective tailored advertising increases perceived differentiation for any given consumer, which is harmful for consumers, as it tends to increase prices. This is in stark contrast to our results with competition but no price discrimination in Section 5. There, wary consumers realized $U^* = \max\left\{U^a, U^b\right\}$ and, thus, benefitted from the increased dispersion in U^m that resulted from tailored advertising. Now, with price discrimination, they are harmed by the increased dispersion.

Note that, as is easily seen, competition is still beneficial to wary consumers, both as it reduces prices and as it makes it more likely that they find a product whose true characteristics provide a good match for their preferences (high values u_n^m). However, given that there is competition, wary consumers would not want firms to gather information about their preferences so as to then tailor their advertising. What is detrimental to consumers is not that a given firm tailors its advertising, but that, by using the information about a consumer's preferences, the firm can gauge also the consumer's expected valuation of the rival firm's product, which under tailored advertising is also more dispersed.

What is key to this argument is that wary consumers make adequate use of the information content of tailored advertising, thereby also updating their beliefs about the attribute that is not disclosed. This does not apply to naive consumers, though. For any given true realization of both firms' attributes, they end up making the same choice as wary consumers, as in the case without price discrimination, but now they pay a strictly lower price. The failure to correctly perceive differences in firms' attributes reduces differentiation and, thereby, induces more price competition. Generally, under competition and with price discrimination the impact of tailored advertising on naive consumers is now ambiguous. For our two working examples of a uniform and an exponential distribution, however, the result is clear cut: Naive consumers are strictly better off under tailored advertising. As we already documented that wary consumers are then strictly worse off, this result provides a striking difference to the case without price discrimination.

Proposition 8 *Suppose there is competition between two firms that can also price discriminate when they gather consumer-specific information. Then, regulation that prohibits such information acquisition and, thereby, makes both tailored advertising and price dis-*

crimination impossible, strictly benefits wary consumers, as it reduces differentiation and intensifies price competition. Also consumer naiveté induces more aggressive price competition when advertising is tailored, so that generally the impact of tailored advertising on naive consumers is ambiguous (though they are strictly better off with tailored advertising and thus strictly worse off under regulation in case of a uniform or an exponential distribution).

Proof. See Appendix.

7 Concluding Remarks

The greater availability of personally identifiable data opens up new opportunities for firms to tailor their advertising message to particular consumers, depending on their perceived preferences. When consumers are sufficiently wary, they will anticipate that firms will display prominently those attributes of a product or service that match their particular needs and taste. Wary consumers, thereby, learn not only about the displayed attributes, but also indirectly about the attributes that firms do not display. This insight leads to one of the core results of our paper, namely that tailored advertising allows wary consumers to make more informed decisions, thereby increasing consumer surplus as well as welfare.

But also consumers who remain naive about firms' capabilities may benefit from tailored advertising, in particular if competition ensures that for any purchase decision they risk overestimating also the value of their alternative choice option. As firms may not necessarily be better off under targeted advertising when this is rationally anticipated by consumers, regulation in the interest of consumers may backfire by providing firms with a commitment not to gather information that would be required to tailor their advertising messages.

There are, however, two cases in our model in which intervention may be warranted. If a monopolistic firm cannot only tailor its message but also price discriminate according to consumers' perceived valuation, then those consumers who are naive about this practice will be exploited. Policy intervention that curbs information gathering then benefits such naive consumers, albeit this works primarily by reducing firms' scope for price discrimination. A more subtle case for policy intervention in the interest of consumers arises in the presence of price discrimination and competition for wary consumers. Then, the

knowledge that firms gather personal data and display their message accordingly leads a priori to greater differentiation among firms, thus dampening price competition. If firms cannot tailor their advertising, consumers are made better off but efficiency is reduced.

8 References

Acquisti, Alessandro, and Hal R. Varian. 2005. Conditioning Prices on Purchase History. Marketing Science 24(3), 367–381.

An, Mark Yuying. 1998. Logconcavity versus Logconvexity: A Complete Characterization. Journal of Economic Theory 80, 350–369.

Athey, Susan, and Joshua Gans. 2010. The Impact of Targeting Technology on Advertising and Media Competition. American Economic Review 100(2), 608–613.

Bagnoli, Mark, and Ted Bergstrom. 2005. Log-concave Probability and its Applications. Economic Theory 26, 445–469.

Campbell, James, Goldfarb, Avi, and Catherine Tucker. 2011. Privacy Regulation and Market Structure. Mimeo.

Fishman, Michael J., and Kathleen M. Hagerty. 1990. The Optimal Amount of Discretion to Allow in Disclosure. Quarterly Journal of Economics 105(2), 427–444.

Federal Trade Commission. 2010. Protecting Consumer Privacy in an Era of Rapid Change. Report.

Glazer, Jacob, and Ariel Rubinstein. 2004. On Optimal Rules of Persuasion. Econometrica 72(6), 1715–1736.

Goldfarb, Avi, and Catherine Tucker. 2011. Privacy Regulation and Online Advertising. Management Science 57(1), 57–71.

Grossman, Sanford. 1981. The Informational Role of Warranties and Private Disclosure about Product Quality. Journal of Law and Economics 24(3), 461–83.

Hermalin, Benjamin, and Michael Katz. 2006. Privacy, Property Rights, and Efficiency: The Economics of Privacy as Secrecy. Quantitative Marketing and Economics 4, 209–239.

Hirshleifer, Jack. 1971. The Private and Social Value of Information and the Reward to Inventive Activity. American Economic Review 61(4), 561–574.

Johnson, Justin P., and David P. Myatt. 2006. On the Simple Economics of Advertising, Marketing, and Product Design. American Economic Review 96(3), 756–84.

Kamenica, Emir, and Matthew Gentzkow. 2011. Bayesian Persuasion. American Economic Review 101(6), 2590–2615.

Lewis, Tracy R., and David E. M. Sappington. 1994. Supplying Information to Facilitate Price Discrimination. International Economic Review 35(2), 309–327.

Milgrom, Paul. 1981. Good News and Bad News: Representation Theorems and Applications. Bell Journal of Economics 12(2), 380–391.

Milgrom, Paul, and John Roberts. 1986. Relying on Information of Interested Parties. RAND Journal of Economics 17(1), 18–32.

Posner, Richard A. 1981. The Economics of Privacy. American Economic Review 71(2), 405–409.

Rayo, Luis, and Ilya Segal. 2010. Optimal Information Disclosure. Journal of Political Economy 118(5), 949–987.

Shapiro, Carl and Hal R. Varian. 1997. US Government Information Policy. Unpublished. University of California, Berkeley.

Stigler, George J. 1980. An Introduction to Privacy in Economics and Politics. The Journal of Legal Studies 9(4), 623–644.

Taylor, Curtis. 2004. Consumer Privacy and the Market for Customer Information. RAND Journal of Economics 35(4), 631–651.

Villas-Boas, Miguel J. 1999. Dynamic Competition with Customer Recognition. RAND Journal of Economics 30(4), 604–631.

9 Appendix: Proofs and Derivations

Proof of Proposition 1. Let us first show that, when f is logconcave, with tailored advertising the distribution of a wary consumer's posterior valuation has more mass in the upper tail, compared to the distribution when advertising is non-tailored also if $\overline{U} = \infty$. Thus, we want to show that $h_N(U)$ goes to zero faster than $h_T(U)$, or equivalently that $lim_{U\to\infty}\left(h_N(U)/h_T(U)\right) < 1$. We have

$$\frac{h_N(U)}{h_T(U)} = \frac{f(U-E[u])}{2F(u_D(U))f(u_D(U))} + \frac{f(U-E[u])u_D(U)}{2F^2(u_D(U))} - \frac{f(U-E[u])E[u \mid u \leq u_D(U)]}{2F^2(u_D(U))}, \tag{9}$$

where $u_D(U)$ solves (3). Now, as $U \to \infty$, we have that $u_D(U) \to \infty$ and, hence, $F(u_D(U)) \to 1$. Further, for $U \to \infty$, $f(U-E[u])$ goes to zero at least exponentially fast.[12] Hence, the third term in (9) goes to zero as $U \to \infty$. For the second term we have

$$\lim_{U\to\infty} \frac{f(U-E[u])u_D(U)}{2F^2(u_D(U))} = \lim_{U\to\infty} \frac{f(U-E[u])\left(U-E[u \mid u \leq u_D(U)]\right)}{2F^2(u_D(U))} = 0,$$

while the limit of the first term is given by

$$\lim_{U\to\infty} \frac{f(U-E[u])}{2F(u_D(U))f(u_D(U))} = \lim_{U\to\infty} \frac{f(u_D(U)+E[u \mid u \leq u_D(U)]-E[u])}{2F(u_D(U))f(u_D(U))} = \frac{1}{2}.$$

Next, we show that (A1) holds in case u is uniformly or exponentially distributed: For the uniform distribution on $[\underline{u}, \overline{u}]$, $H_N(U)$ and $H_T(U)$ are given by

$$H_N(U) = \frac{2U - 3\underline{u} - \overline{u}}{2\left(\overline{u} - \underline{u}\right)} \; and \; H_T(U) = \left(\frac{2\left(U - 2\underline{u}\right)}{3\left(\overline{u} - \underline{u}\right)}\right)^2.$$

Clearly $H_T(U)$ and $H_N(U)$ can have at most two intersections, which are at $\overline{U} = \overline{u} + E[u] = \frac{3\overline{u}+\underline{u}}{2}$ and at $\widetilde{U} = \left(3\overline{u} + 5\underline{u}\right)/4$, with $\underline{u} + E[u] < \widetilde{U} < \overline{u} + E[u]$, so that (A1) is satisfied.

For the exponential distribution with parameter λ, $H_N(U)$ and $H_T(U)$ are given by

$$H_N(U) = 1 - e^{1-\lambda U} \; and \; H_T(U) = \left(1 - e^{-\lambda u_D(U)}\right)^2,$$

where $u_D(U)$ solves

$$U = u_D + \frac{1}{\lambda} - \frac{e^{-\lambda u_D}}{1 - e^{-\lambda u_D}} u_D. \tag{10}$$

[12] First note that $E[u]$ exists as $f(u)$ is log-concave. Further, it is well known that log-concave densities have at most an exponential tail, i.e., $f(u) = o(e^{-\mu u})$ for $u \to \infty$; see An (1998).

To show that $H_T(U)$ and $H_N(U)$ cross exactly once in the interior of both supports, note first, that we can write for $\Delta(U) = H_T(U) - H_N(U)$ the following:

$$\Delta(U) = \exp\{-\lambda u_D(U)\}\left[\exp\{-\lambda u_D(U)\} + \exp\left\{\lambda \frac{e^{-\lambda u_D(U)} u_D(U)}{1 - e^{-\lambda u_D(U)}}\right\} - 2\right],$$

which uses (10). Clearly, as $u_D(U)$ is strictly monotonic in U, $\Delta(U) \to 0$ as $U \to \infty$. So, for bounded U, $\Delta(U) = 0$ if and only if the term in square brackets is equal to zero, i.e., iff u_D solves

$$\exp\{-\lambda u_D\} + \exp\left\{\lambda \frac{e^{-\lambda u_D} u_D}{1 - e^{-\lambda u_D}}\right\} = 2. \tag{11}$$

Let us show that this equation has a unique solution. Taking the derivative of the left-hand side with respect to u_D gives

$$-\lambda \exp\{-\lambda u_D\} - \lambda e^{-\lambda u_D} \frac{e^{-\lambda u_D} + \lambda u_D - 1}{\left(1 - e^{-\lambda u_D}\right)^2} \exp\left\{\lambda u_D \frac{e^{-\lambda u_D}}{1 - e^{-\lambda u_D}}\right\},$$

which is clearly negative if

$$g(u_D) := e^{-\lambda u_D} + \lambda u_D \geq 1.$$

From

$$g'(u_D) = -\lambda e^{-\lambda u_D} + \lambda = \lambda\left(1 - e^{-\lambda u_D}\right),$$

$g(u_D)$ is strictly increasing for $u_D \geq 0$ and thus minimized at $u_D = 0$. Together with $g(u_D = 0) = 1$, it then follows that $g(u_D) \geq 1$ for all $u_D \geq 0$. Hence, (11) has at most one solution. But from $\Delta(U = 1/\lambda) = H_T(1/\lambda) > 0$, together with the fact that $E[U]$ is the same with tailored and non-tailored advertising, cf. (6), we must have a single crossing of $H_T(U)$ and $H_N(U)$ for $\frac{1}{\lambda} < U < \infty$. **Q.E.D.**

Proof of Proposition 4. Take first the case with a uniform distribution. The difference between a naive consumer's ex-ante expected utility without and with tailored advertising, which is

$$\begin{array}{c} \left[F(R - E[u]) - F^2(R - E[u])\right] R \\ + \int_{R-E[u]}^{\overline{u}} \left[u_d + E[u]\right] f(u_d) du_d - \int_{R-E[u]}^{\overline{u}} \left[u_d + E[u \mid u \leq u_d]\right] 2 f(u_d) F(u_d) du_d, \end{array} \tag{12}$$

is then exactly zero, once we substitute for the distribution function.

For the exponential distribution, after some transformations this difference becomes

$$e^{-\lambda\left(R - \frac{1}{\lambda}\right)}\left[R e^{-\lambda\left(R - \frac{1}{\lambda}\right)} - \frac{1}{\lambda}\right].$$

Now note that $Re^{-\lambda\left(R-\frac{1}{\lambda}\right)}$ is maximized at $R=\frac{1}{\lambda}=E[u]$, where it takes value $\frac{1}{\lambda}$. From this we see that a naive customer is always (weakly) better off under tailored advertising, and indifferent only if he purchases always ($R\leq E[u]=\frac{1}{\lambda}$) or never ($R\to\infty$). **Q.E.D.**

Proof of Proposition 6. We need to show that, under (A1), $H_T^*(U^*)$ results from $H_N^*(U^*)$ through a rotation around some interior $\widetilde{U}^*$. The remaining assertions then follow directly from the arguments in the text along with Propositions 2 to 4.

From (A1) it holds for $m=a,b$ that $H_T^m(U^m)=F^2(u_D(U^m))$ results from $H_N^m(U^m)=F(U^m-E[u])$ through a rotation around some interior $\widetilde{U}^m$, where $u_D(U^m)$ solves $U^m=u_D^m+E\left[u|u\leq u_D^m\right]$. By symmetry of the two firms $m=a,b$, we have both $u_D^a(U)=u_D^b(U)=u_D(U)$ as well as $H_N^a(U)=H_N^b(U)=H_N(U)$ and $H_T^a(U)=H_T^b(U)=H_T(U)$. Thus the distribution of $U^*=\max\left\{U^a,U^b\right\}$ is the distribution of the second order statistic of two draws from $H_N(U)$ in case of non-tailored advertising and from $H_T(U)$ with tailored advertising, resulting in

$$\begin{aligned} H_N^*(U^*) &= \left(H_N(U^*)\right)^2=F^2(U^*-E[u]),\\ H_T^*(U^*) &= \left(H_T(U^*)\right)^2=F^4(u_D(U^*)), \end{aligned}$$

showing that $H_T^*(U)-H_N^*(U)$ has the same sign as $H_T(U)-H_N(U)$. Hence, as from (A1) $H_T(U)-H_N(U)$ is positive (negative) for interior $U<\widetilde{U}$ ($U>\widetilde{U}$) and zero at $U=\widetilde{U}$, we have shown that $H_T^*(U)$ results from $H_N^*(U)$ through a rotation around $\widetilde{U}$. **Q.E.D.**

Proof of Proposition 7. With a wary consumer, expected total welfare equals, after integration by parts,

$$\left(\overline{U}-c\right)-\int_{c+R}^{\overline{U}}H_T(U)dU$$

with tailored advertising and

$$\left(\overline{U}-c\right)-\int_{c+R}^{\overline{U}}H_N(U)dU$$

with non-tailored advertising. The difference,

$$\int_{c+R}^{\overline{U}}\left[H_N(U)-H_T(U)\right]dU$$

is then strictly positive under (A.1), as long as $2\underline{u}<c+R<\overline{u}+E[u]$. As the firm sets the price to extract all consumer surplus beyond R, it follows that the firm is strictly better off under tailored advertising (and the wary consumer indifferent).

Consider next the case of a naive consumer. Note first that his expected utility under tailored advertising can now be expressed, after integration by parts, as

$$R - \int_{c+R-E[u]}^{\overline{u}} \left[E[u] - E[u|u \leq u_d]\right] dF^2(u_d)$$

and is thus indeed strictly lower than R. Expected firm profits under tailored advertising can be written as

$$(\overline{u} + E[u] - R - c) - \int_{c+R-E[u]}^{\overline{u}} F^2(u_d) du_d,$$

while with non-tailored advertising expected profits are

$$(\overline{u} + E[u] - R - c) - \int_{c+R-E[u]}^{\overline{u}} F(u_d) du_d.$$

This establishes that the firm is strictly better off under tailored advertising and the naive consumer strictly worse off. Comparing total expected surplus, the difference is

$$\begin{array}{c} \left[F(c+R-E[u]) - F^2(c+R-E[u])\right] R \\ + \int_{c+R-E[u]}^{\overline{u}} \left[u_d + E[u] - c\right] f(u_d) du_d - \int_{c+R-E[u]}^{\overline{u}} \left[u_d + E[u \mid u \leq u_d] - c\right] 2f(u_d) F(u_d) du_d \end{array}.$$

The results for uniform distribution (zero) and exponential distribution (negative) follow then from the analogous results for expression (12). **Q.E.D.**

Proposition of 8. We first show the results for wary customers. When both firms use non-tailored advertising, then U^m, $m \in \{a, b\}$ are distributed according to $H_N(U^m) = F(U^m - E[u])$, when both use tailored advertising, the respective distributions are $H_T(U^m) = F^2(u_D(U^m))$, where $u_D(U)$ solves (3). So, in the non-tailored case, $\breve{U}$ is distributed according to

$$\breve{H}_N(\breve{U}^*) = 1 - \left(1 - H_N^m(\breve{U}^*)\right)^2,$$

for $\underline{u} + E[u] \leq \breve{U}^* \leq \overline{u} + E[u]$ and according to

$$\breve{H}_T(\breve{U}^*) = 1 - \left(1 - H_T^m(\breve{U}^*)\right)^2,$$

for $2\underline{u} \leq \breve{U}^* \leq \overline{u} + E[u]$ with tailored advertising. This implies for the customer's expected utility

$$\begin{aligned} \breve{V}_N &= \int_{\underline{U}}^{\overline{U}} U d\breve{H}_N(U) = \overline{U} - \int_{\underline{U}}^{\overline{U}} \breve{H}_N(U) dU, \\ \breve{V}_T &= \int_{\underline{U}}^{\overline{U}} U d\breve{H}_T(U) = \overline{U} - \int_{\underline{U}}^{\overline{U}} \breve{H}_T(U) dU, \end{aligned}$$

and, denoting $z_T(U) = 1 - H_T^m(U)$ and $z_N(U) = 1 - H_N^m(U)$, we can write

$$\breve{V}_T - \breve{V}_N = \int_{\underline{U}}^{\overline{U}} \left[(z_T(U) + z_N(U)) \, (z_T(U) - z_N(U)) \right] dU.$$

Now, note that $\alpha(U) := (z_T(U) + z_N(U)) \geq 0$ is decreasing in U. Further, from (A1) there exists a unique value $\underline{u} + E[u] < \widetilde{U} < \overline{u} + E[u]$ such that $(z_T(U) - z_N(U)) < 0$ for $2\underline{u} < U < \widetilde{U}$, while $(z_T(U) - z_N(U)) > 0$ for $\widetilde{U} < U < \overline{u} + E[u]$. Hence, we can write

$$\breve{V}_T - \breve{V}_N = \int_{2\underline{u}}^{\widetilde{U}} \alpha(U) \, (z_T(U) - z_N(U)) \, dU + \int_{\widetilde{U}}^{\overline{u}+E[u]} \alpha(U) \, (z_T(U) - z_N(U)) \, dU,$$

where the terms in the first integral are all strictly negative and the terms in the second integral are all strictly positive. Given monotonicity of $\alpha(U)$ we can thus derive the upper bound

$$\begin{aligned} \breve{V}_T - \breve{V}_N &< \int_{2\underline{u}}^{\widetilde{U}} \alpha(\widetilde{U}) \, (z_T(U) - z_N(U)) \, dU + \int_{\widetilde{U}}^{\overline{u}+E[u]} \alpha(\widetilde{U}) \, (z_T(U) - z_N(U)) \, dU \\ &= \alpha(\widetilde{U}) \int_{2\underline{u}}^{\overline{u}+E[u]} (z_T(U) - z_N(U)) \, dU = 0. \end{aligned}$$

Thus, we have shown that a wary customers is worse off under tailored advertising.

Next, let us show that compared to wary customers, naive customers are better off under tailored advertising. To see this suppose that the same values $\left(u_d^a, u_d^b\right)$ are disclosed and assume without loss of generality that $u_d^a \geq u_d^b$, such that $p^a = \widehat{U}^a - \widehat{U}^b$, giving rise to a realized utility of

$$\breve{U}^* = u_d^a + E[u | u \leq u_d^a] - p^a.$$

Substituting for the respective perceived valuations, this implies a realized utility of

$$\min\left\{u_d^a, u_d^b\right\} + E[u | u \leq \max\left\{u_d^a, u_d^b\right\}]$$

for the naive and of

$$\min\left\{u_d^a, u_d^b\right\} + E[u | u \leq \min\left\{u_d^a, u_d^b\right\}]$$

for the wary customer.

Finally, we show that the naive customer is now better off under tailored advertising for both the uniform as well as the exponential distribution. To see this, assume that $u_d^a \geq u_d^b$ and note that the joint distribution of $u_d^b = \min\left\{u_d^a, u_d^b\right\}$ and $u_d^a = \max\left\{u_d^a, u_d^b\right\}$ is given by

$$g(u_d^a, u_d^b) = 2g(u_d^a)g(u_d^b),$$

where

$$g(u_d^m) = 2f(u_d^m)F(u_d^m),$$

such that, in case of tailored advertising, the naive customers' surplus is given by

$$\breve{V}_{Tn} = \int_{\underline{u}}^{\overline{u}} \int_{\underline{u}}^{u_d^a} \left[\left(u_d^b + E[u|u \leq u_d^a] \right) g(u_d^a, u_d^b) \right] du_d^b du_d^a.$$

In case of non-tailored advertising, we obtain

$$\breve{V}_N = \int_{\underline{u}}^{\overline{u}} \left(u_d^b + E[u] \right) 2f(u_d^b) \left(1 - F(u_d^b) \right) du_d^b.$$

After some involved algebra, the differential surplus can be written as

$$\breve{V}_{Tn} - \breve{V}_N = \int_{\underline{u}}^{\overline{u}} \left(\frac{5}{3} F(u) - 3F^2(u) + \frac{4}{3} F^4(u) \right) du,$$

which can be shown to be positive for the uniform as well as for the exponential distribution. **Q.E.D.**